Team notebook

coUNter

November 8, 2019

Contents

1 Data structures

1.1 Centroid decomposition

```
namespace decomposition {
  int cnt[MAX], depth[MAX], f[MAX];
  int dfs (int u, int p = -1) {
    cnt[u] = 1;
    for (int v : g[u])
      if (!depth[v] && v != p)
        cnt[u] += dfs(v, u);
    return cnt[u];
  }
  int get_centroid (int u, int r, int p = -1) {
    for (int v : g[u])
      if (!depth[v] && v != p && cnt[v] > r)
        return get_centroid(v, r, u);
    return u;
  }
  int decompose (int u, int d = 1) {
    int centroid = get_centroid(u, dfs(u) >> 1);
    depth[centroid] = d;
    /// magic function
    for (int v : g[centroid])
      if (!depth[v])
        f[decompose(v, d + 1)] = centroid;
    return centroid;
  }
  int lca (int u, int v) {
    for (; u != v; u = f[u])
      if (depth[v] > depth[u])
        swap(u, v);
    return u;
  }
}
```

1.2 Fenwick tree

```
/// Complexity: log(|N|)
/// Tested: https://tinyurl.com/y88y7ws7
int lower_find(int val) { /// last value < or <= to val
  int idx = 0;
  for(int i = 31-__builtin_clz(n); i >= 0; --i) {
    int nidx = idx | (1 << i);
    if(nidx <= n && bit[nidx] <= val) { /// change <= to <
      val -= bit[nidx];
      idx = nidx;
    }
  }
  return idx;
}
```

1.3 Heavy light decomposition

```
/// Complexity: O(|N|)
/// Tested: https://tinyurl.com/ybdbmbw7(problem L)
int idx;
vector<int> len, hld_child, hld_index, hld_root, up;
void dfs( int u, int p = 0 ) {
```

```
  len[u] = 1;
  up[u] = p;
  for( auto& v : g[u] ) {
    if( v == p ) continue;
    depth[v] = depth[u]+1;
    dfs(v, u);
    len[u] += len[v];
    if( hld_child[u] == -1 || len[hld_child[u]] < len[v] )
      hld_child[u] = v;
  }
}
void build_hld( int u, int p = 0 ) {
  hld_index[u] = idx++;
  narr[hld_index[u]] = arr[u]; /// to initialize the segment tree
  if( hld_root[u] == -1 ) hld_root[u] = u;
  if( hld_child[u] != -1 ) {
    hld_root[hld_child[u]] = hld_root[u];
    build_hld(hld_child[u], u);
  }
  for( auto& v : g[u] ) {
    if( v == p || v == hld_child[u] ) continue;
    build_hld(v, u);
  }
}
void update_hld( int u, int val ) {
  update_tree(hld_index[u], val);
}
data query_hld( int u, int v ) {
  data val = NULL_DATA;
  while( hld_root[u] != hld_root[v] ) {
    if( depth[hld_root[u]] < depth[hld_root[v]] ) swap(u, v);
    val = val+query_tree(hld_index[hld_root[u]], hld_index[u]);
    u = up[hld_root[u]];
  }
  if( depth[u] > depth[v] ) swap(u, v);
  val = val+query_tree(hld_index[u], hld_index[v]);
  return val;
/// when updates are on edges use:
///   if (depth[u] == depth[v]) return val;
///   val = val+query_tree(hld_index[u] + 1, hld_index[v]);
}
void build(int n, int root) {
  len = hld_index = up = depth = vector<int>(n+1);
  hld_child = hld_root = vector<int>(n+1, -1);
  idx = 1; /// segtree index [1, n]
  dfs(root, root); build_hld(root, root);
  /// initialize data structure
}
```

1.4 Mo's

```
/// Complexity: O(|N+Q|*sqrt(|N|)*|ADD/DEL|)
/// Tested: Not yet
// Requires add(), delete() and get_ans()
struct query {
  int l, r, idx;
  query (int l, int r, int idx) : l(l), r(r), idx(idx) {}
};
int S; // s = sqrt(n)
bool cmp (query a, query b) {
  if (a.l/S != b.l/S) return a.l/S < b.l/S;
  return a.r > b.r;
}
S = sqrt(n); // n = size of array
sort(q.begin(), q.end(), cmp);
int l = 0, r = -1;
for (int i = 0; i < q.size(); ++i) {
  while (r < q[i].r) add(++r);
  while (l > q[i].l) add(--l);
  while (r > q[i].r) del(r--);
  while (l < q[i].l) del(l++);
  ans[q[i].idx] = get_ans();
}
```

1.5 Order statistics

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
typedef tree<int, null_type, less<int>, rb_tree_tag,
tree_order_statistics_node_update> ordered_set;
//methods
tree.find_by_order(k) //returns pointer to the k-th smallest element
tree.order_of_key(x) //returns how many elements are smaller than x
//if element does not exist
tree.end() == tree.find_by_order(k) //true
```

1.6 Persistent segment tree

```
/// Complexity: O(|N|*log|N|)
/// Tested: Not yet
struct node {
  node *left, *right;
  int v;
  node() : left(this), right(this), v(0) {}
  node(node *left, node *right, int v) :
    left(left), right(right), v(v) {}
  node* update(int l, int r, int x, int value) {
```

```
    if (l == r) return new node(nullptr, nullptr, v + value);
    int m = (l + r) / 2;
    if (x <= m)
      return new node(left->update(l, m, x, value), right, v + value);
    return new node(left, right->update(m + 1, r, x, value), v + value);
  }
};
```

1.7 Rmq

```
/// Complexity: O(|N|*log|N|)
/// Tested: https://tinyurl.com/y739tcsj
struct rmq {
  vector<vector<int> > table;
  rmq(vector<int> &v) : table(v.size() + 1, vector<int>(20)) {
    int n = v.size()+1;
    for (int i = 0; i < n; i++) table[i][0] = v[i];
    for (int j = 1; (1<<j) <= n; j++)
      for (int i = 0; i + (1<<j-1) < n; i++)
        table[i][j] = max(table[i][j-1], table[i + (1<<j-1)][j-1]);
  }
  int query(int a, int b) {
    int j = 31 - __builtin_clz(b-a+1);
    return max(table[a][j], table[b-(1<<j)+1][j]);
  }
};
```

1.8 Sack

```
/// Complexity: |N|*log(|N|)
/// Tested: https://tinyurl.com/y9fz8vdt
int dfs(int u, int p = -1) {
  who[t] = u; fr[u] = t++;
  pii best = {0, -1};
  int sz = 1;
  for(auto v : g[u])
    if(v != p) {
      int cur_sz = dfs(v, u);
      sz += cur_sz;
      best = max(best, {cur_sz, v});
    }
  to[u] = t-1;
  big[u] = best.second;
  return sz;
}
void add(int u, int x) { /// x == 1 add, x == -1 delete
  cnt[u] += x;
}
void go(int u, int p = -1, bool keep = true) {
  for(auto v : g[u])
    if(v != p && v != big[u])
      go(v, u, 0);
  if(big[u] != -1) go(big[u], u, 1);
  /// add all small
  for(auto v : g[u])
    if(v != p && v != big[u])
      for(int i = fr[v]; i <= to[v]; i++)
        add(who[i], 1);
  add(u, 1);
  ans[u] = get(u);
  if(!keep)
    for(int i = fr[u]; i <= to[u]; i++)
      add(who[i], -1);
}
void solve(int root) {
  t = 0;
  dfs(root);
  go(root);
}
```

1.9 Sqrt Decomposition

```
struct bucket {
  int l, r, lazy;
  bucket(int l, int r) : l(l), r(r), lazy(0) {}
  void build() {
    for(int i = l; i <= r; i++) a[i] += lazy;
    for(int i = l; i <= r; i++) {} /// build DS from scratch
    lazy = 0;
  }
  void update(int L, int R, ll v) {
    if(L == l && R == r) lazy += v;
    else { /// handle by hand
      for(int i = L; i <= R; i++) a[i] += v;
      build();
    }
  }
  int query(int L, int R) {
    int ans = INT_MIN;
    if(L == l && R == r) ans = ds.get_max(x);
    else { /// handle by hand
      for(int i = L; i <= R; i++) ans = max(ans, abs(a[i] + x) * b[i]);
    }
    return ans;
  }
};
{ /// at main(), update from a to b, len is the size of bucket
  int l = a / len, r = b / len;
  for(int i = l i <= r; i++) { /// in theory, all are complete
```

```
    int x = max(l, i*len);
    int y = min(r, (i+1)*len-1);
    bucket[i].operation(x, y);
  }
}
```

1.10 Treap

```
/// Complexity: O(|N|*log|N|)
/// Tested: Not yet
mt19937_64 rng64(chrono::steady_clock::now().time_since_epoch().count());
uniform_int_distribution<ll> dis64(0, 1ll<<60);
template <typename T>
class treap {
private:
	struct item;
	typedef struct item * pitem;
	pitem root = NULL;
	struct item {
		ll prior; int cnt, rev;
		T key, add, fsum;
		pitem l, r;
		item(T x, ll p) {
			add = 0*x; key = fsum = x;
			cnt = 1; rev = 0;
			l = r = NULL; prior = p;
		}
	};
	int cnt(pitem it) { return it ? it->cnt : 0; }
	void upd_cnt(pitem it) {
		if(it) it->cnt = cnt(it->l) + cnt(it->r) + 1;
	}
	void upd_sum(pitem it) {
		if(it) {
			it->fsum = it->key;
			if(it->l) it->fsum += it->l->fsum;
			if(it->r) it->fsum += it->r->fsum;
		}
	}
	void update(pitem t, T add, int rev) {
		if(!t) return;
		t->add = t->add + add;
		t->rev = t->rev ^ rev;
		t->key = t->key + add;
		t->fsum = t->fsum + cnt(t) * add;
	}
	void push(pitem t) {
		if(!t || (t->add == 0*T() && t->rev == 0)) return;
		update(t->l, t->add, t->rev);
		update(t->r, t->add, t->rev);
		if(t->rev) swap(t->l,t->r);
		t->add = 0*T(); t->rev = 0;
	}
	void merge(pitem & t, pitem l, pitem r) {
		push(l); push(r);
		if(!l || !r) t = l ? l : r;
		else if(l->prior > r->prior) merge(l->r, l->r, r), t = l;
		else merge(r->l, l, r->l), t = r;
		upd_cnt(t); upd_sum(t);
	}
	void split(pitem t, pitem & l, pitem & r, int index) { // split index = how many
	    elements
		if(!t) return void(l = r = 0);
		push(t);
		if(index <= cnt(t->l)) split(t->l, l, t->l, index), r = t;
		else split(t->r, t->r, r, index - 1 - cnt(t->l)), l = t;
		upd_cnt(t); upd_sum(t);
	}
	void insert(pitem & t, pitem it, int index) { // insert at position
		push(t);
		if(!t) t = it;
		else if(it->prior > t->prior) split(t, it->l, it->r, index), t = it;
		else if(index <= cnt(t->l)) insert(t->l, it, index);
		else insert(t->r, it, index-cnt(t->l)-1);
		upd_cnt(t);   upd_sum(t);
	}
	void erase(pitem & t, int index) {
		push(t);
		if(cnt(t->l) == index) merge(t, t->l, t->r);
		else if(index < cnt(t->l)) erase(t->l, index);
		else erase(t->r, index - cnt(t->l) - 1);
		upd_cnt(t);   upd_sum(t);
	}
	T get(pitem t, int index) {
		push(t);
		if(index < cnt(t->l)) return get(t->l, index);
		else if(index > cnt(t->l)) return get(t->r, index - cnt(t->l) - 1);
		return t->key;
	}
  T query_sum (pitem &t, int l, int r) {
		pitem l1, r1;
		split (t, l1, r1, r + 1);
		pitem l2, r2;
		split (l1, l2, r2, l);
		T ret = r2->fsum;
		pitem t2;
		merge (t2, l2, r2);
		merge (t, t2, r1);
		return ret;
	}
public:
	int size() { return cnt(root); }
	void insert(int pos, T x) {
		if(pos > size()) return;
```

```
            pitem it = new item(x, dis64(rng64));
            insert(root, it, pos);
        }
        void erase(int pos) {
            if(pos >= size()) return;
            erase(root, pos);
        }
  T sum(int left, int right) {
    return query_sum(root, left, right);
        }
        T operator[](int index) { return get(root, index); }
};
```

2 Dp optimization

2.1 Convex hull trick dynamic

```
/// Complexity: O(|N|*log(|N|))
/// Tested: Not yet
typedef ll T;
const T is_query = -(1LL<<62); // special value for query
struct line {
  T m, b;
  mutable multiset<line>::iterator it, end;
  const line* succ(multiset<line>::iterator it) const {
    return (++it == end ? nullptr : &*it);
  }
  bool operator < (const line& rhs) const {
    if(rhs.b != is_query) return m < rhs.m;
    const line *s = succ(it);
    if(!s) return 0;
    return b-s->b < (s->m-m)*rhs.m;
  }
};
struct hull_dynamic : public multiset<line> { // for maximum
  bool bad(iterator y) {
    iterator z = next(y);
    if(y == begin()){
      if(z == end()) return false;
      return y->m == z->m && y->b <= z->b;
    }
    iterator x = prev(y);
    if(z == end()) return y->m == x->m && y->b <= x->b;
    return (x->b - y->b)*(z->m - y->m) >=
           (y->b - z->b)*(y->m - x->m);
  }
  iterator next(iterator y){ return ++y; }
  iterator prev(iterator y){ return --y; }
  void add(T m, T b){
    iterator y = insert((line){m, b});
    y->it = y; y->end = end();
    if(bad(y)){ erase(y); return; }
    while(next(y) != end() && bad(next(y))) erase(next(y));
    while(y != begin() && bad(prev(y))) erase(prev(y));
  }
  T eval(T x){
    line l = *lower_bound((line){x, is_query});
    return l.m*x+l.b;
  }
};
```

2.2 Convex hull trick

