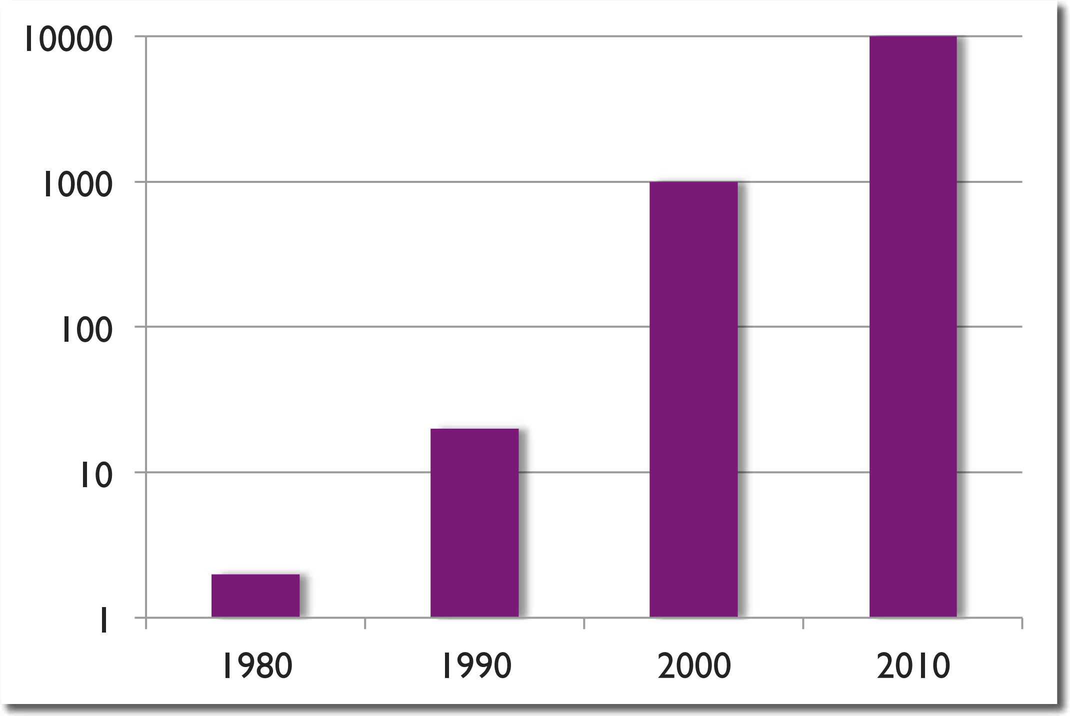
Aerial Robotics

Unmanned Aerial Vehicles in 2010

Number of UAVs worldwide



Predictions of a $10B industry

- Military: Surveillance, force protection, warfare
- Civilian commercial: Transport, environment
- Civilian private: DIY Drones

FAA predicts 15,000 civilian drones by 2020

Unmanned Aerial Vehicles in 2015

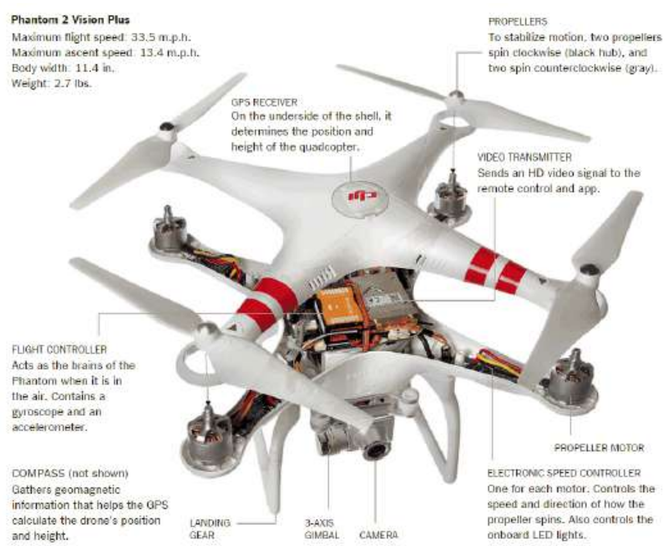
- Over 15,000 drones sold in the US every month
- $15B industry, projected to grow to $25B by 2020
- Expectations for leading industry applications
  - Agriculture
  - Infrastructure inspection
  - Border patrols
  - Photography
  - Construction
  - Film production

Penn Engineering

Unmanned Aerial Vehicles

Remotely Piloted Vehicles (RPVs)

Aerial Robots



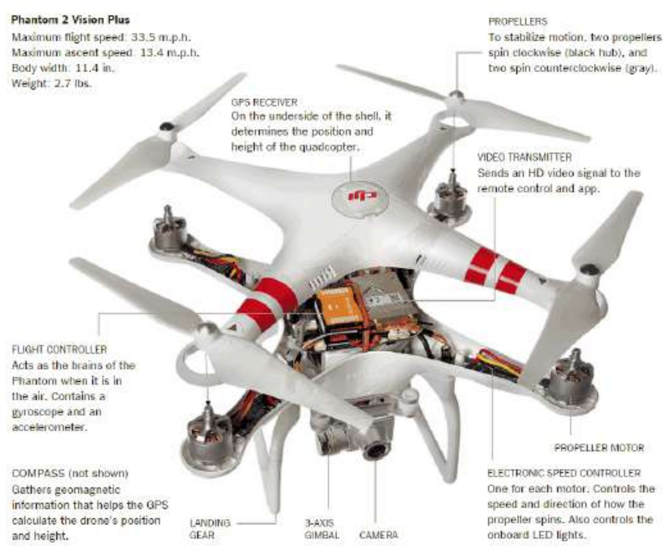
Drones

Drones mischaracterize what these things are. They're not dumb. Nor are they unmanned, actually. They're remotely piloted aircraft. - Gen. Norton Schwarz, August 10, 2012

UAVs = RPVs = Aerial Robots = Drones

Remotely Piloted Vehicles (RPVs)

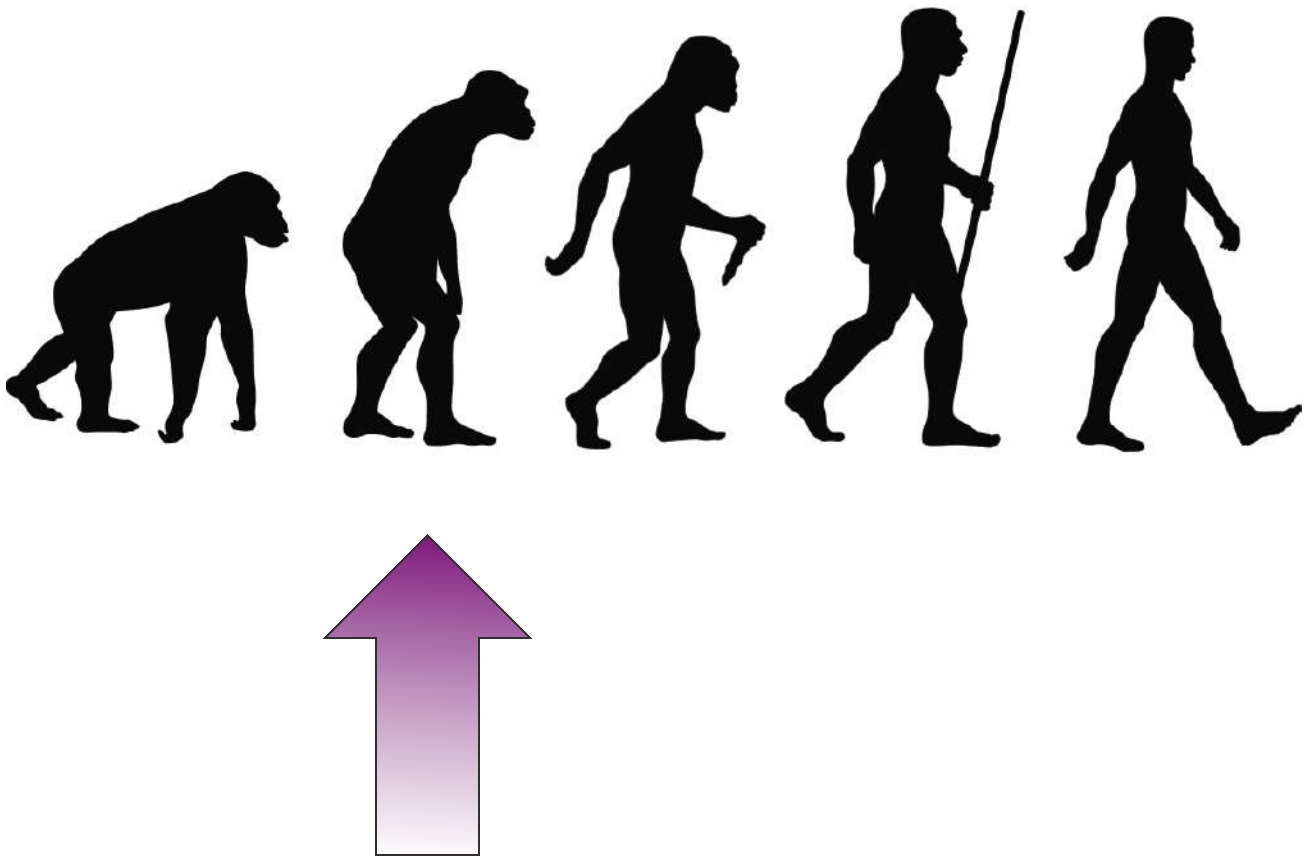
Aerial Robots



Drones

Penn Engineering

Aerial Robotics



We are here!

THE WALL STREET JOURNAL.

U.S. NEWS

FAA Authorizes Commercial-Drone Testing

Six Operators Selected to Conduct Research, Setting Stage for Eventual Widespread Use

By ANDY PASZTOR

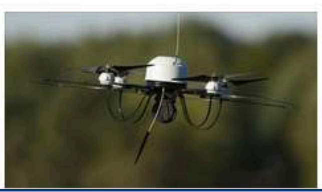
Updated Dec. 30, 2013 7:42 p.m. ET

Officials eventually want to allow widespread use of private drones. Above, an unarmed government Predator. *Getty Images*

Aviation officials on Monday selected a handful of universities and state agencies to operate sites for drone testing commercial unmanned aircraft into the U.S. aviation system.

Under the six operators chosen by the Federal Aviation Administration, research will be conducted by industry experts and academics on the safe operation of drones, or unmanned aerial vehicles, across a broad array of geographical areas, climates and types of airspace. The work is expected to target everything from federal certification of the safety of commercial drones tothe reliability of air-to-ground communication links to verifying a generation of new, lower-cost sensors designed to avoid midair collisions.

The FAA, however, stopped short of committing itself to a specific timetable for permitting widespread use of unmanned commercial aircraft across U.S. skies.



Currently, law-enforcement agencies, universities and some environmental organizations are allowed to fly remotely piloted aircraft in clearly delineated corridors or swaths of U.S. airspace. The FAA has been facing escalating industry and congressional pressure to move more quickly to open up additional flight regions.

The winning applicants were the commerce department of North Dakota; the state of Nevada; a public airport some 250 miles north of New York City; the University of Alaska; Texas A&M University in Corpus Christi; and a partnership between Virginia Tech and Rutgers University. The first site is expected to begin operating within six months.

For Immediate Release

September 15, 2015

WASHINGTON – As the Papal visit approaches, the U.S. Department of Transportation's Federal Aviation Administration (FAA) is reminding residents of and visitors to Washington, DC, New York, and Philadelphia that these cities and the surrounding communities are No Drone Zones from September 22 through September 27, 2015.

"If you plan to attend any of the Papal visit events, please leave your drone at home," said FAA Administrator Michael Huerta. "Anyone flying a drone within the designated restricted areas may be subject to civil and criminal charges."

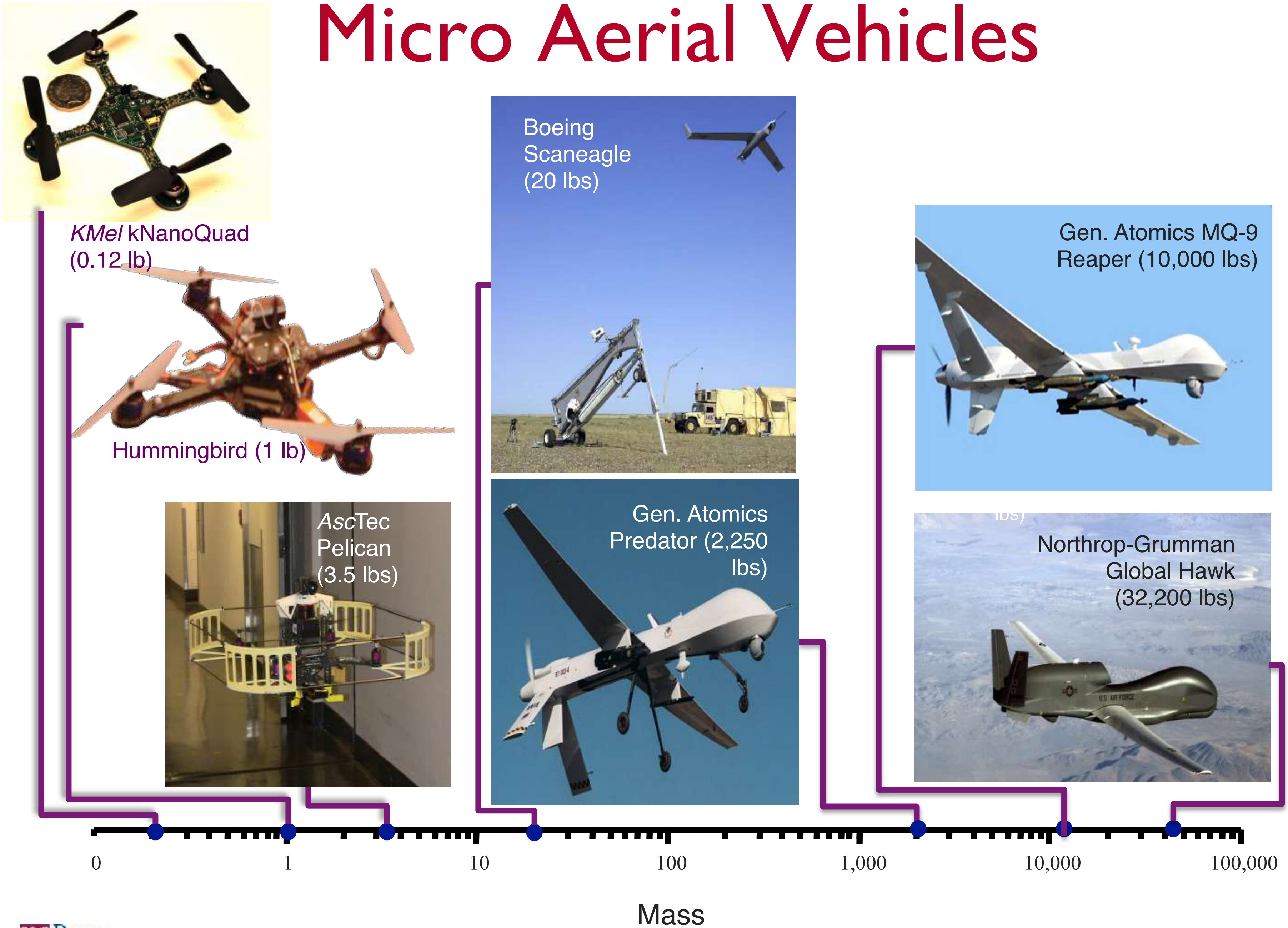
The FAA has flight restrictions in place in and around Washington, DC, New York and Philadelphia (Notice 1,Notice 2). Flying a drone anywhere Pope Francis will visit is against the law. Any unmanned aircraft (UAS) – including radio-controlled model aircraft/UAS – are subject to FAA requirements.

The Skies will be Abuzz with Drones!

Gartner's 2015 Hype Cycle for Emerging Technologies

acquired by

Micro Aerial Vehicles



Images from www.af.mil

Types of Micro Air Vehicles

- Fixed wing



- Flapping wing
  - Insect flight
  - Avian flight

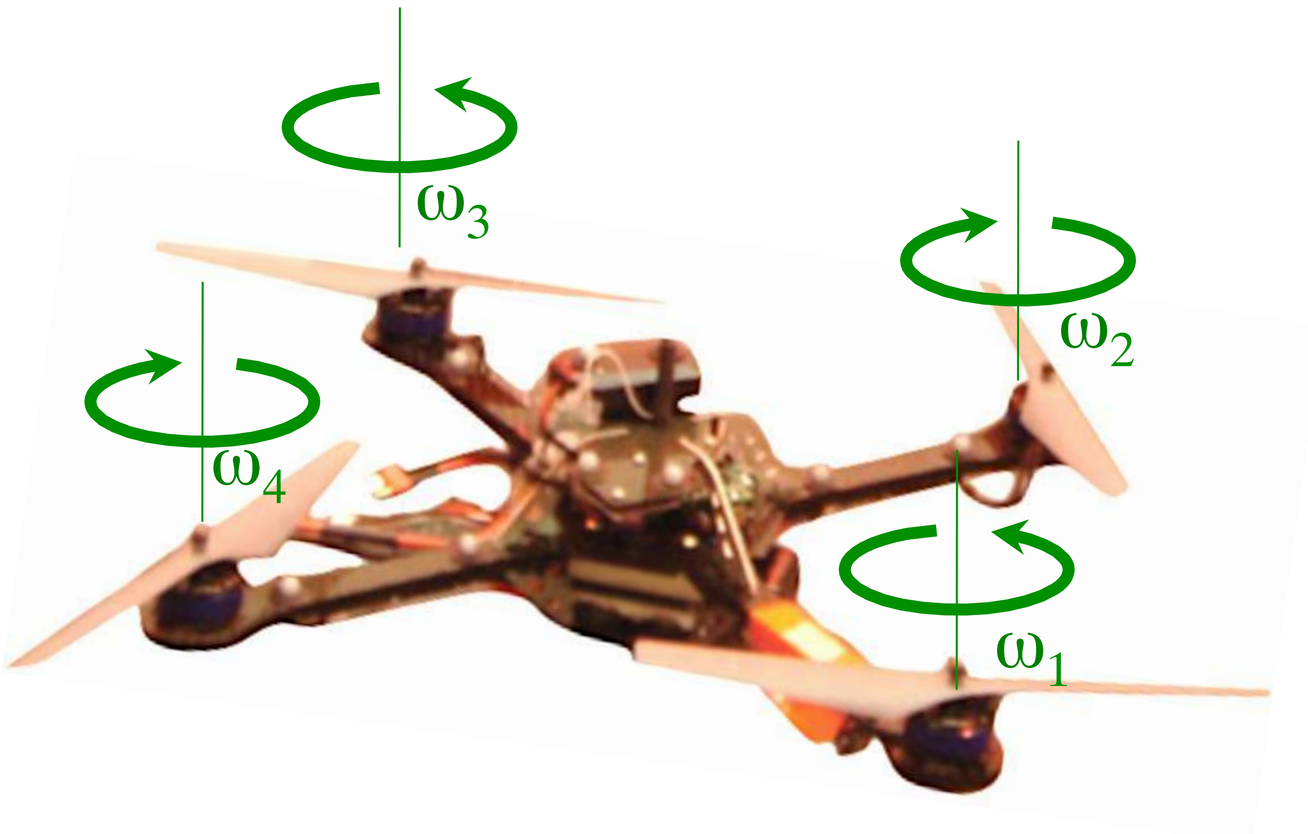
- Rotor crafts
  - Helicopter
  - Ducted fan
  - Co-axial
  - Quadrotor
  - Hexrotor


Quadrotor

Quadrotor

Quadrotor

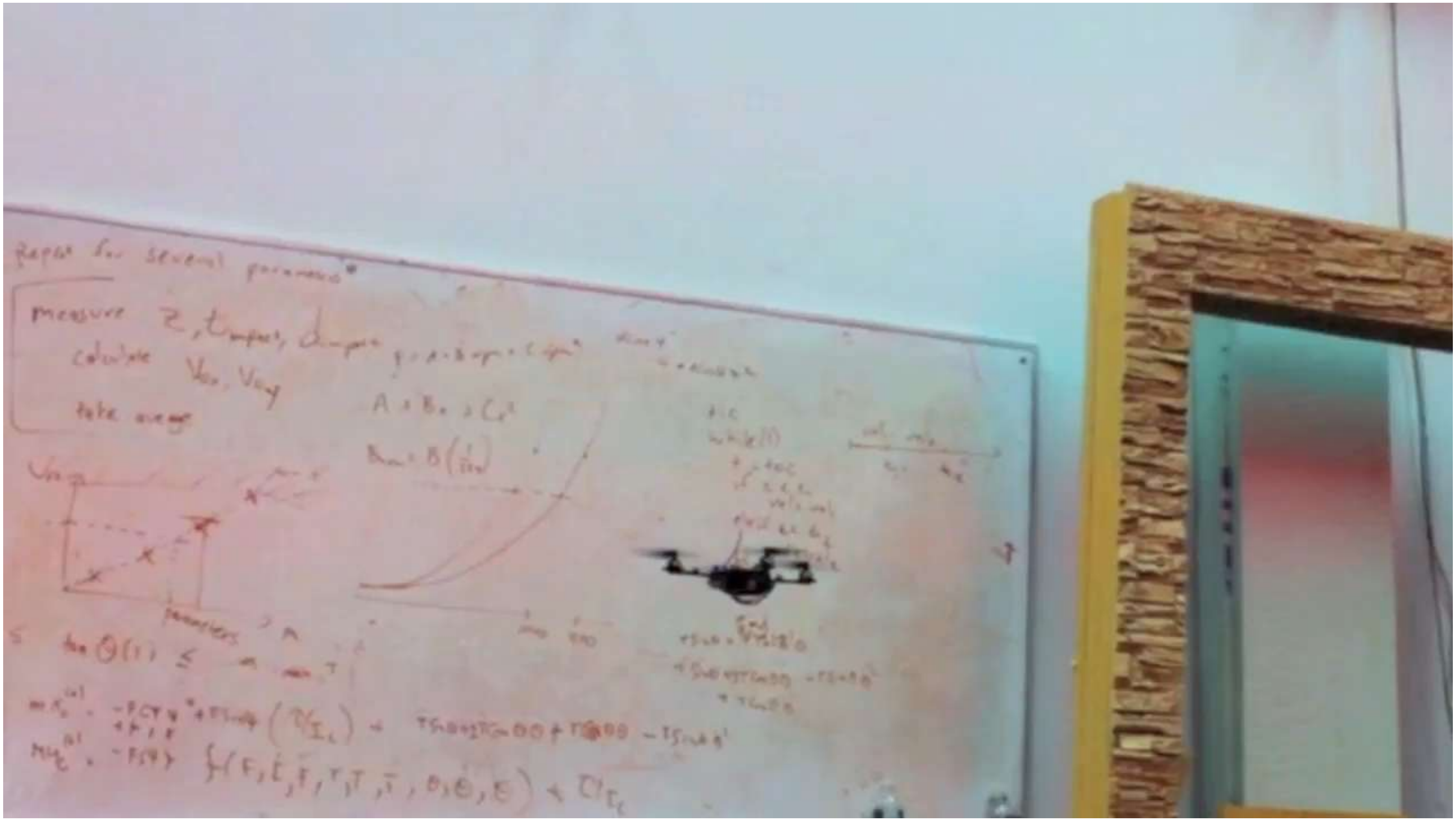
Quadrotor



Vary the speeds of the rotors to control the position and orientation of the robot

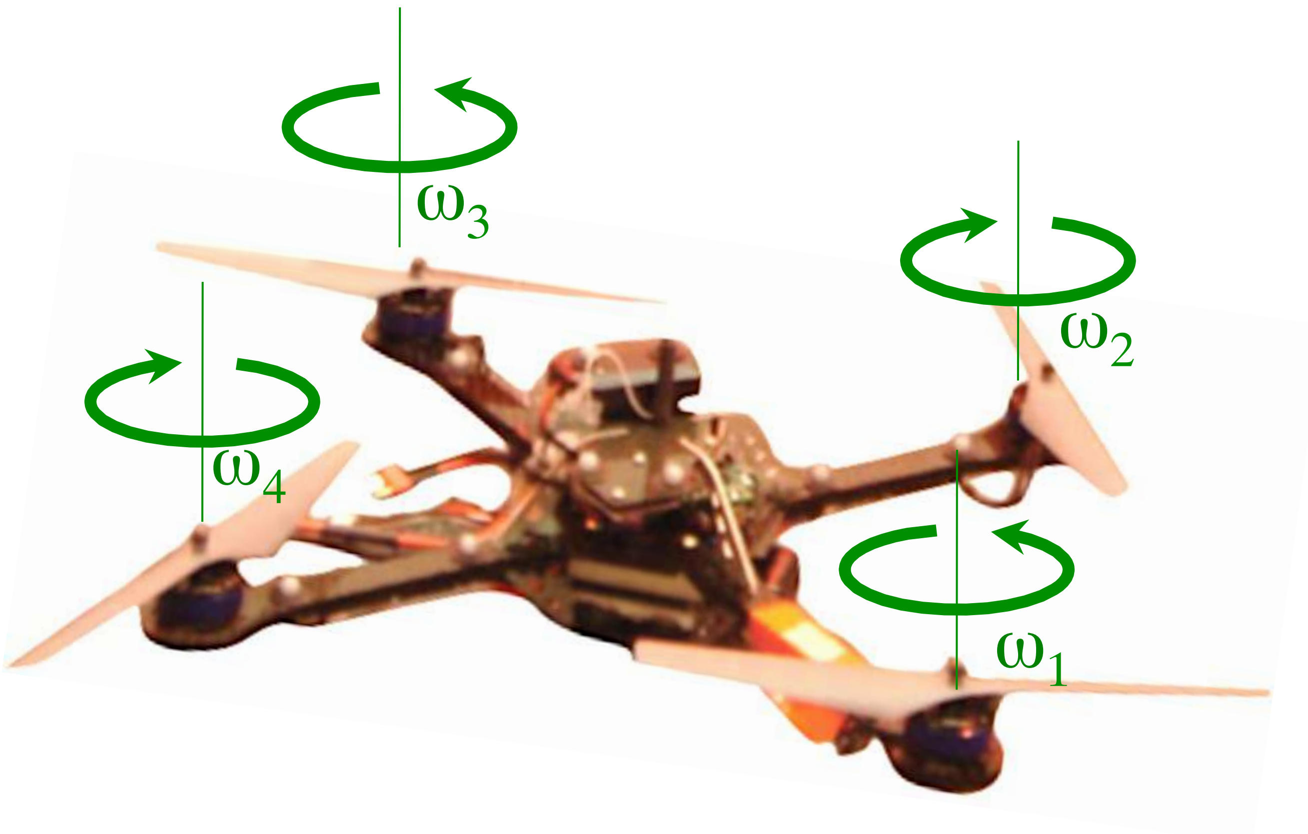
Penn Engineering

Roll and Pitch



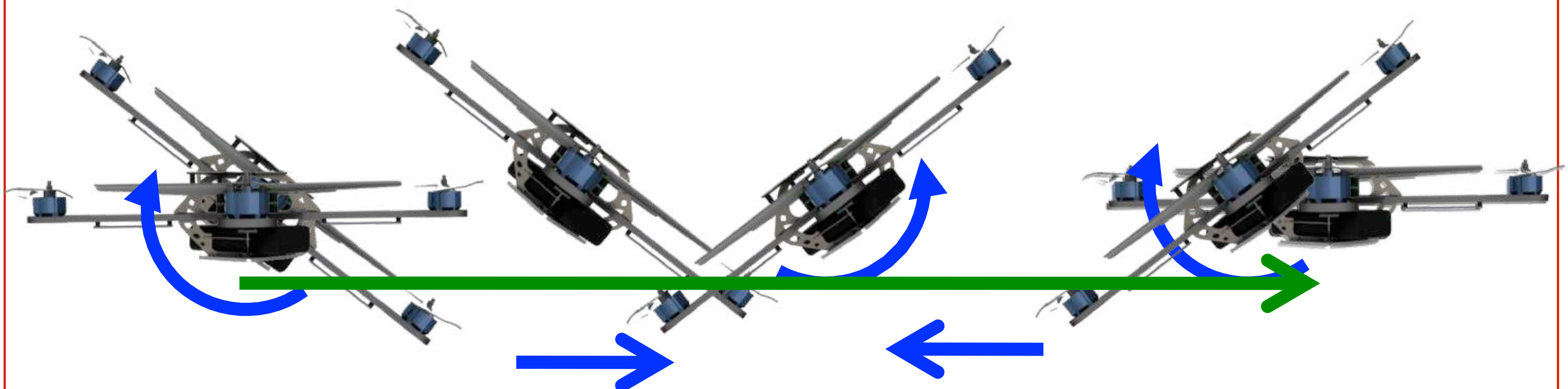
KMel Nano
Maximum pitch velocity of 2000 deg/sec

