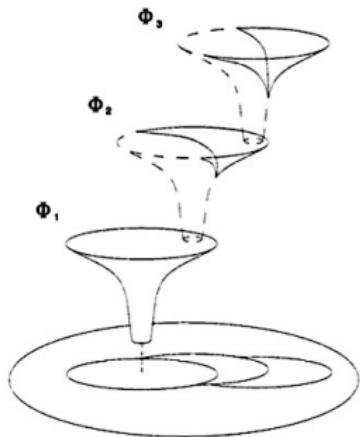
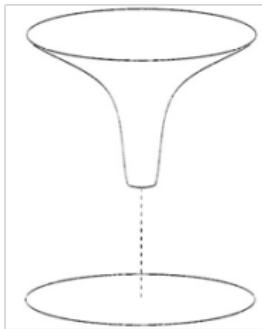


## Sequential Composition: Back-Chaining Basins



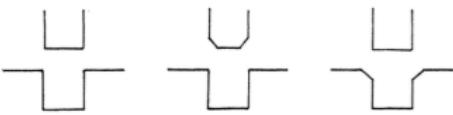
Burridge et.al. [1996]—Juggling obstacle avoidance



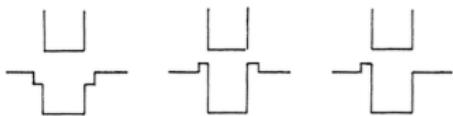
<https://www.youtube.com/watch?v=NfdG8ZZdtg>

# Sequential Composition: Examples from the Literature

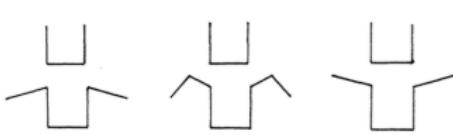
Lozano-Perez et. al. [1984]—peg in hole



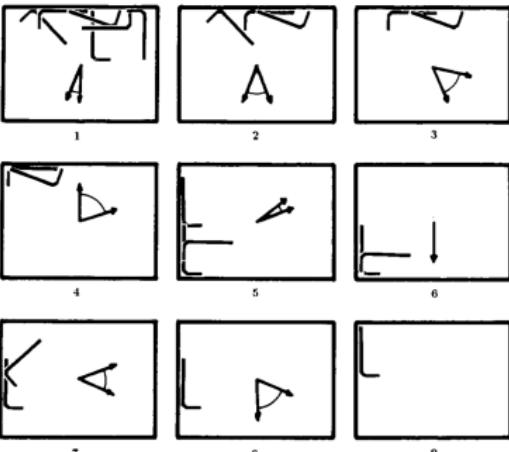
Erdmann and Mason [1988]—sensorless tray tilting manipulation



Majumdar and Tedrake [2013]—Funnel libraries



<https://www.youtube.com/watch?v=cESFpLgSb50>



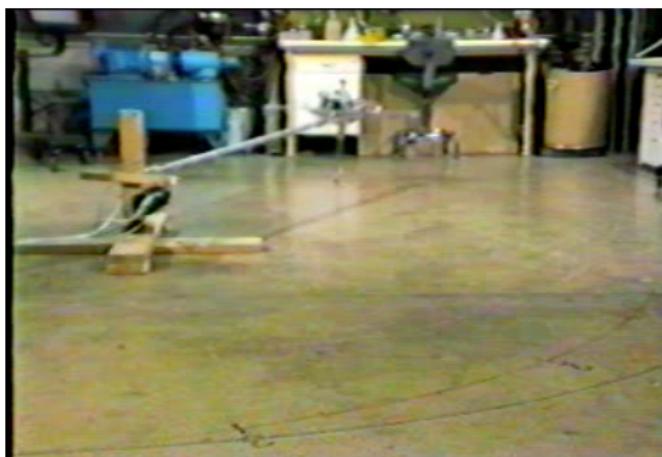
# Parallel Composition: Idea and Examples

Two juggle—Rizzi and Koditschek  
[1994]

Raibert [1986]



<https://www.youtube.com/watch?v=u8l7EXXgTvk>

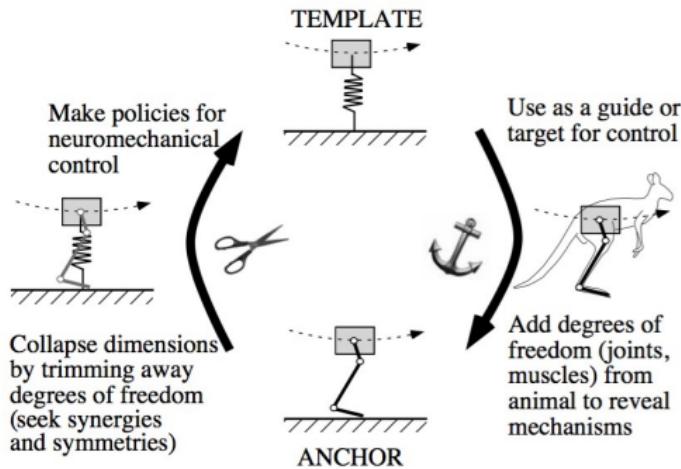


## Summary

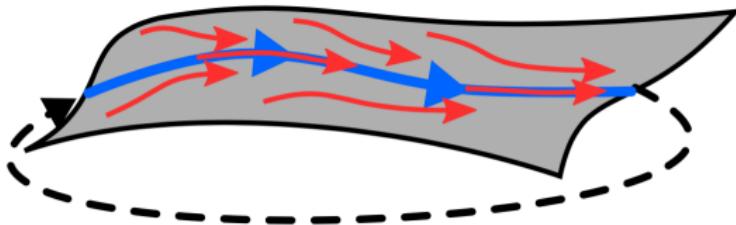
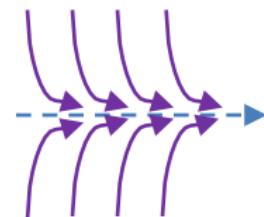
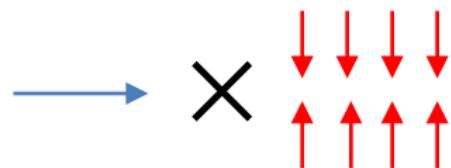
- Robot control can be thought of as flowing “downhill” on artificial energy functions
- Sequential composition formalizes a sequence of tasks using back-chaining of basins
  - Sequencing coarse manipulation: juggling to palming
  - Assembly tasks
  - ...
- Parallel composition describes satisfying multiple objectives at once
  - Controlling multiple DoFs for hopping etc.
  - Juggling two balls
  - ... (more coming in 4.2)

