Práctica 4

jueves, 25 de abril de 2024 10:41 a. m.

Para los incisos a) y b).

1. Calcula de manera analítica, en tu cuaderno, la función de autocorrelación de los procesos dados, para los primeros 5 periodos: $\rho_0, \rho_1, \rho_2, \rho_3$ y ρ_4.
2. Compara estos valores que calculaste analíticamente con los estimados por R con la función acf.

a) $X_t = 0.4W_{t-1} + W_t$

$\rho_0 = 1$

$\rho_1 = \frac{\gamma_1}{\gamma_0} \rightarrow \gamma_0 = Cov(X_t, X_t)$

$\gamma_0 = Cov(0.4W_{t-1} + W_t, 0.4W_{t-1} + W_t)$

$= Cov(0.4W_{t-1}, 0.4W_{t-1}) + Cov(W_t, W_t)$

$= 0.16 + 1 = 1.16$

$\gamma_1 = Cov(X_t, X_{t+1})$

$= Cov(0.4W_{t-1} + W_t, 0.4W_t + W_{t+1})$

$= Cov(W_t, 0.4W_t) = 0.4$

$\rho_1 = \frac{0.4}{1.16} = \frac{10}{29} \approx 0.3448$

$\rho_2 = \frac{\gamma_2}{\gamma_0} \rightarrow \gamma_2 = Cov(X_t, X_{t+2})$

$\gamma_2 = Cov(0.4W_{t-1} + W_t, 0.4W_{t+1} + W_{t+2})$

$= 0$

$\rho_2 = 0$

$\rho_3 = 0$

$\rho_4 = 0$

b) $X_t = 0.3W_{t-2} - 0.5W_{t-1} + W_t$

$\rho_0 = 1$

$\rho_1 = \frac{\gamma_1}{\gamma_0} \rightarrow \gamma_0 = Cov(0.3W_{t-2} - 0.5W_{t-1} + W_t, 0.3W_{t-2} - 0.5W_{t-1} + W_t)$

$\gamma_0 = Cov(0.3W_{t-2}, 0.3W_{t-2}) + Cov(-0.5W_{t-1}, -0.5W_{t-1}) + Cov(W_t, W_t)$

$\gamma_0 = 0.9 + 0.25 + 1 = 2.15$

$\gamma_1 = Cov(0.3W_{t-2} - 0.5W_{t-1} + W_t, 0.3W_{t-1} - 0.5W_t + W_{t+1})$

$\gamma_1 = Cov(-0.5W_{t-1}, 0.3W_{t-1}) + Cov(W_t, -0.5W_t)$

$= -0.15 - 0.5 = -0.65$

$\rho_1 = \frac{-0.65}{2.15} = -0.3023$

$\rho_2 = \frac{\gamma_2}{\gamma_0} \rightarrow \gamma_2 = Cov(0.3W_{t-2} - 0.5W_{t-1} + W_t, 0.3W_t - 0.5W_{t+1} + W_{t+2})$

$\gamma_2 = Cov(W_t, 0.3W_t) = 0.3$

$\rho_2 = \frac{0.3}{2.15} = 0.1395$

$\rho_3 = \frac{\gamma_3}{\gamma_0} \rightarrow \gamma_3 = Cov(X_t, X_{t+3}) = 0$

$\rho_3 = 0$

$\rho_4 = 0$