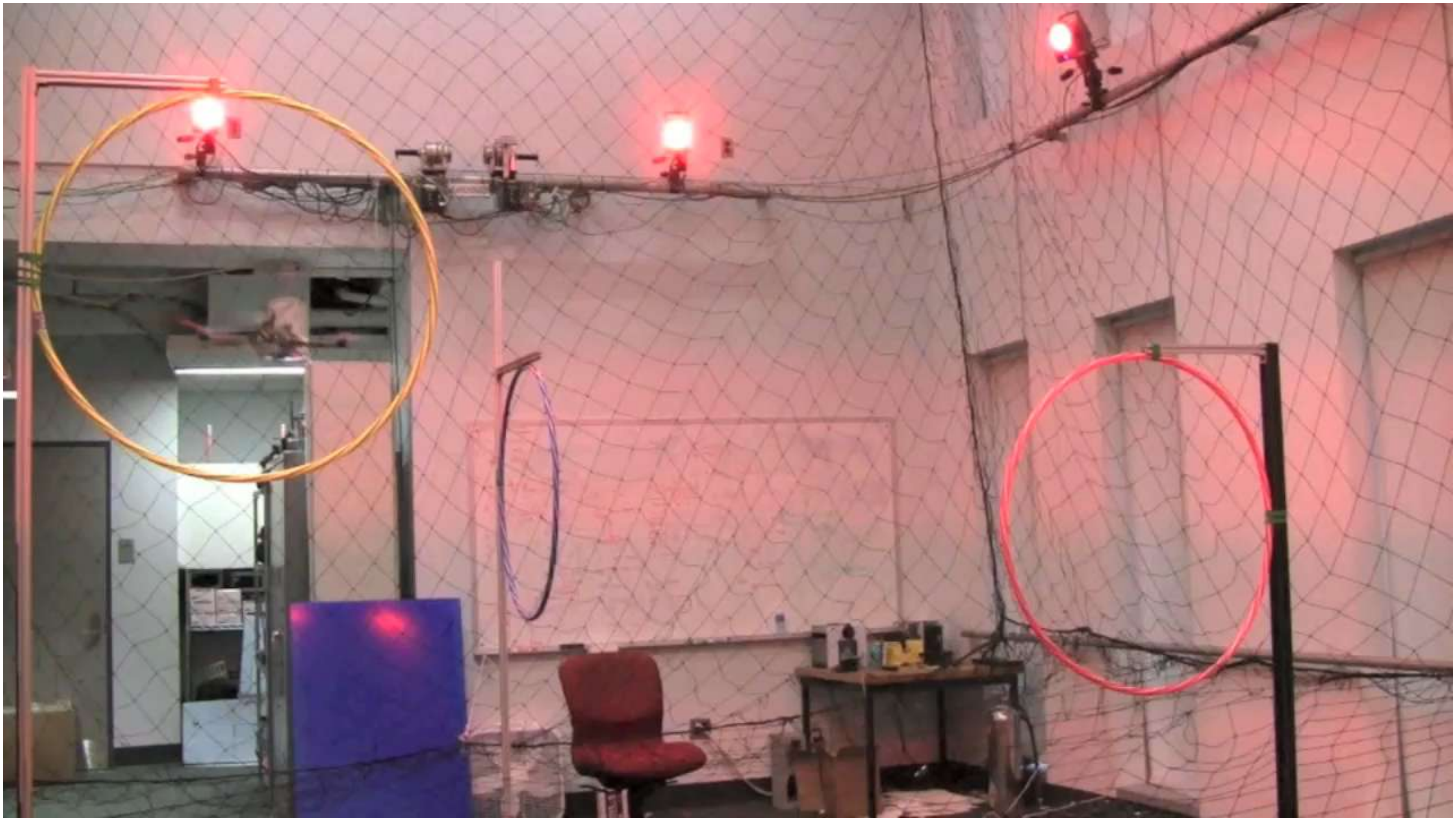
Yaw



How do you get the robot to yaw (steer)?

Translation





The robot has six degrees of freedom!

How many different ways can you rotate or translate the robot?

Key Components for Autonomous Flight

In any system, ask how the following components work!

- State Estimation
- Control
- Mapping
- Planning

In any system, ask how the following components work!

- State Estimation
- Control
- Mapping
- Planning

estimate the position and velocity (including rotation and angular velocity of the robot)

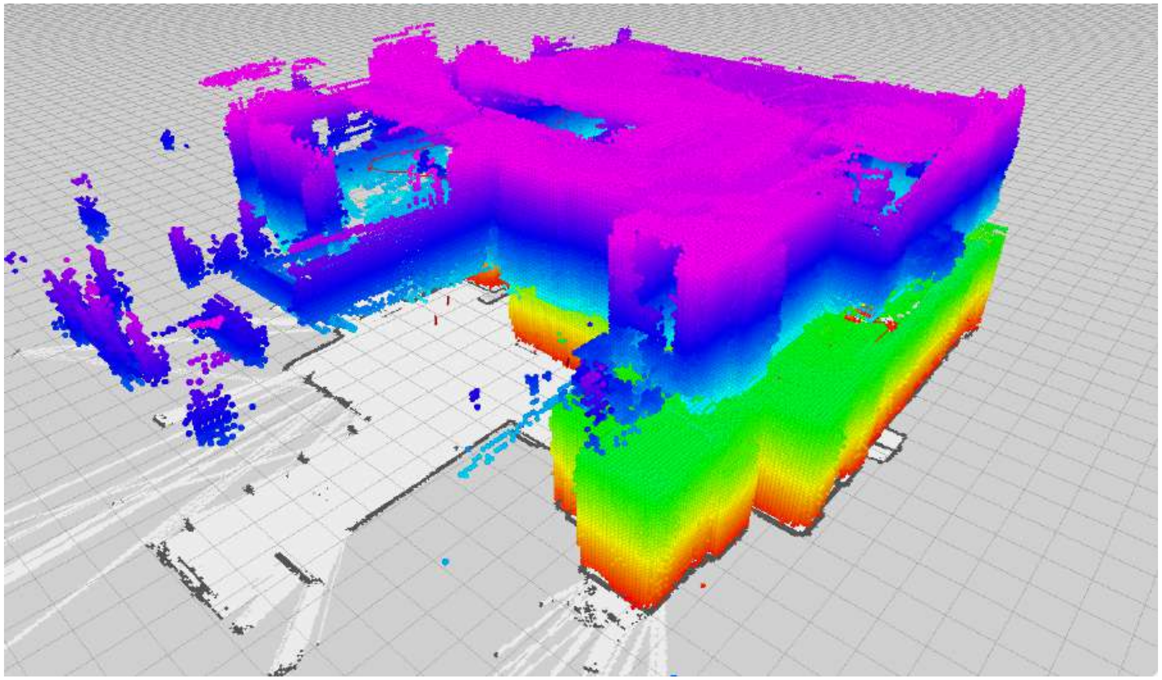
In any system, ask how the following components work!

- State Estimation
- Control
- Mapping
- Planning

command motors and produce desired actions in order to navigate to desired state

In any system, ask how the following components work!

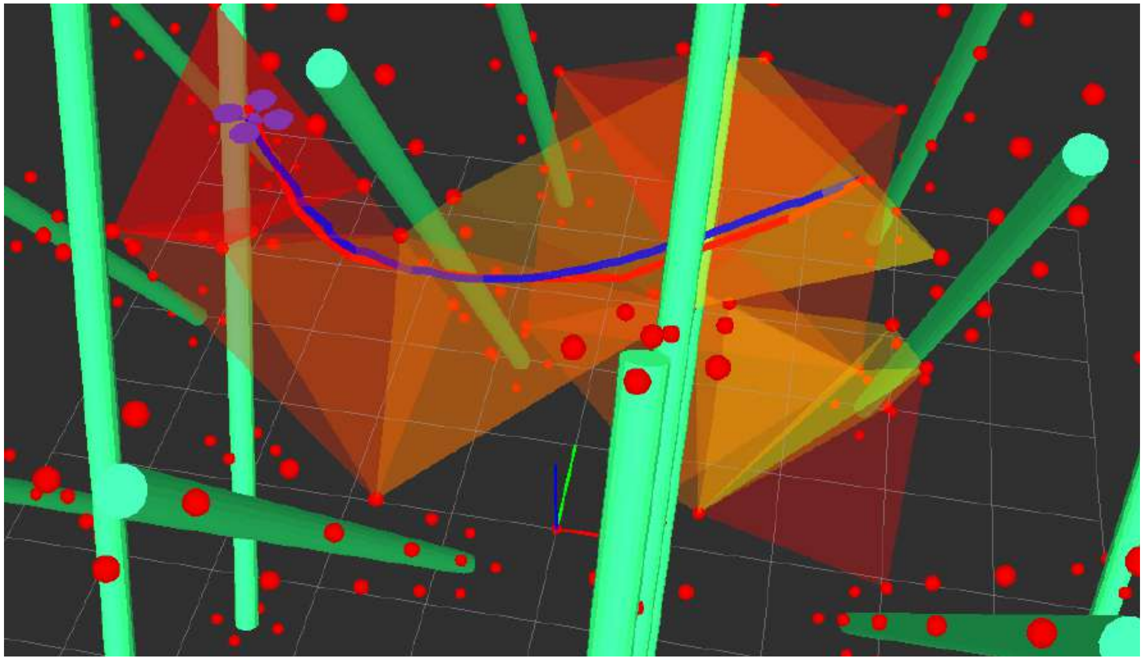
- State Estimation
- Control
- Mapping
- Planning



Penn Engineering

In any system, ask how the following components work!

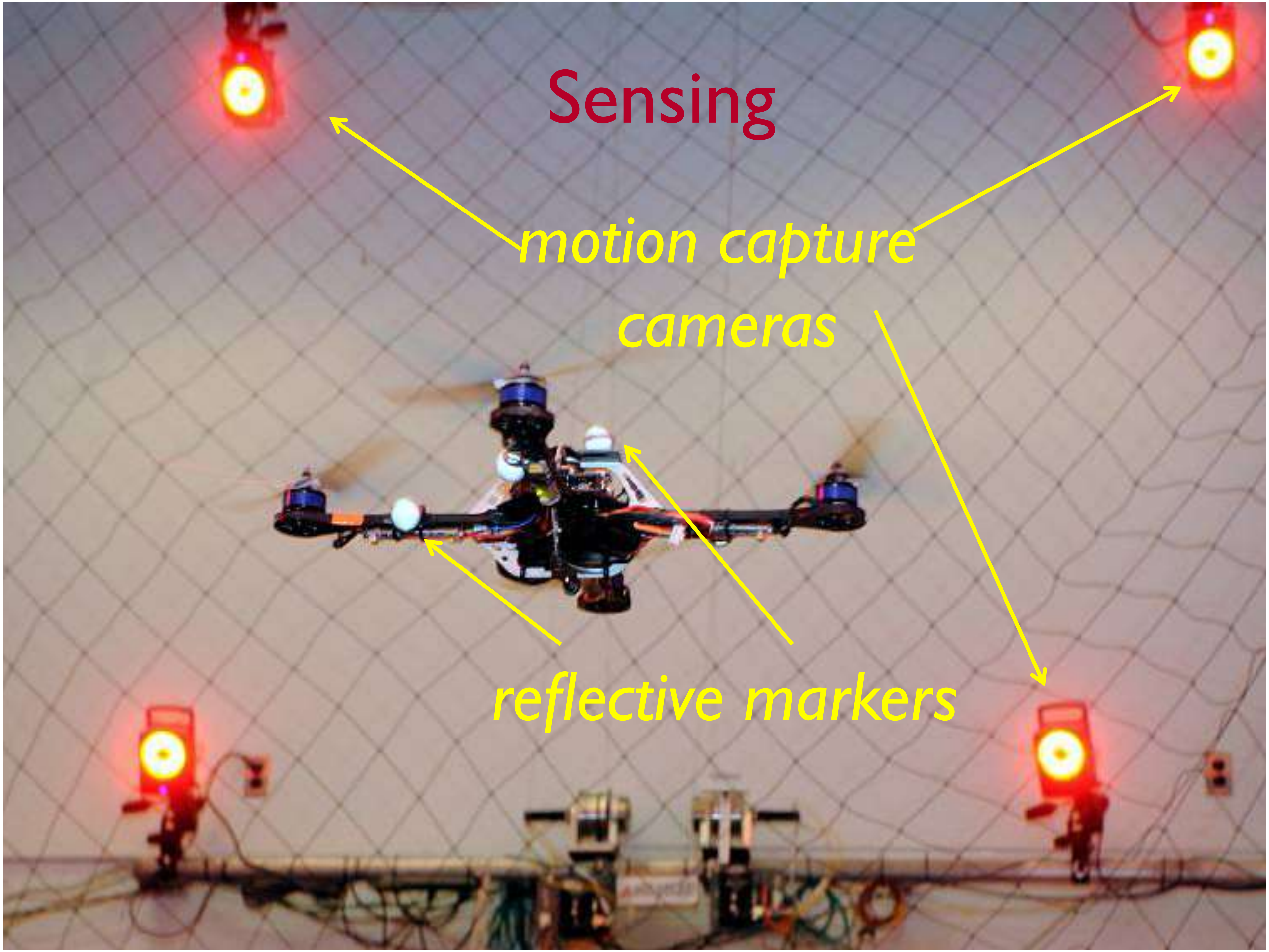
- State Estimation
- Control
- Mapping
- Planning



Penn Engineering

State Estimation

obtain reliable estimates of position and velocity


Sensing
motion capture cameras
reflective markers

X-47B Wins Popular Mechanics Magazine 2013 Breakthrough Award

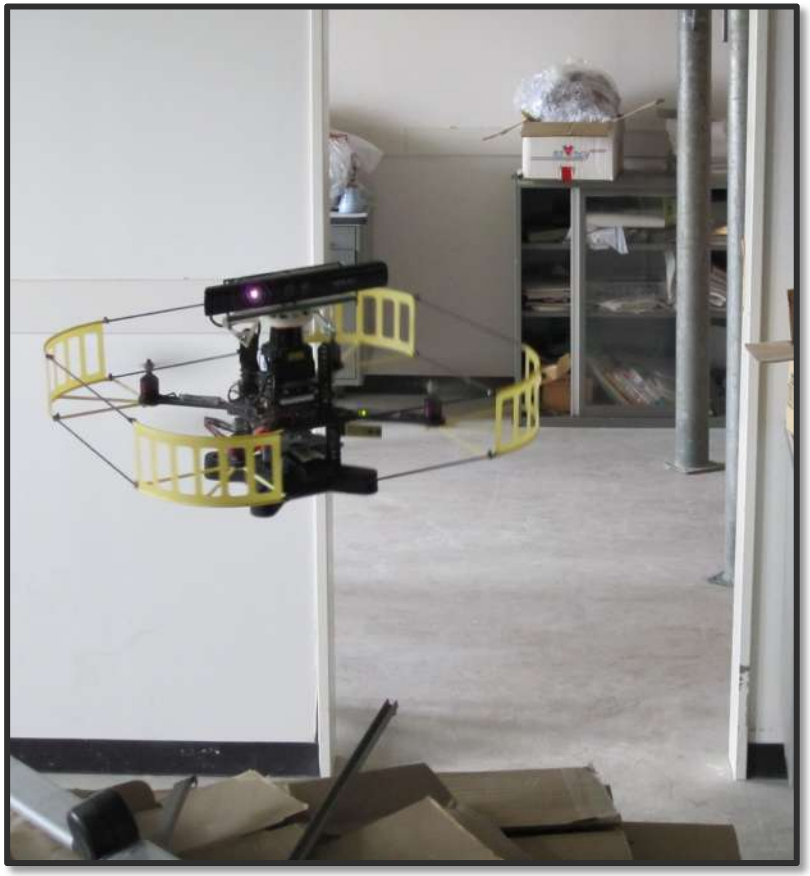
Photograph by Joe McNally


Unreliable GPS

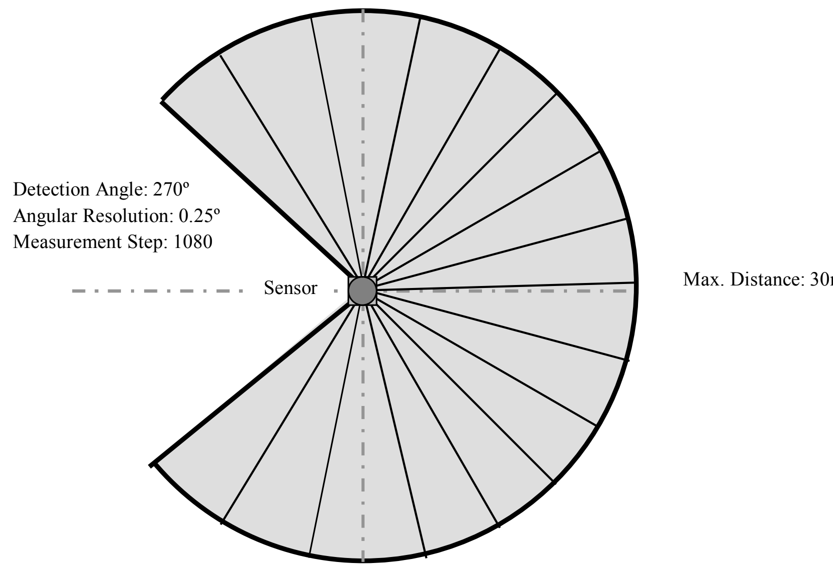

Operate indoors and outdoors
No GPS
Small, maneuverable
Agile, fast

How to navigate without GPS or external motion capture cameras?

Penn Engineering



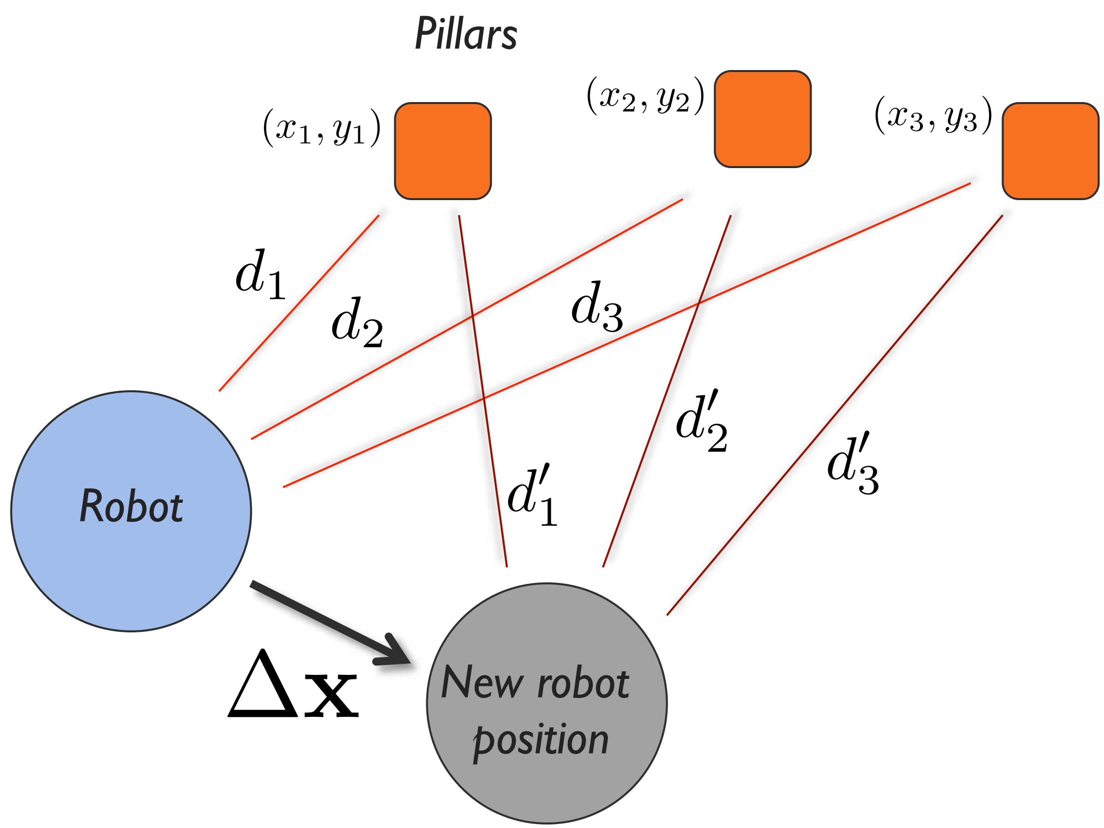
Hokuyo UTM-30LX
Scanning Laser
Rangefinder



mvBlueFox USB camera

Kinect sensor for XBOX 360

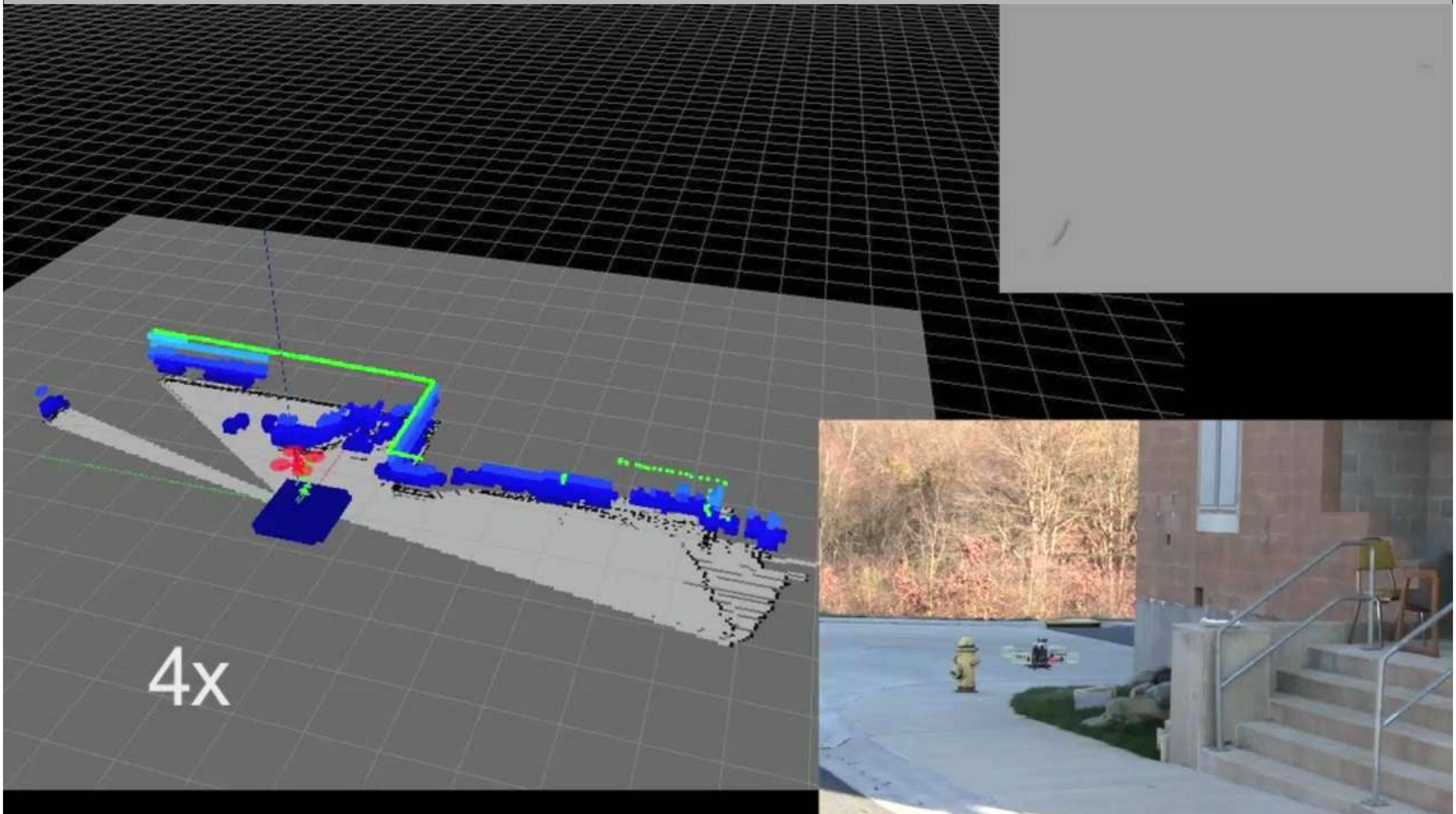
Simultaneous Localization And Mapping



Concurrently estimate

Locations of pillars (6)

Displacement of the robot (2)