```
struct line {
  ll m, b;
  ll eval (ll x) { return m*x + b; }
};
struct cht {
  vector<line> lines;
  vector<lf> inter;
  int n;
  lf get_inter(line &a, line &b) { return lf(b.b - a.b) / (a.m - b.m); }
  inline bool ok(line &a, line &b, line &c) {
    return lf(a.b-c.b) / (c.m-a.m) > lf(a.b-b.b) / (b.m-a.m);
  }
  void add(line l) {
    n = lines.size();
    if(n && lines.back().m == l.m && lines.back().b >= l.b) return;
    if(n == 1 && lines.back().m == l.m && lines.back().b < l.b) lines.pop_back(), n--;
    while(n >= 2 && !ok(lines[n-2], lines[n-1], l)) {
      n--;
      lines.pop_back(); inter.pop_back();
    }
    lines.push_back(l); n++;
    if(n >= 2) inter.push_back(get_inter(lines[n-1], lines[n-2]));
  }
  ll get_max(lf x) {
    if(lines.size() == 0) return LLONG_MIN;
    if(lines.size() == 1) return lines[0].eval(x);
    int pos = lower_bound(inter.begin(), inter.end(), x) - inter.begin();
    return lines[pos].eval(x);
  }
};
```

2.3 Divide and conquer

```
/// Complexity: O(|N|*|K|*log|N|))
/// ******* Theory *******
```

```
/// dp[k][i]=min(dp[k1][j]+C[i][j]), j < i
/// opt[k][i]   opt[k][i+1].
/// A sufficient (but not necessary) condition for above is
/// C[a][c]  + C  [b][d]    C[a][d]  + C  [b][c] where  a <   b   <   c   < d .
void go(int k, int l, int r, int opl, int opr) {
  if(l > r) return;
  int mid = (l + r) / 2, op = -1;
  ll &best = dp[mid][k];
  best = INF;
  for(int i = min(opr, mid); i >= opl; i--) {
    ll cur = dp[i][k-1] + cost(i+1, mid);
    if(best > cur) {
      best = cur;
      op = i;
    }
  }
  go(k, l, mid-1, opl, op);
  go(k, mid+1, r, op, opr);
}
```

2.4 Knuth

```
/// Complexity: O(|N|^2))
/// Tested: https://tinyurl.com/y6ofp8wb
/// ******* Theory *******
/// dp[i][j]= min(dp[i][k]+dp[k][j])+C[i][j], i<k<j
/// where opt[i][ j1 ]     opt[i][j]    opt[i+1][j].
/// sufficient (but not necessary) condition for above is
/// C[a][c]  + C  [b][d]    C[a][d]  + C  [b][c] and C[b][c]   C[a][d] where
              abcd            .
for(int i = 1; i <= n; i++) {
  opt[i][i] = i;
  dp[i][i] = sum[i] - sum[i-1];
}
for(int len = 2; len <= n; len++)
  for(int l = 1; l+len-1 <= n; l++) {
    int r = l+len-1;
    dp[l][r] = oo;
    for(int i = opt[l][r-1]; i <= opt[l+1][r]; i++) {
      ll cur = dp[l][i-1] + dp[i+1][r] + sum[r] - sum[l-1];
      if(cur < dp[l][r]) {
        dp[l][r] = cur;
        opt[l][r] = i;
      }
    }
  }
```

3 Formulas

3.1 2-SAT rules

- $p \to q \equiv \neg p \lor q$
- $p \to q \equiv \neg q \to \neg p$
- $p \lor q \equiv \neg p \to q$
- $p \land q \equiv \neg(p \to \neg q)$
- $\neg(p \to q) \equiv p \land \neg q$
- $(p \to q) \land (p \to r) \equiv p \to (q \land r)$
- $(p \to q) \lor (p \to r) \equiv p \to (q \lor r)$
- $(p \to r) \land (q \to r) \equiv (p \land q) \to r$
- $(p \to r) \lor (q \to r) \equiv (p \lor q) \to r$
- $(p \land q) \lor (r \land s) \equiv (p \lor r) \land (p \lor s) \land (q \lor r) \land (q \lor s)$

3.2 Burnside's lemma

$\#orbitas = \frac{1}{|G|} \sum_{g \in G} |fix(g)|$

1. **G**: Las acciones que se pueden aplicar sobre un elemento, incluyendo la identidad, eg. Shift 0 veces, Shift 1 veces...
2. **Fix(g)**: Es el número de elementos que al aplicar g vuelven a ser ser ellos mismos
3. **Órbita**: El conjunto de elementos que pueden ser iguales entre si al aplicar alguna de las acciones de G

3.3 Catalan Numbers

- $C_n = \frac{1}{n+1}\binom{2n}{n} = \frac{(2n)!}{(n+1)!n!}$ con $n \geq 0$, $C_0 = 1$ y $C_{n+1} = \frac{2(2n+1)}{n+2} C_n$
- 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, 208012, 742900, 2674440, 9694845, 35357670

3.4 Combinatorics

- Distribute N objects among K people
  $\left(\binom{n}{k}\right) = \binom{n+k-1}{k} = \frac{(n+k-1)!}{k!(n-1)!}$
- Hockey-stick identity
  $\sum_{i=r}^{n} \binom{i}{r} = \binom{n+1}{r+1}$

3.5 Compound Interest

- N is the initial population, it grows at a rate of R. So, after X years the popularion will be $N \times (1+R)^X$

3.6 DP optimization theory

Name	Original Recurrence	Sufficient Condition		
CH 1	$dp[i] = min_{j<i}\{dp[j]+b[j]*a[i]\}$	$b[j] \geq b[j+1]$Optionally $a[i] \leq a[i+1]$	$O(n^2)$	$O(n)$
CH 2	$dp[i][j] = min_{k<j}\{dp[i-1][k] + b[k]*a[j]\}$	$b[k] \geq b[k+1]$ Optionally $a[j] \leq a[j+1]$	$O(kn^2)$	$O(kn)$
D&Q	$dp[i][j] = min_{k<j}\{dp[i-1][k] + C[k][j]\}$	$A[i][j] \leq A[i][j+1]$	$O(kn^2)$	$O(kn\log n)$
Knuth	$dp[i][j] = min_{i<k<j}\{dp[i][k] + dp[k][j]\} + C[i][j]$	$A[i,j-1] \leq A[i,j] \leq A[i+1,j]$	$O(n^3)$	$O(n^2)$

Notes:

- $A[i][j]$ - the smallest k that gives the optimal answer, for example in $dp[i][j] = dp[i-1][k] + C[k][j]$
- $C[i][j]$ - some given cost function
- We can generalize a bit in the following way $dp[i] = \min_{j<i}\{F[j] + b[j]*a[i]\}$, where $F[j]$ is computed from $dp[j]$ in constant time

3.7 Euler Totient properties

- $\phi(p) = p-1$
- $\phi(p^e) = p^e(1-\frac{1}{p})$
- $\phi(n*m) = \phi(n)*\phi(m)$ si $gcd(n,m) = 1$
- $\phi(n) = n(1-\frac{1}{p_1})(1-\frac{1}{p_2})...(1-\frac{1}{p_k})$ donde p_i es primo y divide a n

3.8 Fermat's theorem

Let m be a prime and x and m coprimes, then:

- $x^{m-1} \mod m = 1$
- $x^k \mod m = x^{k \mod (m-1)} \mod m$
- $x^{\phi(m)} \mod m = 1$

3.9 Great circle distance or geographical distance

Great circle distance or geographical distance

- d = great distance, ϕ = latitude, λ = longitude, Δ = difference (all the values in radians)
- σ = central angle, angle form for the two vector
- $d = r*\sigma$, $\sigma = 2*\arcsin(\sqrt{\sin^2(\frac{\Delta\phi}{2}) + \cos(\phi_1)\cos(\phi_2)\sin^2(\frac{\Delta\lambda}{2})})$

3.10 Heron's Formula

- $s = \frac{a+b+c}{2}$
- $Area = \sqrt{s(s-a)(s-b)(s-c)}$
- a, b, c there are the lenghts of the sides

3.11 Interesting theorems

- $a^d \equiv a^{d \mod \phi(n)} \mod n$
  if $a \in Z^{n*}$ or $a \notin Z^{n*}$ and $d \mod \phi(n) \neq 0$
- $a^d \equiv a^{\phi(n)} \mod n$
  if $a \notin Z^{n*}$ and $d \mod \phi(n) = 0$
- thus, for all a, n and d (with $d \geq \log_2(n)$)
  $a^d \equiv a^{\phi(n)+d \mod \phi(n)} \mod n$

3.12 Law of sines and cosines

- a, b, c: lenghts, A, B, C: opposite angles, d: circumcircle
- $\frac{a}{sin(A)} = \frac{b}{sin(B)} = \frac{c}{sin(C)} = d$
- $c^2 = a^2 + b^2 - 2ab\cos(C)$

3.13 Number of divisors

- $\tau(n) = \prod_{i=1}^{k}(\alpha_i + 1)$

3.14 Product of divisors of a number

$\mu(n) = n^{\frac{\tau(n)}{2}}$

- if p is a prime, then: $\mu(p^k) = p^{\frac{k(k+2)}{2}}$
- if a and b are coprimes, then: $\mu(ab) = \mu(a)^{\tau(b)}\mu(b)^{\tau(a)}$

3.15 Pythagorean triples ($a^2 + b^2 = c^2$)

- Given an arbitrary pair of integers m and n with $m > n > 0$:
  $a = m^2 - n^2$, $b = 2mn$, $c = m^2 + n^2$
- The triple generated by Euclid's formula is primitive if and only if m and n are coprime and not both odd.
- To generate all Pythagorean triples uniquely:
  $a = k(m^2 - n^2)$, $b = k(2mn)$, $c = k(m^2 + n^2)$
- If m and n are two odd integer such that $m > n$, then:
  $a = mn$, $b = \frac{m^2-n^2}{2}$, $c = \frac{m^2+n^2}{2}$
- If $n = 1$ or 2 there are no solutions. Otherwise
  n is even: $((\frac{n^2}{4} - 1)^2 + n^2 = (\frac{n^2}{4} + 1)^2)$
  n is odd: $((\frac{n^2-1}{2})^2 + n^2 = (\frac{n^2+1}{2})^2)$

3.16 Simplex Rules

The simplex algorithm operated on linear programs in standard form:
Maximixe : $c^T \cdot x$
Subject to : $Ax \leq b, x_i \geq 0$

- $x = (x_1, ..., x_n)$ the variables of the problem
- $c = (c_1, ..., c_n)$ are the coefficients of the objective function
- A is a $p \times n$ matrix and $b = (b_1, ..., b_p)$ constants with $b_j \geq 0$

3.17 Sum of divisors of a number

- $\sigma(n) = \prod_{i=1}^{k}(1 + p_i + ... + p_i^{\alpha_i}) = \prod_{i=1}^{k} \frac{p_i^{\alpha_i+1}-1}{p_i-1}$

3.18 Summations

- $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$
- $\sum_{i=1}^{n} i^3 = (\frac{n(n+1)}{2})^2$
- $\sum_{i=1}^{n} i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$
- $\sum_{i=1}^{n} i^5 = \frac{(n(n+1))^2(2n^2+2n-1)}{12}$
- $\sum_{i=0}^{n} x^i = \frac{x^{n+1}-1}{x-1}$ para $x \neq 1$

3.19 Theorems

- There is always a prime between numbers n^2 and $(n+1)^2$, where n is any positive integer
- There is an infinite number of pairs of the from $\{p, p+2\}$ where both p and $p+2$ are primes.
- Every even integer greater than 2 can be expressed as the sum of two primes.
- Every integer greater than 2 can be written as the sum of three primes.

4 Geometry

4.1 3D

```
typedef double T;
struct p3 {
  T x, y, z;
  // Basic vector operations
  p3 operator + (p3 p) { return {x+p.x, y+p.y, z+p.z }; }
  p3 operator - (p3 p) { return {x - p.x, y - p.y, z - p.z}; }
  p3 operator * (T d) { return {x*d, y*d, z*d}; }
  p3 operator / (T d) { return {x / d, y / d, z / d}; } // only for floating point
  // Some comparators
  bool operator == (p3 p) { return tie(x, y, z) == tie(p.x, p.y, p.z); }
  bool operator != (p3 p) { return !operator == (p); }
};
p3 zero {0, 0, 0 };
T operator | (p3 v, p3 w) { /// dot
  return v.x*w.x + v.y*w.y + v.z*w.z;
}
p3 operator * (p3 v, p3 w) { /// cross
  return { v.y*w.z - v.z*w.y, v.z*w.x - v.x*w.z, v.x*w.y - v.y*w.x };
}
T sq(p3 v) { return v | v; }
double abs(p3 v) { return sqrt(sq(v)); }
p3 unit(p3 v) { return v / abs(v); }
double angle(p3 v, p3 w) {
  double cos_theta = (v | w) / abs(v) / abs(w);
  return acos(max(-1.0, min(1.0, cos_theta)));
}
T orient(p3 p, p3 q, p3 r, p3 s) { /// orient s, pqr form a triangle
  return (q - p) * (r - p) | (s - p);
}
T orient_by_normal(p3 p, p3 q, p3 r, p3 n) { /// same as 2D but in n-normal direction
  return (q - p) * (r - p) | n;
}
struct plane {
  p3 n; T d;
  /// From normal n and offset d
  plane(p3 n, T d): n(n), d(d) {}
```