# Templates and anchors (recap from 2.1)

From Full and Koditschek [1999]

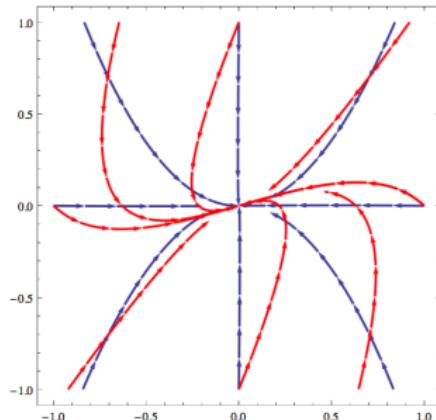
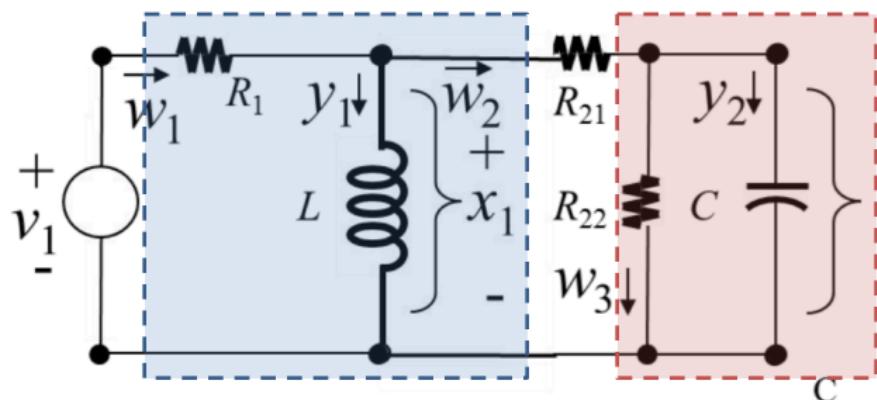


Anchoring is a parallel composition!



Template  
Anchoring dynamics

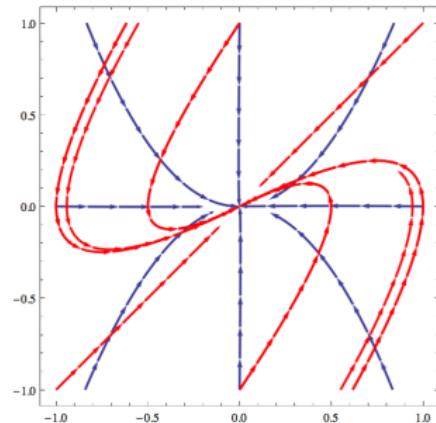
## Decoupled vs. coupled systems



$$R_{21} \rightarrow \infty$$

- Physical decoupling (circuit disconnected)
- Solutions decoupled (diagonal  $A$ )

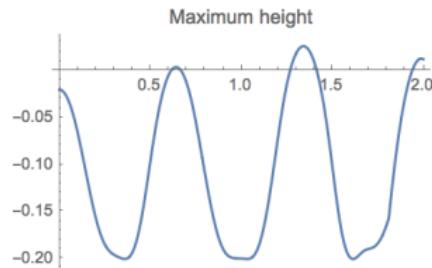
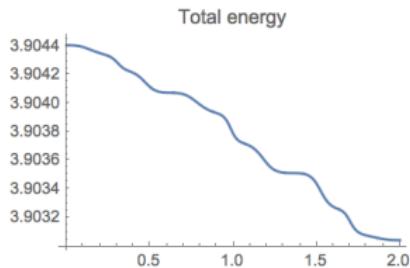
$$\dot{x} = Ax = \begin{bmatrix} A_{11} & 0 \\ 0 & A_{22} \end{bmatrix} x$$



# Cross-talk in mechanical systems

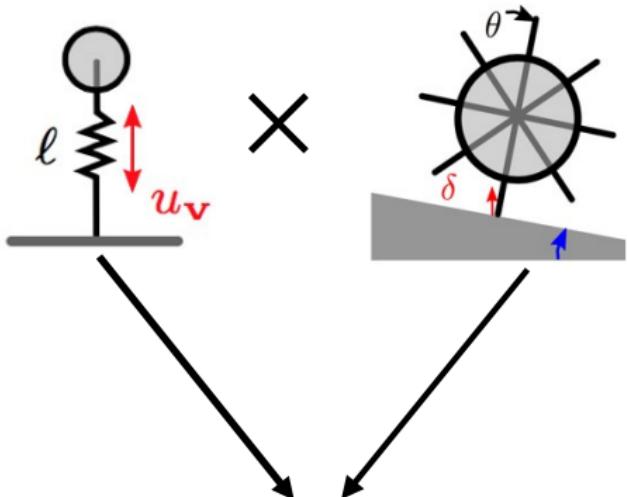
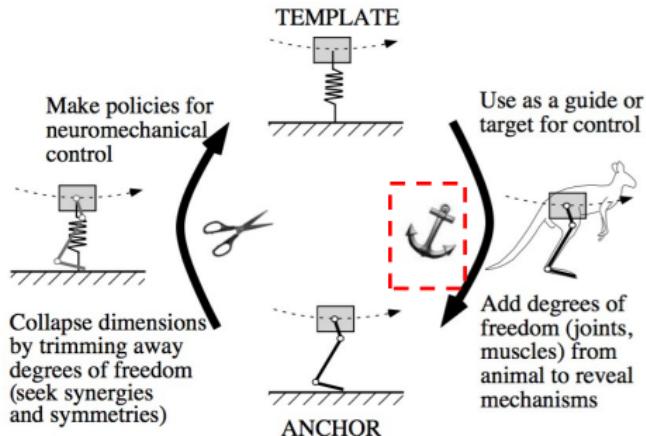
- Double pendulum
- Controller is gravity + natural damping
- “Template:” decaying oscillator