4x

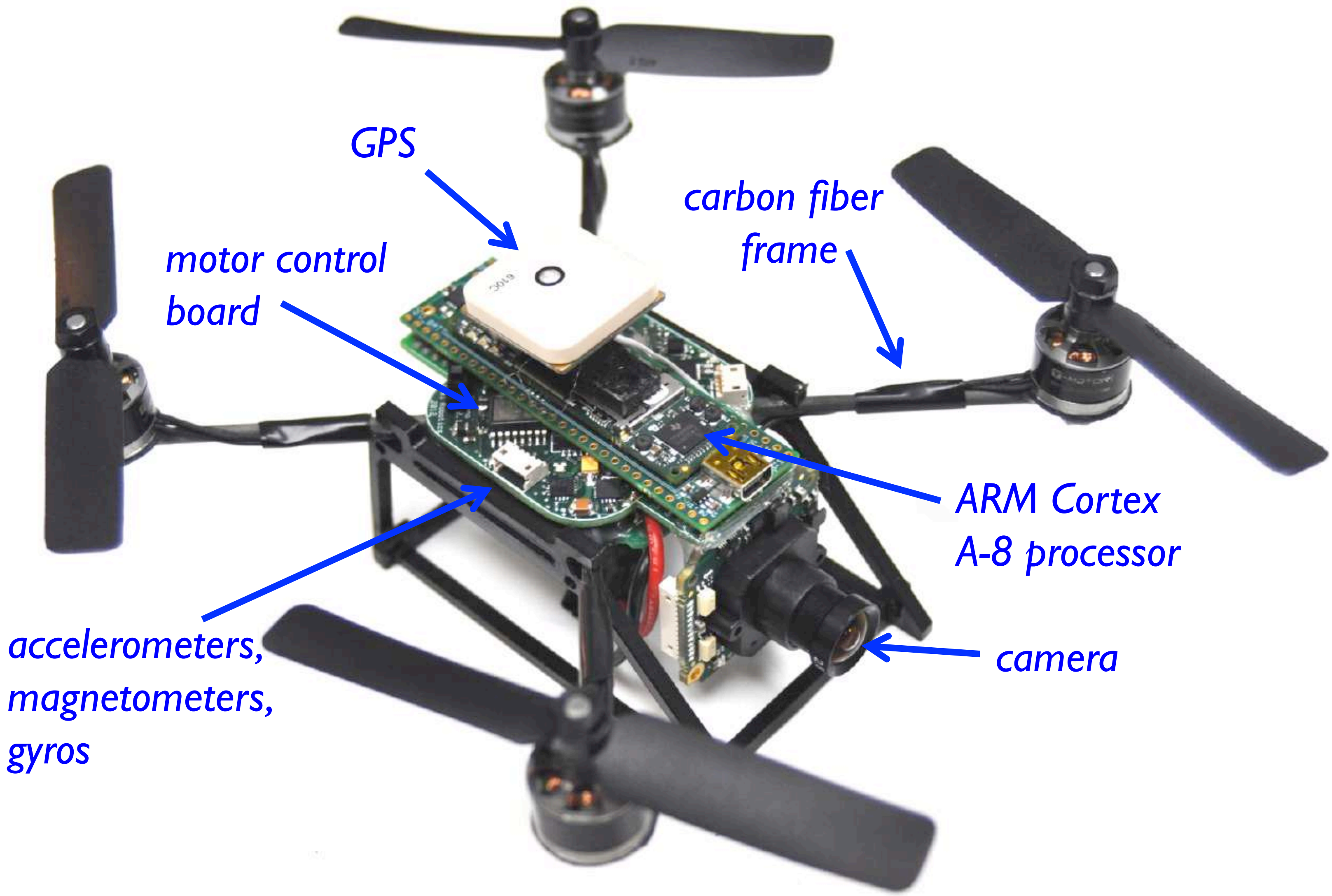
1750 g (laser, 3 cameras, GPS, IMU)

650 g (camera, IMU)

740 g (2 cameras, IMU)

1800 g (laser, Kinect, IMU)

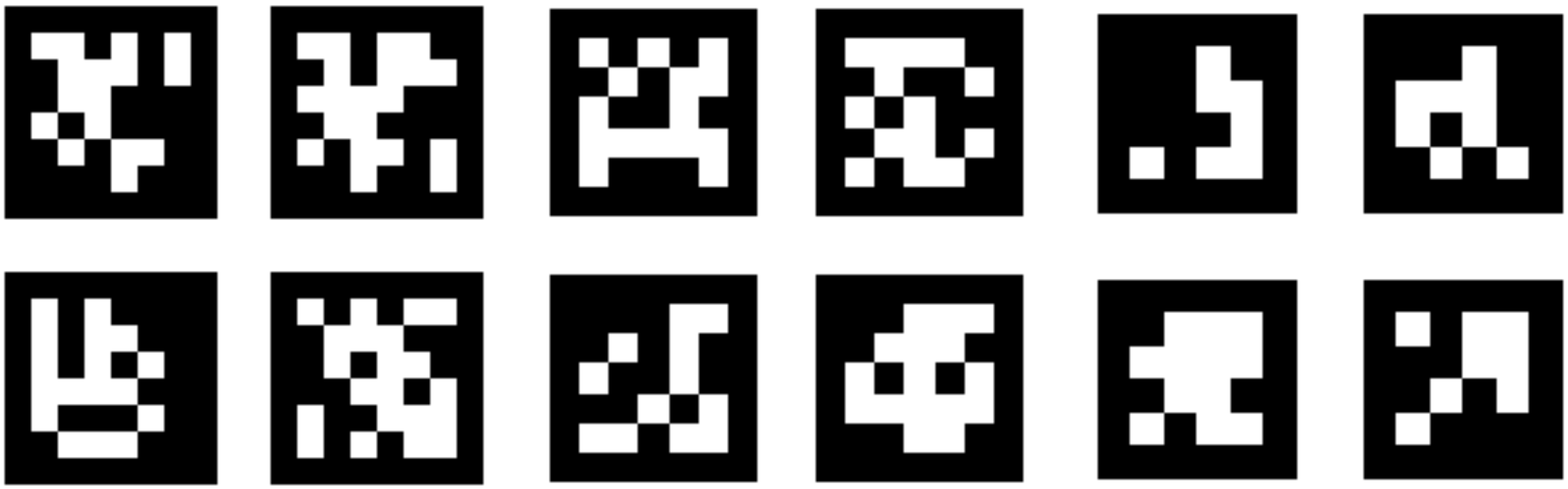
Experimental Platform



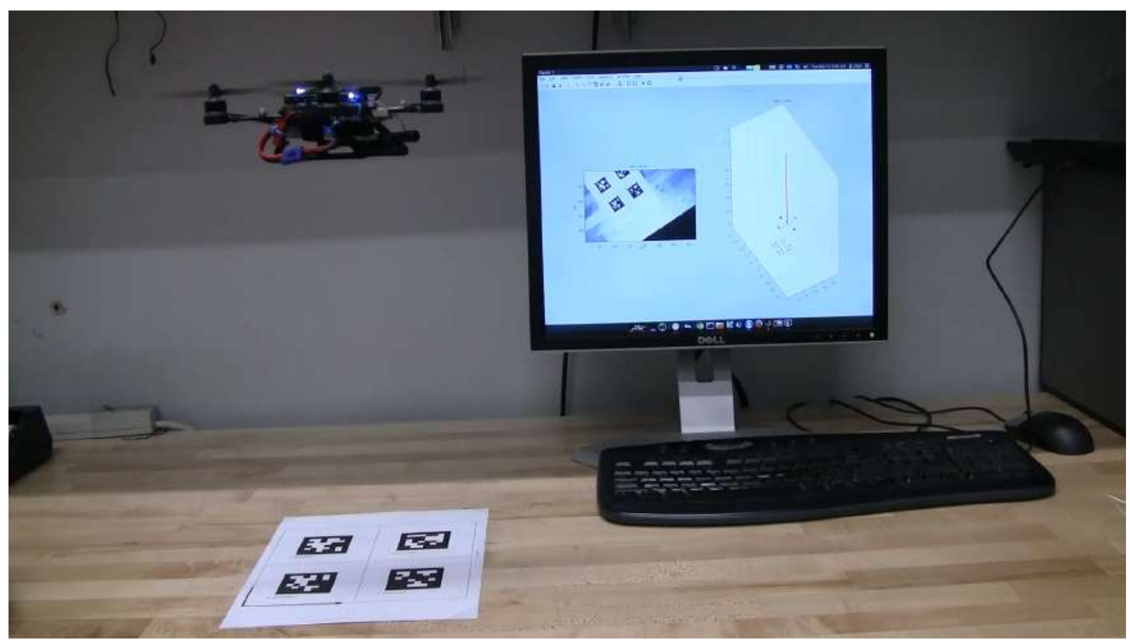
KMel Robotics

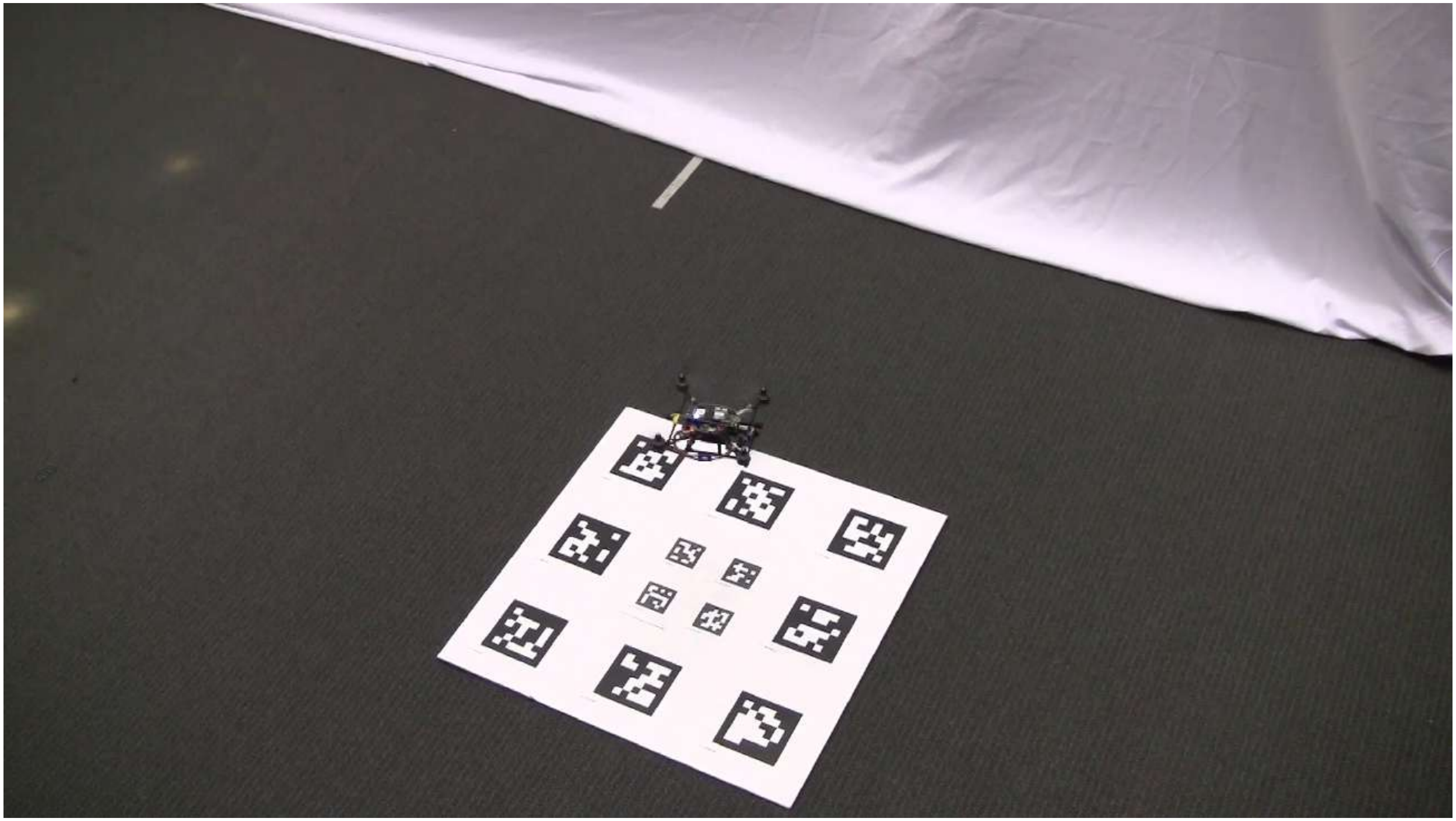
www.kmelrobotics.com

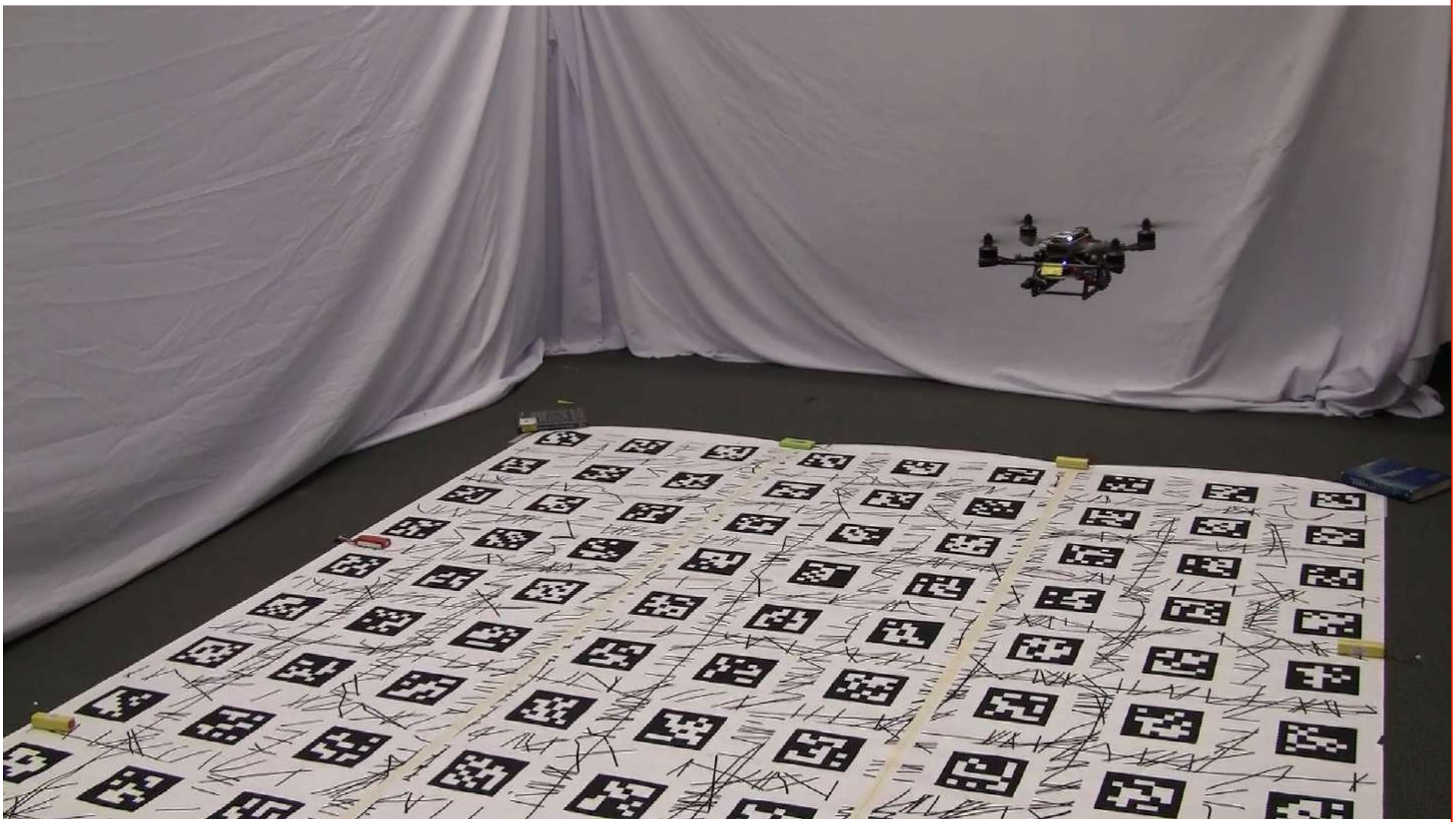
Beacons that are easy to recognize



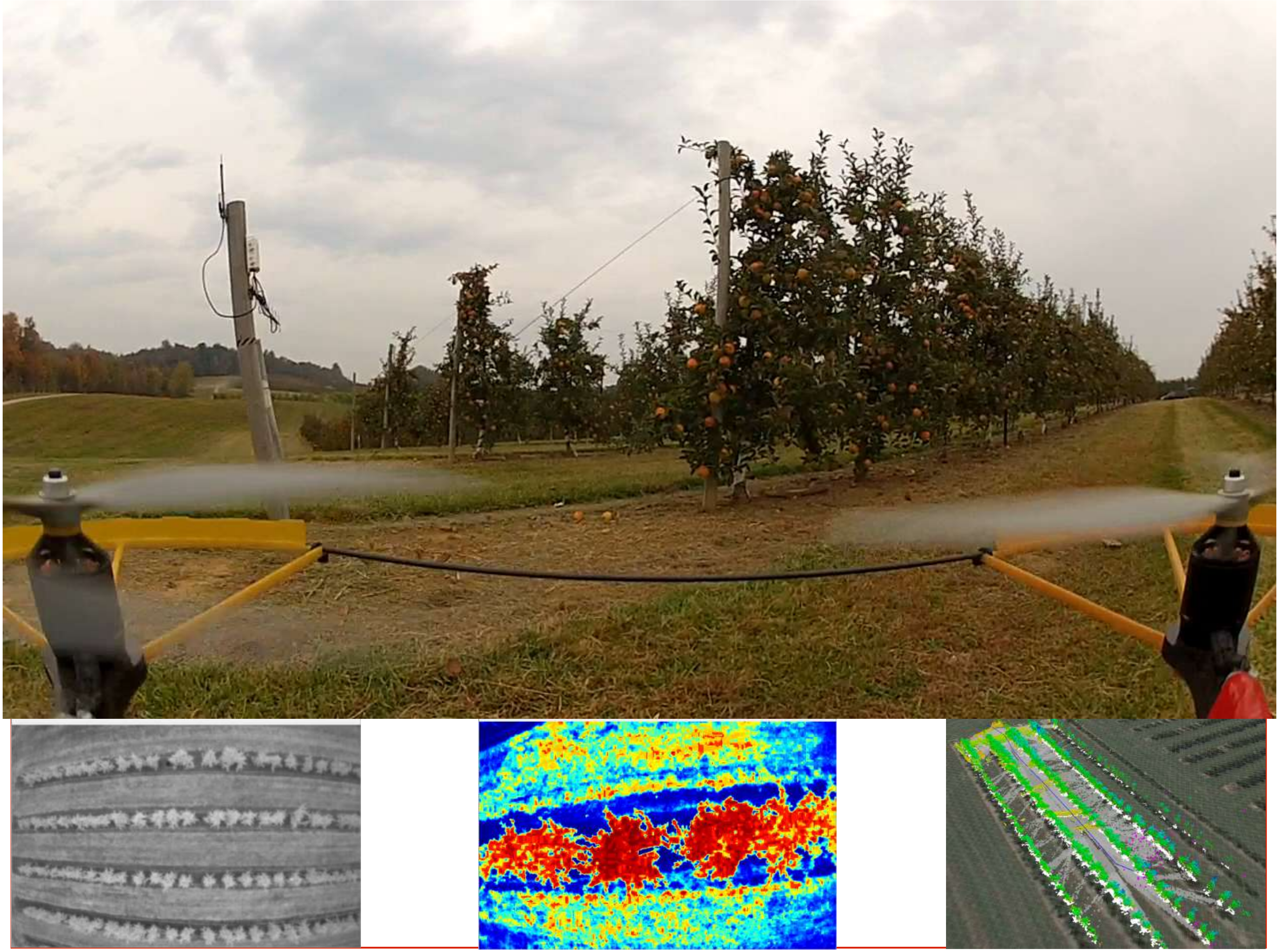
http://april.eecs.umich.edu/wiki/images/9/94/Tagsampler.png








Precision Farming

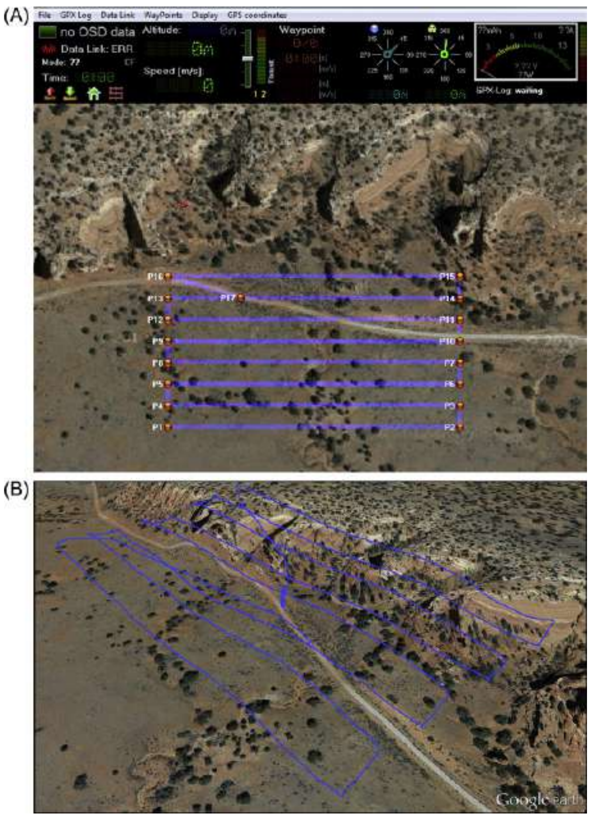


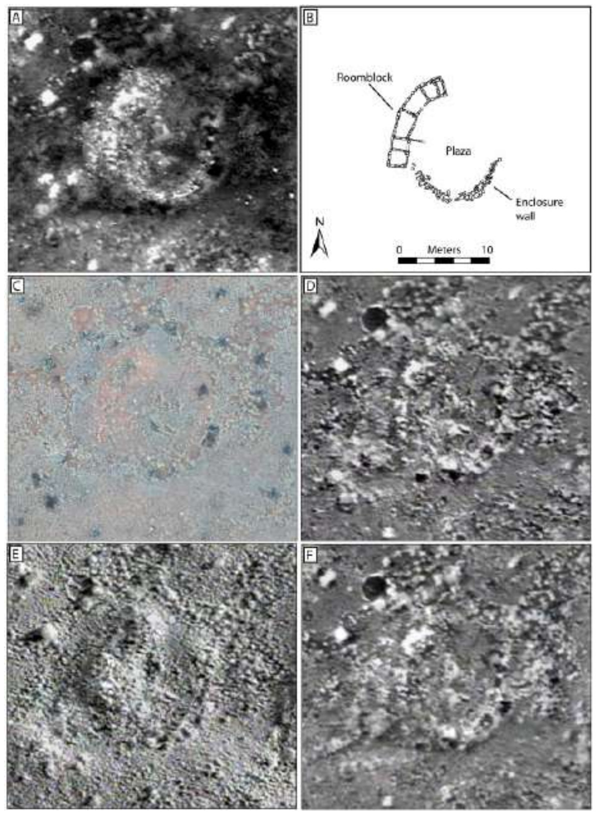
Construction

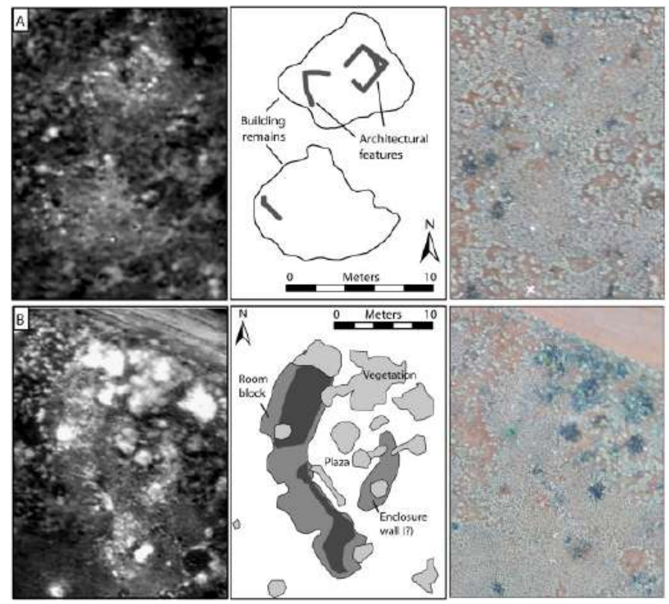


Penn Engineering

Archeology


(A)
(B)
Google earth


A
B
Roomblock
Plaza
Enclosure wall
N
0 Meters 10
C
D
E
F

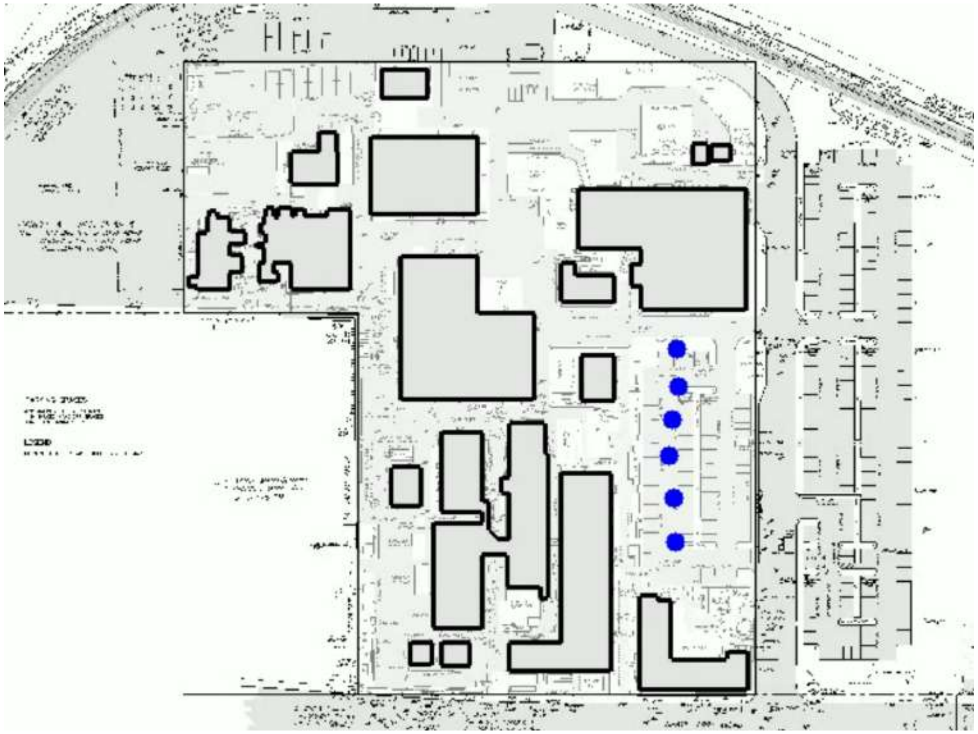

A
Building remains
Architectural features
N
0 Meters 10
B
Room block
Vegetation
Plaza
Enclosure wall (?)