```
  /// From normal n and point P
  plane(p3 n, p3 p): n(n), d(n | p) {}
  /// From three non-collinear points P,Q,R
  plane(p3 p, p3 q, p3 r): plane((q - p) * (r - p), p) {}
  /// - these work with T = int
  T side(p3 p) { return (n | p) - d; }
  double dist(p3 p) { return abs(side(p)) / abs(n); }
  plane translate(p3 t) {return {n, d + (n | t)}; }
  /// - these require T = double
  plane shift_up(double dist) { return {n, d + dist * abs(n)}; }
  p3 proj(p3 p) { return p - n * side(p) / sq(n); }
  p3 refl(p3 p) { return p - n * 2 * side(p) / sq(n); }
};

struct line3d {
  p3 d, o;
  /// From two points P, Q
  line3d(p3 p, p3 q): d(q - p), o(p) {}
  /// From two planes p1, p2 (requires T = double)
  line3d(plane p1, plane p2) {
    d = p1.n * p2.n;
    o = (p2.n * p1.d - p1.n * p2.d) * d / sq(d);
  }
  /// - these work with T = int
  double sq_dist(p3 p) { return sq(d * (p - o)) / sq(d); }
  double dist(p3 p) { return sqrt(sq_dist(p)); }
  bool cmp_proj(p3 p, p3 q) { return (d | p) < (d | q); }
  /// - these require T = double
  p3 proj(p3 p) { return o + d * (d | (p - o)) / sq(d); }
  p3 refl(p3 p) { return proj(p) * 2 - p; }
  p3 inter(plane p) { return o - d * p.side(o) / (p.n | d); }
};

double dist(line3d l1, line3d l2) {
  p3 n = l1.d * l2.d;
  if(n == zero) // parallel
    return l1.dist(l2.o);
  return abs((l2.o - l1.o) | n) / abs(n);
}
p3 closest_on_line1(line3d l1, line3d l2) { /// closest point on l1 to l2
  p3 n2 = l2.d * (l1.d * l2.d);
  return l1.o + l1.d * ((l2.o - l1.o) | n2) / (l1.d | n2);
}
double small_angle(p3 v, p3 w) { return acos(min(abs(v | w) / abs(v) / abs(w), 1.0)); }
double angle(plane p1, plane p2) { return small_angle(p1.n, p2.n); }
bool is_parallel(plane p1, plane p2) { return p1.n * p2.n == zero; }
bool is_perpendicular(plane p1, plane p2) { return (p1.n | p2.n) == 0; }
double angle(line3d l1, line3d l2) { return small_angle(l1.d, l2.d); }
bool is_parallel(line3d l1, line3d l2) { return l1.d * l2.d == zero; }
bool is_perpendicular(line3d l1, line3d l2) { return (l1.d | l2.d) == 0; }
double angle(plane p, line3d l) { return _pI / 2 - small_angle(p.n, l.d); }
bool is_parallel(plane p, line3d l) { return (p.n | l.d) == 0; }
bool is_perpendicular(plane p, line3d l) { return p.n * l.d == zero; }
line3d perp_through(plane p, p3 o) { return line(o, o + p.n); }
plane perp_through(line3d l, p3 o) { return plane(l.d, o); }
```

4.2 General

```
const lf eps = 1e-9;
typedef double T;
struct pt {
  T x, y;
  pt operator + (pt p) { return {x+p.x, y+p.y}; }
  pt operator - (pt p) { return {x-p.x, y-p.y}; }
  pt operator * (pt p) { return {x*p.x-y*p.y, x*p.y+y*p.x}; }
  pt operator * (T d) { return {x*d, y*d}; }
  pt operator / (lf d) { return {x/d, y/d}; } /// only for floating point
  bool operator == (pt b) { return x == b.x && y == b.y; }
  bool operator != (pt b) { return !(*this == b); }
  bool operator < (const pt &o) const { return y < o.y || (y == o.y && x < o.x); }
  bool operator > (const pt &o) const { return y > o.y || (y == o.y && x > o.x); }
};
int cmp (lf a, lf b) { return (a + eps < b ? -1 :(b + eps < a ? 1 : 0)); }
/** Already in complex **/
T norm(pt a) { return a.x*a.x + a.y*a.y; }
lf abs(pt a) { return sqrt(norm(a)); }
lf arg(pt a) { return atan2(a.y, a.x); }
ostream& operator << (ostream& os, pt &p) {
  return os << "("<< p.x << "," << p.y << ")";
}
/***/
istream &operator >> (istream &in, pt &p) {
    T x, y; in >> x >> y;
    p = {x, y};
    return in;
}
T dot(pt a, pt b) { return a.x*b.x + a.y*b.y; }
T cross(pt a, pt b) { return a.x*b.y - a.y*b.x; }
T orient(pt a, pt b, pt c) { return cross(b-a,c-a); }
//pt rot(pt p, lf a) { return {p.x*cos(a) - p.y*sin(a), p.x*sin(a) + p.y*cos(a)}; }
//pt rot(pt p, double a) { return p * polar(1.0, a); } /// for complex
//pt rotate_to_b(pt a, pt b, lf ang) { return rot(a-b, ang)+b; }
pt rot90ccw(pt p) { return {-p.y, p.x}; }
pt rot90cw(pt p) { return {p.y, -p.x}; }
pt translate(pt p, pt v) { return p+v; }
pt scale(pt p, double f, pt c) { return c + (p-c)*f; }
bool are_perp(pt v, pt w) { return dot(v,w) == 0; }
int sign(T x) { return (T(0) < x) - (x < T(0)); }
pt unit(pt a) { return a/abs(a); }

bool in_angle(pt a, pt b, pt c, pt x) {
  assert(orient(a,b,c) != 0);
  if (orient(a,b,c) < 0) swap(b,c);
  return orient(a,b,x) >= 0 && orient(a,c,x) <= 0;
```

```
}

//lf angle(pt a, pt b) { return acos(max(-1.0, min(1.0, dot(a,b)/abs(a)/abs(b)))); }
//lf angle(pt a, pt b) { return atan2(cross(a, b), dot(a, b)); }
/// returns vector to transform points
pt get_linear_transformation(pt p, pt q, pt r, pt fp, pt fq) {
  pt pq = q-p, num{cross(pq, fq-fp), dot(pq, fq-fp)};
  return fp + pt{cross(r-p, num), dot(r-p, num)} / norm(pq);
}

bool half(pt p) { /// true if is in (0, 180]
  assert(p.x != 0 || p.y != 0); /// the argument of (0,0) is undefined
  return p.y > 0 || (p.y == 0 && p.x < 0);
}
bool half_from(pt p, pt v = {1, 0}) {
  return cross(v,p) < 0 || (cross(v,p) == 0 && dot(v,p) < 0);
}
bool polar_cmp(const pt &a, const pt &b) {
  return make_tuple(half(a), 0) < make_tuple(half(b), cross(a,b));
}

struct line {
  pt v; T c;
  line(pt v, T c) : v(v), c(c) {}
  line(T a, T b, T c) : v({b,-a}), c(c) {}
  line(pt p, pt q) : v(q-p), c(cross(v,p)) {}
  T side(pt p) { return cross(v,p)-c; }
  lf dist(pt p) { return abs(side(p)) / abs(v); }
  lf sq_dist(pt p) { return side(p)*side(p) / (lf)norm(v); }
  line perp_through(pt p) { return {p, p + rot90ccw(v)}; }
  bool cmp_proj(pt p, pt q) { return dot(v,p) < dot(v,q); }
  line translate(pt t) { return {v, c + cross(v,t)}; }
  line shift_left(double d) { return {v, c + d*abs(v)}; }
  pt proj(pt p) { return p - rot90ccw(v)*side(p)/norm(v); }
  pt refl(pt p) { return p - rot90ccw(v)*2*side(p)/norm(v); }
};

bool inter_ll(line l1, line l2, pt &out) {
  T d = cross(l1.v, l2.v);
  if (d == 0) return false;
  out = (l2.v*l1.c - l1.v*l2.c) / d;
  return true;
}
line bisector(line l1, line l2, bool interior) {
  assert(cross(l1.v, l2.v) != 0); /// l1 and l2 cannot be parallel!
  lf sign = interior ? 1 : -1;
  return {l2.v/abs(l2.v) + l1.v/abs(l1.v) * sign,
          l2.c/abs(l2.v) + l1.c/abs(l1.v) * sign};
}

bool in_disk(pt a, pt b, pt p) {
  return dot(a-p, b-p) <= 0;
}
bool on_segment(pt a, pt b, pt p) {
  return orient(a,b,p) == 0 && in_disk(a,b,p);
}
bool proper_inter(pt a, pt b, pt c, pt d, pt &out) {
  T oa = orient(c,d,a),
  ob = orient(c,d,b),
  oc = orient(a,b,c),
  od = orient(a,b,d);
  /// Proper intersection exists iff opposite signs
  if (oa*ob < 0 && oc*od < 0) {
    out = (a*ob - b*oa) / (ob-oa);
    return true;
  }
  return false;
}
set<pt> inter_ss(pt a, pt b, pt c, pt d) {
  pt out;
  if (proper_inter(a,b,c,d,out)) return {out};
  set<pt> s;
  if (on_segment(c,d,a)) s.insert(a);
  if (on_segment(c,d,b)) s.insert(b);
  if (on_segment(a,b,c)) s.insert(c);
  if (on_segment(a,b,d)) s.insert(d);
  return s;
}
lf pt_to_seg(pt a, pt b, pt p) {
  if(a != b) {
    line l(a,b);
    if (l.cmp_proj(a,p) && l.cmp_proj(p,b)) /// if closest to projection
      return l.dist(p); /// output distance to line
  }
  return min(abs(p-a), abs(p-b)); /// otherwise distance to A or B
}
lf seg_to_seg(pt a, pt b, pt c, pt d) {
  pt dummy;
  if (proper_inter(a,b,c,d,dummy)) return 0;
  return min({pt_to_seg(a,b,c), pt_to_seg(a,b,d),
             pt_to_seg(c,d,a), pt_to_seg(c,d,b)});
}

enum {IN, OUT, ON};
struct polygon {
  vector<pt> p;
  polygon(int n) : p(n) {}
  int top = -1, bottom = -1;
  void delete_repetead() {
    vector<pt> aux;
    sort(p.begin(), p.end());
    for(pt &i : p)
      if(aux.empty() || aux.back() != i)
        aux.push_back(i);
    p.swap(aux);
  }
```

```
bool is_convex() {
  bool pos = 0, neg = 0;
  for (int i = 0, n = p.size(); i < n; i++) {
    int o = orient(p[i], p[(i+1)%n], p[(i+2)%n]);
    if (o > 0) pos = 1;
    if (o < 0) neg = 1;
  }
  return !(pos && neg);
}
lf area(bool s = false) {
  lf ans = 0;
  for (int i = 0, n = p.size(); i < n; i++)
    ans += cross(p[i], p[(i+1)%n]);
  ans /= 2;
  return s ? ans : abs(ans);
}
lf perimeter() {
  lf per = 0;
  for(int i = 0, n = p.size(); i < n; i++)
    per += abs(p[i] - p[(i+1)%n]);
  return per;
}
bool above(pt a, pt p) { return p.y >= a.y; }
bool crosses_ray(pt a, pt p, pt q) {
  return (above(a,q)-above(a,p))*orient(a,p,q) > 0;
}
int in_polygon(pt a) {
  int crosses = 0;
  for(int i = 0, n = p.size(); i < n; i++) {
    if(on_segment(p[i], p[(i+1)%n], a)) return ON;
    crosses += crosses_ray(a, p[i], p[(i+1)%n]);
  }
  return (crosses&1 ? IN : OUT);
}
void normalize() { /// polygon is CCW
  bottom = min_element(p.begin(), p.end()) - p.begin();
  vector<pt> tmp(p.begin()+bottom, p.end());
  tmp.insert(tmp.end(), p.begin(), p.begin()+bottom);
  p.swap(tmp);
  bottom = 0;
  top = max_element(p.begin(), p.end()) - p.begin();
}
int in_convex(pt a) {
  assert(bottom == 0 && top != -1);
  if(a < p[0] || a > p[top]) return OUT;
  T orientation = orient(p[0], p[top], a);
  if(orientation == 0) {
    if(a == p[0] || a == p[top]) return ON;
    return top == 1 || top + 1 == p.size() ? ON : IN;
  } else if (orientation < 0) {
    auto it = lower_bound(p.begin()+1, p.begin()+top, a);
    T d = orient(*prev(it), a, *it);
    return d < 0 ? IN : (d > 0 ? OUT: ON);
  }
  else {
    auto it = upper_bound(p.rbegin(), p.rend()-top-1, a);
    T d = orient(*it, a, it == p.rbegin() ? p[0] : *prev(it));
    return d < 0 ? IN : (d > 0 ? OUT: ON);
  }
}
polygon cut(pt a, pt b) {
  line l(a, b);
  polygon new_polygon(0);
  for(int i = 0, n = p.size(); i < n; ++i) {
    pt c = p[i], d = p[(i+1)%n];
    lf abc = cross(b-a, c-a), abd = cross(b-a, d-a);
    if(abc >= 0) new_polygon.p.push_back(c);
    if(abc*abd < 0) {
      pt out; inter_ll(l, line(c, d), out);
      new_polygon.p.push_back(out);
    }
  }
  return new_polygon;
}
void convex_hull() {
  sort(p.begin(), p.end());
  vector<pt> ch;
  ch.reserve(p.size()+1);
  for(int it = 0; it < 2; it++) {
    int start = ch.size();
    for(auto &a : p) {
      /// if colineal are needed, use < and remove repeated points
      while(ch.size() >= start+2 && orient(ch[ch.size()-2], ch.back(), a) <= 0)
        ch.pop_back();
      ch.push_back(a);
    }
    ch.pop_back();
    reverse(p.begin(), p.end());
  }
  if(ch.size() == 2 && ch[0] == ch[1]) ch.pop_back();
  /// be careful with CH of size < 3
  p.swap(ch);
}
vector<pii> antipodal() {
  vector<pii> ans;
  int n = p.size();
  if(n == 2) ans.push_back({0, 1});
  if(n < 3) return ans;
  auto nxt = [&](int x) { return (x+1 == n ? 0 : x+1); };
  auto area2 = [&](pt a, pt b, pt c) { return cross(b-a, c-a); };
  int b0 = 0;
  while(abs(area2(p[n - 1], p[0], p[nxt(b0)])) >
        abs(area2(p[n - 1], p[0], p[b0])))
    ++b0;
  for(int b = b0, a = 0; b != 0 && a <= b0; ++a) {
    ans.push_back({a, b});
```