$t = 0.000$

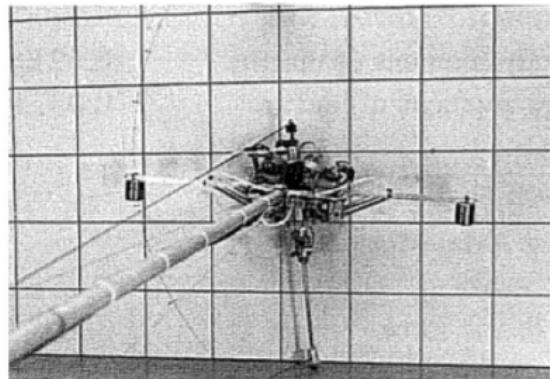


# Synthetic view of anchoring

From Full and Koditschek [1999]



- Map controllers  $T \rightarrow A$
- Anchor multiple templates in parallel
- Decoupled controllers
- Conditions for correctness



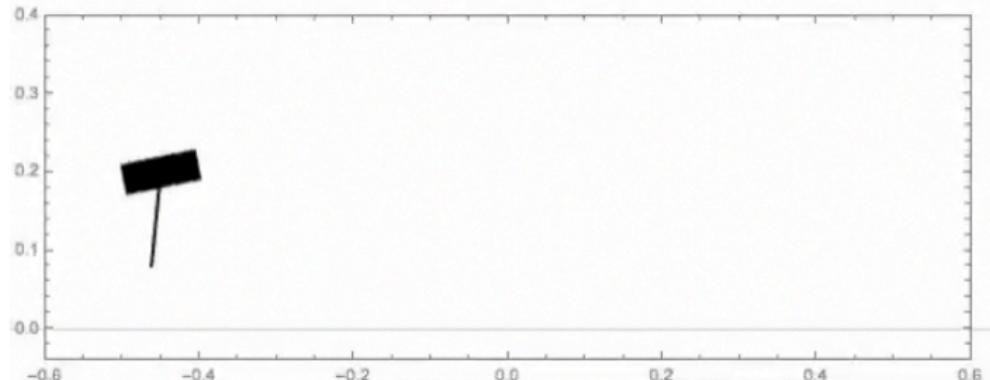
# Empirical parallel compositions – Raibert hopper

From Raibert [1986]



Body:leg inertia

- 14:1 (Raibert)
- 7:1 (10 g)
- 1:1 (70 g)

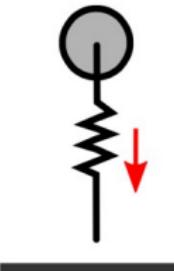
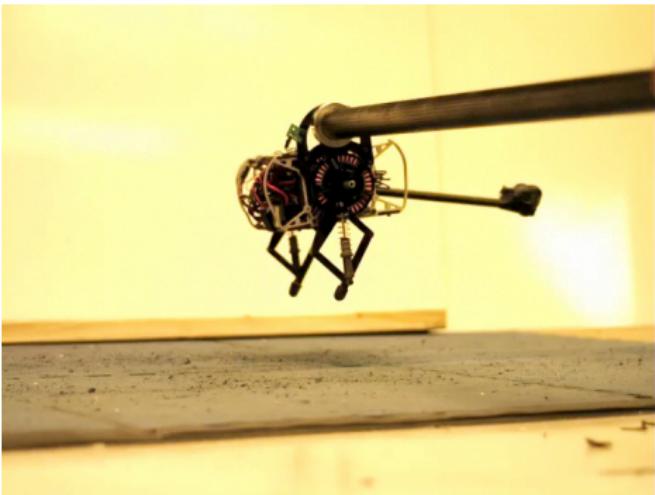


## Summary

- On a high DOF robot, there is no alternative to thinking about either parallel composition, or preflexes
- Coupling forces try to screw us up
- Parallel composition = “anchoring multiple templates simultaneously with decoupled controls”
- Verification is important, but also difficult

## Controlled 1DOF vertical hopper (VH)

- From De and Koditschek [2015b],



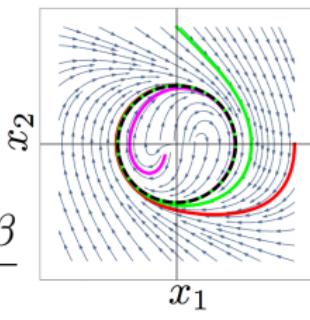
- Phase-energy EoM (De and Koditschek [2015a])

$$\dot{a} = \cos(\psi)u, \quad \dot{\psi} = \omega - \frac{1}{\omega a} \sin(\psi)u$$

- Oscillatory energization, natural damping

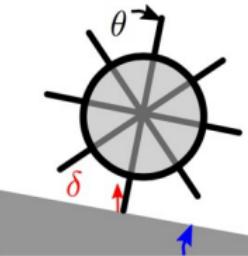
$$u = (k - a\beta) \cos(\psi)$$

- On average (De and Koditschek [2015a])  $\frac{da}{d\psi} \stackrel{\text{avg}}{=} \frac{k - a\beta}{2}$



# Controlled 1DOF Active Rimless Wheel

- Rimless wheel (McGeer [1990])



- Add liftoff impulse

- Simplistic view:

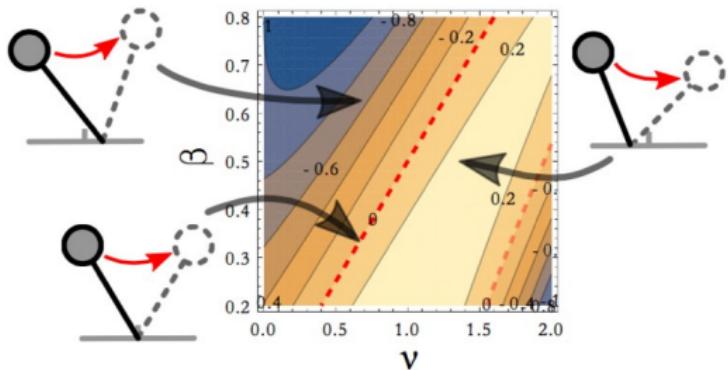
$$F(a_{LO}) = a_{LO} + \gamma$$

$$R(a_{TD}) = \delta a_{TD}, \quad 0 < \delta < 1$$

$$P(a) = \delta(a + \gamma)$$

- Equivalent to stepping (Raibert [1986])