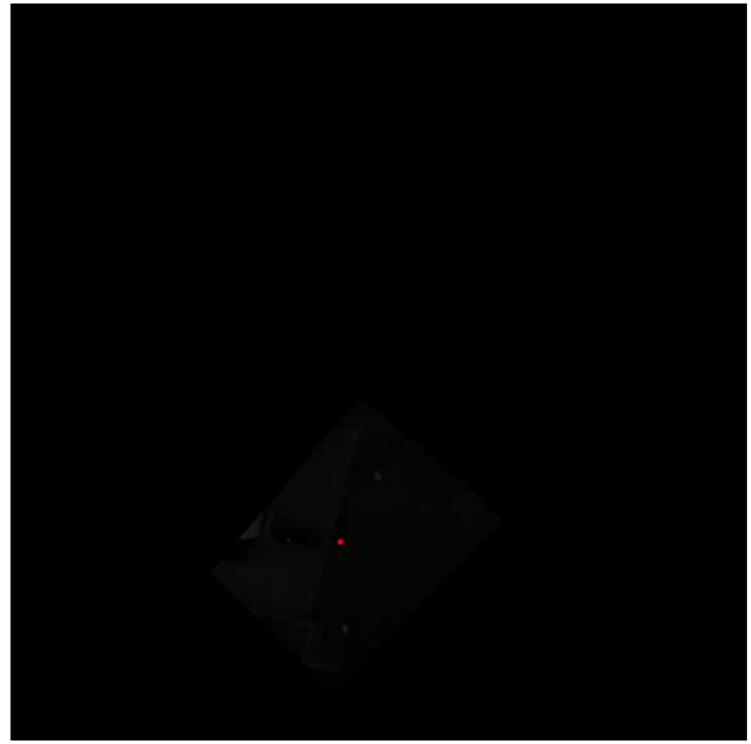
Photography

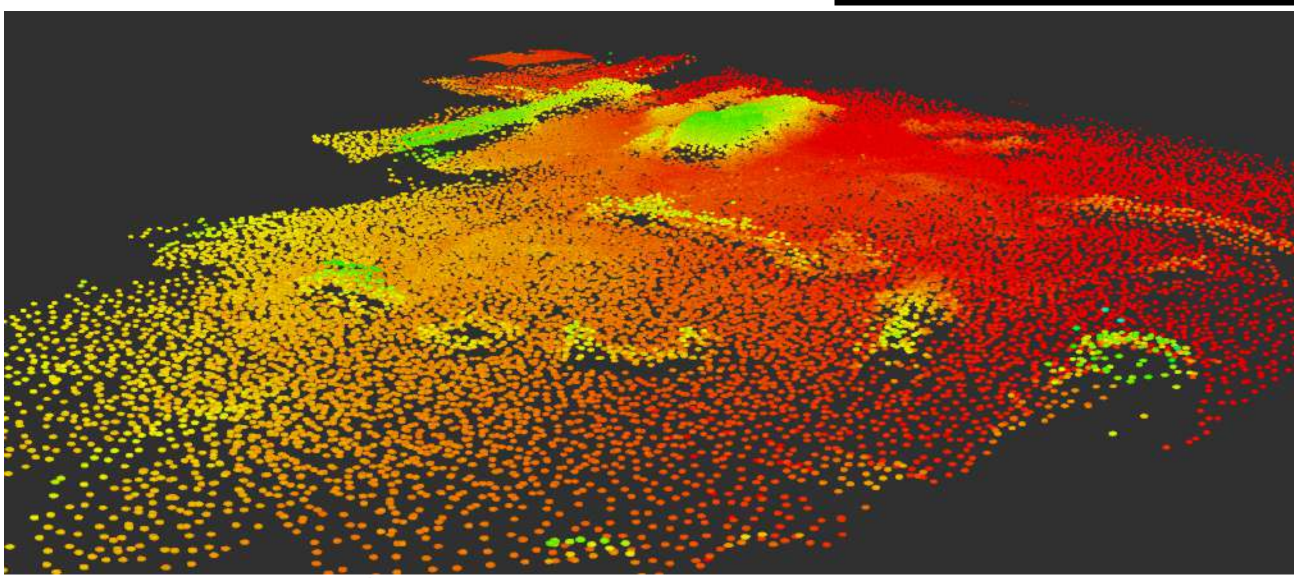
Robot First Responders

Kartik Mohta, Matthew Turpin, Alex Kushleyev, Daniel Mellinger, Nathan Michael, and Vijay Kumar, "QuadCloud: A Rapid Response Force with Quadrotor Teams," *Int. Symp. on Experimental Robotics* (ISER), 2014.

Penn Engineering





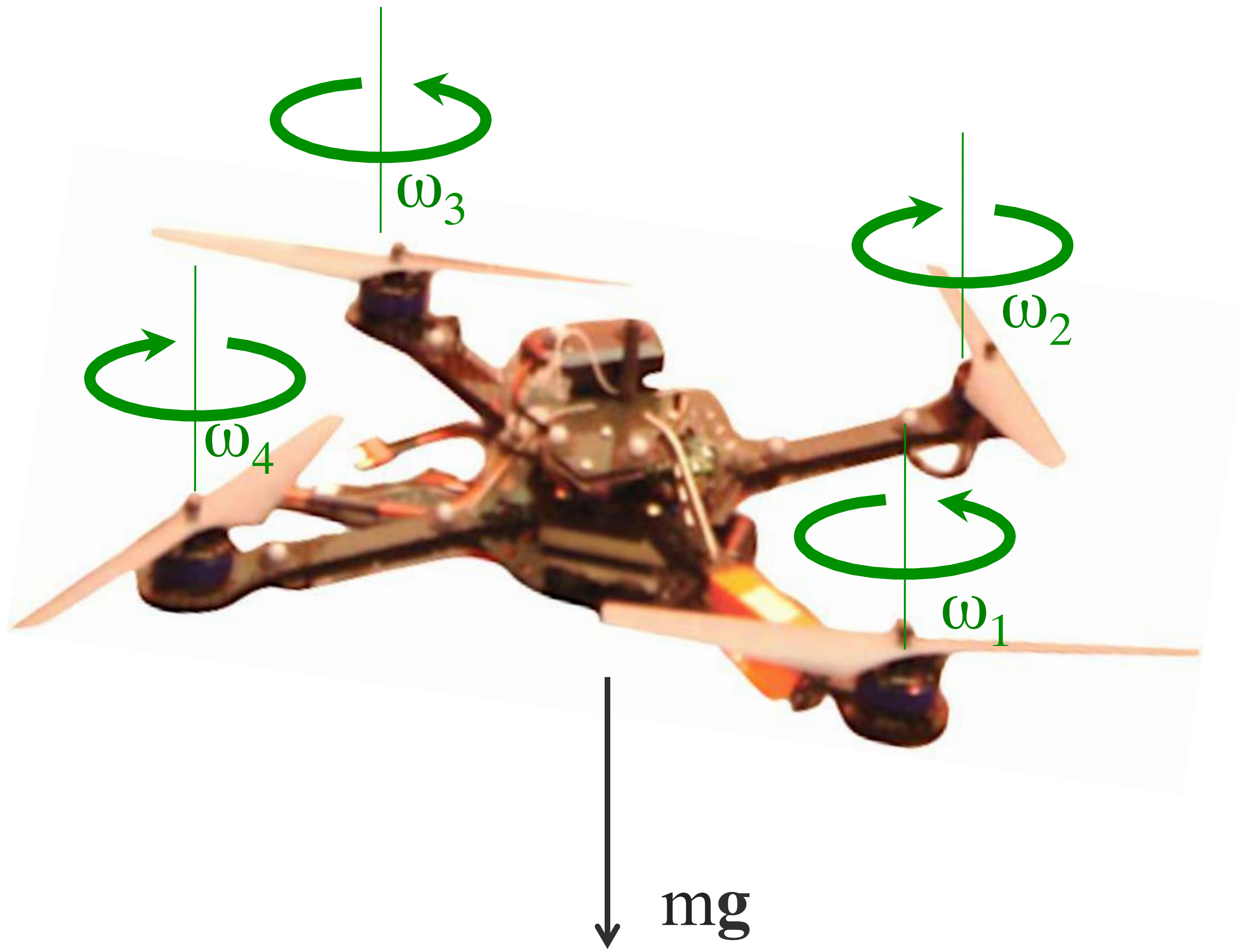


Kartik Mohta, Matthew Turpin, Alex Kushleyev, Daniel Mellinger, Nathan Michael, and Vijay Kumar, "QuadCloud: A Rapid Response Force with Quadrotor Teams," *Int. Symp. on Experimental Robotics* (ISER), 2014.

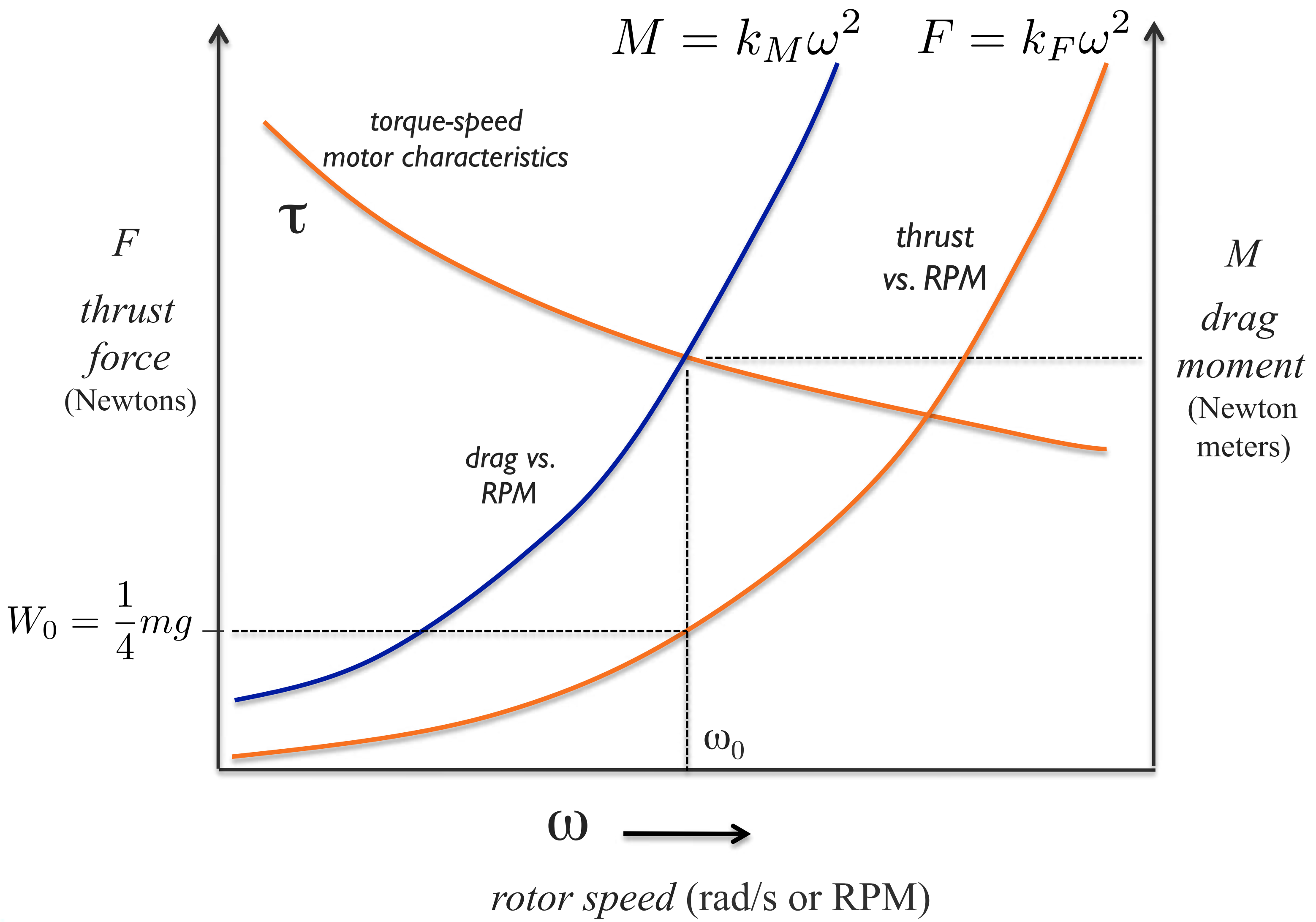
Goals

- Basic mechanics
- Control
- Design considerations
- Agility
- Component selection
- Effects of size

Basic Mechanics

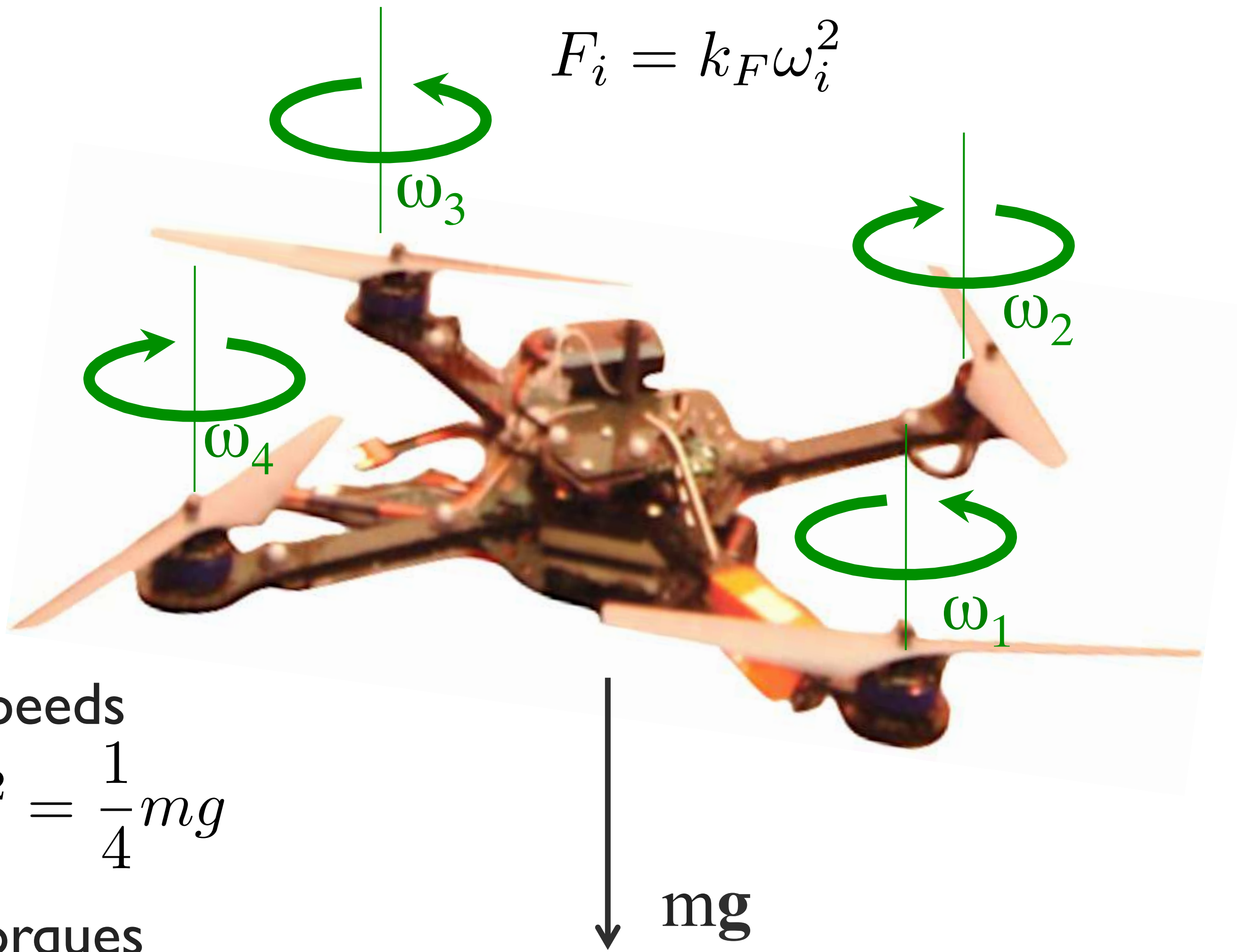


Rotor Physics



Basic Mechanics (Hover)

$$F_i = k_F \omega_i^2$$



Motor Speeds

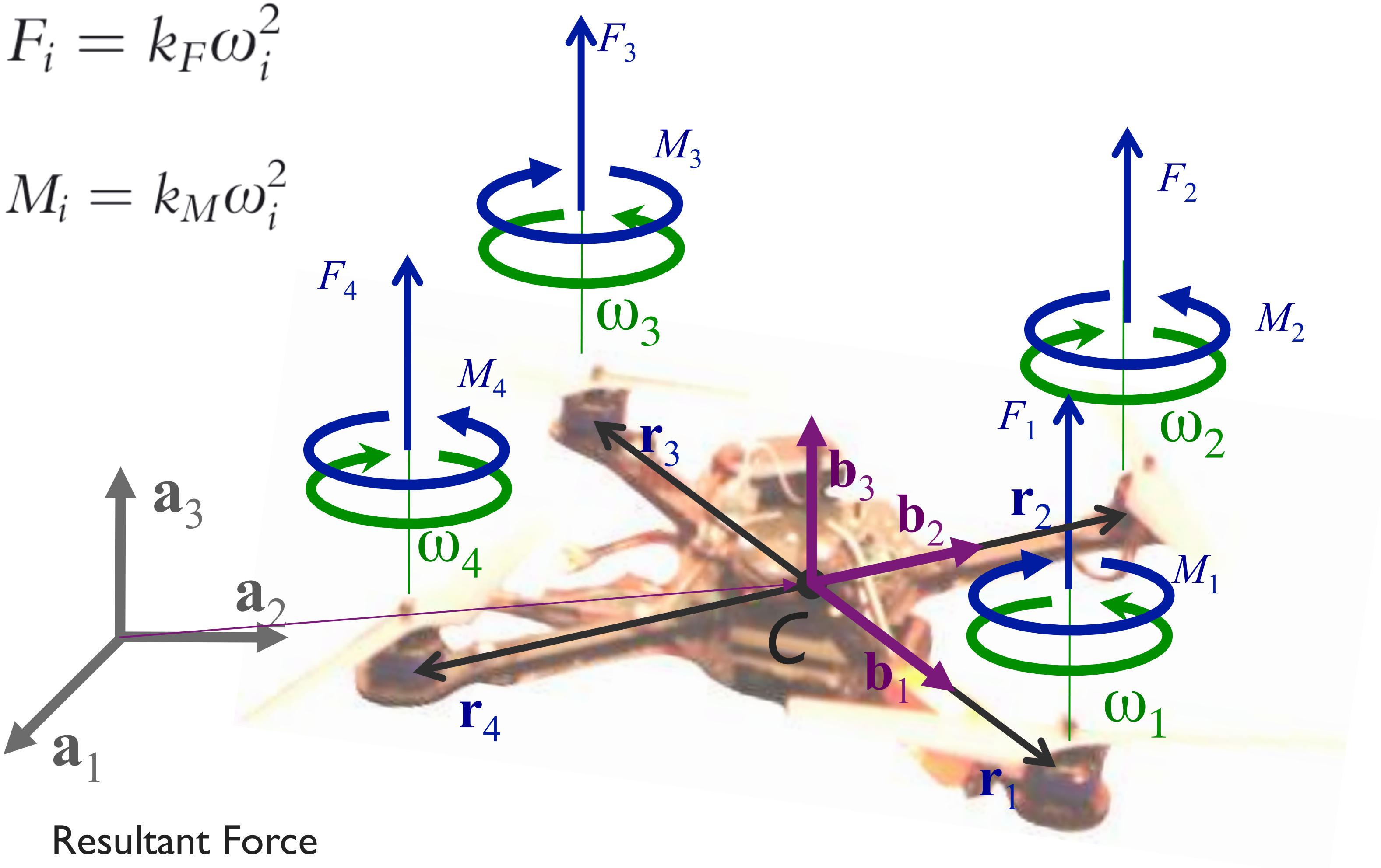
$$k_F \omega_i^2 = \frac{1}{4} mg$$

Motor Torques

$$\tau_i = k_M \omega_i^2$$

$$F_i = k_F \omega_i^2$$

$$M_i = k_M \omega_i^2$$



Resultant Force

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4 - mg\mathbf{a}_3$$

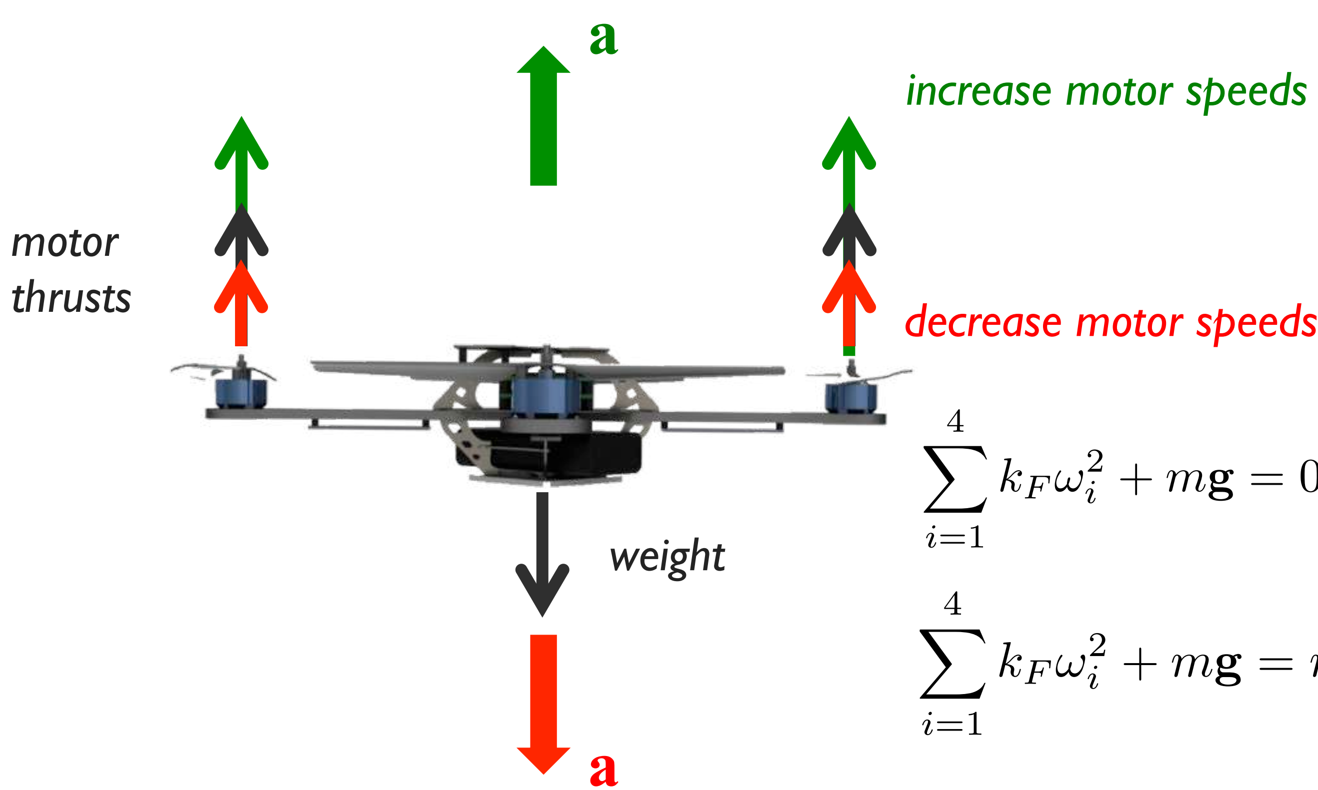
Resultant Moment

$$\mathbf{M} = \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2 + \mathbf{r}_3 \times \mathbf{F}_3 + \mathbf{r}_4 \times \mathbf{F}_4$$
$$+\mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3 + \mathbf{M}_4$$

Penn Engineering

Acceleration

(in the vertical direction)



$$\sum_{i=1}^{4} k_F \omega_i^2 + m\mathbf{g} = 0$$

$$\sum_{i=1}^{4} k_F \omega_i^2 + m\mathbf{g} = m\mathbf{a}$$

Goals

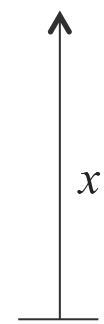
- Basic mechanics
- Control
- Design considerations
- Agility
- Component selection
- Effects of size

Control of height

$$\boxed{\sum_{i=1}^{4} k_F \omega_i^2 + m\mathbf{g}} = m\mathbf{a} \qquad a = \frac{d^2x}{dt^2} = \ddot{x}$$

Input $\quad u = \dfrac{1}{m}\left[\sum_{i=1}^{4} k_F \omega_i^2 + m\mathbf{g}\right]$

Second order dynamic system $\quad u = \ddot{x}$



What input drives the robot to the desired position?

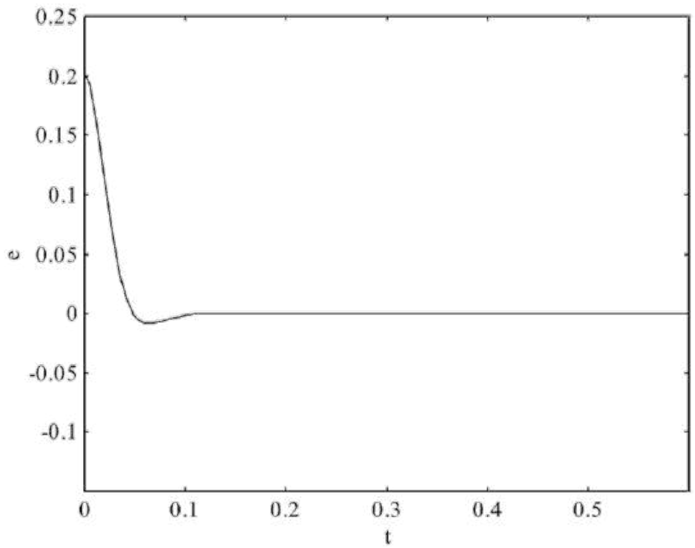
Control of a linear second-order system

Problem

State, input $x,\ u\ \in \mathbb{R}$

Plant model $\ddot{x} = u$

Want x to follow the desired trajectory $x^{des}(t)$



General Approach

Define error, $e(t)=x^{des}(t)- x(t)$

Want $e(t)$ to converge exponentially to zero

Strategy

Find u such that

$$\ddot{e} + K_v \dot{e} + K_p e = 0 \qquad K_p,\ K_v\ >\ 0$$

$$u(t) = \ddot{x}^{\text{des}}(t) + K_v \dot{e}(t) + K_p e(t)$$

Feedforward Derivative Proportional

Control for trajectory tracking in a simple second-order system

PD control

$$u(t) = \ddot{x}^{\mathrm{des}}(t) + K_v \dot{e}(t) + K_p e(t)$$

Proportional control acts like a spring (capacitance) response

Derivative control is a viscous dashpot (resistance) response

Large derivative gain makes the system overdamped and the system converges slowly

PID control

In the presence of disturbances (e.g., wind) or modeling errors (e.g. unknown mass), it is often advantageous to use PID control

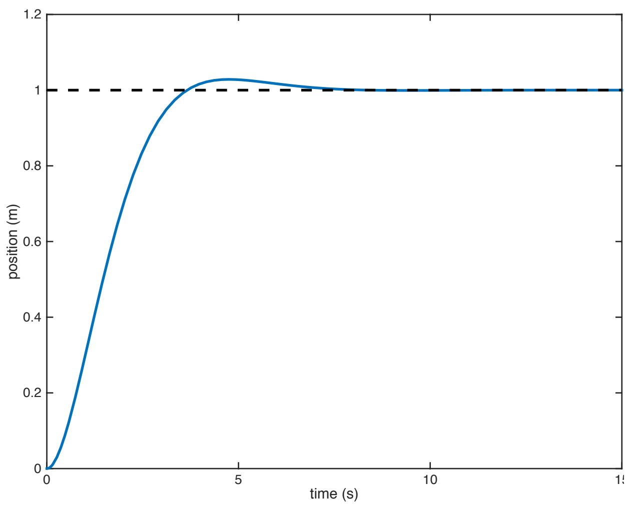
$$u(t) = \ddot{x}^{\mathrm{des}}(t) + K_v \dot{e}(t) + K_p e(t) + K_i \underbrace{\int_0^t e(\tau) d\tau}_{\text{Integral}}$$

