This document presents a solution to two different questions related to mathematical foundations.

# Problem 1: Guarantee of Addition Result

Show that if any six numbers from {0,1, . . .9} then two of them will add up to 9

## Solution for Problem 1

1. Check for the pairs of distinct numbers that add up to 9.
   1. We can count them manually, as follows.
      1. (0,9)
      2. (1,8)
      3. (2,7)
      4. (3,6)
      5. (4,5)
2. Count them.
   1. There are 5 pairs that add up to 9. Other pairs of numbers, such as (0,1), (0,2), and so on, do not add up to 9.
3. Apply the Pigeonhole Principle.
   1. The Pigeonhole Principle states that if n items are put into m containers, where n > m, then at least one container must contain one item.
      1. Containers = Pairs.
      2. Items = Numbers, to be chosen from the set.
      3. We have 5 pairs and 6 numbers.
      4. 6 items > 5 containers.
      5. 6 – 5 = 1 excess item.
   2. If we choose 6 numbers, by the Pigeonhole Principle, at least one pair must have both numbers chosen.

Therefore, since at least one of these pairs must have both numbers chosen, at least two of the chosen numbers must add up to 9.

# Problem 2: Sides of a Triangle

Consider an equilateral triangle whose sides have length 1. Show that five points are chosen on or insider the triangle, then two of them will be no more than ½ unit apart

## Solution for Problem 2

1. Divide the triangle into smaller equilateral triangles, by connecting the midpoints of the sides of the original triangle.
   1. Four smaller equilateral triangles are formed.
   2. Each of these have sides with length of ½ unit.
2. We now have: 4 triangles, and 5 points to place in them.
3. Apply the Pigeonhole Principle.
   1. The Pigeonhole Principle states that if n items are put into m containers, where n > m, then at least one container must contain one item.
   2. In our scenario,
      1. Items = points, and
      2. Containers = triangles.
   3. 5 points > 4 triangles.
4. The maximum distance between any two points inside an equilateral triangle of side length ½ is equal to the side length of that triangle.

Since at least two of the five points must be within the same smaller triangle, therefore, they are no more than ½ unit apart.