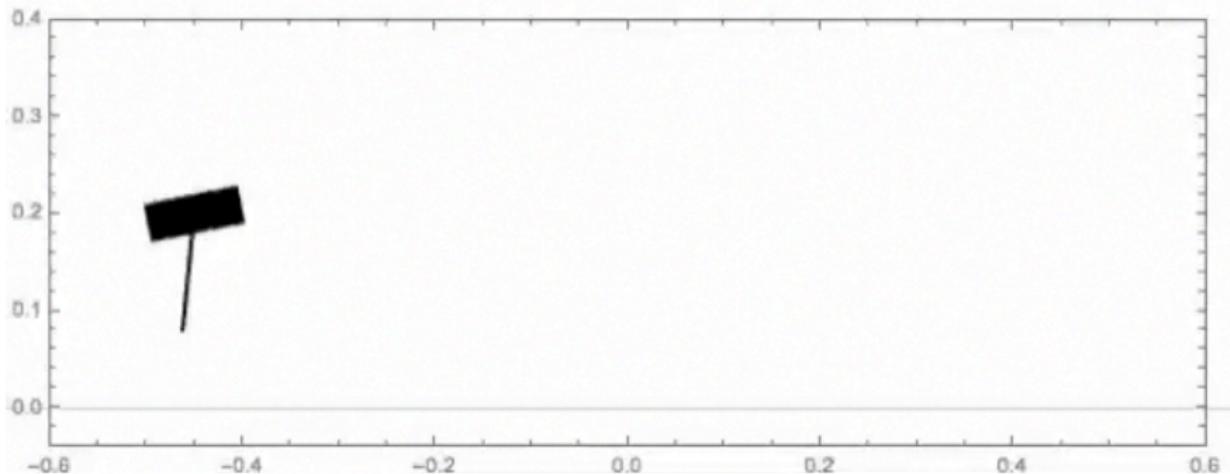
From De and Koditschek [2015a],



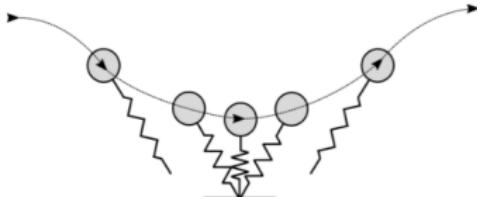
Bhounsule et al. [2012]

## SLIP as a composition

Clearly works in practice



## SLIP as a composition—verification

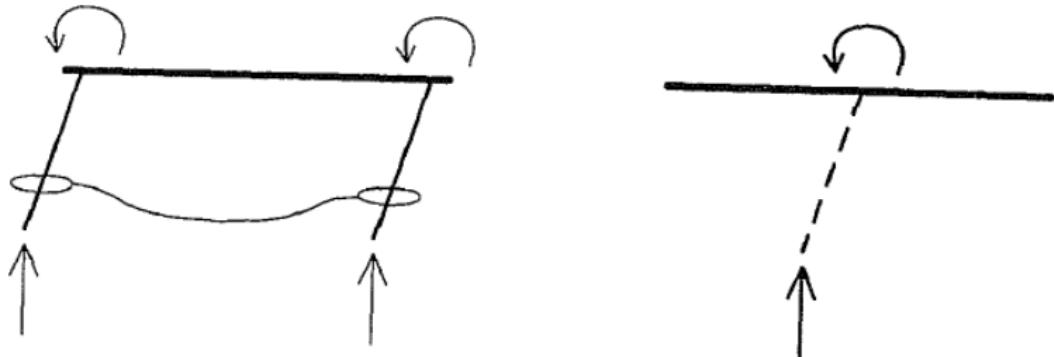


- Does SLIP anchor a VH?
  - i.e. the dynamics are “the same” on an invariant (attracting) submanifold—Full and Koditschek [1999]
  - Radial stance dynamics:  $m\ddot{r} = -\nabla\varphi(r) - g \cos\theta - r\dot{\theta}^2$
  - Say  $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} := \begin{bmatrix} r \\ \dot{r} \end{bmatrix}$
  - Then
- $$f(x) = \begin{bmatrix} x_2 \\ -\nabla\varphi(x_1) - g \cos\theta - x_1\dot{\theta}^2 \end{bmatrix} \quad f_{\text{VH}}(x) = \begin{bmatrix} x_2 \\ -\nabla\varphi(x_1) - g \end{bmatrix}$$
- Clearly  $f|_{\theta=0, \dot{\theta}=0} = f_{\text{VH}}$   
...but not otherwise
  - New research:  $\bar{f} \sim \bar{f}_{\text{VH}}$  —De and Koditschek [2015a]

## Summary

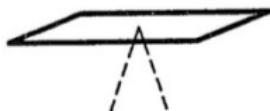
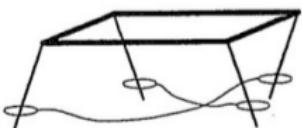
- SLIP can be decomposed
- A vertical hopper can be controlled by active damping
- A rimless wheel can be controlled with active liftoff impulse
- SLIP as a composition empirically works
- But exact anchoring cannot be shown in general

## Virtual leg

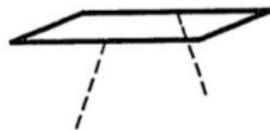
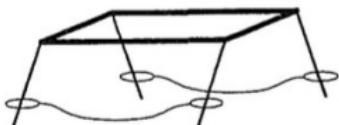


- From Raibert [1986]
- Intra-group control
  - Toe positioning (flight)
  - Touchdown synchronization
  - Virtual hip positioning (stance)
- Inter-group control

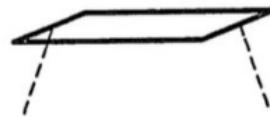
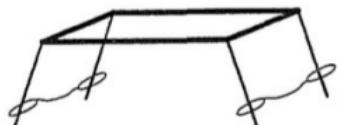
## Virtual bipeds (trot, bound, pace)



TROT



PACE



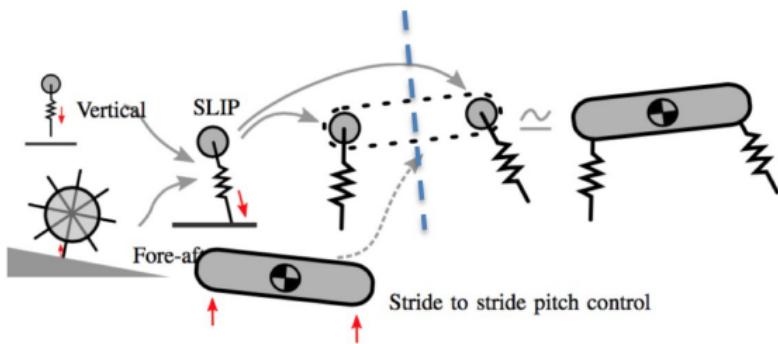
BOUND

- From Raibert [1986]
- Control action “mirrored,” repeated step-to-step
- Location of virtual hip
- Move using internal forces in stance