PID control generates a third-order closed-loop system

Integral control makes the steady-state error go to zero

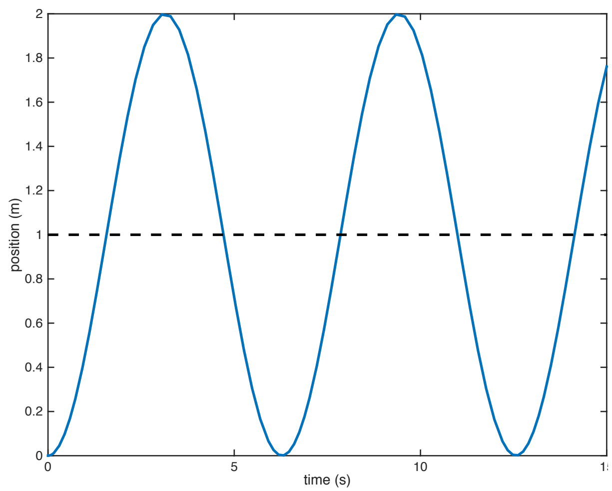
Penn Engineering

Effects of Gains for a PD Control System



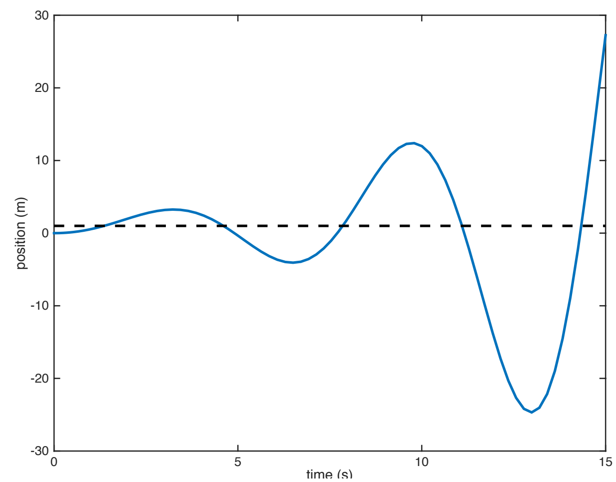
Stable

$K_p,\ K_v\ >\ 0$



Marginally Stable

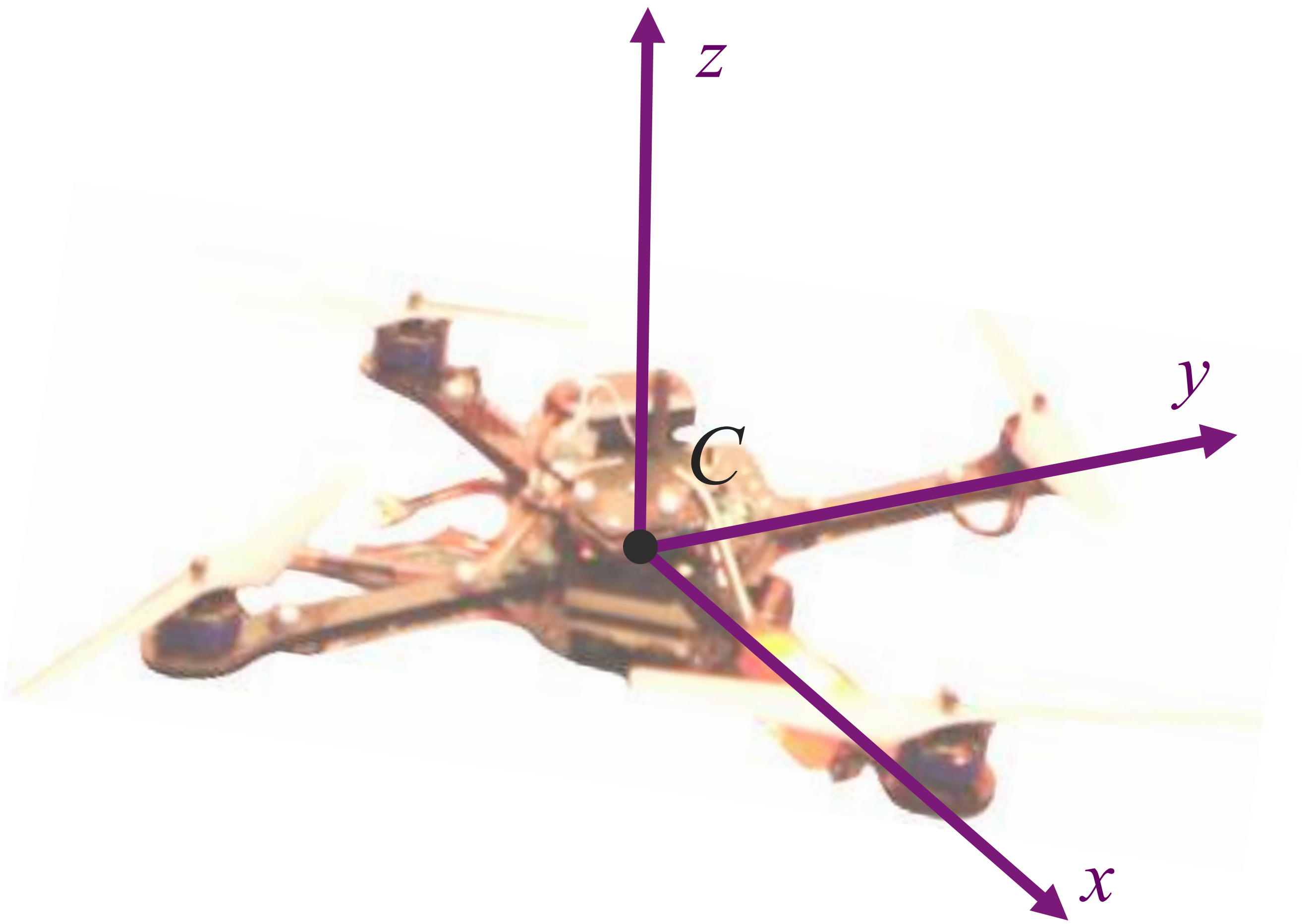
$K_p >\ 0,\quad K_v\ = 0$



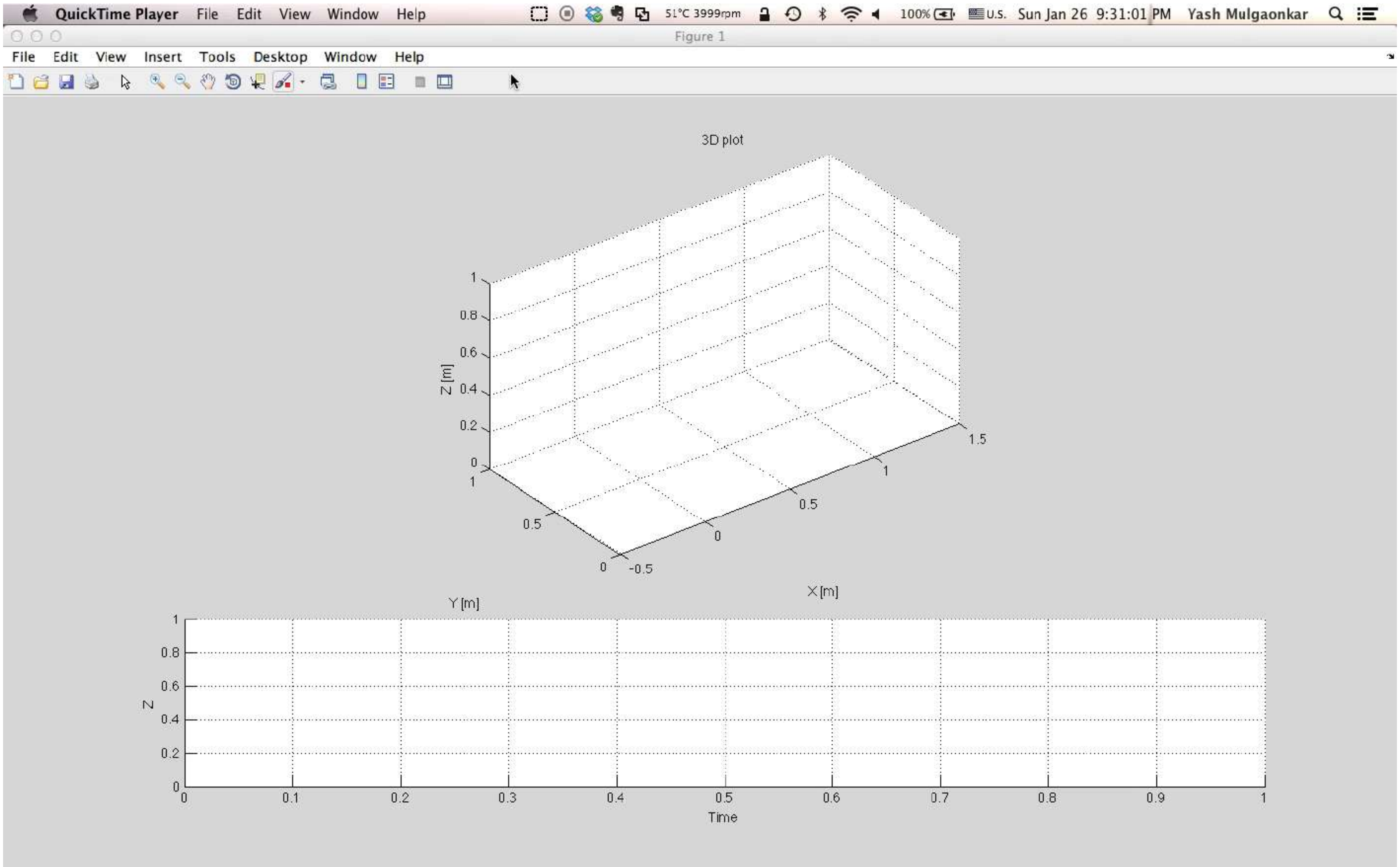
Unstable

K_p or $K_v\ < 0$

Control of quadrotor height

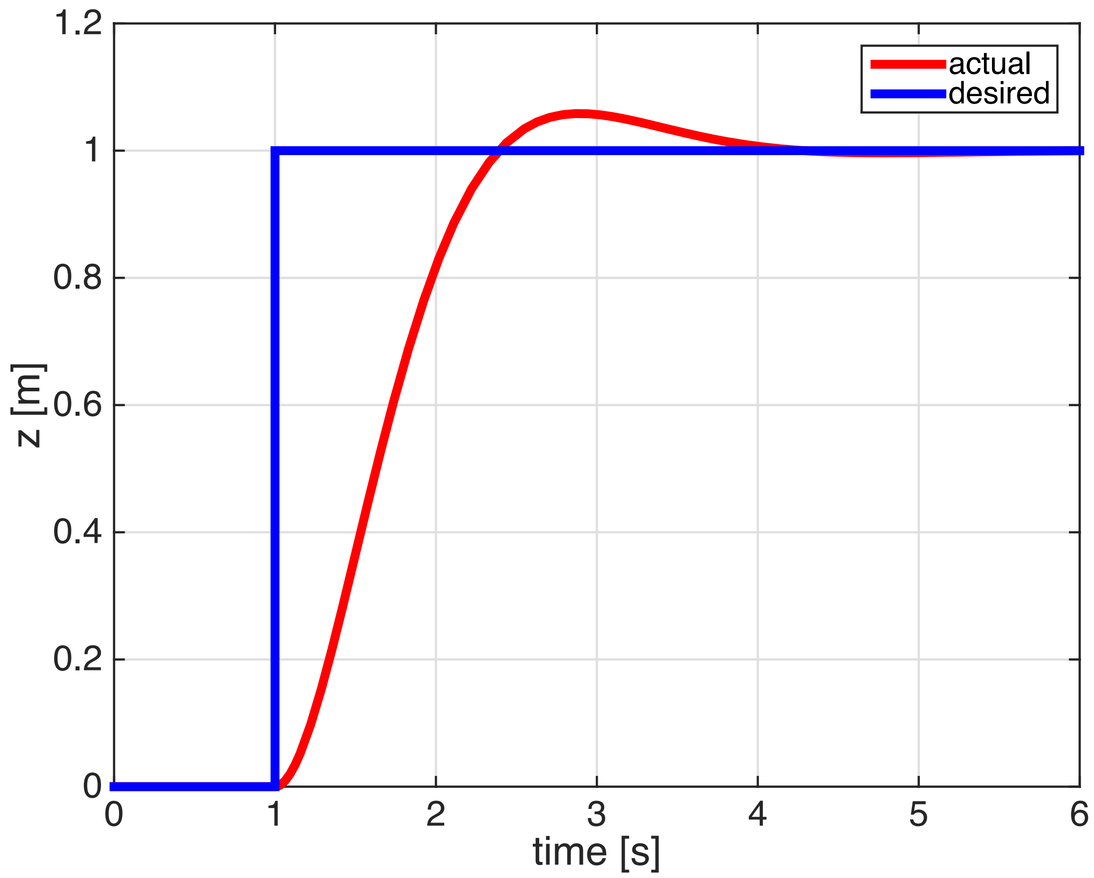


Simulation - PD Control of height

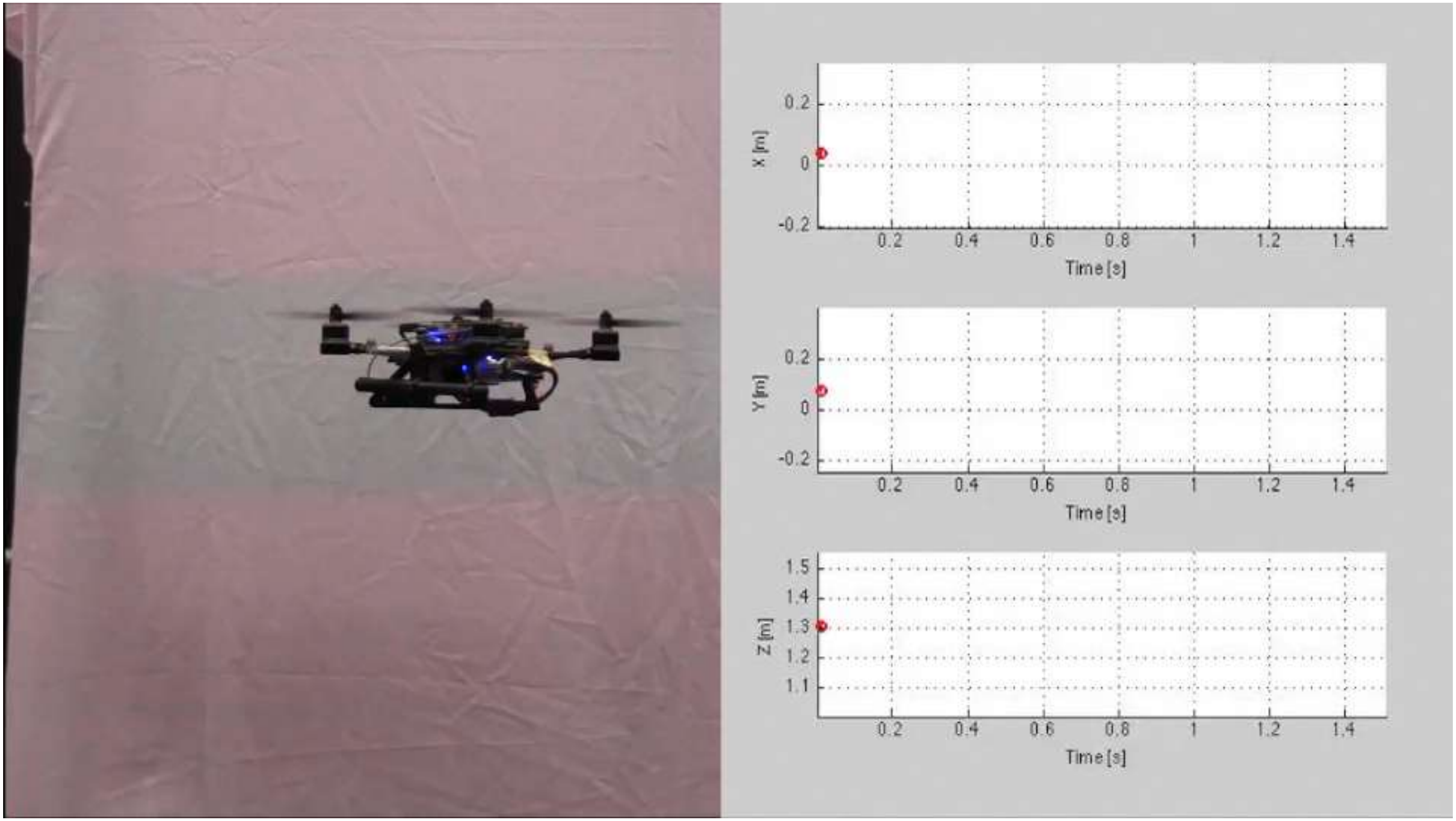


Penn Engineering

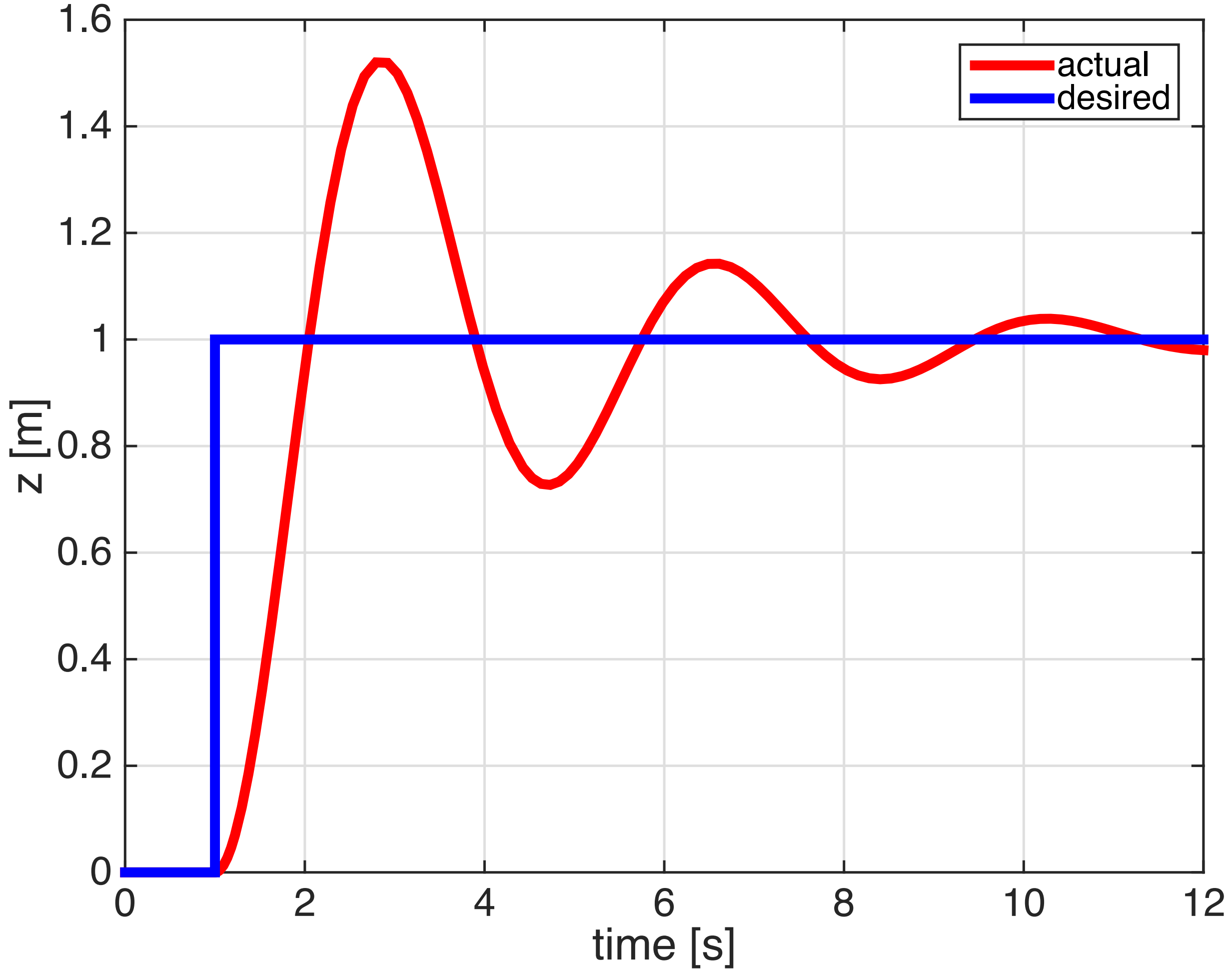
PD Controller



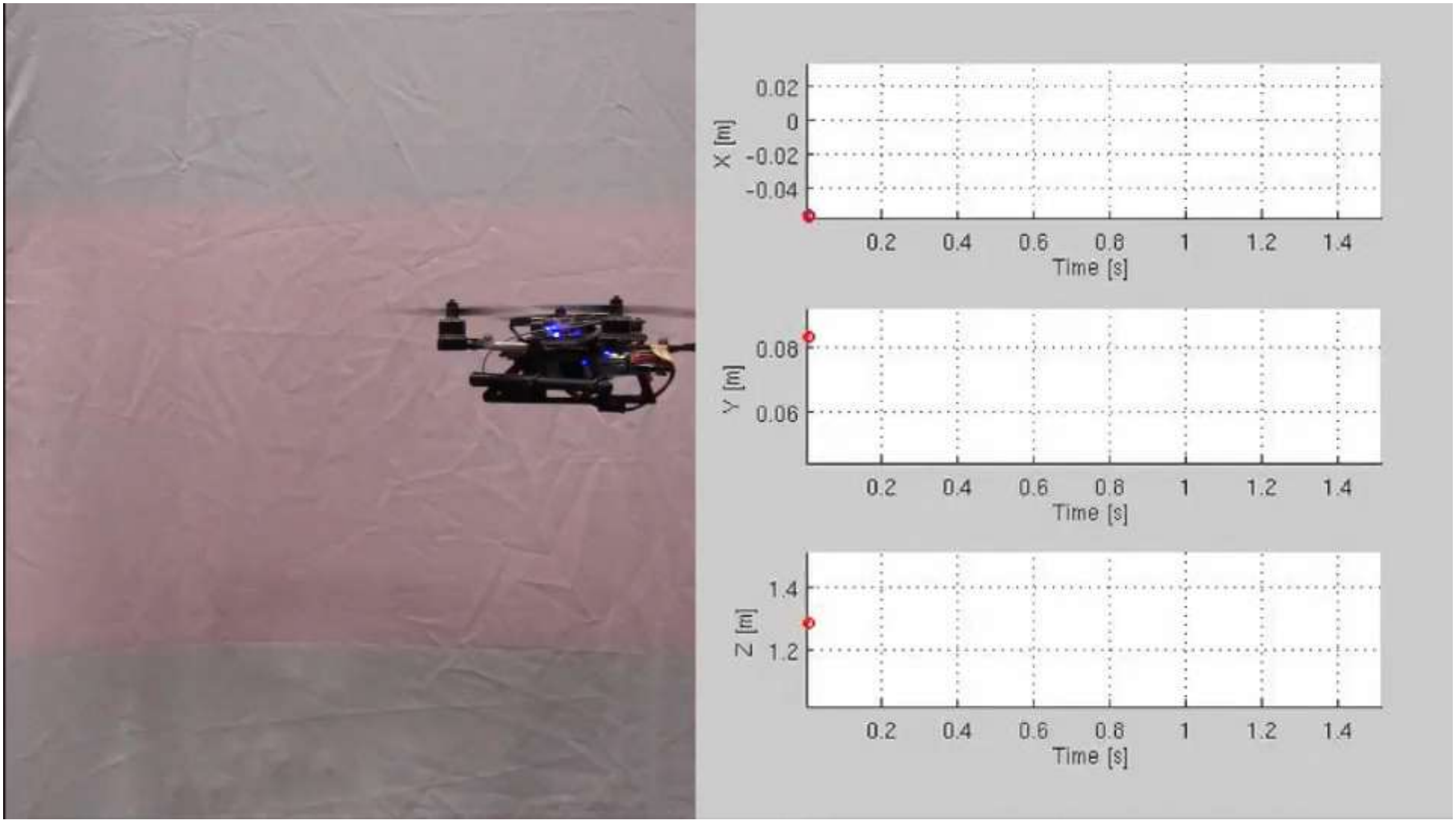
PD Controller



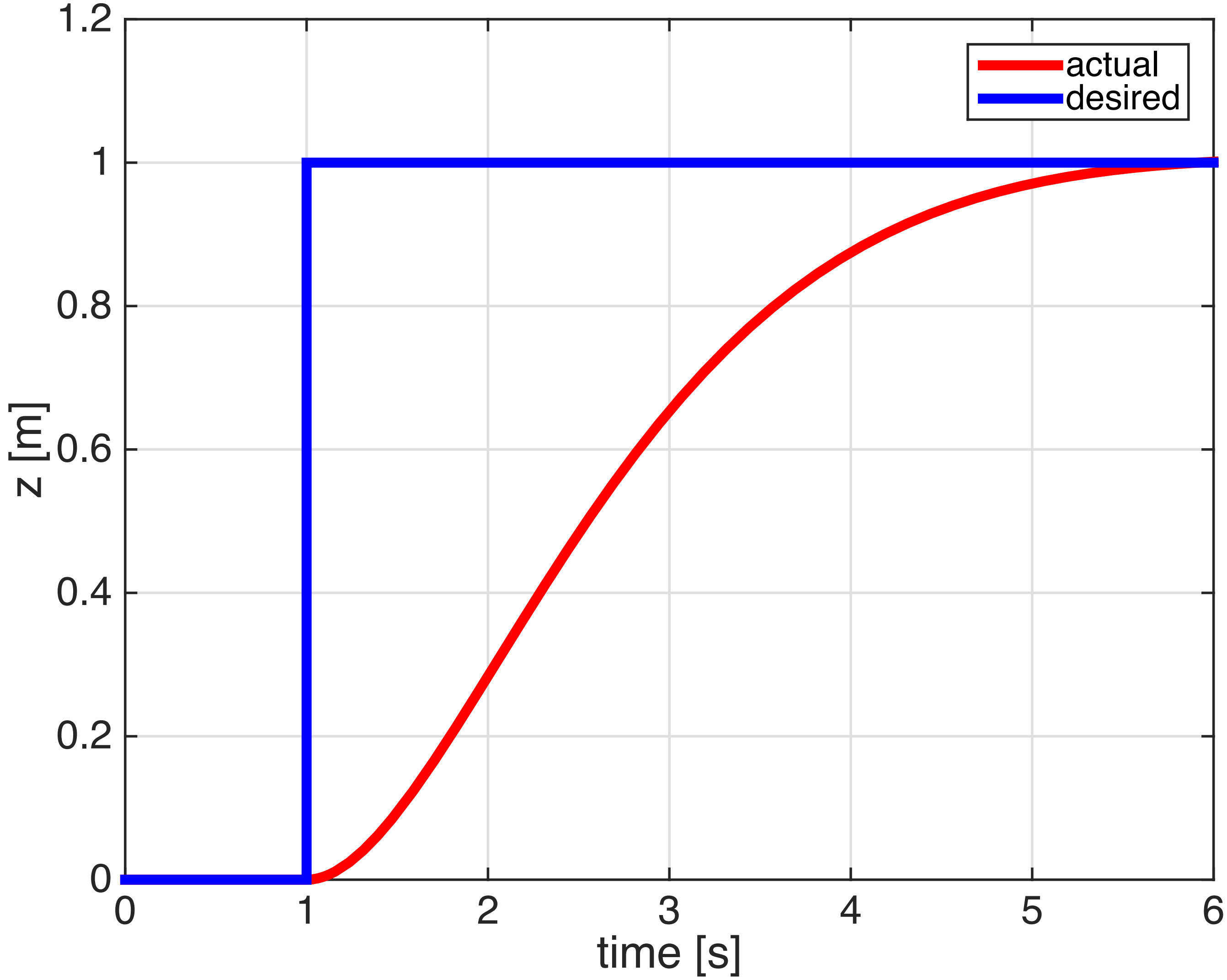
High K_p



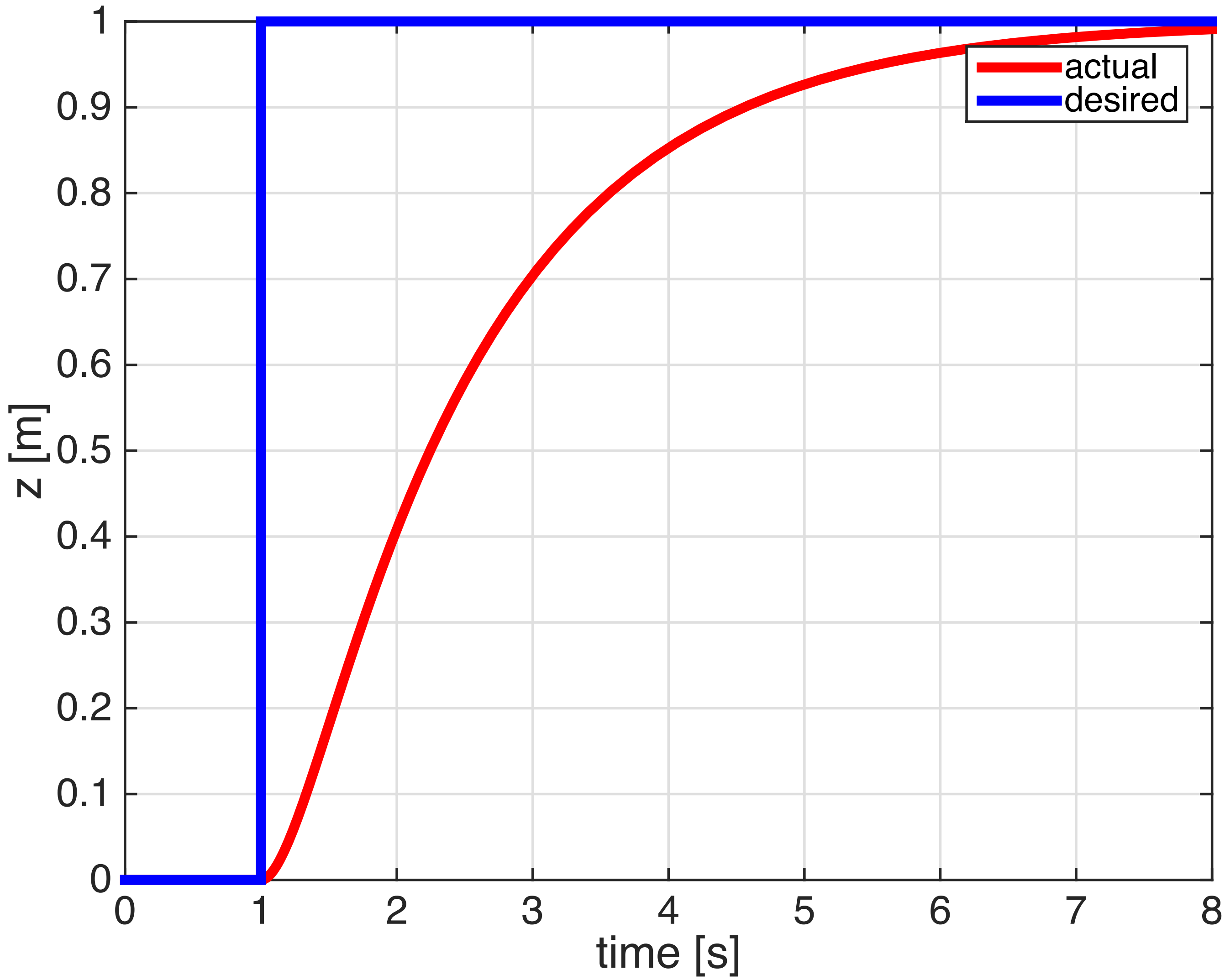
Penn Engineering


0.02
0
-0.02
-0.04
X [m]
0.2
0.4
0.6
0.8
1
1.2
1.4
Time [s]
0.08
0.06
Y [m]
Time [s]
1.4
1.2
Z [m]
Time [s]