```
      while (abs(area2(p[a], p[nxt(a)], p[nxt(b)])) >
             abs(area2(p[a], p[nxt(a)], p[b]))) {
        b = nxt(b);
        if(a != b0 || b != 0) ans.push_back({ a, b });
        else return ans;
      }
      if(abs(area2(p[a], p[nxt(a)], p[nxt(b)])) ==
         abs(area2(p[a], p[nxt(a)], p[b]))) {
        if(a != b0 || b != n-1) ans.push_back({ a, nxt(b) });
        else ans.push_back({ nxt(a), b });
      }
    }
    return ans;
  }
  pt centroid() {
    pt c{0, 0};
    lf scale = 6. * area(true);
    for(int i = 0, n = p.size(); i < n; ++i) {
      int j = (i+1 == n ? 0 : i+1);
      c = c + (p[i] + p[j]) * cross(p[i], p[j]);
    }
    return c / scale;
  }
  ll pick() {
    ll boundary = 0;
    for(int i = 0, n = p.size(); i < n; i++) {
      int j = (i+1 == n ? 0 : i+1);
      boundary += __gcd((ll)abs(p[i].x - p[j].x), (ll)abs(p[i].y - p[j].y));
    }
    return area() + 1 - boundary/2;
  }
  pt& operator[] (int i){ return p[i]; }
};

struct circle {
  pt c; T r;
};

circle center(pt a, pt b, pt c) {
  b = b-a, c = c-a;
  assert(cross(b,c) != 0); /// no circumcircle if A,B,C aligned
  pt cen = a + rot90ccw(b*norm(c) - c*norm(b))/cross(b,c)/2;
  return {cen, abs(a-cen)};
}
int inter_cl(circle c, line l, pair<pt, pt> &out) {
  lf h2 = c.r*c.r - l.sq_dist(c.c);
  if(h2 >= 0) {
    pt p = l.proj(c.c);
    pt h = l.v*sqrt(h2)/abs(l.v);
    out = {p-h, p+h};
  }
  return 1 + sign(h2);
}
int inter_cc(circle c1, circle c2, pair<pt, pt> &out) {
  pt d=c2.c-c1.c; double d2=norm(d);
  if(d2 == 0) { assert(c1.r != c2.r); return 0; } // concentric circles
  double pd = (d2 + c1.r*c1.r - c2.r*c2.r)/2; // = |O_1P| * d
  double h2 = c1.r*c1.r - pd*pd/d2; // = h2
  if(h2 >= 0) {
    pt p = c1.c + d*pd/d2, h = rot90ccw(d)*sqrt(h2/d2);
    out = {p-h, p+h};
  }
  return 1 + sign(h2);
}

int tangents(circle c1, circle c2, bool inner, vector<pair<pt,pt>> &out) {
  if(inner) c2.r = -c2.r;
  pt d = c2.c-c1.c;
  double dr = c1.r-c2.r, d2 = norm(d), h2 = d2-dr*dr;
  if(d2 == 0 || h2 < 0) { assert(h2 != 0); return 0; }
  for(double s : {-1,1}) {
    pt v = (d*dr + rot90ccw(d)*sqrt(h2)*s)/d2;
    out.push_back({c1.c + v*c1.r, c2.c + v*c2.r});
  }
  return 1 + (h2 > 0);
}

int tangent_through_pt(pt p, circle c, pair<pt, pt> &out) {
  double d = abs(p - c.c);
      if(d < c.r) return 0;
  pt base = c.c-p;
  double w = sqrt(norm(base) - c.r*c.r);
  pt a = {w, c.r}, b = {w, -c.r};
  pt s = p + base*a/norm(base)*w;
  pt t = p + base*b/norm(base)*w;
  out = {s, t};
  return 1 + (abs(c.c-p) == c.r);
}
```

4.3 nonTested

```
lf part(pt a, pt b, T r) {
  lf l = abs(a-b);
  pt p = (b-a)/l;
  lf c = dot(a, p), d = 4.0 * (c*c - dot(a, a) + r*r);
  if (d < eps) return angle(a, b) * r * r * 0.5;
  d = sqrt(d) * 0.5;
  lf s = -c - d, t = -c + d;
  if (s < 0.0) s = 0.0; else if (s > l) s = l;
  if (t < 0.0) t = 0.0; else if (t > l) t = l;
  pt u = a + p*s, v = a + p*t;
  return (cross(u, v) + (angle(a, u) + angle(v, b)) * r * r) * 0.5;
}
```

```
lf inter_cp(circle c, polygon p) {
  lf ans = 0;
  int n = p.p.size();
  for (int i = 0; i < n; i++) {
    ans += part(p[i]-c.c, p[(i+1)%4]-c.c, c.r);
  }
  return abs(ans);
}
struct circle{
    point center; double r;
    bool contain(point &p) { return abs(center - p) < r + eps;}
};
T cross(point a, point b) { return a.x*b.y - a.y*b.x; }
point rot90ccw(point p) { return {-p.y, p.x}; }
point get(point a, point b, point c) {
  b = b-a, c = c-a;
  //assert(cross(b,c) != 0); /// no circumcircle if A,B,C aligned
  point cen = a + rot90ccw(b*norm(c) - c*norm(b))/cross(b,c)/2;
  return cen;
}

circle min_circle(vector<point> &cloud, int a, int b){
    point center = (cloud[a] + cloud[b]) / double(2.);
    double rat = abs(center - cloud[a]);
    circle C = {center, rat};
    for (int i = 0; i < b; ++i){
        point x = cloud[i];
        if (C.contain(x)) continue;
        center = get( cloud[a], cloud[b], cloud[i] );
        rat = abs(center - cloud[a]);
        C = {center, rat};
    }
    return C;
}

circle min_circle(vector<point> &cloud, int a){
    point center = (cloud[a] + cloud[0]) / double(2.);
    double rat = abs(center - cloud[a]);
    circle C = {center, rat};
    for (int i = 0; i < a; ++i){
        point x = cloud[i];
        if (C.contain(x)) continue;
        C = min_circle(cloud, a, i);
    }
    return C;
}
circle min_circle(vector<point> cloud){
    // random_shuffle(cloud.begin(), cloud.end());
    int n = (int)cloud.size();
    for (int i = 1; i < n; ++i){
        int u = rand() % i;
        swap(cloud[u], cloud[i]);
    }
    point center = (cloud[0] + cloud[1]) / double(2.);
    double rat = abs(center - cloud[0]);
    circle C = {center, rat};
    for (int i = 2; i < n; ++i){
        point x = cloud[i];
        if (C.contain(x)) continue;
        C = min_circle(cloud, i);
    }
    return C;
}
```

5 Graphs

5.1 2-satisfiability

```
/// Complexity: O(|N|)
/// Tested: https://tinyurl.com/y8qhbzn4
struct sat2 {
  int n;
  vector<vector<vector<int>>> g;
  vector<int> tag;
  vector<bool> seen, value;
  stack<int> st;
  sat2(int n) : n(n), g(2, vector<vector<int>>(2*n)), tag(2*n), seen(2*n), value(2*n) {
      }
  int neg(int x) { return 2*n-x-1; }
  void add_or(int u, int v) { implication(neg(u), v); }
  void make_true(int u) { add_edge(neg(u), u); }
  void make_false(int u) { make_true(neg(u)); }
  void eq(int u, int v) {
    implication(u, v);
    implication(v, u);
  }
  void diff(int u, int v) { eq(u, neg(v)); }
  void implication(int u, int v) {
    add_edge(u, v);
    add_edge(neg(v), neg(u));
  }
  void add_edge(int u, int v) {
    g[0][u].push_back(v);
    g[1][v].push_back(u);
  }
  void dfs(int id, int u, int t = 0) {
    seen[u] = true;
    for(auto& v : g[id][u])
      if(!seen[v])
        dfs(id, v, t);
    if(id == 0) st.push(u);
    else tag[u] = t;
  }
```

```
  void kosaraju() {
    for(int u = 0; u < n; u++) {
      if(!seen[u]) dfs(0, u);
      if(!seen[neg(u)]) dfs(0, neg(u));
    }
    fill(seen.begin(), seen.end(), false);
    int t = 0;
    while(!st.empty()) {
      int u = st.top(); st.pop();
      if(!seen[u]) dfs(1, u, t++);
    }
  }
  bool satisfiable() {
    kosaraju();
    for(int i = 0; i < n; i++) {
      if(tag[i] == tag[neg(i)]) return false;
      value[i] = tag[i] > tag[neg(i)];
    }
    return true;
  }
};
```

5.2 Erdos–Gallai theorem

```
/// Complexity: O(|N|*log|N|)
/// Tested: https://tinyurl.com/yb5v9bau
/// Theorem: it gives a necessary and sufficient condition for a finite sequence
///         of natural numbers to be the degree sequence of a simple graph
bool erdos(vector<int> &d) {
  ll sum = 0;
  for(int i = 0; i < d.size(); ++i) sum += d[i];
  if(sum & 1) return false;
  sort(d.rbegin(), d.rend());
  ll l = 0, r = 0;
  for(int k = 1, i = d.size() - 1; k <= d.size(); ++k) {
    l += d[k-1];
    if(k > i) r -= d[++i];
    while (i >= k && d[i] < k+1) r += d[i--];
    if(l > 1ll*k*(k-1) + 1ll*k*(i-k+1) + r)
      return false;
  }
  return true;
}
```

5.3 Eulerian path

```
/// Complexity: O(|N|)
/// Tested: https://tinyurl.com/y85t8e83
bool eulerian(vector<int> &tour) { /// directed graph
  int one_in = 0, one_out = 0, start = -1;
  bool ok = true;
  for (int i = 0; i < n; i++) {
    if(out[i] && start == -1) start = i;
    if(out[i] - in[i] == 1) one_out++, start = i;
    else if(in[i] - out[i] == 1) one_in++;
    else ok &= in[i] == out[i];
  }
  ok &= one_in == one_out && one_in <= 1;
  if (ok) {
    function<void(int)> go = [&](int u) {
      while(g[u].size()) {
        int v = g[u].back();
        g[u].pop_back();
        go(v);
      }
      tour.push_back(u);
    };
    go(start);
    reverse(tour.begin(), tour.end());
    if(tour.size() == edges + 1) return true;
  }
  return false;
}
```

5.4 Lowest common ancestor

```
/// Complexity: O(|N|*log|N|)
/// Tested: https://tinyurl.com/y9g2ljv9, https://tinyurl.com/y87q3j93
int lca(int a, int b) {
  if(depth[a] < depth[b]) swap(a, b);
  //int ans = INT_MAX;
  for(int i = LOG2-1; i >= 0; --i)
    if(depth[ dp[a][i] ] >= depth[b]) {
      //ans = min(ans, mn[a][i]);
      a = dp[a][i];
    }
  //if (a == b) return ans;
  if(a == b) return a;
  for(int i = LOG2-1; i >= 0; --i)
    if(dp[a][i] != dp[b][i]) {
      //ans = min(ans, mn[a][i]);
      //ans = min(ans, mn[b][i]);
      a = dp[a][i],
      b = dp[b][i];
    }
  //ans = min(ans, mn[a][0]);
  //ans = min(ans, mn[b][0]);
  //return ans;
  return dp[a][0];
```

```
}
void dfs(int u, int d = 1, int p = -1) {
  depth[u] = d;
  for(auto v : g[u]) {
    //int v = x.first;
    //int w = x.second;
    if(v != p) {
      dfs(v, d + 1, u);
      dp[v][0] = u;
      //mn[v][0] = w;
    }
  }
}
void build(int n) {
  for(int i = 0; i <= n; i++) dp[i][0] = -1;
  for(int i = 0; i < n, i++) {
    if(dp[i][0] == -1) {
      dp[i][0] = i;
      //mn[i][0] = INT_MAX;
      dfs(i);
    }
  }

  for(int j = 0; j < LOG2-1; ++j)
    for(int i = 0; i <= n; ++i) { // nodes
      dp[i][j+1] = dp[ dp[i][j] ][j];
      //mn[i][j+1] = min(mn[ dp[i][j] ][j], mn[i][j]);
    }
}
```

5.5 Number of spanning trees

```
/// Tested: not yet
///A -> adjacency matrix
///It is necessary to compute the D-A matrix, where D is a diagonal matrix
///that contains the degree of each node.
///To compute the number of spanning trees it's necessary to compute any
///D-A cofactor
///C(i, j) = (-1)^(i+j) * Mij
///Where Mij is the matrix determinant after removing row i and column j
double mat[MAX][MAX];
///call determinant(n - 1)
double determinant(int n) {
  double det = 1.0;
  for(int k = 0; k < n; k++) {
    for(int i = k+1; i < n; i++) {
      assert(mat[k][k] != 0);
      long double factor = mat[i][k]/mat[k][k];
      for(int j = 0; j < n; j++) {
        mat[i][j] = mat[i][j] - factor*mat[k][j];
      }
    }
    det *= mat[k][k];
  }
  return round(det);
}
```

5.6 Scc

```
/// Complexity: O(|N|)
/// Tested: https://tinyurl.com/y8ujj3ws
int scc(int n) {
  vector<int> dfn(n+1), low(n+1), in_stack(n+1);
  stack<int> st;
  int tag = 0;
  function<void(int, int&)> dfs = [&](int u, int &t) {
    dfn[u] = low[u] = ++t;
    st.push(u);
    in_stack[u] = true;
    for(auto &v : g[u]) {
      if(!dfn[v]) {
        dfs(v, t);
        low[u] = min(low[u], low[v]);
      } else if(in_stack[v])
        low[u] = min(low[u], dfn[v]);
    }
    if (low[u] == dfn[u]) {
      int v;
      do {
        v = st.top(); st.pop();
//        id[v] = tag;
        in_stack[v] = false;
      } while (v != u);
      tag++;
    }
  };
  for(int u = 1, t; u <= n; ++u) {
    if(!dfn[u]) dfs(u, t = 0);
  }
  return tag;
}
```

5.7 Tarjan tree