# Minitaur bounding



Pitch coordination:  
antiphase

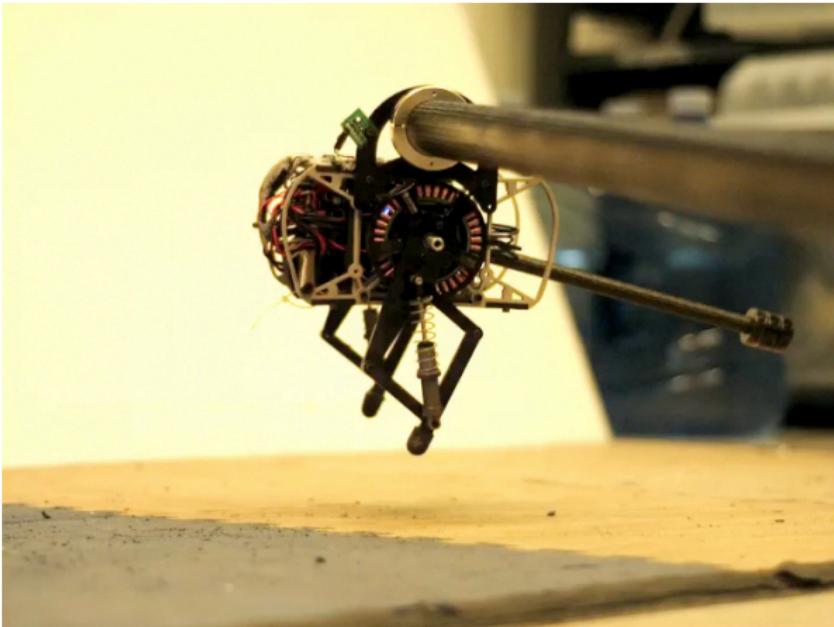
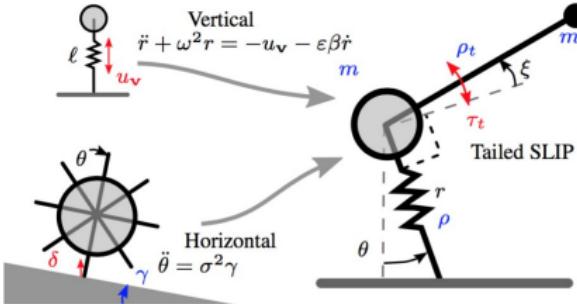


## Summary

- Multilegged robots can be controlled as compositions of monopeds
- Virtual legs allow you to think of groups of legs as a single leg
- A virtual biped with a leg-to-leg symmetric control action is a composition of identical monopeds
- Phase coordination can be active or passive

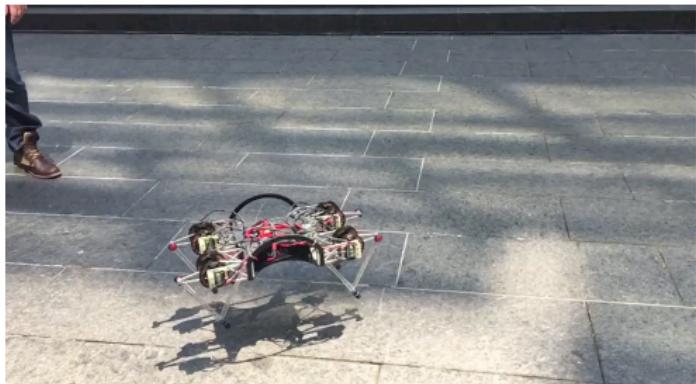
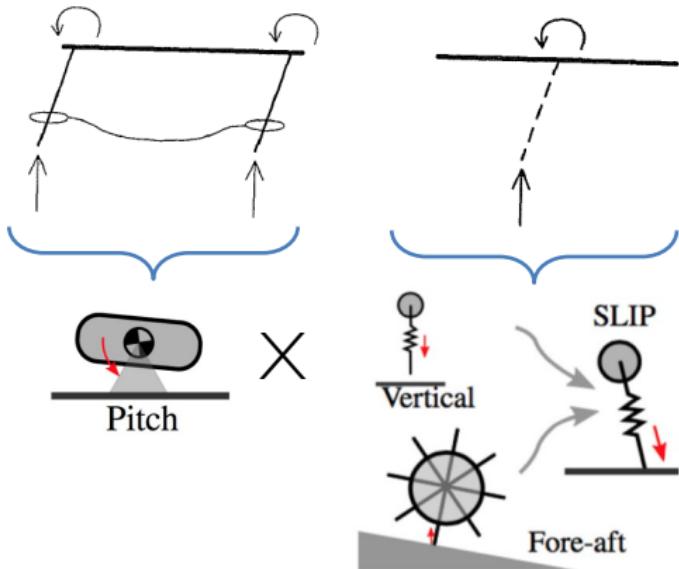
# Jerboa planar hopping

- Tail-instantiated vertical hopping—De and Koditschek [2015a,b]