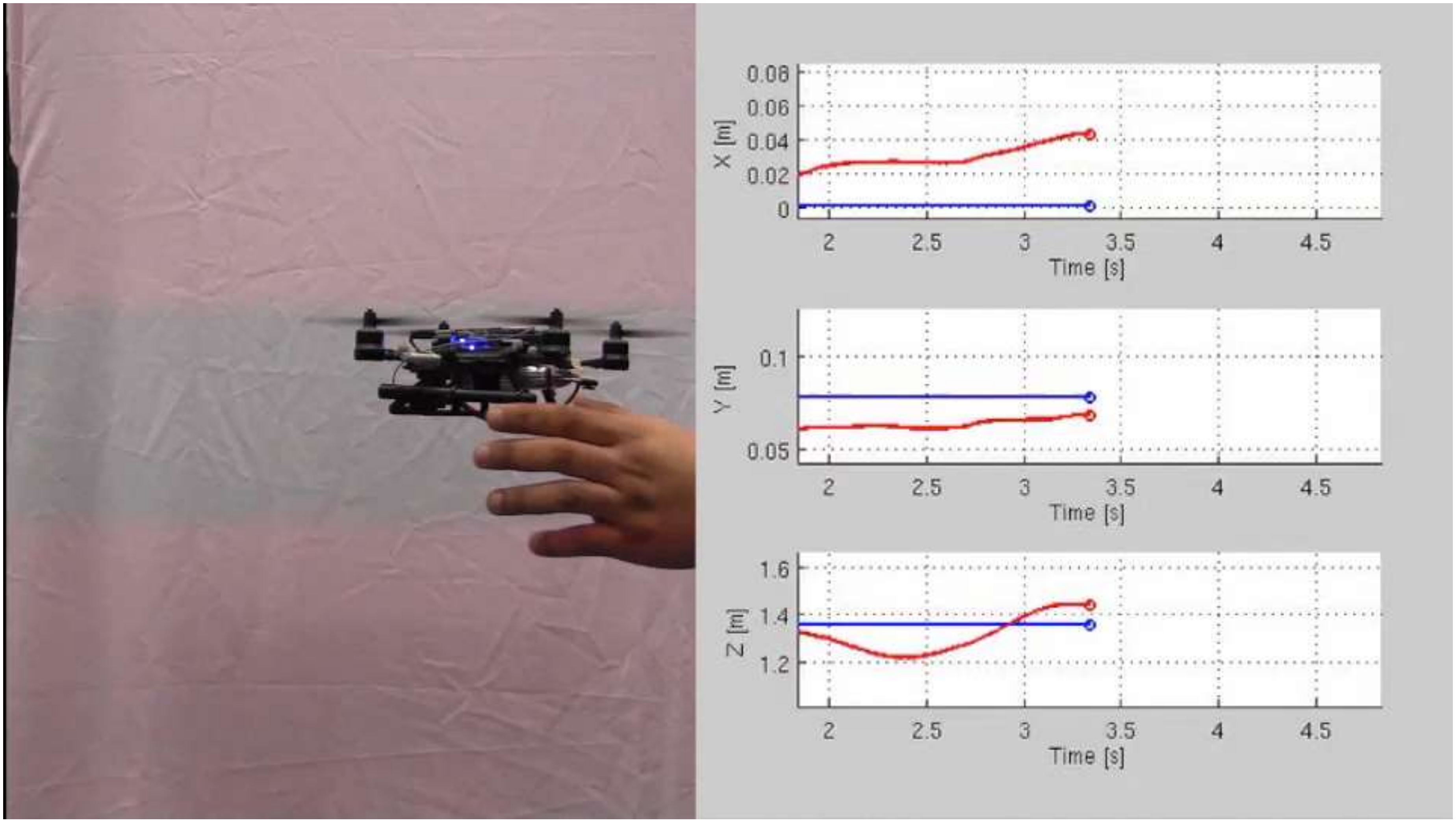
Low K_p (soft response)



High K_v (overdamped)

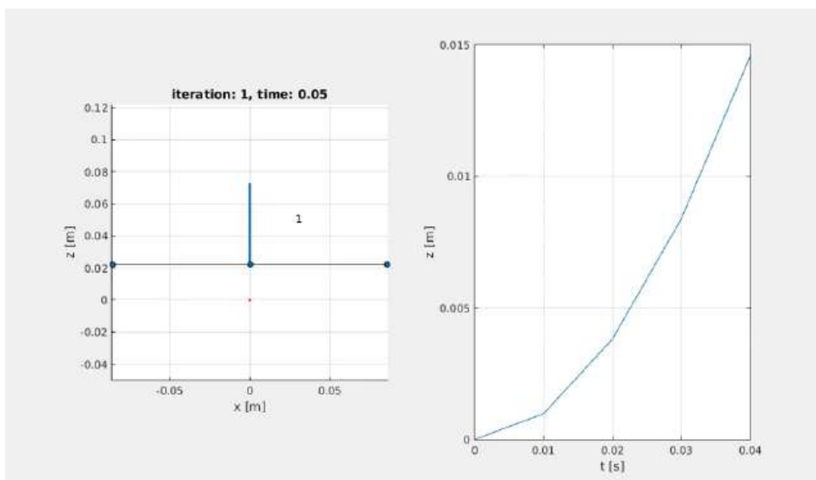


Penn Engineering


0.08
0.06
0.04
0.02
0
X [m]
2
2.5
3
3.5
4
4.5
Time [s]
0.1
0.05
Y [m]
2
2.5
3
3.5
4
4.5
Time [s]
1.6
1.4
1.2
Z [m]
2
2.5
3
3.5
4
4.5
Time [s]

Exercise

- You are given a simulator which models a PD controller for the height of a quadrotor.
- The aim of the exercise is to tune the proportional gain (Kp) of the controller in order to get a desired response from the system. The derivative gain (Kd) is kept constant.
- You should aim to get a response which has a rise time of less than 1s and a maximum overshoot of less than 5% similar to the one shown in the video below.

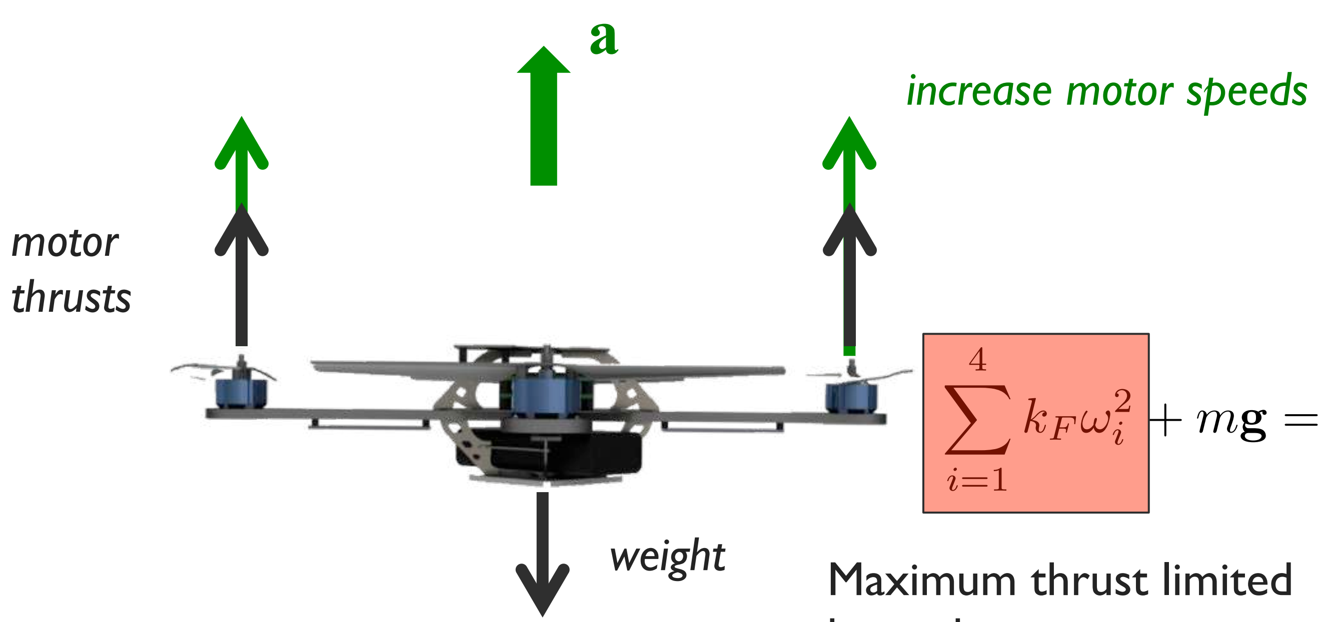


Goals

- Basic mechanics
- Control
- Design considerations
- Agility
- Component selection
- Effects of size

Penn Engineering

Effect of Maximum Thrust



$$\boxed{\sum_{i=1}^{4} k_F \omega_i^2} + m\mathbf{g} = m\mathbf{a}$$

Maximum thrust limited by peak motor torque

Maximum thrust T_{max}

Maximum acceleration a_{max}

Control with Thrust Limitations

Effect of Maximum Thrust on Input

$$u = \frac{1}{m}\left[\sum_{i=1}^{4} k_F \omega_i^2 + m\mathbf{g}\right]$$
$$= \frac{1}{m}\left[T + m\mathbf{g}\right]$$

Input, defined in terms of the thrust

$$T_{max}$$

Maximum thrust, as determined by peak motor torque

$$u_{max} = \frac{1}{m}\left[T_{max} + m\mathbf{g}\right]$$

Maximum input, as determined by maximum thrust

PD control

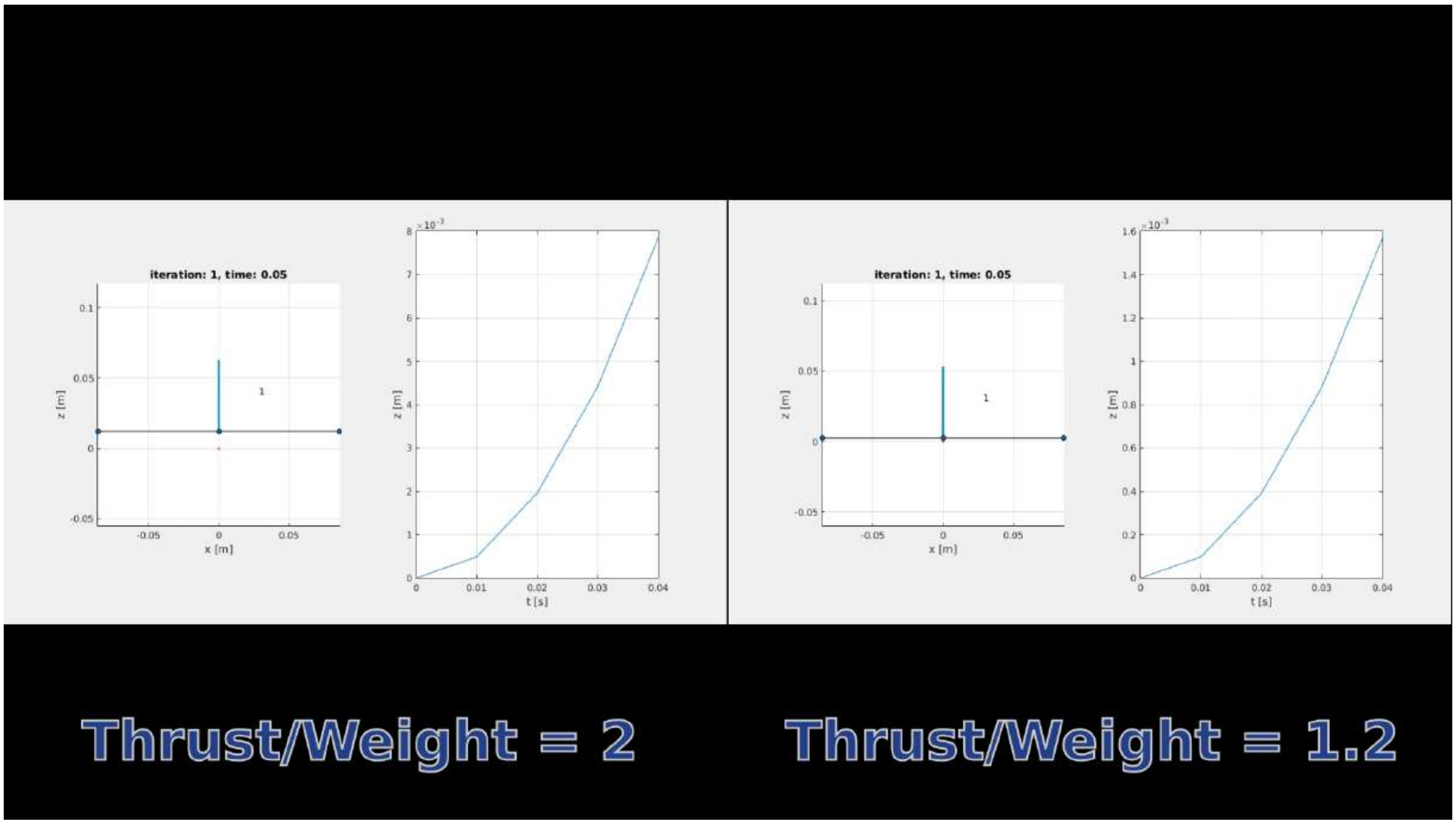
$$u(t) = \min(\ddot{x}^{des}(t) + K_V \dot{e}(t) + K_P e(t), u_{max})$$

PID control

$$u(t) = \min(\ddot{x}^{des}(t) + K_V \dot{e}(t) + K_P e(t) + K_I \int_0^t e(\tau)d\tau, u_{max})$$

Effect of the Thrust/Weight Ratio

What happens if the payload of the robot is increased (with the same motors and propellers)?



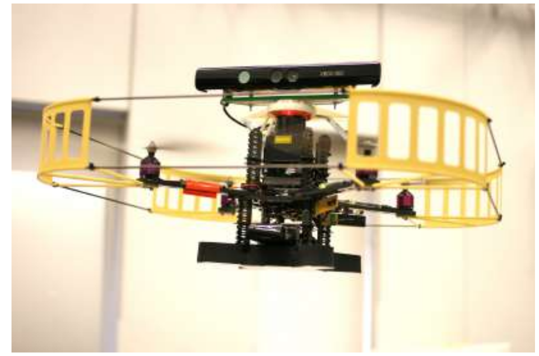
Exercise

- We'll use the same height controller from the previous exercise but now we have a limit on the max thrust for the robot.
- In this exercise, you will explore how the thrust/weight ratio affects the control response of the quadrotor. Change the mass (payload) of the robot and see how the response changes.
- We may be interested in the maximum payload that the robot can carry before the response time is degraded significantly. Using the simulation determine the maximum mass for which the rise time is less than 1s?

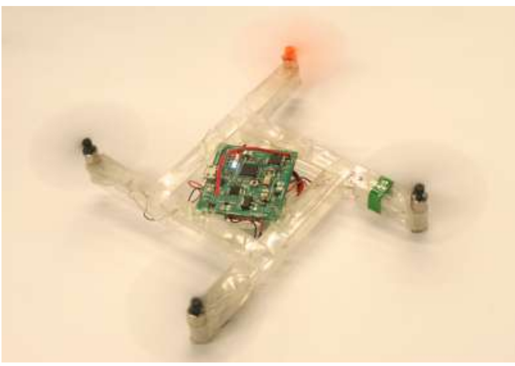
Power and Thrust

AscTec Hummingbird[1]
http://www.asctec.de/

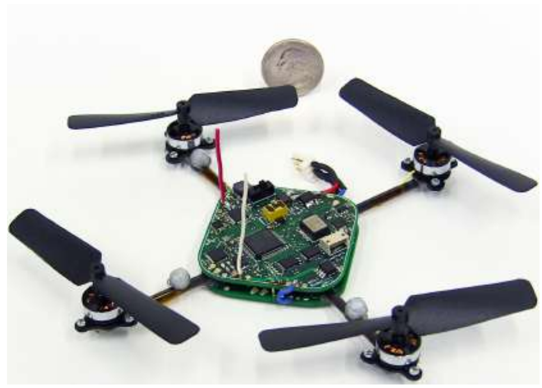
AscTec Pelican (unloaded)
http://www.asctec.de/



AscTec Pelican (loaded)[2]
http://www.asctec.de/



PPR Folded Quadrotor[3]



KMel Nano[4]

KMel kQuad 500[5]

[1]Daniel Mellinger, Nathan Michael, and Vijay Kumar. Trajectory Generation and Control for Precise Aggressive Maneuvers with Quadrotors. *International Journal of Robotics Research*, Apr. 2012.

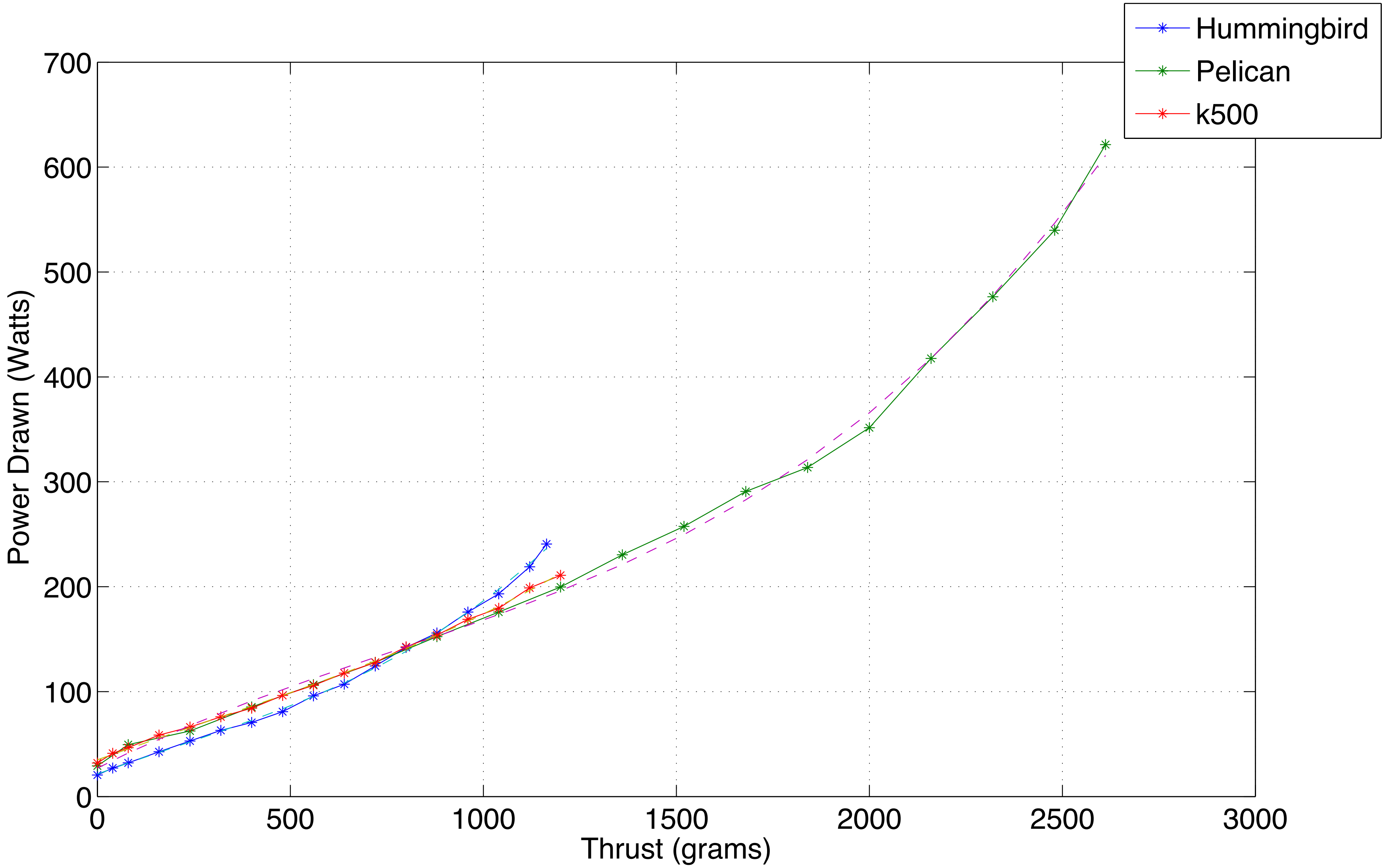
[2]S. Shen, N. Michael, and V. Kumar, "Stochastic differential equation-based exploration algorithm for autonomous indoor 3D exploration with a micro-aerial vehicle," Intl. J. Robot. Research, Vol. 31, No. 12, pp. 1431-1444, 2012.

[3]A. Mehta, D. Rus, K. Mohta, Y. Mulgaonkar, M. Piccoli, and V. Kumar, "A Scripted Printable Quadrotor: Rapid Design and Fabrication of a Folded MAV" Proc. 16th International Symposium of Robotics Research, Singapore, Dec. 2013.

[4]Aleksandr Kushleyev, Daniel Mellinger, Caitlin Powers, and Vijay Kumar, "Towards a swarm of agile micro quadrotors," *Autonomous Robots*, Vol. 35, No. 4, Pg. 287-300, 2013.