```
/// Complexity: O(|N|)
/// Tested: https://tinyurl.com/y9g2ljv9, https://tinyurl.com/y87q3j93
struct tarjan_tree {
  int n;
  vector<vector<int>> g, comps;
```

```
vector<pii> bridge;
vector<int> id, art;
tarjan_tree(int n) : n(n), g(n+1), id(n+1), art(n+1) {}
void add_edge(vector<vector<int>> &g, int u, int v) { /// nodes from [1, n]
  g[u].push_back(v);
  g[v].push_back(u);
}
void add_edge(int u, int v) { add_edge(g, u, v); }
void tarjan(bool with_bridge) {
  vector<int> dfn(n+1), low(n+1);
  stack<int> st;
  comps.clear();
  function<void(int, int, int&)> dfs = [&](int u, int p, int &t) {
    dfn[u] = low[u] = ++t;
    st.push(u);
    int cntp = 0;
    for(int v : g[u]) {
      cntp += v == p;
      if(!dfn[v])   {
        dfs(v, u, t);
        low[u] = min(low[u], low[v]);
        if(with_bridge && low[v] > dfn[u]) {
          bridge.push_back({min(u,v), max(u,v)});
          comps.push_back({});
          for(int w = -1; w != v; )
            comps.back().push_back(w = st.top()), st.pop();
        }
        if(!with_bridge && low[v] >= dfn[u]) {
          art[u] = (dfn[u] > 1 || dfn[v] > 2);
          comps.push_back({u});
          for(int w = -1; w != v; )
            comps.back().push_back(w = st.top()), st.pop();
        }
      }
      else if(v != p || cntp > 1) low[u] = min(low[u], dfn[v]);
    }
    if(p == -1 && ( with_bridge || g[u].size() == 0 )) {
      comps.push_back({});
      for(int w = -1; w != u; )
        comps.back().push_back(w = st.top()), st.pop();
    }
  };
  for(int u = 1, t; u <= n; ++u)
    if(!dfn[u]) dfs(u, -1, t = 0);
}
vector<vector<int>> build_block_cut_tree() {
  tarjan(false);
  int t = 0;
  for(int u = 1; u <= n; ++u)
    if(art[u]) id[u] = t++;
  vector<vector<int>> tree(t+comps.size());
  for(int i = 0; i < comps.size(); ++i)
    for(int u : comps[i]) {
      if(!art[u]) id[u] = i+t;
      else add_edge(tree, i+t, id[u]);
    }
  return tree;
}
vector<vector<int>> build_bridge_tree() {
  tarjan(true);
  vector<vector<int>> tree(comps.size());
  for(int i = 0; i < comps.size(); ++i)
    for(int u : comps[i]) id[u] = i;
  for(auto &b : bridge)
    add_edge(tree, id[b.first], id[b.second]);
  return tree;
}
};
```

5.8 Tree binarization

```
/// Complexity: O(|N|)
/// Tested: Not yet
void add(int u, int v, int w) { ng[u].push_back({v, w}); }
void binarize(int u, int p = -1) {
  int last = u, f = 0;
  for(auto x : g[u]) {
    int v = x.first, w = x.second, node = ng.size();
    if(v == p) continue;
    if(f++) {
      ng.push_back({});
      add(last, node, 0);
      add(node, v, w);
      last = node;
    } else add(u, v, w);
    binarize(v, u);
  }
}
```

5.9 Yen

```
/// Complexity: O( |K|*|N|^3 )
/// Tested: not yet
int n;
vector<int> graph[ MAXN ];
int cost[ MAXN ][ MAXN ], dist[ MAXN ], connect[ MAXP ], path[ MAXN ];
ll vis = 0, mark = 0, edge[ MAXN ];
vector<int> emp;
struct Path {
  int w;
  vector<int> p;
```

```
  Path( ) : w(0) { }
  Path( int w ) : w(w) { }
  Path( int w, vector<int> p ) : w(w), p(p) { }
  bool operator < ( const Path& other )const {
    if( w == other.w ) {
      return lexicographical_compare( p.begin(), p.end(), other.p.begin(), other.p.end()
           );
    }
    return w < other.w;
  }
  bool operator > ( const Path& other )const {
    if( w == other.w ){
      return lexicographical_compare( other.p.begin(), other.p.end(), p.begin(), p.end()
           );
    }
    return w > other.w;
  }
};

void add_edge( int u, int v, int w ) {
  cost[u][v] = w;
  edge[u] |= ( 1LL<<v );
  graph[u].push_back( v );
}

Path dijkstra( int s, int t ) {
  priority_queue< pii, vector<pii>, greater<pii> > pq;
  fill( dist, dist+n+1, INF );
  pq.push( {0,s} );
  dist[s] = 0;
  while( !pq.empty() ) {
    int u = pq.top().second, c = pq.top().first;
    pq.pop();
    if( u == t ) break;
    if( ((vis>>u)&1) && s != u )
      continue;
    vis |= ( 1LL<<u );
    for( int i = 0; i < graph[u].size(); ++i ) {
      int v = graph[u][i];
      if( ((vis>>v)&1) || ( s == u && !((mark>>v)&1)) ) {
        continue;
      }
      if( cost[u][v] != INF && dist[v] >= c+cost[u][v] ) {
        if( dist[v] > c+cost[u][v] || ( dist[v] == c+cost[u][v] && u < path[v] ) ) {
          dist[v] = c+cost[u][v];
          path[v] = u;
          pq.push( {dist[v], v} );
        }
      }
    }
  }
  if( dist[t] == INF ) {
    return Path();
  }
  Path ret( dist[t] );
  for( int u = t; u != s; u = path[u] ) {
    ret.p.push_back( u );
  }
  ret.p.push_back( s );
  reverse( ret.p.begin(), ret.p.end() );
  return ret;
}

vector<int> yen( int s, int t, int k ) {
  priority_queue< Path, vector<Path>, greater<Path> > B;
  vector<vector<int>> A( MAXP );
  vis = 0;
  mark = edge[s];
  A[0] = dijkstra( s, t ).p;
  if( A[0].size() == 0 ) {
    return A[0];
  }
  for( int it = 1; it < k; ++it ){
    Path root_path;
    memset( connect, -1, sizeof(connect) );
    vis = 0;
    bool F = true;
    for( int i = 0; i < A[it-1].size()-1; ++i ) {
      bool flag = false;
      if( F && it > 2 && A[it-1].size() > i+1 &&
          A[it-2].size() > i+1 && A[it-1][i+1] == A[it-2][i+1] ) flag = true;
      else F = false;
      if( i >= A[it-1].size()-1 ) continue;
      int spur_node = A[it-1][i];
      mark = edge[ spur_node ];
      root_path.w += ( i ? cost[ A[it-1][i-1] ][ spur_node ] : 0 );
      root_path.p.push_back( spur_node );
      vis |= ( 1LL<<spur_node );
      for( int j = 0; j < it; ++j ) {
        if( connect[j] == i-1 && A[j].size() > i && A[j][i] == spur_node ) {
          connect[j] = i;
          if( A[j].size() > i+1 ) {
            mark &= ~( 1LL<<A[j][i+1] );
          }
        }
      }
      if( flag ) continue;
      ll prev_vis = vis;
      Path spur_path = dijkstra( spur_node, t );
      vis = prev_vis;
      if( spur_path.p.empty() ) continue;
      Path cur_path = root_path;
      cur_path.w += spur_path.w;
      for( int j = 1; j < spur_path.p.size(); ++j ) {
        cur_path.p.push_back( spur_path.p[j] );
      }
```

```
      B.push( cur_path );
    }
    if( B.empty() ) return emp;
    A[ it ] = B.top().p;
    while( !B.empty() && B.top().p == A[it] ) {
      B.pop();
    }
  }
  return A[ k-1 ];
}
```

6 Math

6.1 Berlekamp-Massey

```
#include<bits/stdc++.h>

using namespace std;

const int mod = 998244353;

inline int pw(int a, int b) {
  int ans = 1;
  while (b) {
    if (b & 1) ans = 1 LL * ans * a % mod;
    a = 1 LL * a * a % mod;
    b >>= 1;
  }
  return ans;
}

namespace linear_seq {
  int m;
  // a = first m terms
  // p = dependence, length is m
  vector < int > p, a;

  inline vector < int > BM(vector < int > x) { // finds shortest linear recurrence
      given first x terms in O(x^2)
    //ls = last s' recurrence
    vector < int > ls, cur;
    //ld  = last t' found
    //lf delta of last found
    int lf, ld;
    for (int i = 0; i < (int) x.size(); ++i) {
      int t = 0;
      //evaluate at position i
      for (int j = 0; j < (int) cur.size(); ++j)
        t = (t + 1 LL * x[i - j - 1] * cur[j]) % mod;
      if ((t - x[i]) % mod == 0) continue;

      if (!cur.size()) { //first non-zero element
        cur.resize(i + 1);
        lf = i;
        ld = (t - x[i]) % mod;
        continue;
      }
      int k = 1 LL * (t - x[i]) * pw(ld, mod - 2) % mod;
      vector < int > c(i - lf - 1); //add zeroes in front
      c.push_back(k); //add '1'
      for (int j = 0; j < (int) ls.size(); ++j) //add minus previous s'
        c.push_back(-1 LL * ls[j] * k % mod);

      if (c.size() < cur.size()) c.resize(cur.size());

      for (int j = 0; j < (int) cur.size(); ++j)
        c[j] = (c[j] + cur[j]) % mod;

      if (i + lf + (int) ls.size() >= (int) cur.size())
        ls = cur, lf = i, ld = (t - x[i]) % mod;
      cur = c;
    }
    for (int i = 0; i < (int) cur.size(); ++i)
      cur[i] = (cur[i] % mod + mod) % mod;

    m = cur.size();
    p.resize(m), a.resize(m);
    for (int i = 0; i < m; ++i)
      p[i] = cur[i], a[i] = x[i];
    return cur;
  }

  inline vector < int > mul(vector < int > & a, vector < int > & b) { // a * b mod f; f
       = x ** m - sum{1..m} x**(m-j) * pj
    //may be optimized using FFT, NTT
    vector < int > r(2 * m);
    for (int i = 0; i < m; ++i)
      if (a[i])
        for (int j = 0; j < m; ++j)
          r[i + j] = (r[i + j] + 1 LL * a[i] * b[j]) % mod;

    for (int i = 2 * m - 1; i >= m; --i)
      if (r[i])
        for (int j = m - 1; j >= 0; --j)
          r[i - j - 1] = (r[i - j - 1] + 1 LL * p[j] * r[i]) % mod;

    r.resize(m);
    return r;
  }
  // O(m*m*log(k)) with Fourier O(m*log(m)*log(k))
  inline int calc(long long k) { // res = G[ x**k ] = G[ x ** k mod f]
    if (m == 0) return 0;
```

```
    vector < int > bs(m), r(m);

    if (m == 1) bs[0] = p[0];
    else bs[1] = 1;

    r[0] = 1;

    while (k) {
      if (k & 1) r = mul(r, bs);
      bs = mul(bs, bs);
      k >>= 1;
    }
    int res = 0;
    for (int i = 0; i < m; ++i)
      res = (res + 1 LL * r[i] * a[i]) % mod;
    return res;
  }
}
```

6.2 Chinese remainder theorem

```
/// Complexity: |N|*log(|N|)
/// Tested: Not yet.
/// finds a suitable x that meets: x is congruent to a_i mod n_i
/** Works for non-coprime moduli.
 Returns {-1,-1} if solution does not exist or input is invalid.
 Otherwise, returns {x,L}, where x is the solution unique to mod L = LCM of mods
*/
pair<int, int> chinese_remainder_theorem( vector<int> A, vector<int> M ) {
  int n = A.size(), a1 = A[0], m1 = M[0];
  for(int i = 1; i < n; i++) {
    int a2 = A[i], m2 = M[i];
    int g = __gcd(m1, m2);
    if( a1 % g != a2 % g ) return {-1,-1};
    int p, q;
    eea(m1/g, m2/g, &p, &q);
    int mod = m1 / g * m2;
    q %= mod; p %= mod;
    int x = ((1ll*(a1%mod)*(m2/g))%mod*q + (1ll*(a2%mod)*(m1/g))%mod*p) % mod; // if WA
         there is overflow
    a1 = x;
    if (a1 < 0) a1 += mod;
    m1 = mod;
  }
  return {a1, m1};
}
```

6.3 Constant modular inverse

```
/// Complexity: O(|P|)
/// Tested: not yet
/// Find the multiplicative inverse of all 2<=i<p, module p
inv[1] = 1;
for(int i = 2; i < p; ++i)
        inv[i] = (p - (p / i) * inv[p % i] % p) % p;
```

6.4 Extended euclides

```
/// Complexity: O(log(|N|))
/// Tested: https://tinyurl.com/y8yc52gv
ll eea(ll a, ll b, ll& x, ll& y) {
  ll xx = y = 0; ll yy = x = 1;
  while (b) {
    ll q = a / b; ll t = b; b = a % b; a = t;
    t = xx; xx = x - q * xx; x = t;
    t = yy; yy = y - q * yy; y = t;
  }
  return a;
}
ll inverse(ll a, ll n) {
  ll x, y;
  ll g = eea(a, n, x, y);
  if(g > 1)
    return -1;
  return (x % n + n) % n;
}
```

6.5 Fast Fourier transform module

```
/// Complexity: O(|N|*log(|N|))
/// Tested: https://tinyurl.com/yagvw3on
const int mod = 7340033; /// mod = c*2^k+1
/// find g = primitive root of mod.
const int root = 2187; /// (g^c)%mod
const int root_1 = 4665133; /// inverse of root
const int root_pw = 1 << 19; /// 2^k

pii find_c_k(int mod) {
  pii ans;
  for(int k = 1; (1<<k) < mod; k++) {
    int pot = 1<<k;
    if((mod - 1) % pot == 0)
      ans = {(mod-1) / pot, k};
  }
  return ans;
}
```