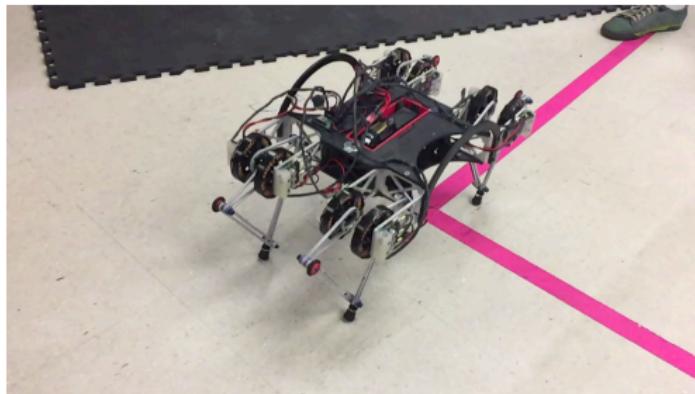
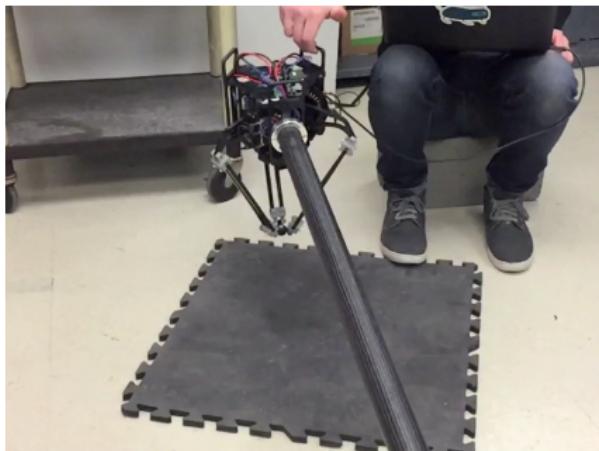
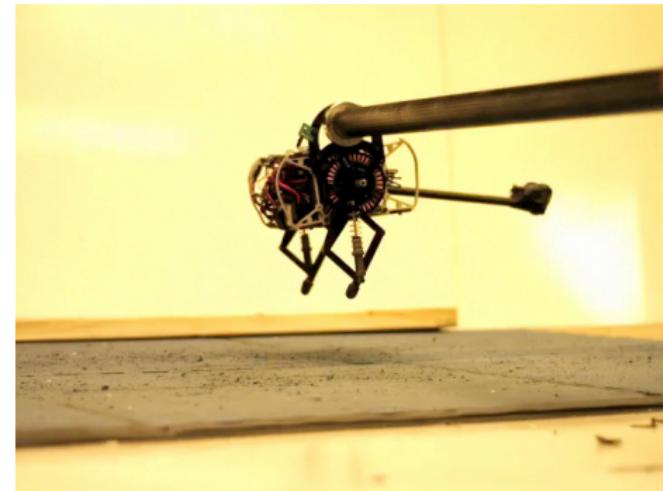


# Minitaur pronking

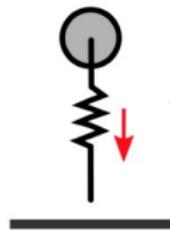
- Monoped control: SLIP as parallel composition (same controllers as Jerboa)
- Intra-group synchronization  $\approx$  attitude control



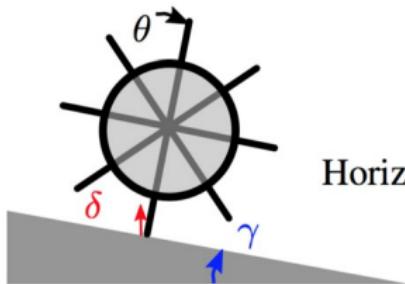
# Reuse



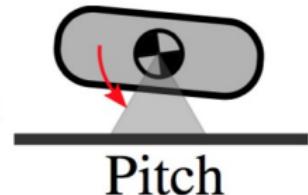
## Generate



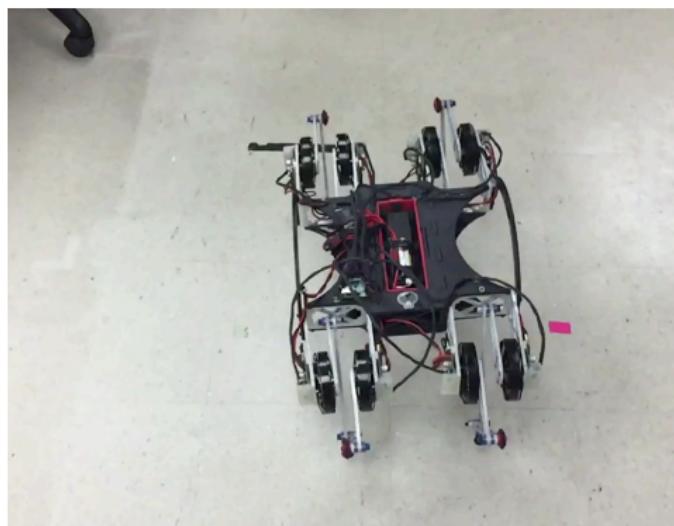
Vertical



Horizontal



Pitch



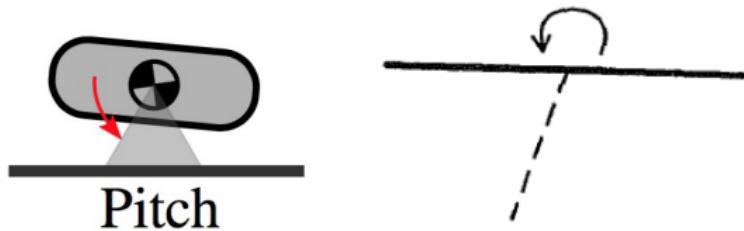
# Where do compositional techniques lie?

	Holistic	Compositional
Stability guarantees	Methods exist	Not yet, but progress e.g. De and Koditschek [2015a]
Underactuation	Principled methods exist (but need very good models)	✗
Optimality guarantees	Possible	✗
Modeling/calibration	Must be precise	Can be crude
Controller synthesis (e.g. optimization dims)	$(n \text{ inputs}, n \text{ outputs}) =$ $x^n$	$(1 \text{ input}, 1 \text{ output}) \times n$ $= n$
Reuse	✗	✓
Generate	✗	✓

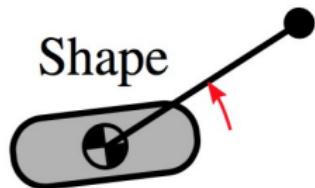
## Summary

- A compositional approach to synthesis allows us to reuse templates on different robots
- ...and also the generate new combinations

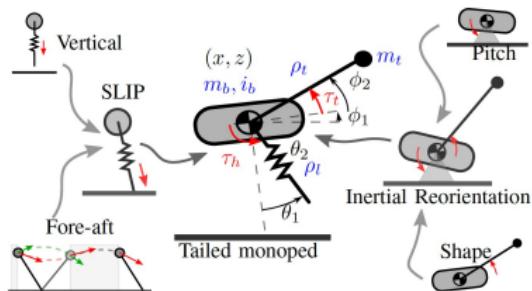
## Attitude control templates



Inertial reorientation Johnson et al. [2012]