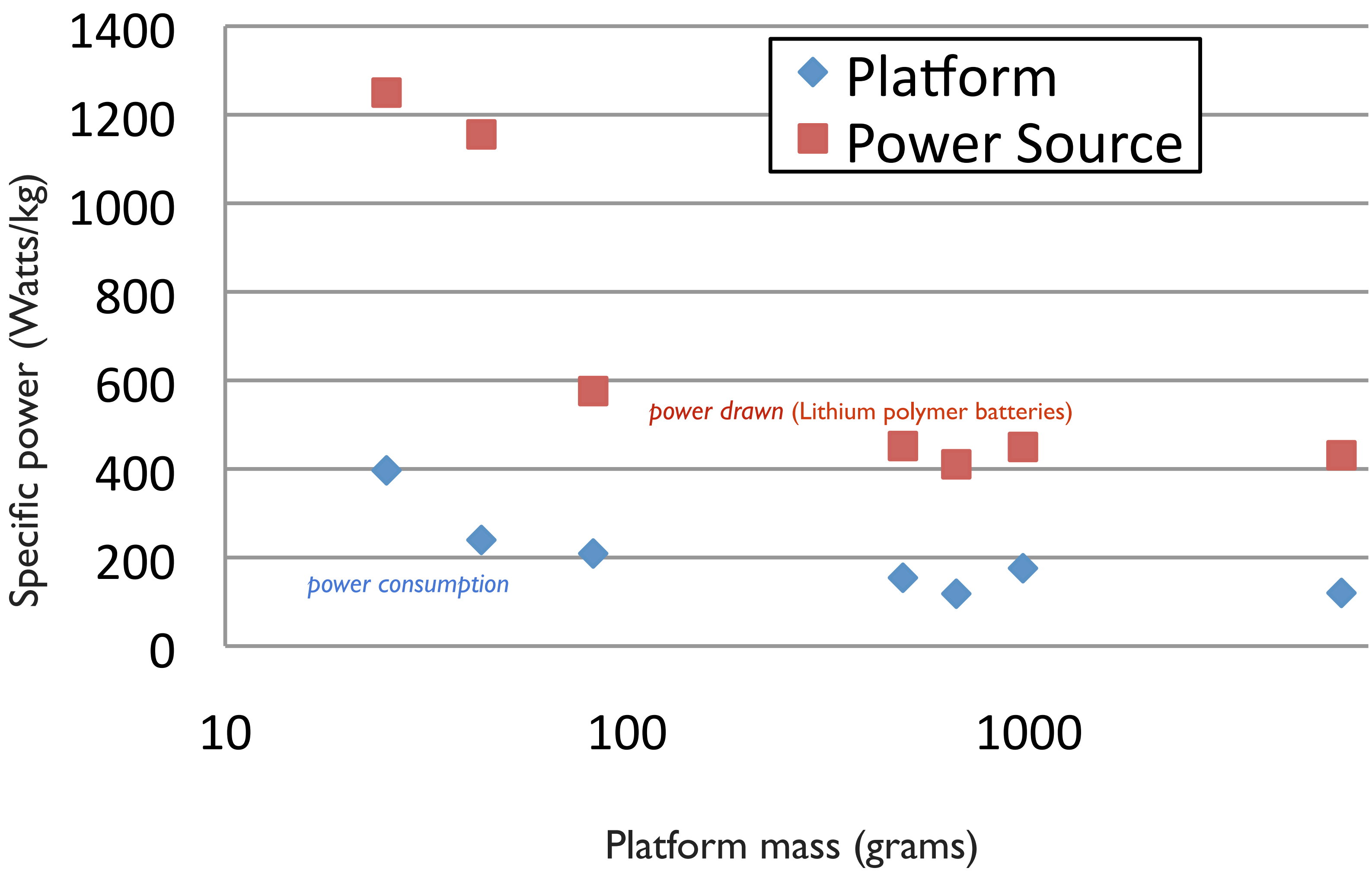
[5]K. Mohta, M. Turpin, A. Kushleyev, D. Mellinger, N. Michael, and Vijay Kumar, "QuadCloud: A Rapid Response Force with Quadrotor Teams," *International Symposium on Experimental Robotics*, Morocco, 2014

Power Consumption



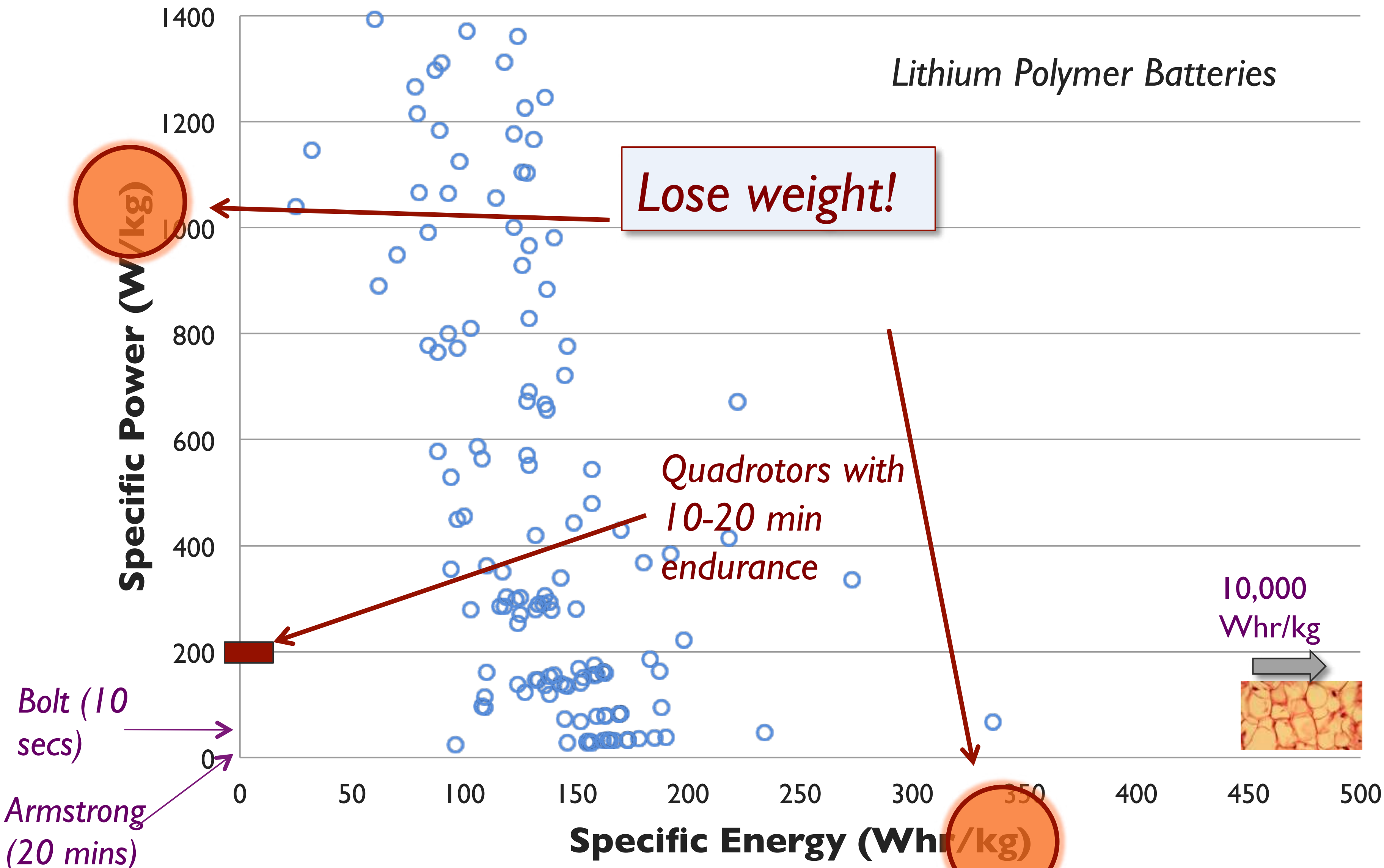
Yash Mulgaonkar, Michael Whitzer, Brian Morgan, Christopher M. Kroninger, Aaron M. Harrington, Vijay Kumar, Power and weight considerations in small, agile quadrotors, in Micro- and Nanotechnology Sensors, Systems, and Applications VI, Thomas George, M. Saif Islam; Achyut K. Dutta, Editors, Proceedings of SPIE Vol. 9083, May 2014.

Power Consumption



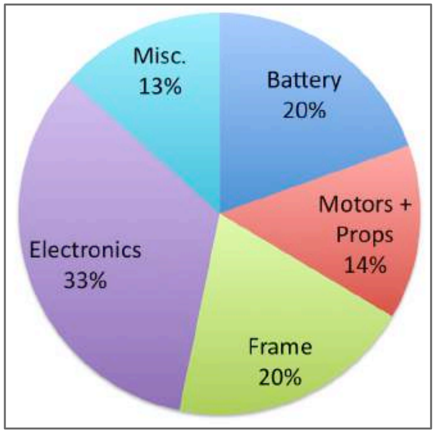
Penn Engineering

Power and Energy

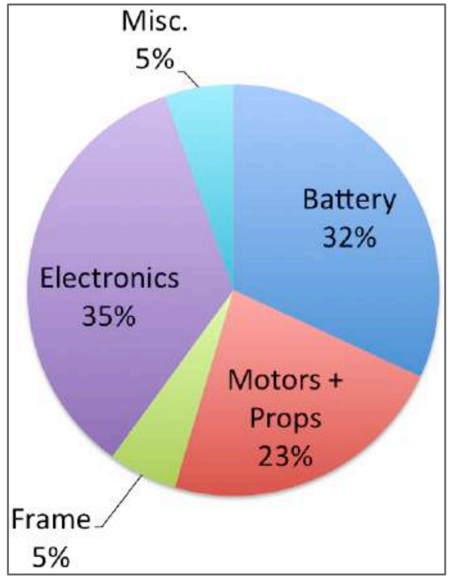


Yash Mulgaonkar, Michael Whitzer, Brian Morgan, Christopher M. Kroninger, Aaron M. Harrington, Vijay Kumar, Power and weight considerations in small, agile quadrotors, in Micro- and Nanotechnology Sensors, Systems, and Applications VI, Thomas George, M. Saif Islam; Achyut K. Dutta, Editors, Proceedings of SPIE Vol. 9083, May 2014.

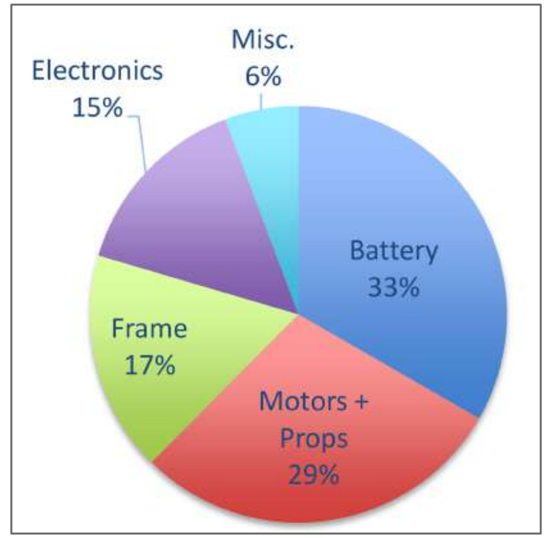
Mass Distribution



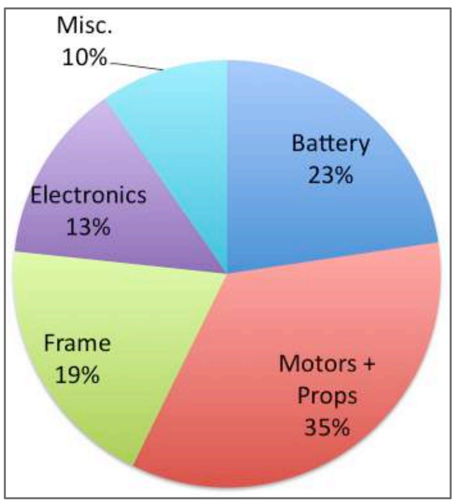
Ascending Tech.
Pelican (1937 g)



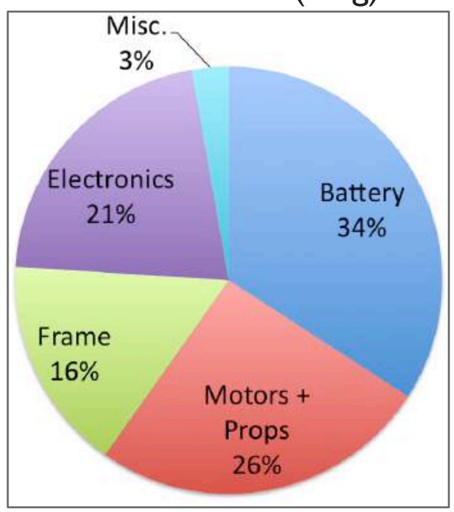
KMel Nano (82 g)



Pico



KMel kQuad (920 g)



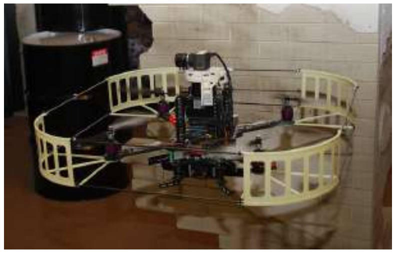
Ascending Tech.
Hummingbird (486 g)

Batteries ~ 33% mass

Motors ~ 25% mass

Penn Engineering

Sensors and Power



- Laser scanner

    270 gm

    10 W for operation plus 50-60 W for mobility

    Range 30 m

- Cameras

    80 gm (including frame, each camera 25 g)

    1.5 W for operation plus 15 W for mobility

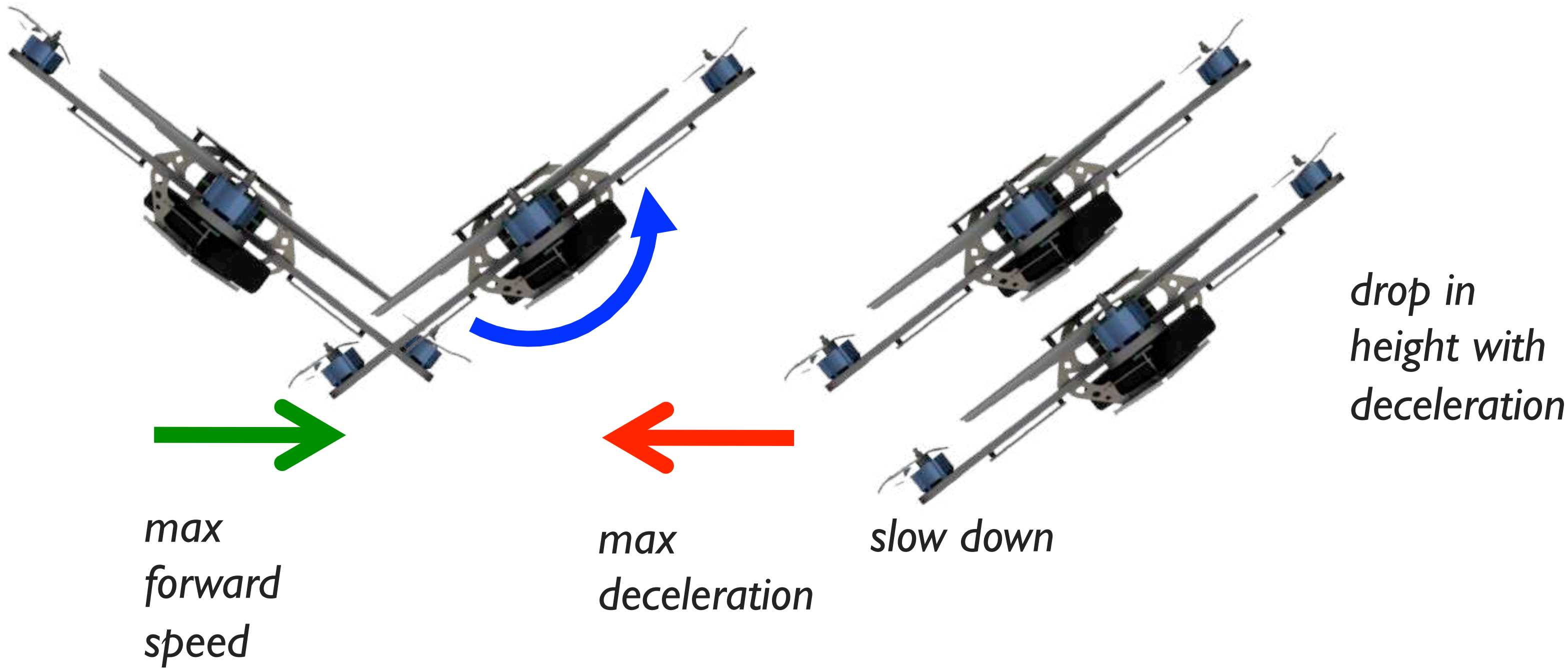
    Range 10-15 m

Goals

- Basic mechanics
- Control
- Design considerations
- Agility
- Component selection
- Effects of size

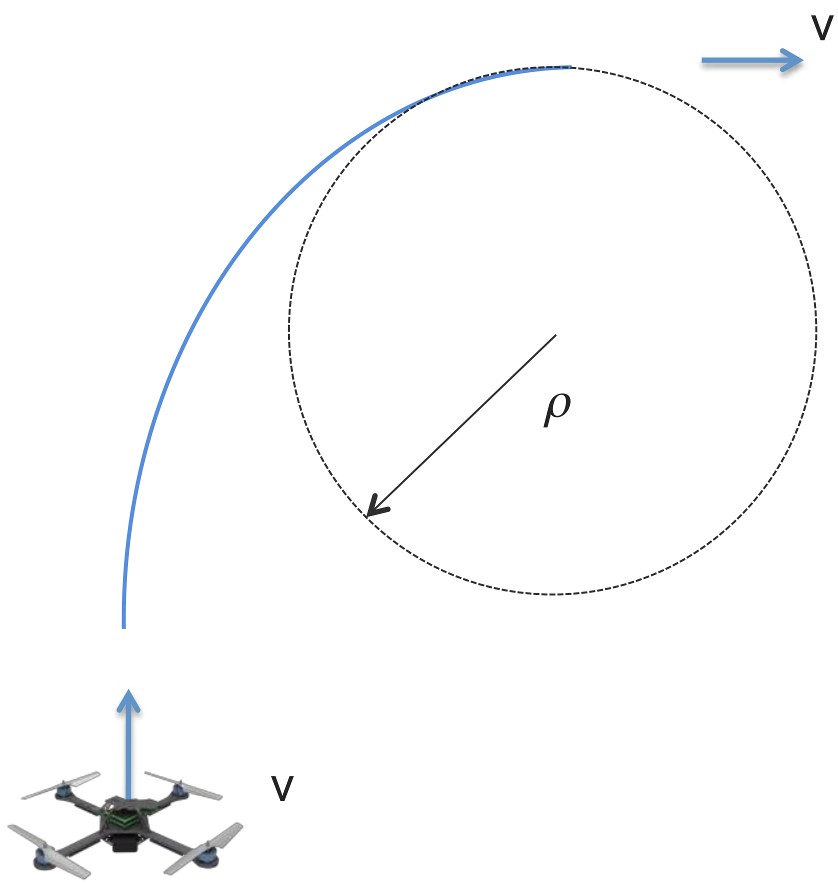
Agility

A. Maximum Velocity to Rest



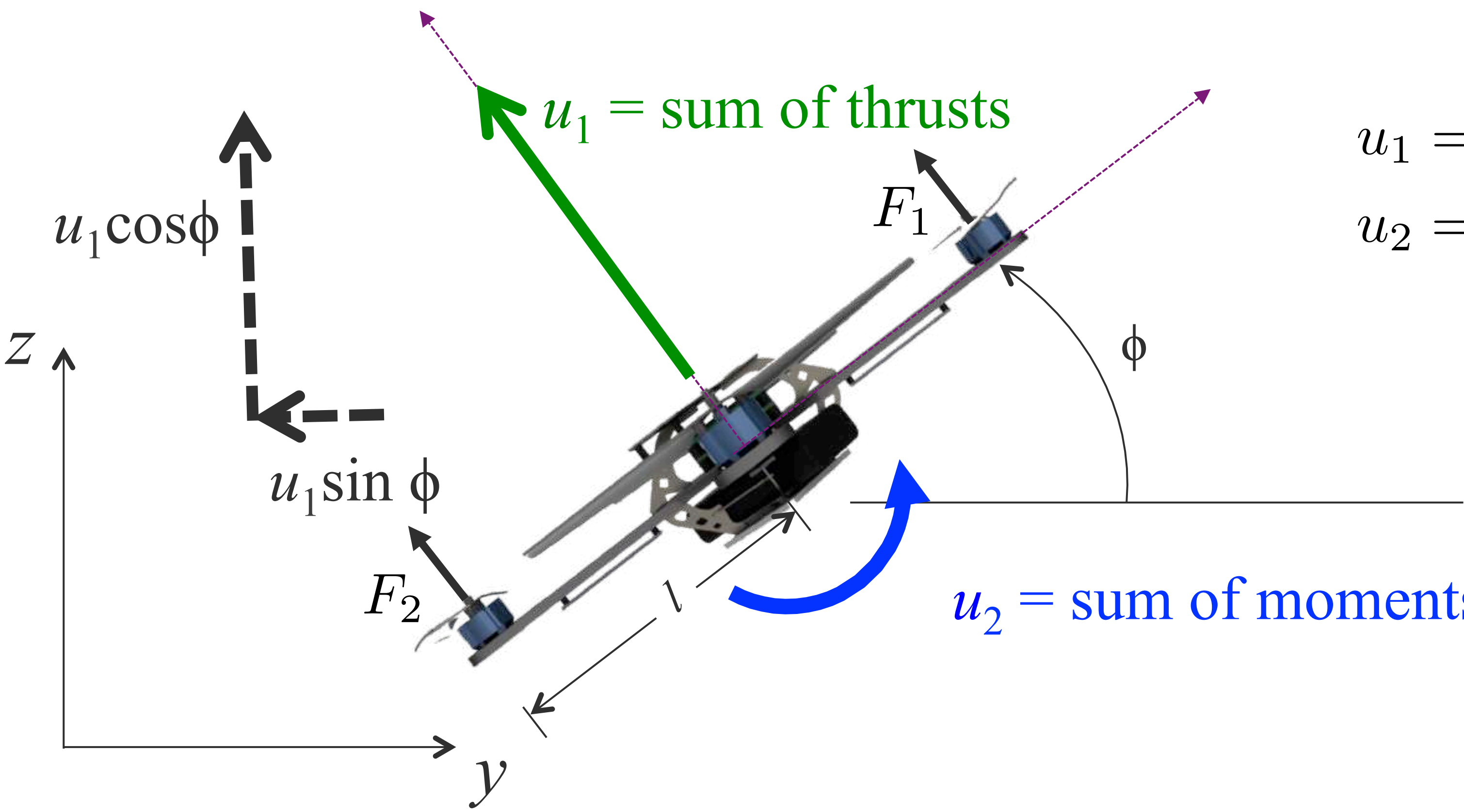
Maximize Agility: Minimize stopping distance

B. Turn Quickly without Slowing Down



Maximize Agility: Minimize minimum turning radius

Quadrotor in a Vertical Plane



$$u_1 = F_1 + F_2$$
$$u_2 = (F_1 - F_2)l$$

linear acceleration, a

angular acceleration, α

$$\begin{bmatrix} \ddot{y} \\ \ddot{z} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} 0 \\ -g \\ 0 \end{bmatrix} + \begin{bmatrix} -\frac{1}{m}\sin\phi & 0 \\ \frac{1}{m}\cos\phi & 0 \\ 0 & \frac{1}{I_{xx}} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Agility

Two key ideas

- Accelerate quickly

$$\text{maximize} \quad a_{max}$$

linear acceleration

$$\boxed{\text{maximize} \quad \frac{u_{1,max}}{W}}$$

- Roll/pitch quickly

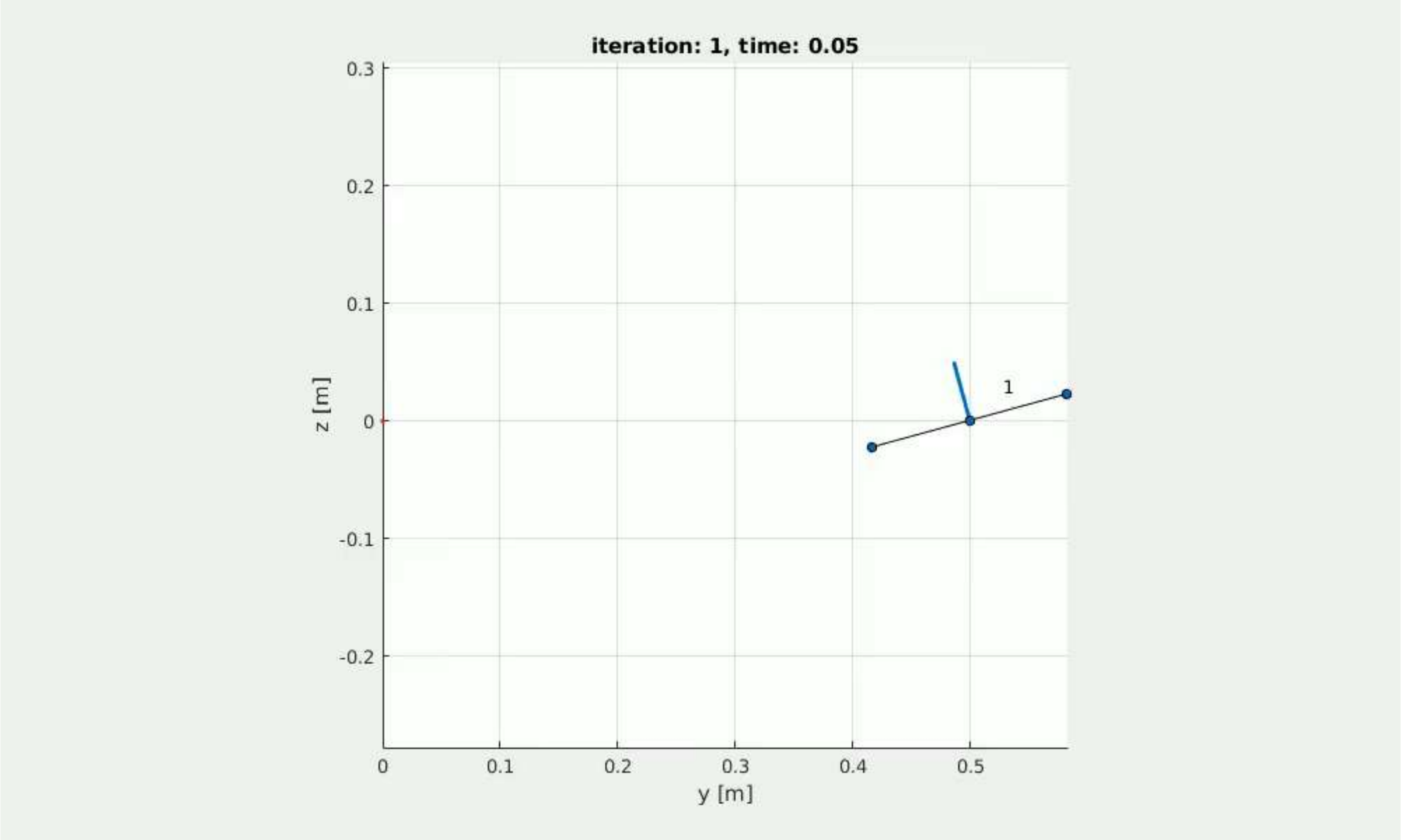
$$\text{maximize} \quad \alpha_{max}$$

angular acceleration

$$\boxed{\text{maximize} \quad \frac{u_{2,max}}{I_{xx}}}$$

Simulation

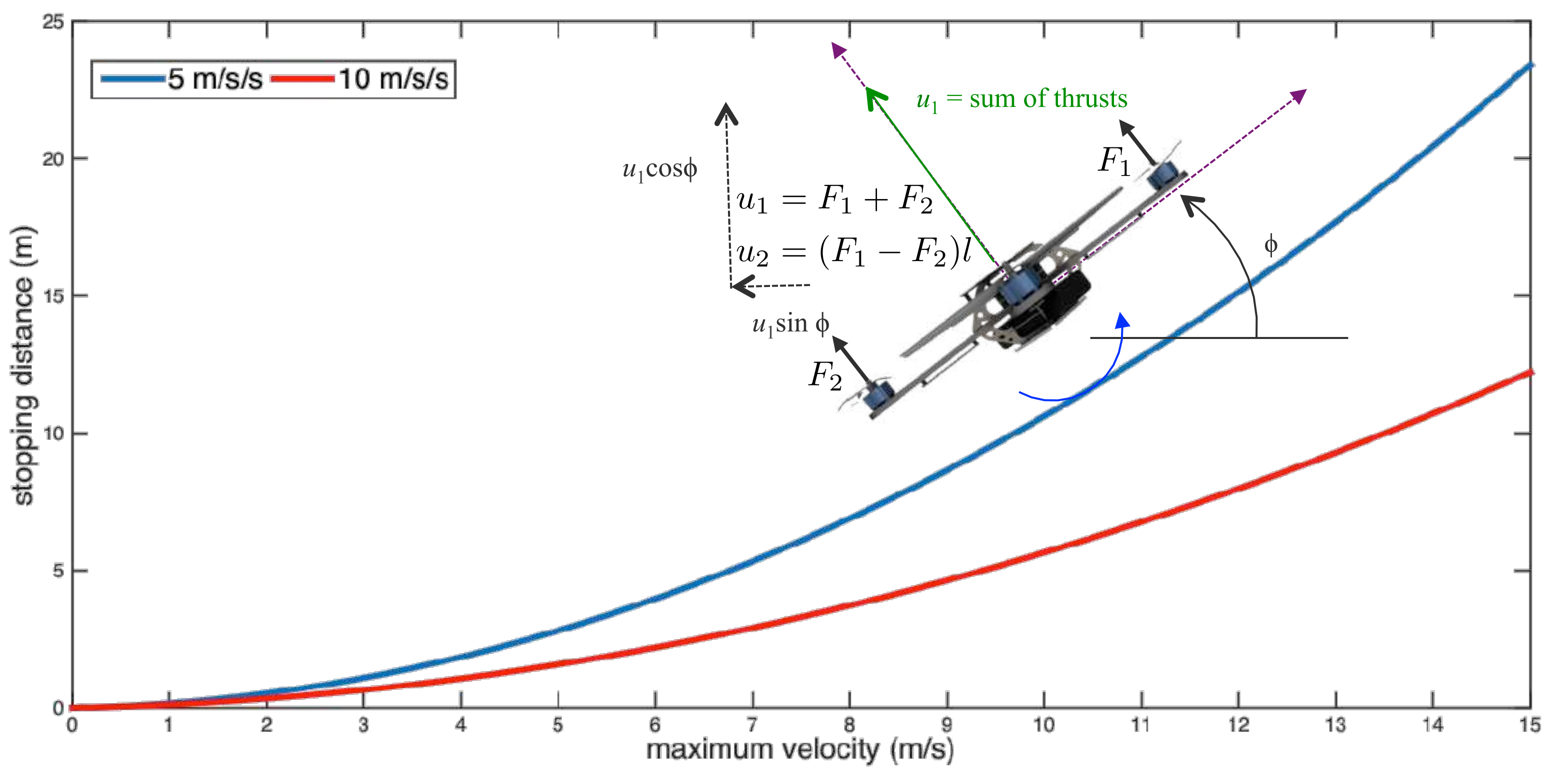
Max forward speed to zero speed



Stopping Distance

Assumptions

Thrust/weight ratio = 2
Assume robot can drop in height while turning
0 to 90 deg ~ 0.25 sec (1), 0.5 sec (2)
Conventional technology (e.g., dc motors, carbon fiber frame, li-po batteries)



Matlab Exercise

- In this exercise, we'll study how the initial velocity affects the stopping distance
- The robot is moving horizontally with the given initial velocity and it is commanded to stop
- You can change the initial velocity of the robot and run the simulation to find out the distance required* for stopping.
- What is the maximum initial velocity for which the stopping distance is less than 6m?