```
int find_primitive_root(int mod) {
  vector<bool> seen(mod);
  for(int r = 2; ; r++) {
    fill(seen.begin(), seen.end(), 0);
    int cur = 1, can = 1;
    for(int i = 0; i <= mod-2 && can; i++) {
      if(seen[cur]) can = 0;
      seen[cur] = 1;
      cur = (1ll*cur*r) % mod;
    }
    if(can)
      return r;
  }
  assert(false);
}

void fft(vector<int> &a, bool inv = 0) {
  int n = a.size();
  for(int i = 1, j = 0; i < n; i++) {
    int c = n >> 1;
    for (; j >= c; c >>= 1) j -= c;
    j += c;
    if(i < j) swap(a[i], a[j]);
  }
  for (int len = 2; len <= n; len <<= 1) {
    int wlen = inv ? root_1 : root;
    for(int i = len; i < root_pw; i <<= 1) wlen = (1 LL * wlen * wlen) % mod;
    for(int i = 0; i < n; i += len) {
      int w = 1;
      for(int j = 0; j < (len >> 1); j++) {
        int u = a[i + j], v = (a[i + j + (len >> 1)] * 1 LL * w) % mod;
        a[i + j] = u + v < mod ? u + v : u + v - mod;
        a[i + j + (len >> 1)] = u - v >= 0 ? u - v : u - v + mod;
        w = (w * 1 LL * wlen) % mod;
      }
    }
  }
  if (inv) {
    int nrev = pow(n);
    for(int i = 0; i < n; i++) a[i] = (a[i] * 1 LL * nrev) % mod;
  }
}
vector<int> mul(const vector <int> a, const vector <int> b) {
  vector<int> fa(a.begin(), a.end()), fb(b.begin(), b.end());
  int n = 1;
  while (n < max(a.size(), b.size())) n <<= 1;
  n <<= 1;
  fa.resize(n); fb.resize(n);
  fft(fa); fft(fb);
  for (int i = 0; i < n; i++) fa[i] = (1ll * fa[i] * fb[i]) % mod;
  fft(fa, 1);
  return fa;
}
```

6.6 Fast fourier transform

```
/// Complexity: O(|N|*log(|N|))
/// Tested: https://tinyurl.com/y8g2q66b
namespace fft {
  typedef long long ll;
  const double PI = acos(-1.0);
  vector<int> rev;
  struct pt {
    double r, i;
    pt(double r = 0.0, double i = 0.0) : r(r), i(i) {}
    pt operator + (const pt & b) { return pt(r + b.r, i + b.i); }
    pt operator - (const pt & b) { return pt(r - b.r, i - b.i); }
    pt operator * (const pt & b) { return pt(r * b.r - i * b.i, r * b.i + i * b.r); }
  };
  void fft(vector<pt> &y, int on) {
    int n = y.size();
    for(int i = 1; i < n; i++) if(i < rev[i]) swap(y[i], y[rev[i]]);
    for(int m = 2; m <= n; m <<= 1) {
      pt wm(cos(-on * 2 * PI / m), sin(-on * 2 * PI / m));
      for(int k = 0; k < n; k += m) {
        pt w(1, 0);
        for(int j = 0; j < m / 2; j++) {
          pt u = y[k + j];
          pt t = w * y[k + j + m / 2];
          y[k + j] = u + t;
          y[k + j + m / 2] = u - t;
          w = w * wm;
        }
      }
    }
    if(on == -1)
      for(int i = 0; i < n; i++) y[i].r /= n;
  }
  vector<ll> mul(vector<ll> &a, vector<ll> &b) {
    int n = 1, la = a.size(), lb = b.size(), t;
    for(n = 1, t = 0; n <= (la+lb+1); n <<= 1, t++); t = 1<<(t-1);
    vector<pt> x1(n), x2(n);
    rev.assign(n, 0);
    for(int i = 0; i < n; i++) rev[i] = rev[i >> 1] >> 1 |(i & 1 ? t : 0);
    for(int i = 0; i < la; i++) x1[i] = pt(a[i], 0);
    for(int i = 0; i < lb; i++) x2[i] = pt(b[i], 0);
    fft(x1, 1); fft(x2, 1);
    for(int i = 0; i < n; i++) x1[i] = x1[i] * x2[i];
    fft(x1, -1);
    vector<ll> sol(n);
    for(int i = 0; i < n; i++) sol[i] = x1[i].r + 0.5;
    return sol;
  }
```

```
}
```

6.7 Gauss jordan

```
/// Complexity: O(|N|^3)
/// Tested: https://tinyurl.com/y23sh38k
const int EPS = 1;
int gauss (vector<vector<int>> a, vector<int> &ans) {
  int n = a.size(), m = a[0].size()-1;
  vector<int> where(m, -1);
  for(int col = 0, row = 0; col < m && row < n; ++col) {
    int sel = row;
    for(int i = row; i < n; ++i)
      if(abs(a[i][col]) > abs(a[sel][col])) sel = i;
    if(abs(a[sel][col]) < EPS) continue;
    swap(a[sel], a[row]);
    where[col] = row;
    for(int i = 0; i < n; ++i)
      if(i != row) {
        int c = divide(a[i][col], a[row][col]); /// precalc inverses
        for(int j = col; j <= m; ++j)
          a[i][j] = sub(a[i][j], mul(a[row][j], c));
      }
    ++row;
  }
  ans.assign(m, 0);
  for(int i = 0; i < m; ++i)
    if(where[i] != -1) ans[i] = divide(a[where[i]][m], a[where[i]][i]);
  for(int i = 0; i < n; ++i) {
    int sum = 0;
    for(int j = 0; j < m; ++j)
      sum = add(sum, mul(ans[j], a[i][j]));
    if(sum != a[i][m]) return 0;
  }
  for(int i = 0; i < m; ++i)
    if(where[i] == -1) return -1; /// infinite solutions
  return 1;
}
```

6.8 Integral

- Simpsons rule: $\int_a^b f(x)dx \approx \frac{b-a}{6}[f(a) + 4f(\frac{a+b}{2}) + f(b)]$
- Arc length: $s = \int_a^b \sqrt{1 + [f'(x)]^2}dx$
- Area of a surface of revolution: $A = 2\pi \int_a^b f(x)\sqrt{1 + [f'(x)]^2}dx$
- Volume of a solid of revolution: $V = \pi \int_a^b f(x)^2 dx$
- Note: In case of multiple functions such as *g(x) h(x)* for a solid of revolution then $f(x) = g(x) - h(x)$
- $f'(x) \approx \frac{f(x+h)-f(x-h)}{2h}$
- $f'(x) \approx \frac{f(x-2h)-8f(x-h)+8f(x+h)-f(x+2h)}{12h}$
- $f''(x) \approx \frac{f(x+h)-2f(x)+f(x-h)}{h^2}$

6.9 Lagrange Interpolation

```
/// Complexity: O(|N|^2)
/// Tested: https://tinyurl.com/y23sh38k
vector<lf> X, F;
lf f(lf x) {
  lf answer = 0;
  for(int i = 0; i < (int)X.size(); i++) {
    lf prod = F[i];
    for(int j = 0; j < (int)X.size(); j++) {
      if(i == j) continue;
      prod = mul(prod, divide(sub(x, X[j]), sub(X[i], X[j])));
    }
    answer = add(answer, prod);
  }
  return answer;
}
```

6.10 Linear diophantine

```
/// Complexity: O(log(|N|))
/// Tested: https://tinyurl.com/y8yc52gv
bool diophantine(ll a, ll b, ll c, ll &x, ll &y, ll &g) {
  x = y = 0;
  if(a == 0 && b == 0) return c == 0;
  if(b == 0) swap(a, b), swap(x, y);
  g = eea(abs(a), abs(b), x, y);
  if(c % g) return false;
  a /= g; b /= g; c /= g;
  if(a < 0) x *= -1;
  x = (x % b) * (c % b) % b;
  if(x < 0) x += b;
  y = (c - a*x) / b;
  return true;
}
///finds the first k | x + b * k / gcd(a, b) >= val
ll greater_or_equal_than(ll a, ll b, ll x, ll val, ll g) {
  lf got = 1.0 * (val - x) * g / b;
  return b > 0 ? ceil(got) : floor(got);
}
```

```
void get_xy (ll a, ll b, ll &x, ll &y, ll k, ll g) { /// if for y, change the order to
    b,a y,x
  x = x + b / g * k;
  y = y - a / g * k;
}
```

6.11 Matrix multiplication

```
const int MOD = 1e9+7;
struct matrix {
  const int N = 2;
  int m[N][N], r, c;
  matrix(int r = N, int c = N, bool iden = false) : r(r), c(c) {
    memset(m, 0, sizeof m);
    if(iden)
      for(int i = 0; i < r; i++) m[i][i] = 1;
  }
  matrix operator * (const matrix &o) const {
    matrix ret(r, o.c);
    for(int i = 0; i < r; ++i)
      for(int j = 0; j < o.c; ++j) {
        ll &r = ret.m[i][j];
        for(int k = 0; k < c; ++k)
          r = (r + 1ll*m[i][k]*o.m[k][j]) % MOD;
      }
    return ret;
  }
};
```

6.12 Miller rabin

```
/// Complexity: ???
/// Tested: A lot.. but no link
ll mul (ll a, ll b, ll mod) {
  ll ret = 0;
  for(a %= mod, b %= mod; b != 0;
    b >>= 1, a <<= 1, a = a >= mod ? a - mod : a) {
    if (b & 1) {
      ret += a;
      if (ret >= mod) ret -= mod;
    }
  }
  return ret;
}
ll fpow (ll a, ll b, ll mod) {
  ll ans = 1;
  for (; b; b >>= 1, a = mul(a, a, mod))
    if (b & 1)
      ans = mul(ans, a, mod);
  return ans;
}
bool witness (ll a, ll s, ll d, ll n) {
  ll x = fpow(a, d, n);
  if (x == 1 || x == n - 1) return false;
  for (int i = 0; i < s - 1; i++) {
    x = mul(x, x, n);
    if (x == 1) return true;
    if (x == n - 1) return false;
  }
  return true;
}
ll test[] = {2, 3, 5, 7, 11, 13, 17, 19, 23, 0};
bool is_prime (ll n) {
  if (n < 2) return false;
  if (n == 2) return true;
  if (n % 2 == 0) return false;
  ll d = n - 1, s = 0;
  while (d % 2 == 0) ++s, d /= 2;
  for (int i = 0; test[i] && test[i] < n; ++i)
    if (witness(test[i], s, d, n))
      return false;
  return true;
}
```

6.13 Pollard's rho

```
/// Complexity: ???
/// Tested: Not yet
ll pollard_rho(ll n, ll c) {
  ll x = 2, y = 2, i = 1, k = 2, d;
  while (true) {
    x = (mul(x, x, n) + c);
    if (x >= n) x -= n;
    d = __gcd(x - y, n);
    if (d > 1) return d;
    if (++i == k) y = x, k <<= 1;
  }
  return n;
}
void factorize(ll n, vector<ll> &f) {
  if (n == 1) return;
  if (is_prime(n)) {
    f.push_back(n);
    return;
  }
  ll d = n;
  for (int i = 2; d == n; i++)
    d = pollard_rho(n, i);
  factorize(d, f);
```

```
  factorize(n/d, f);
}
```

6.14 Simplex

```
/// Complexity: O(|N|^2 * |M|) N variables, N restrictions
/// Tested: https://tinyurl.com/ybphh57p
const double EPS = 1e-6;
typedef vector<double> vec;
namespace simplex {
  vector<int> X, Y;
  vector<vec> a;
  vec b, c;
  double z;
  int n, m;
  void pivot(int x, int y) {
    swap(X[y], Y[x]);
    b[x] /= a[x][y];
    for(int i = 0; i < m; i++)
      if(i != y)
        a[x][i] /= a[x][y];
    a[x][y] = 1 / a[x][y];
    for(int i = 0; i < n; i++)
      if(i != x && abs(a[i][y]) > EPS) {
      b[i] -= a[i][y] * b[x];
      for(int j = 0; j < m; j++)
        if(j != y)
          a[i][j] -= a[i][y] * a[x][j];
      a[i][y] =- a[i][y] * a[x][y];
    }
    z += c[y] * b[x];
    for(int i = 0; i < m; i++)
      if(i != y)
        c[i] -= c[y] * a[x][i];
    c[y] =- c[y] * a[x][y];
  }
  /// A is a vector of 1 and 0. B is the limit restriction. C is the factors of O.F.
  pair<double, vec> simplex(vector<vec> &A, vec &B, vec &C) {
    a = A; b = B; c = C;
    n = b.size(); m = c.size(); z = 0.0;
    X = vector<int>(m);
    Y = vector<int>(n);
    for(int i = 0; i < m; i++) X[i] = i;
    for(int i = 0; i < n; i++) Y[i] = i + m;
    while(1) {
      int x = -1, y = -1;
      double mn = -EPS;
      for(int i = 0; i < n; i++)
        if(b[i] < mn)
          mn = b[i], x = i;
      if(x < 0) break;
      for(int i = 0; i < m; i++)
        if(a[x][i] < -EPS) { y = i; break; }
      assert(y >= 0); // no sol
      pivot(x, y);
    }
    while(1) {
      double mx = EPS;
      int x = -1,y = -1;
      for(int i = 0; i < m; i++)
        if(c[i] > mx)
          mx = c[i], y = i;
      if(y < 0) break;
      double mn = 1e200;
      for(int i = 0; i < n; i++)
        if(a[i][y] > EPS && b[i] / a[i][y] < mn)
          mn = b[i] / a[i][y], x = i;
      assert(x >= 0); // unbound
      pivot(x, y);
    }
    vec r(m);
    for(int i = 0; i < n; i++)
      if(Y[i] < m)
        r[ Y[i] ] = b[i];
    return make_pair(z, r);
  }
}
```

6.15 Simpson