# Four-way composition for tail-energized planar hopping

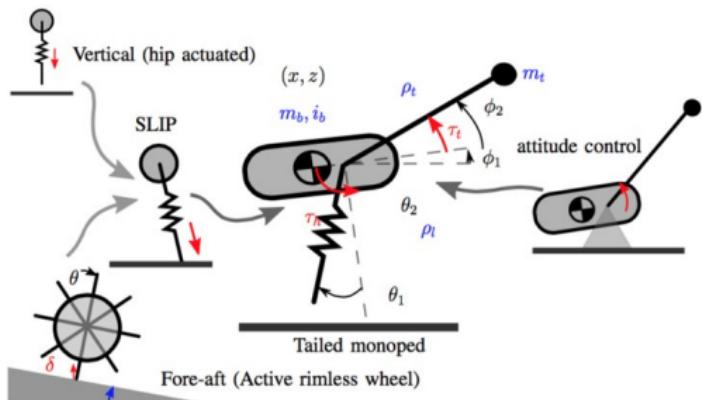


From De and Koditschek [2015b]

Experiments WIP



# Hip-energized planar hopping



## Alternate solutions

- Trade-off: speed vs. traction
- Morphological reduction (Libby et al. [2015])—constraint on CoM location

$t = 0.0000$

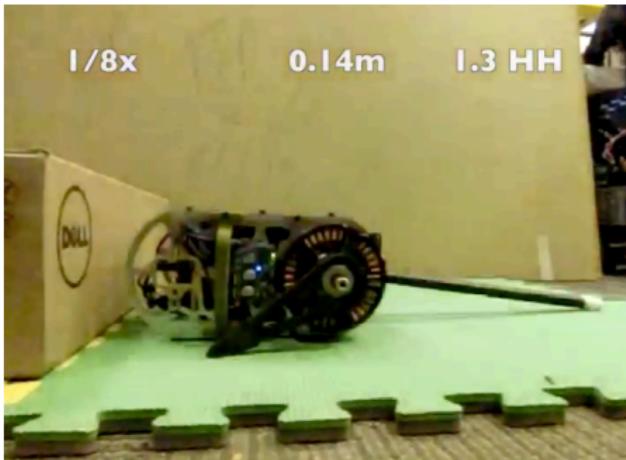
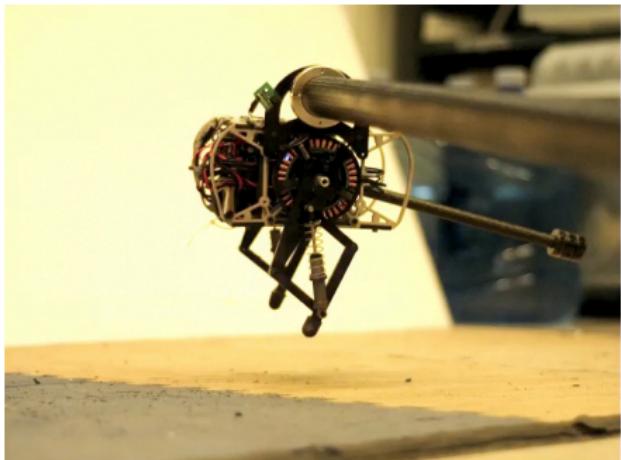
T



## Summary

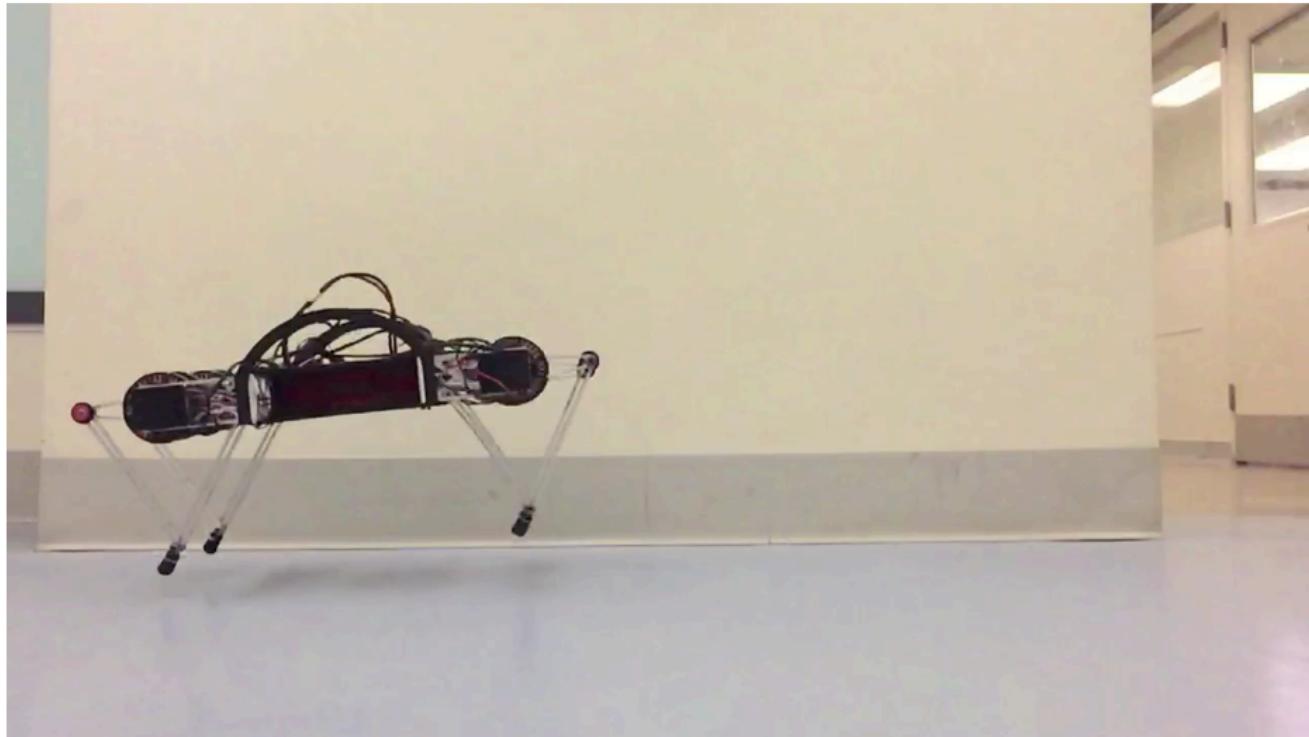
- We can have different *anchorings* for the same *templates*
- Can be thought of as “alternate solutions” for the same behaviors
- Allow freedom to select the most efficient/most robust/etc. for the specific operating conditions

