T

JDC

Tarea No. 4

Sean las matrices de rotación:

$$^{0}\boldsymbol{R}_1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{1}$$

y

$$^{1}\boldsymbol{R}_2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

obtener la matriz $^{0}\boldsymbol{R}_2$ si

$$^{0}\boldsymbol{R}_2 = {}^{0}\boldsymbol{R}_1 {}^{1}\boldsymbol{R}_2. \tag{3}$$

Para el resultado final, considerar las identidades de suma y resta de dos ángulos:

$$\begin{aligned} \sin(\alpha \pm \beta) &= \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \\ \cos(\alpha \pm \beta) &= \cos\alpha\cos\beta \pm \sin\alpha\sin\beta \\ \tan(\alpha \pm \beta) &= \frac{\tan\alpha \pm \tan\beta}{1 \pm \tan\alpha\tan\beta}. \end{aligned}$$

Resultado:

$$^{0}\boldsymbol{R}_2 = {}^{0}\boldsymbol{R}_1 {}^{1}\boldsymbol{R}_2 = \tag{4}$$

$$^{0}\boldsymbol{R}_2 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \tag{5}$$

$$= \begin{bmatrix} [(\cos\theta_1)(\cos\theta_2) + (-\sin\theta_1)(\sin\theta_2) + 0] & [(\cos\theta_1)(-\sin\theta_2) + (-\sin\theta_1)(\cos\theta_2) + 0] & 0 \\ [(\sin\theta_1)(\cos\theta_2) + (\cos\theta_1)(\sin\theta_2) + 0] & [(\sin\theta_1)(-\sin\theta_2) + (\cos\theta_1)(\cos\theta_2) + 0] & 0 \\ 0 & 0 & 1 \end{bmatrix} = \tag{6}$$

$$= \begin{bmatrix} [\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2] & [-\cos\theta_1\sin\theta_2 - \sin\theta_1\cos\theta_2] & 0 \\ [\sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2] & [-\sin\theta_1\sin\theta_2 + \cos\theta_1\cos\theta_2] & 0 \\ 0 & 0 & 1 \end{bmatrix} = \tag{7}$$

$$= \begin{bmatrix} [\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2] & -[\cos\theta_1\sin\theta_2 + \sin\theta_1\cos\theta_2] & 0 \\ [\sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2] & [\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2] & 0 \\ 0 & 0 & 1 \end{bmatrix} = \tag{8}$$

$$\boxed{^{0}\boldsymbol{R}_2 = \begin{bmatrix} \cos(\theta_1 - \theta_2) & -\sin(\theta_1 + \theta_2) & 0 \\ \sin(\theta_1 + \theta_2) & \cos(\theta_1 - \theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}} \tag{9}$$

Matriz de rotación (articulación 2 respecto a articulación 0)