```
/// Complexity: ?????
/// Tested: Not yet
inline lf simpson(lf fl, lf fr, lf fmid, lf l, lf r) {
  return (fl + fr + 4.0 * fmid) * (r - l) / 6.0;
}
lf rsimpson (lf slr, lf fl, lf fr, lf fmid, lf l, lf r) {
        lf mid = (l + r) * 0.5;
        lf fml = f((l + mid) * 0.5);
        lf fmr = f((mid + r) * 0.5);
        lf slm = simpson(fl, fmid, fml, l, mid);
        lf smr = simpson(fmid, fr, fmr, mid, r);
        if (fabs(slr - slm - smr) < eps) return slm + smr;
        return rsimpson(slm, fl, fmid, fml, l, mid) + rsimpson(smr, fmid, fr, fmr, mid,
             r);
}
lf integrate(lf l,lf r) {
        lf mid = (l + r) * .5, fl = f(l), fr = f(r), fmid = f(mid);
        return rsimpson(simpson(fl, fr, fmid, l, r), fl, fr, fmid, l, r);
}
```

6.16 Totient and divisors

```
vector<int> count_divisors_sieve() {
  bitset<mx> is_prime; is_prime.set();
  vector<int> cnt(mx, 1);
  is_prime[0] = is_prime[1] = 0;
  for(int i = 2; i < mx; i++) {
    if(!is_prime[i]) continue;
    cnt[i]++;
    for(int j = i+i; j < mx; j += i) {
      int n = j, c = 1;
      while( n%i == 0 ) n /= i, c++;
      cnt[j] *= c;
      is_prime[j] = 0;
    }
  }
  return cnt;
}
vector<int> euler_phi_sieve() {
  bitset<mx> is_prime; is_prime.set();
  vector<int> phi(mx);
  iota(phi.begin(), phi.end(), 0);
  is_prime[0] = is_prime[1] = 0;
  for(int i = 2; i < mx; i++) {
    if(!is_prime[i]) continue;
    for(int j = i; j < mx; j += i) {
      phi[j] -= phi[j]/i;
      is_prime[j] = 0;
    }
  }
  return phi;
}
ll euler_phi(ll n) {
  ll ans = n;
  for(ll i = 2; i * i <= n; ++i) {
    if(n % i == 0) {
      ans -= ans / i;
      while(n % i == 0) n /= i;
    }
  }
  if(n > 1) ans -= ans / n;
  return ans;
}
```

7 Network flows

7.1 Blossom

```
/// Complexity: O(|E||V|^2)
/// Tested: https://tinyurl.com/oe5rnpk
struct network {
  struct struct_edge { int v; struct_edge * n; };
  typedef struct_edge* edge;
  int n;
  struct_edge pool[MAXE]; ///2*n*n;
  edge top;
  vector<edge> adj;
  queue<int> q;
  vector<int> f, base, inq, inb, inp, match;
  vector<vector<int>> ed;
  network(int n) : n(n), match(n, -1), adj(n), top(pool), f(n), base(n),
                   inq(n), inb(n), inp(n), ed(n, vector<int>(n)) {}
  void add_edge(int u, int v) {
    if(ed[u][v]) return;
    ed[u][v] = 1;
    top->v = v, top->n = adj[u], adj[u] = top++;
    top->v = u, top->n = adj[v], adj[v] = top++;
  }
  int get_lca(int root, int u, int v) {
    fill(inp.begin(), inp.end(), 0);
    while(1) {
      inp[u = base[u]] = 1;
      if(u == root) break;
      u = f[ match[u] ];
    }
    while(1) {
      if(inp[v = base[v]]) return v;
      else v = f[ match[v] ];
    }
  }
  void mark(int lca, int u) {
    while(base[u] != lca) {
      int v = match[u];
      inb[ base[u ]] = 1;
      inb[ base[v] ] = 1;
      u = f[v];
      if(base[u] != lca) f[u] = v;
    }
  }
  void blossom_contraction(int s, int u, int v) {
    int lca = get_lca(s, u, v);
    fill(inb.begin(), inb.end(), 0);
    mark(lca, u); mark(lca, v);
    if(base[u] != lca) f[u] = v;
    if(base[v] != lca) f[v] = u;
    for(int u = 0; u < n; u++)
      if(inb[base[u]]) {
        base[u] = lca;
        if(!inq[u]) {
            inq[u] = 1;
            q.push(u);
        }
```

```
    }
  }
  int bfs(int s) {
    fill(inq.begin(), inq.end(), 0);
    fill(f.begin(), f.end(), -1);
    for(int i = 0; i < n; i++) base[i] = i;
    q = queue<int>();
    q.push(s);
    inq[s] = 1;
   while(q.size()) {
      int u = q.front(); q.pop();
      for(edge e = adj[u]; e; e = e->n) {
        int v = e->v;
        if(base[u] != base[v] && match[u] != v) {
          if((v == s) || (match[v] != -1 && f[match[v]] != -1))
            blossom_contraction(s, u, v);
          else if(f[v] == -1) {
            f[v] = u;
            if(match[v] == -1) return v;
            else if(!inq[match[v]]) {
              inq[match[v]] = 1;
              q.push(match[v]);
            }
          }
        }
      }
    }
    return -1;
  }
  int doit(int u) {
    if(u == -1) return 0;
    int v = f[u];
    doit(match[v]);
    match[v] = u; match[u] = v;
    return u != -1;
  }
  /// (i < net.match[i]) => means match
  int maximum_matching() {
    int ans = 0;
    for(int u = 0; u < n; u++)
      ans += (match[u] == -1) && doit(bfs(u));
    return ans;
  }
};
```

7.2 Dinic

```
/// Complexity: O(|E|*|V|^2)
/// Tested: https://tinyurl.com/ya9rgoyk
struct edge { int v, cap, inv, flow; };
struct network {
  int n, s, t;
  vector<int> lvl;
  vector<vector<edge>> g;
  network(int n) : n(n), lvl(n), g(n) {}
  void add_edge(int u, int v, int c) {
    g[u].push_back({v, c, g[v].size(), 0});
    g[v].push_back({u, 0, g[u].size()-1, c});
  }
  bool bfs() {
    fill(lvl.begin(), lvl.end(), -1);
    queue<int> q;
    lvl[s] = 0;
    for(q.push(s); q.size(); q.pop()) {
      int u = q.front();
      for(auto &e : g[u]) {
        if(e.cap > 0 && lvl[e.v] == -1) {
          lvl[e.v] = lvl[u]+1;
          q.push(e.v);
        }
      }
    }
    return lvl[t] != -1;
  }
  int dfs(int u, int nf) {
    if(u == t) return nf;
    int res = 0;
    for(auto &e : g[u]) {
      if(e.cap > 0 && lvl[e.v] == lvl[u]+1) {
        int tf = dfs(e.v, min(nf, e.cap));
        res += tf; nf -= tf; e.cap -= tf;
        g[e.v][e.inv].cap += tf;
        g[e.v][e.inv].flow -= tf;
        e.flow += tf;
        if(nf == 0) return res;
      }
    }
    if(!res) lvl[u] = -1;
    return res;
  }
  int max_flow(int so, int si, int res = 0) {
    s = so; t = si;
    while(bfs()) res += dfs(s, INT_MAX);
    return res;
  }
};
```

7.3 Hopcroft karp

```
/// Complexity: O(|E|*sqrt(|V|))
/// Tested: https://tinyurl.com/yad2g9g9
struct mbm {
```

```
  vector<vector<int>> g;
  vector<int> d, match;
  int nil, l, r;
  /// u -> 0 to l, v -> 0 to r
  mbm(int l, int r) : l(l), r(r), nil(l+r), g(l+r),
                      d(1+l+r, INF), match(l+r, l+r) {}
  void add_edge(int a, int b) {
    g[a].push_back(l+b);
    g[l+b].push_back(a);
  }
  bool bfs() {
    queue<int> q;
    for(int u = 0; u < l; u++) {
      if(match[u] == nil) {
        d[u] = 0;
        q.push(u);
      } else d[u] = INF;
    }
    d[nil] = INF;
    while(q.size()) {
      int u = q.front(); q.pop();
      if(u == nil) continue;
      for(auto v : g[u]) {
        if(d[ match[v] ] == INF) {
          d[ match[v] ] = d[u]+1;
          q.push(match[v]);
        }
      }
    }
    return d[nil] != INF;
  }
  bool dfs(int u) {
    if(u == nil) return true;
    for(int v : g[u]) {
      if(d[ match[v] ] == d[u]+1 && dfs(match[v])) {
        match[v] = u; match[u] = v;
        return true;
      }
    }
    d[u] = INF;
    return false;
  }
  int max_matching() {
    int ans = 0;
    while(bfs()) {
      for(int u = 0; u < l; u++) {
        ans += (match[u] == nil && dfs(u));
      }
    }
    return ans;
  }
};
```

7.4 Maximum bipartite matching

```
/// Complexity: O(|E|*|V|)
/// Tested: https://tinyurl.com/yad2g9g9
struct mbm {
  int l, r;
  vector<vector<int>> g;
  vector<int> match, seen;
  mbm(int l, int r) : l(l), r(r), seen(r), match(r), g(l) {}
  void add_edge(int l, int r) { g[l].push_back(r); }
  bool dfs(int u) {
    for(auto v : g[u]) {
      if(seen[v]++) continue;
      if(match[v] == -1 || dfs(match[v])) {
        match[v] = u;
        return true;
      }
    }
    return false;
  }
  int max_matching() {
    int ans = 0;
    fill(match.begin(), match.end(), -1);
    for(int u = 0; u < l; ++u) {
      fill(seen.begin(), seen.end(), 0);
      ans += dfs(u);
    }
    return ans;
  }
};
```

7.5 Maximum flow minimum cost

```
/// Complexity: O(|V|*|E|^2*log(|E|))
/// Tested: https://tinyurl.com/ycgpp47z
template <class type>
struct mcmf {
  struct edge { int u, v, cap, flow; type cost; };
  int n;
  vector<edge> ed;
  vector<vector<int>> g;
  vector<int> p;
  vector<type> d, phi;
  mcmf(int n) : n(n), g(n), p(n), d(n), phi(n) {}
  void add_edge(int u, int v, int cap, type cost) {
    g[u].push_back(ed.size());
    ed.push_back({u, v, cap, 0, cost});
    g[v].push_back(ed.size());
    ed.push_back({v, u, 0, 0, -cost});
  }
```

```
  bool dijkstra(int s, int t) {
    fill(d.begin(), d.end(), INF);
    fill(p.begin(), p.end(), -1);
    set<pair<type, int>> q;
    d[s] = 0;
    for(q.insert({d[s], s}); q.size();) {
      int u = (*q.begin()).second; q.erase(q.begin());
      for(auto v : g[u]) {
        auto &e = ed[v];
        type nd = d[e.u]+e.cost+phi[e.u]-phi[e.v];
        if(0 < (e.cap-e.flow) && nd < d[e.v]) {
          q.erase({d[e.v], e.v});
          d[e.v] = nd; p[e.v] = v;
          q.insert({d[e.v], e.v});
        }
      }
    }
    for(int i = 0; i < n; i++) phi[i] = min(INF, phi[i]+d[i]);
    return d[t] != INF;
  }
  pair<int, type> max_flow(int s, int t) {
    type mc = 0;
    int mf = 0;
    fill(phi.begin(), phi.end(), 0);
    while(dijkstra(s, t)) {
      int flow = INF;
      for(int v = p[t]; v != -1; v = p[ ed[v].u ])
        flow = min(flow, ed[v].cap-ed[v].flow);
      for(int v = p[t]; v != -1; v = p[ ed[v].u ]) {
        edge &e1 = ed[v];
        edge &e2 = ed[v^1];
        mc += e1.cost*flow;
        e1.flow += flow;
        e2.flow -= flow;
      }
      mf += flow;
    }
    return {mf, mc};
  }
};
```

7.6 Stoer Wagner

```
/// Complexity: O(|V|^3)
/// Tested: https://tinyurl.com/y8eu433d
 struct stoer_wagner {
  int n;
  vector<vector<int>> g;
  stoer_wagner(int n) : n(n), g(n, vector<int>(n)) {}
  void add_edge(int a, int b, int w) { g[a][b] = g[b][a] = w; }
  pair<int, vector<int>> min_cut() {
    vector<int> used(n);
    vector<int> cut, best_cut;
    int best_weight = -1;
    for(int p = n-1; p >= 0; --p) {
      vector<int> w = g[0];
      vector<int> added = used;
      int prv, lst = 0;
      for(int i = 0; i < p; ++i) {
        prv = lst; lst = -1;
        for(int j = 1; j < n; ++j)
          if(!added[j] && (lst == -1 || w[j] > w[lst]))
            lst = j;
        if(i == p-1) {
          for(int j = 0; j < n; j++)
            g[prv][j] += g[lst][j];
          for(int j = 0; j < n; j++)
            g[j][prv] = g[prv][j];
          used[lst] = true;
          cut.push_back(lst);
          if(best_weight == -1 || w[lst] < best_weight) {
            best_cut = cut;
            best_weight = w[lst];
          }
        } else {
          for(int j = 0; j < n; j++)
            w[j] += g[lst][j];
          added[lst] = true;
        }
      }
    }
    return {best_weight, best_cut}; /// best_cut contains all nodes in the same set
  }
 };
```

7.7 Weighted matching

```
/// Complexity: O(|V|^3)
/// Tested: https://tinyurl.com/ycpq8eyl problem G
typedef int type;
struct matching_weighted {
  int l, r;
  vector<vector<type>> c;
  matching_weighted(int l, int r) : l(l), r(r), c(l, vector<type>(r)) {
    assert(l <= r);
  }
  void add_edge(int a, int b, type cost) { c[a][b] = cost; }
  type matching() {
    vector<type> v(r), d(r); // v: potential
    vector<int> ml(l, -1), mr(r, -1); // matching pairs
    vector<int> idx(r), prev(r);
    iota(idx.begin(), idx.end(), 0);
```