# Hybrid self-manipulation systems

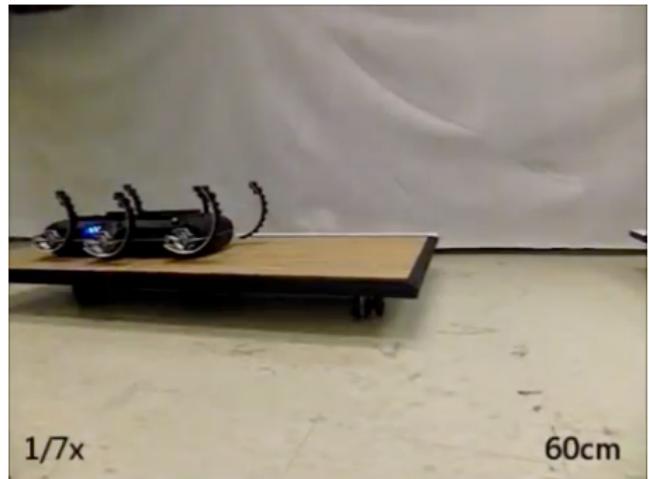


Johnson et al. [2015], Burden et al. [2015].

# Minitaur leaping



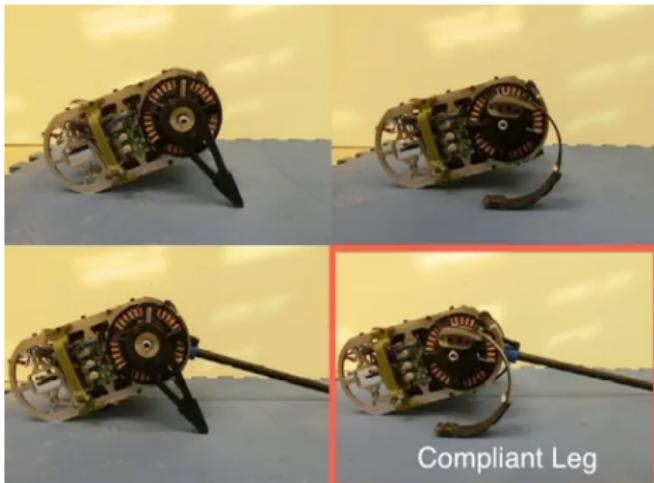
# RHex and Jerboa leaping



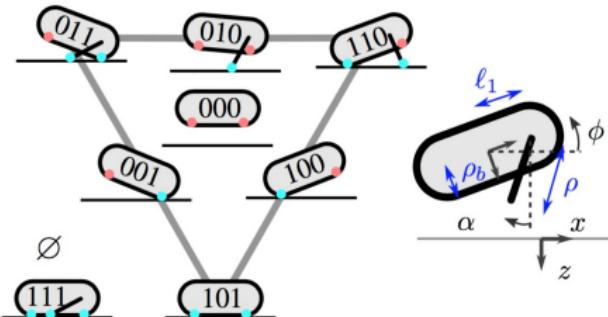
<https://www.youtube.com/watch?v=kV9J-oayCBU>

From Johnson and Koditschek [2013],  
Brill et al. [2015],

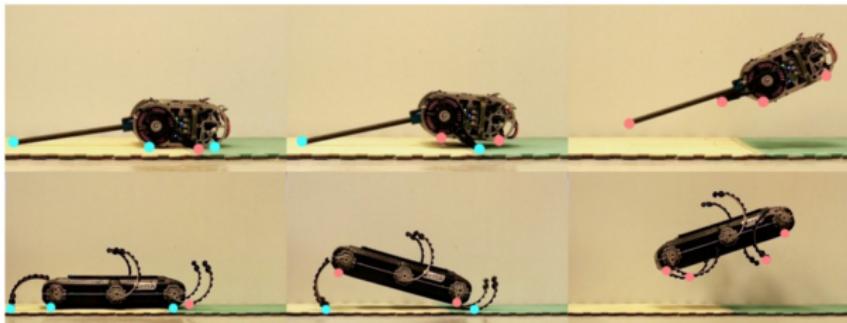
- Hybrid system with many modes, simultaneous transitions
- Organize into “ground reaction complex”



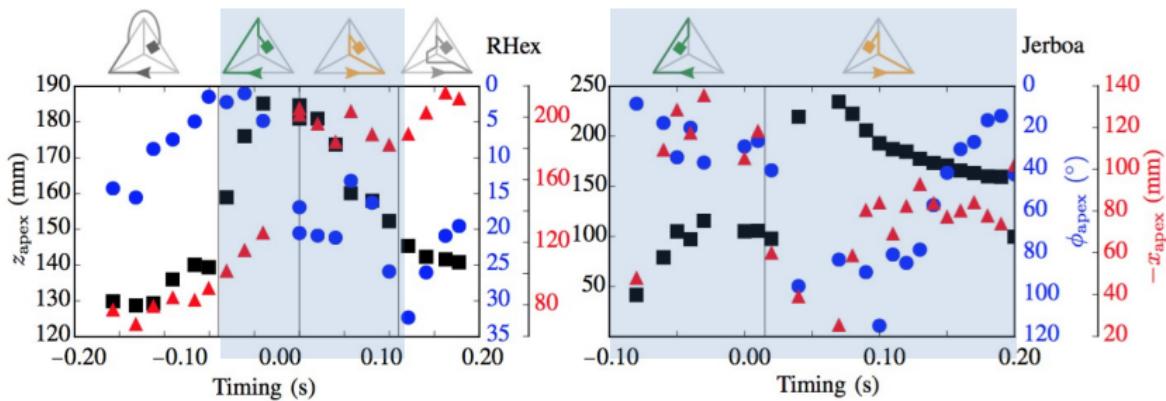
<https://www.youtube.com/watch?v=0p-y6bSqbC>



# Leaping: stability and convergence



When do RHex, Jerboa leap “the same”?—Brill et al. [2015]



## Summary

- Transitions go through various hybrid modes
- Some transitions are nicer than others
- Minitaur leaps using parallel composition
- RHex and Jerboa leap using the ground reaction complex