```
auto residue = [&](int i, int j) { return c[i][j]-v[j]; };
for(int f = 0; f < l; ++f) {
  for(int j = 0; j < r; ++j) {
    d[j] = residue(f, j);
    prev[j] = f;
  }
  type w;
  int j, l;
  for (int s = 0, t = 0;;) {
    if(s == t) {
      l = s;
      w = d[ idx[t++] ];
      for(int k = t; k < r; ++k) {
        j = idx[k];
        type h = d[j];
        if (h <= w) {
          if (h < w) t = s, w = h;
          idx[k] = idx[t];
          idx[t++] = j;
        }
      }
      for (int k = s; k < t; ++k) {
        j = idx[k];
        if (mr[j] < 0) goto aug;
      }
    }
    int q = idx[s++], i = mr[q];
    for (int k = t; k < r; ++k) {
      j = idx[k];
      type h = residue(i, j) - residue(i, q) + w;
      if (h < d[j]) {
        d[j] = h;
        prev[j] = i;
        if(h == w) {
          if(mr[j] < 0) goto aug;
          idx[k] = idx[t];
          idx[t++] = j;
        }
      }
    }
  }
  aug: for (int k = 0; k < l; ++k)
    v[ idx[k] ] += d[ idx[k] ] - w;
  int i;
  do {
    mr[j] = i = prev[j];
    swap(j, ml[i]);
  } while (i != f);
}
type opt = 0;
for (int i = 0; i < l; ++i)
  opt += c[i][ml[i]]; // (i, ml[i]) is a solution
return opt;
  }
};
```

8 Strings

8.1 Aho corasick

```
/// Complexity: O(|text|+SUM(|pattern_i|)+matches)
/// Tested: https://tinyurl.com/y2zq594p
const static int alpha = 26;
int trie[N*alpha][alpha], fail[N*alpha], nodes;
void add(string &s, int i) {
  int cur = 0;
  for(char c : s) {
    int x = c-'a';
    if(!trie[cur][x]) trie[cur][x] = ++nodes;
    cur = trie[cur][x];
  }
  //cnt_word[cur]++;
  //end_word[cur] = i; // for i > 0
}
void build() {
  queue<int> q; q.push(0);
  while(q.size()) {
    int u = q.front(); q.pop();
    for(int i = 0; i < alpha; ++i) {
      int v = trie[u][i];
      if(!v) continue;
      q.push(v);
      if(!u) continue;
      fail[v] = fail[u];
      while(fail[v] && !trie[ fail[v] ][i]) fail[v] = fail[ fail[v] ];
      fail[v] = trie[ fail[v] ][i];
      //fail_out[v] = end_word[ fail[v] ] ? fail[v] : fail_out[ fail[v] ];
      //cnt_word[v] += cnt_word[ fail[v] ]; // obtener informacion del fail_padre
    }
  }
}
```

8.2 Hashing

```
/// Tested: https://tinyurl.com/y8qstx97
/// 1000234999, 1000567999, 1000111997, 1000777121
const int MODS[] = { 1001864327, 1001265673 };
const mint BASE(256, 256), ZERO(0, 0), ONE(1, 1);
inline int add(int a, int b, const int& mod) { return a+b >= mod ? a+b-mod : a+b; }
inline int sbt(int a, int b, const int& mod) { return a-b < 0 ? a-b+mod : a-b; }
```

```cpp
inline int mul(int a, int b, const int& mod) { return 1ll*a*b%mod; }
inline ll operator ! (const mint a) { return (ll(a.first)<<32)|ll(a.second); }
inline mint operator + (const mint a, const mint b) {
  return {add(a.first, b.first, MODS[0]), add(a.second, b.second, MODS[1])};
}
inline mint operator - (const mint a, const mint b) {
  return {sbt(a.first, b.first, MODS[0]), sbt(a.second, b.second, MODS[1])};
}
inline mint operator * (const mint a, const mint b) {
  return {mul(a.first, b.first, MODS[0]), mul(a.second, b.second, MODS[1])};
}
mint base[MAXN];
void prepare() {
  base[0] = ONE;
  for(int i = 1; i < MAXN; i++) base[i] = base[i-1]*BASE;
}
template <class type>
struct hashing {
  vector<mint> code;
  hashing(type &t) {
    code.resize(t.size()+1);
    code[0] = ZERO;
    for (int i = 1; i < code.size(); ++i)
      code[i] = code[i-1]*BASE + mint{t[i-1], t[i-1]};
  }
  mint query(int l, int r) {
    return code[r+1] - code[l]*base[r-l+1];
  }
};
```

8.3 Kmp automaton

```cpp
/// Complexity: O(|N|*alphabet)
/// Tested: not yet
const int alpha = 256;
int aut[102][alpha];
void kmp_automaton(string &t) {
  vector<int> phi = get_phi(t);
  for(int i = 0; i <= t.size(); ++i) {
    for(int c = 0; c < alpha; ++c) {
      if(i == t.size() || (i > 0 && c != t[i])) aut[i][c] = aut[ phi[i-1] ][c];
      else aut[i][c] = i + (c == t[i]);
    }
  }
}
```

8.4 Kmp

```cpp
/// Complexity: O(|N|)
/// Tested: https://tinyurl.com/y7svn3kr
vector<int> get_phi(string &p) {
  vector<int> phi(p.size());
  phi[0] = 0;
  for(int i = 1, j = 0; i < p.size(); ++i ) {
    while(j > 0 && p[i] != p[j] ) j = phi[j-1];
    if(p[i] == p[j]) ++j;
    phi[i] = j;
  }
  return phi;
}
int get_match(string &t, string &p) {
  vector<int> phi = get_phi(p);
  int matches = 0;
  for(int i = 0, j = 0; i < t.size(); ++i ) {
    while(j > 0 && t[i] != p[j] ) j = phi[j-1];
    if(t[i] == p[j]) ++j;
    if(j == p.size()) {
      matches++;
      j = phi[j-1];
    }
  }
  return matches;
}
```

8.5 Manacher

```cpp
/// Complexity: O(|N|)
/// Tested: https://tinyurl.com/y6upxbpa
///to = i - from[i];
///len = to - from[i] + 1 = i - 2 * from[i] + 1;
vector<int> manacher(string &s) {
  int n = s.size(), p = 0, pr = -1;
  vector<int> from(2*n-1);
  for(int i = 0; i < 2*n-1; ++i) {
    int r = i <= 2*pr ? min(p - from[2*p - i], pr) : i/2;
    int l = i - r;
    while(l > 0 && r < n-1 && s[l-1] == s[r+1]) --l, ++r;
    from[i] = l;
    if (r > pr) {
      pr = r;
      p = i;
    }
  }
  return from;
}
```

8.6 Minimun expression

```
/// Complexity: O(|N|)
/// Tested: https://tinyurl.com/y6gfzgsm
int minimum_expression(string s) {
  s = s+s;
  int len = s.size(), i = 0, j = 1, k = 0;
  while(i+k < len && j+k < len) {
    if(s[i+k] == s[j+k]) k++;
    else if(s[i+k] > s[j+k]) i = i+k+1, k = 0;
    else j = j+k+1, k = 0;
    if(i == j) j++;
  }
  return min(i, j);
}
```

8.7 Suffix array

```
/// Complexity: O(|N|*log(|N|))
/// Tested: https://tinyurl.com/y8wdubdw
struct suffix_array {
  const static int alpha = 300;
  int mx, n;
  string s;
  vector<int> pos, tpos, sa, tsa, lcp;
  suffix_array(string t) {
    s = t+"$"; n = s.size(); mx = max(alpha, n)+2;
    pos = tpos = tsa = sa = lcp = vector<int>(n);
  }
  bool check(int i, int gap) {
    if(pos[ sa[i-1] ] != pos[ sa[i] ]) return true;
    if(sa[i-1]+gap < n && sa[i]+gap < n)
      return (pos[ sa[i-1]+gap ] != pos[ sa[i]+gap ]);
    return true;
  }
  void radix_sort(int k) {
    vector<int> cnt(mx);
    for(int i = 0; i < n; i++)
      cnt[(i+k < n) ? pos[i+k]+1 : 1]++;
    for(int i = 1; i < mx; i++)
      cnt[i] += cnt[i-1];
    for(int i = 0; i < n; i++)
      tsa[cnt[(sa[i]+k < n) ? pos[sa[i]+k] : 0]++] = sa[i];
    sa = tsa;
  }
  void build_sa() {
    for(int i = 0; i < n; i++) {
      sa[i] = i;
      pos[i] = s[i];
    }
    for(int gap = 1; gap < n; gap <<= 1) {
      radix_sort(gap);
      radix_sort(0);
      tpos[ sa[0] ] = 0;
      for(int i = 1; i < n; i++)
        tpos[ sa[i] ] = tpos[ sa[i-1] ] + check(i, gap);
      pos = tpos;
      if(pos[ sa[n-1] ] == n-1) break;
    }
  }
  void build_lcp() {
    int k = 0;
    lcp[0] = 0;
    for(int i = 0; i < n; i++) {
      if(pos[i] == 0) continue;
      while(s[i+k] == s[ sa[ pos[i]-1 ]+k ]) k++;
      lcp[ pos[i] ] = k;
      k = max(0, k-1);
    }
  }
  int& operator[] ( int i ){ return sa[i]; }
};
```

8.8 Suffix automaton

```
/// Complexity: O(|N|*log(|alphabet|))
/// Tested: https://tinyurl.com/y7cevdeg
struct suffix_automaton {
  struct node {
    int len, link; bool end;
    map<char, int> next;
  };
  vector<node> sa;
  int last;
  suffix_automaton() {}
  suffix_automaton(string s) {
    sa.reserve(s.size()*2);
    last = add_node();
    sa[last].len = 0;
    sa[last].link = -1;
    for(int i = 0; i < s.size(); ++i)
      sa_append(s[i]);
    ///t0 is not suffix
    for(int cur = last; cur; cur = sa[cur].link)
      sa[cur].end = 1;
  }
  int add_node() {
    sa.push_back({});
    return sa.size()-1;
  }
  void sa_append(char c) {
```

```
    int cur = add_node();
    sa[cur].len = sa[last].len + 1;
    int p = last;
    while(p != -1 && !sa[p].next[c] ){
      sa[p].next[c] = cur;
      p = sa[p].link;
    }
    if(p == -1) sa[cur].link = 0;
    else {
      int q = sa[p].next[c];
      if(sa[q].len == sa[p].len+1) sa[cur].link = q;
      else {
        int clone = add_node();
        sa[clone] = sa[q];
        sa[clone].len = sa[p].len+1;
        sa[q].link = sa[cur].link = clone;
        while(p != -1 && sa[p].next[c] == q) {
          sa[p].next[c] = clone;
          p = sa[p].link;
        }
      }
    }
    last = cur;
  }
  node& operator[](int i) { return sa[i]; }
};
```

8.9 Z algorithm

```
/// Complexity: O(|N|)
/// Tested: https://tinyurl.com/yc3rjh4p
vector<int> z_algorithm (string s) {
  int n = s.size();
  vector<int> z(n);
  int x = 0, y = 0;
  for(int i = 1; i < n; ++i) {
    z[i] = max(0, min(z[i-x], y-i+1));
    while (i+z[i] < n && s[z[i]] == s[i+z[i]])
      x = i, y = i+z[i], z[i]++;
  }
  return z;
}
```

9 Utilities

9.1 Hash STL

```
/// Complexity: -
/// Tested: https://tinyurl.com/y8orp8t2
struct Hash {
  size_t operator()(const pii &x) const {
    return (size_t) x.first * 37U + (size_t) x.second;
  }
      size_t operator()(const vector<int> &v)const{
              size_t s = 0;
              for(auto &e : v)
                      s ^= hash<int>()(e)+0x9e3779b9+(s<<6)+(s>>2);
              return s;
      }
};
unordered_map<pii, T, Hash> mp;
mp.reserve(1024); /// power of 2
mp.max_load_factor(0.25);
```

9.2 Pragma optimizations

```
#pragma GCC optimize ("O3")
#pragma GCC target ("sse4")
#pragma GCC target ("avx,tune=native")
```

9.3 Random

```
// Declare number generator
  mt19937 / mt19937_64 rng(chrono::steady_clock::now().time_since_epoch().count())
  // or
  random_device rd
  mt19937 / mt19937_64 rng(rd())

// Use it to shuffle a vector
shuffle(permutation.begin(), permutation.end(), rng)

// Use it to generate a random number between [fr, to]
uniform_int_distribution<T> / uniform_real_distribution<T> dis(fr, to);
  dis(rng)
```

9.4 template

```
#include <bits/stdc++.h>
using namespace std;

#define ff first
#define ss second
```

```
#define mp make_pair
#define pb push_back

typedef long long ll;
typedef double lf;
typedef pair<int,int> pii;

const int N = 1e5+10;
const int oo = 1e9;

int main () {
  ios::sync_with_stdio(0);
  cin.tie(0);
  #ifdef LOCAL
    freopen("input.txt", "r", stdin);
  #else
    #define endl '\n'
  #endif

  return 0;
}
```

9.5 vimsrc

```
//vimrc
noremap <F5> :w <bar> !g++ -DLOCAL -std=c++14 -static -Wall -Wno-unused-result -O2
     %:r.cpp -o %:r<CR>
noremap <F6> :w <bar> !g++ -DLOCAL -std=c++14 -static -Wall -Wno-unused-result -O2
     %:r.cpp -o %:r && ./%:r<in<CR>
noremap <F9> :<C-U> !./%:r<CR>
set number
set shiftwidth=2
set tabstop=2
set autoindent
set expandtab
//judge
submit() { boca-submit-run TEAM PASSWORD "$1" C++14 "$1.cpp"; }
```