**Note that during this maneuver, the robot will also lose height.*

Goals

- Basic mechanics
- Control
- Design considerations
- Agility
- Component selection
- Effects of size

Penn Engineering

Frame, motors and propellors

	FRAME + PIXHAWK + PROPULSION	BATTERY	PAYLOAD	TOTAL WEIGHT	MAX THRUST	THRUST/ WEIGHT	Propeller
3DR X8+	1855	817	600	3272	10560	3.227	11” x 4.7”
DJI F550 + E600	1494	721	600	2815	9600	3.410	12” x 4.2”
DJI F450 + E310	826	400	600	1826	3200	1.752	9.4” x 5”
DJI F450 + E600	970	721	600	2291	6400	2.794	12” x 4.2”
DJI F550 + E310	1278	400	600	2278	4800	2.107	9.4” x 5”
DJI F550 + E310 @ 4 cell	1278	600	600	2478	5316	2.145	9.4” x 5”
DJI F550 + E305 @ 4 cell	1134	600	600	2334	5100	2.185	9.4” x 5”

DJI F450 frame

http://www.dji.com/

Penn Engineering

Basic Hardware

Pixhawk

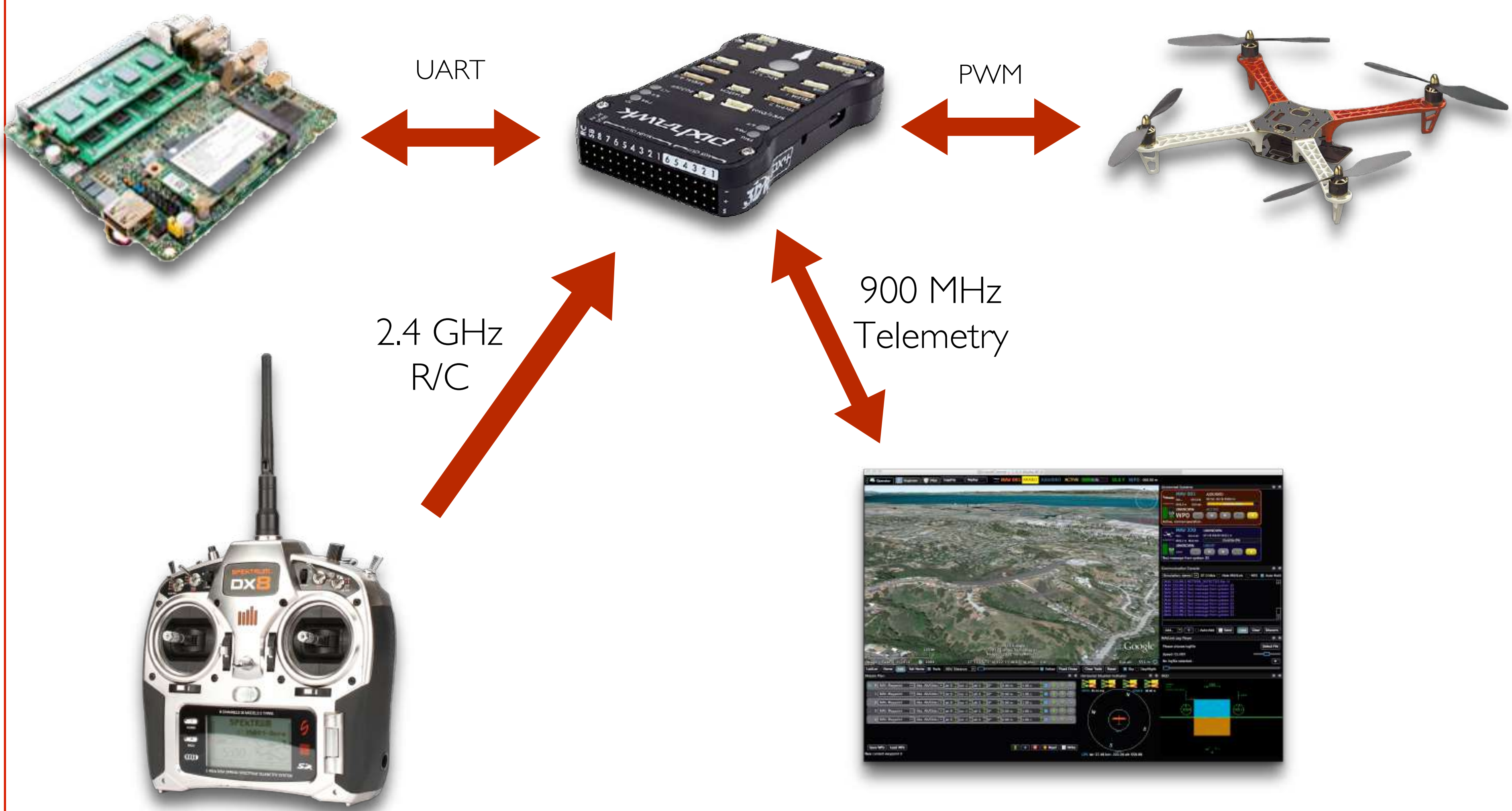
- $200, 38g
- 168 MHz / 252 MIPS Cortex-M4F
- Sensors: 3D ACC / Gyro / MAG / Baro
- Integrated backup, override and failsafe processor with mixing
- microSD slot, 5 UARTs, CAN, I2C, SPI, ADC, etc

Intel NUC i7

- $480, 200g
- 5th Generation Intel Core i7-5557U processor, 3.1 Ghz
- Max memory 16GB
- 4 x USB3, 2 x USB2 ports
- Internal support for M.2 SSD card & SATA3 for 2.5" HDD/SSD
- 12V DC

Penn Engineering

Processing and Communication



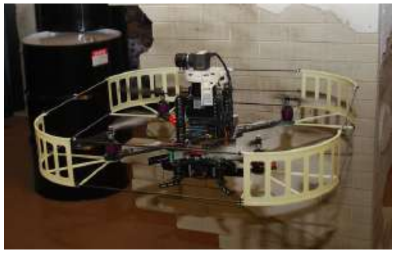
Outdoor Platform

Penn Engineering

Outdoor Test

DJI F450 platform + E600 motors + 600 gram payload + 721 gram battery
Thrust/Weight ratio greater than 2.7

Sensors and Power



- Laser scanner
  - 270 gm
  - 10 W for operation plus 50-60 W for mobility
  - Range 30 m

- Cameras
  - 80 gm (including frame, each camera 25 g)
  - 1.5 W for operation plus 15 W for mobility
  - Range 10-15 m

Examples

1750 g (laser, 3 cameras, GPS, IMU)

650 g (camera, IMU)

740 g (2 cameras, IMU)

1800 g (laser, Kinect, IMU)

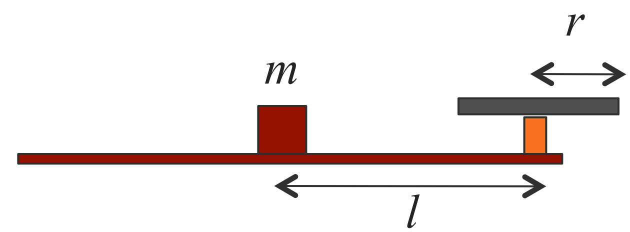
Penn Engineering

Goals

- Basic mechanics
- Control
- Design considerations
- Agility
- Component selection
- Effects of size

Agility with Scaling

- mass, inertia



$$m \sim l^3,\ I \sim l^5$$

since $r \sim l$

- thrust

$$F \sim \pi r^2 \times (\omega r)^2 \qquad \boxed{F \sim l^2 v^2}$$

rotor angular speed

$$a \sim \frac{F}{m} \quad \sim l^3$$

- moment

$$M \sim Fl \qquad \boxed{M \sim l^3 v^2}$$

$$\alpha \sim \frac{M}{I} \quad \sim l^5$$

maximum accelerations

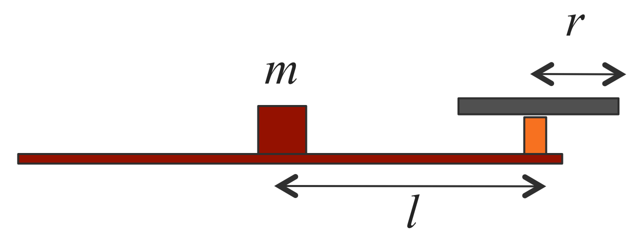
$$a \sim \frac{v^2}{l}$$

$$\alpha \sim \frac{v^2}{l^2}$$

Penn Engineering

Agility with Scaling

- mass, inertia



$$m \sim l^3, \ I \sim l^5$$

since $r \sim l$

- thrust

$$F \sim r^2 v^2$$

blade tip speed

$$\boxed{F \sim l^2 v^2}$$

$$a \sim \frac{F}{m} \sim l^3$$

- moment

$$M \sim Fl$$

$$\boxed{M \sim l^3 v^2}$$

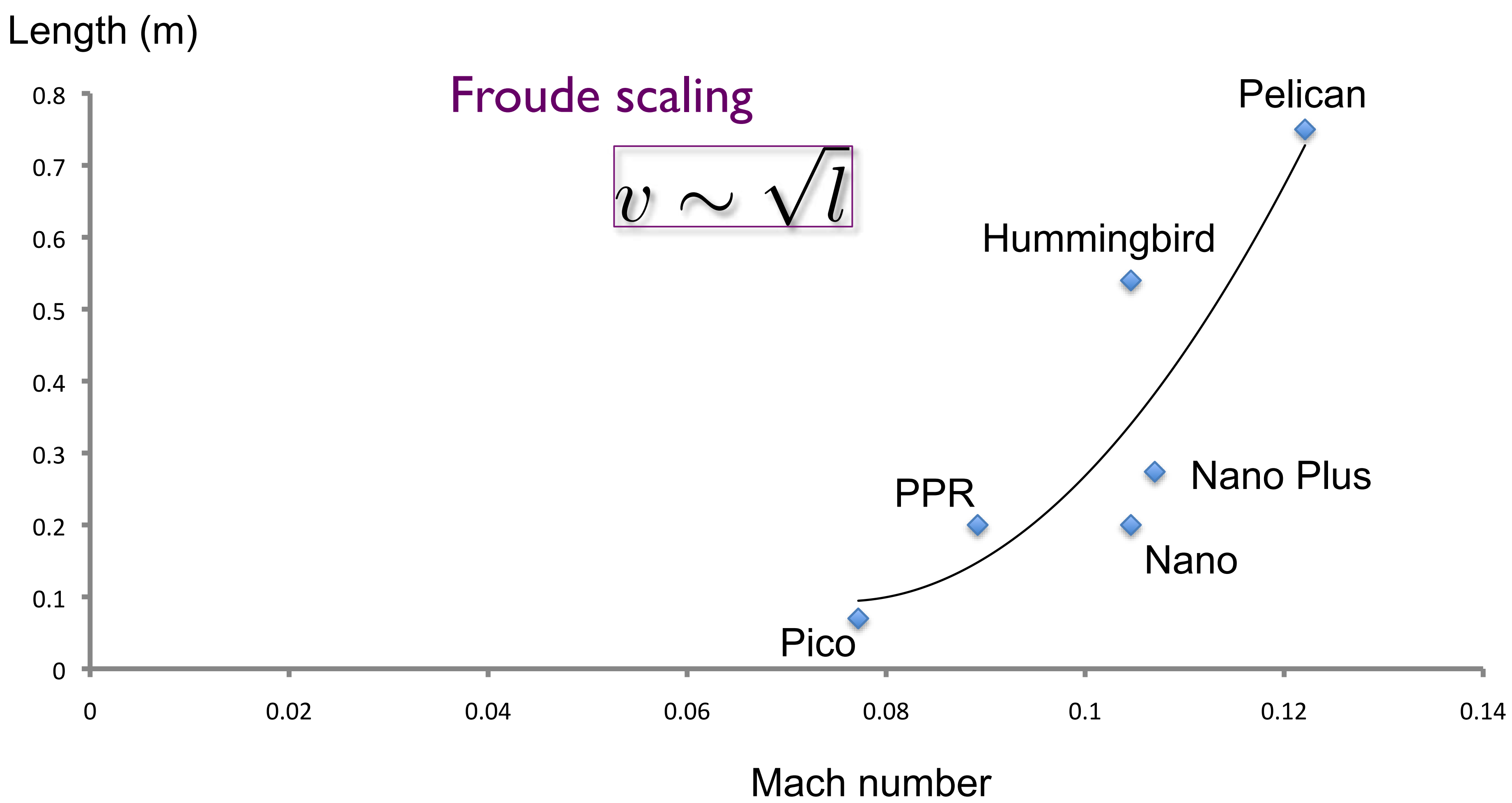
$$\alpha \sim \frac{M}{I} \sim l^5$$

maximum accelerations

$$a \sim \frac{v^2}{l}$$

$$\alpha \sim \frac{v^2}{l^2}$$

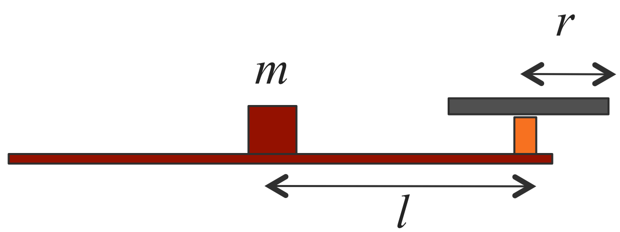
Scaling Experiments



Yash Mulgaonkar, Michael Whitzer, Brian Morgan, Christopher M. Kroninger, Aaron M. Harrington, Vijay Kumar, Power and weight considerations in small, agile quadrotors, in Micro- and Nanotechnology Sensors, Systems, and Applications VI, Thomas George, M. Saif Islam; Achyut K. Dutta, Editors, Proceedings of SPIE Vol. 9083, May 2014.

Agility with Scaling

- Froude scaling



$$v \sim \sqrt{l}$$

$$F \sim l^3$$

$$\boxed{a \sim 1,\ \alpha \sim \frac{1}{l}}$$

- Mach scaling

$$v \sim 1$$

$$F \sim l^2$$

$$\boxed{a \sim \frac{1}{l},\ \alpha \sim \frac{1}{l^2}}$$

The maximum angular acceleration tends to be higher from smaller vehicles

Dynamical Systems

Dynamical Systems

Systems where the effects of actions do not occur immediately

State: a collection of variables that completely characterizes the motion of a system

Dynamical Systems

Systems where the effects of actions do not occur immediately

State: a collection of variables that completely characterizes the motion of a system

- $x(t)$ gives the values of these states over time

Penn Engineering

Dynamical Systems

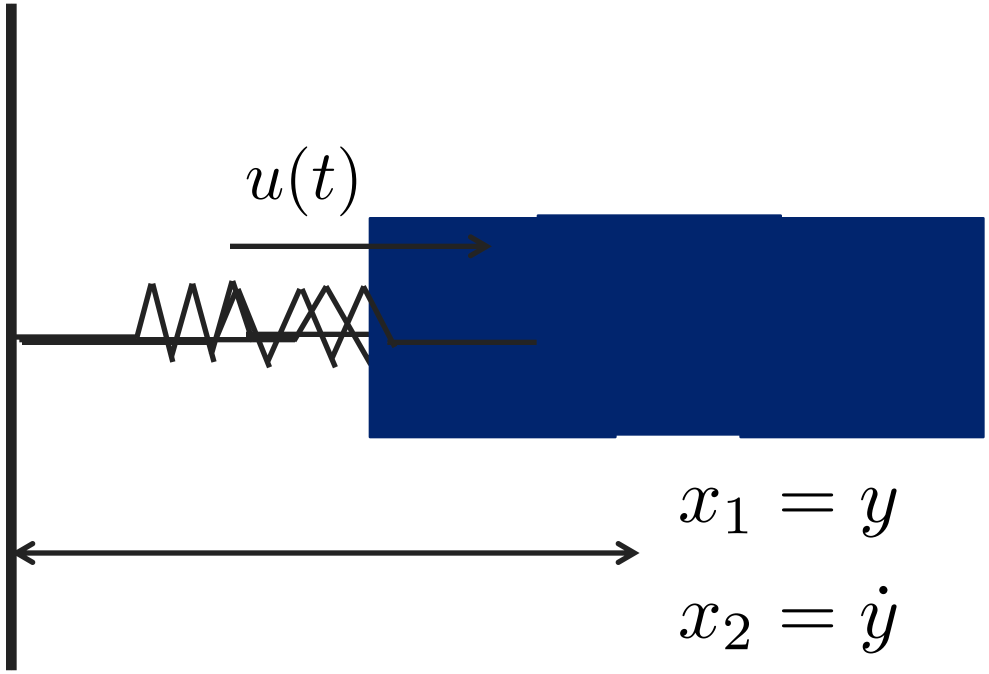
Evolution of these states over time is often given by a set of governing ordinary differential equations

- Order: highest derivative that appears in the equations

$$\boxed{\ddot{x}(t)} = u(t)$$

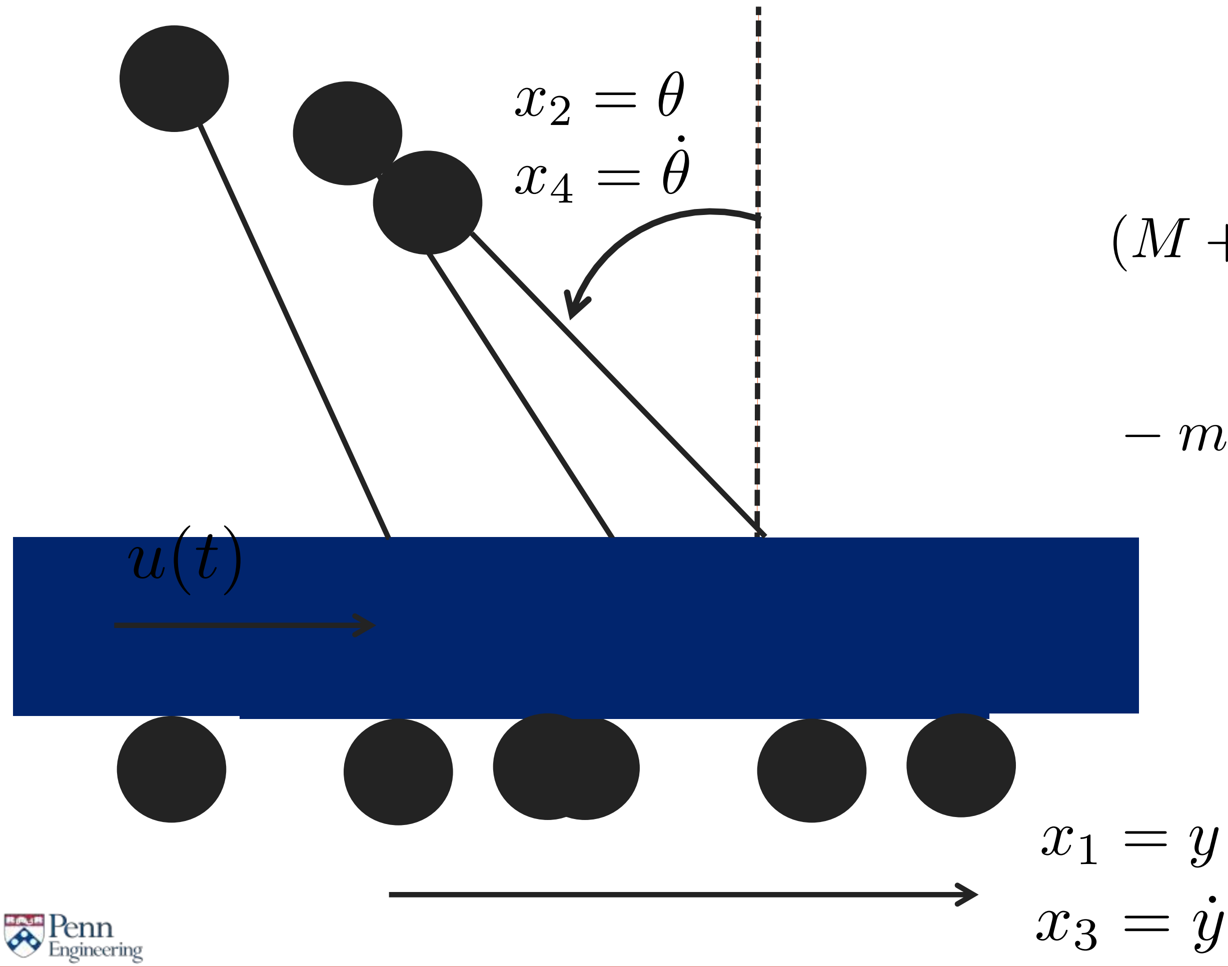
Second-order system

Example 1: Mass-Spring System



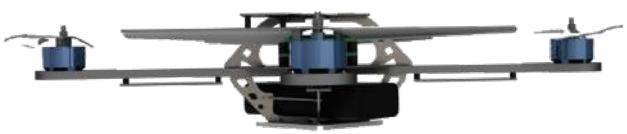
$$m\ddot{y}(t) + ky(t) = u(t)$$

Example 2: Pendulum on a Cart



$$
\begin{aligned}
(M+m)\ddot{y}(t) - ml\cos(\theta(t))\ddot{\theta}(t) \\
+ ml\sin(\theta(t))\dot{\theta}(t)^2 = u(t) \\
-ml\cos(\theta(t))\ddot{y}(t) + (J+ml^2)\ddot{\theta}(t) \\
-mgl\sin(\theta(t)) = 0
\end{aligned}
$$

Example 3: Quadrotor



$$x_1 = x \qquad x_7 = \dot{x}$$
$$x_2 = y \qquad x_8 = \dot{y}$$
$$x_3 = z \qquad x_9 = \dot{z}$$
$$x_4 = \phi \qquad x_{10} = p$$
$$x_5 = \theta \qquad x_{11} = q$$
$$x_6 = \psi \qquad x_{12} = r$$

Penn Engineering

Rates of Convergence

Feedback Control

Recall the control problem

Determine the appropriate input that will cause the error between the desired state and the actual state of a dynamical system to eventually reach 0.

$$e(t) = x^{des}(t) - x(t) \rightarrow 0 \text{ as } t \rightarrow \infty$$

Rates of Convergence

How fast do we want this error to go to 0?

- The error *exponentially converges* to 0 if there exists constants α and β and time t_0, such that for all $t \geq t_0$:

$$\|e(t)\| \leq \alpha e^{-\beta t}$$

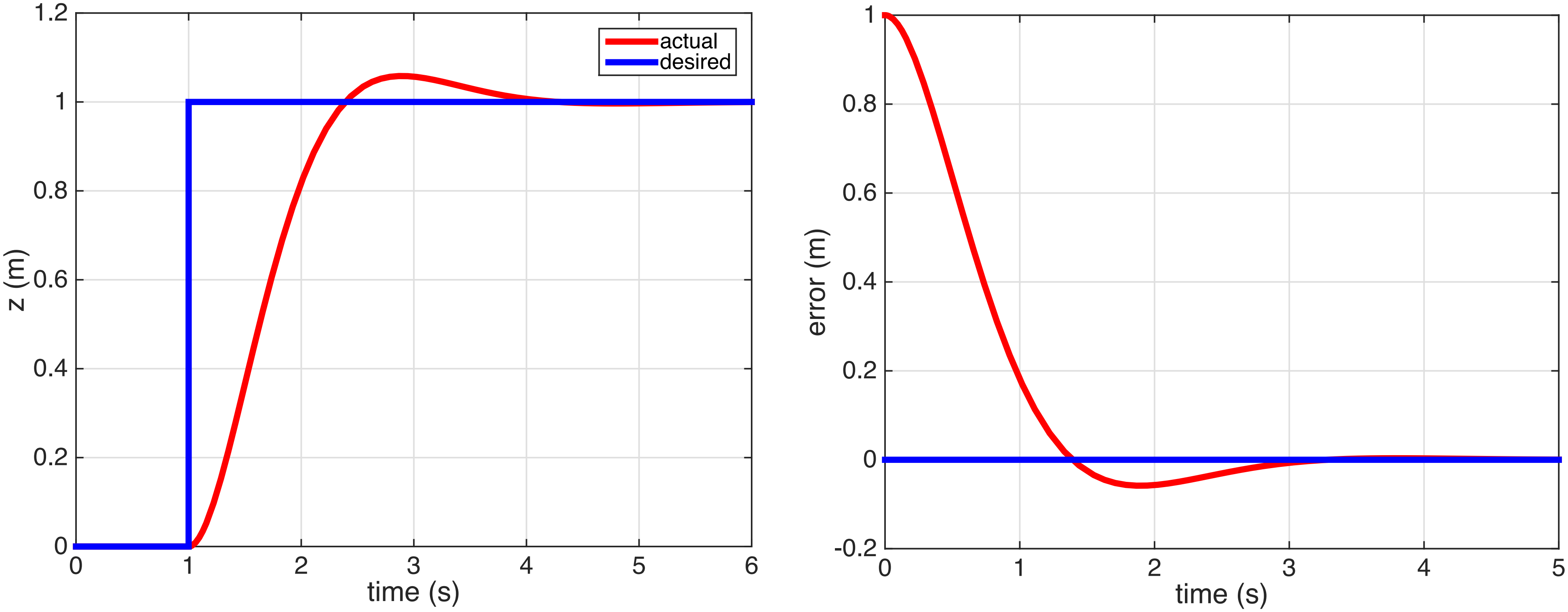
Feedback Control

Here we will accomplish this using a PD (or PID) controller.

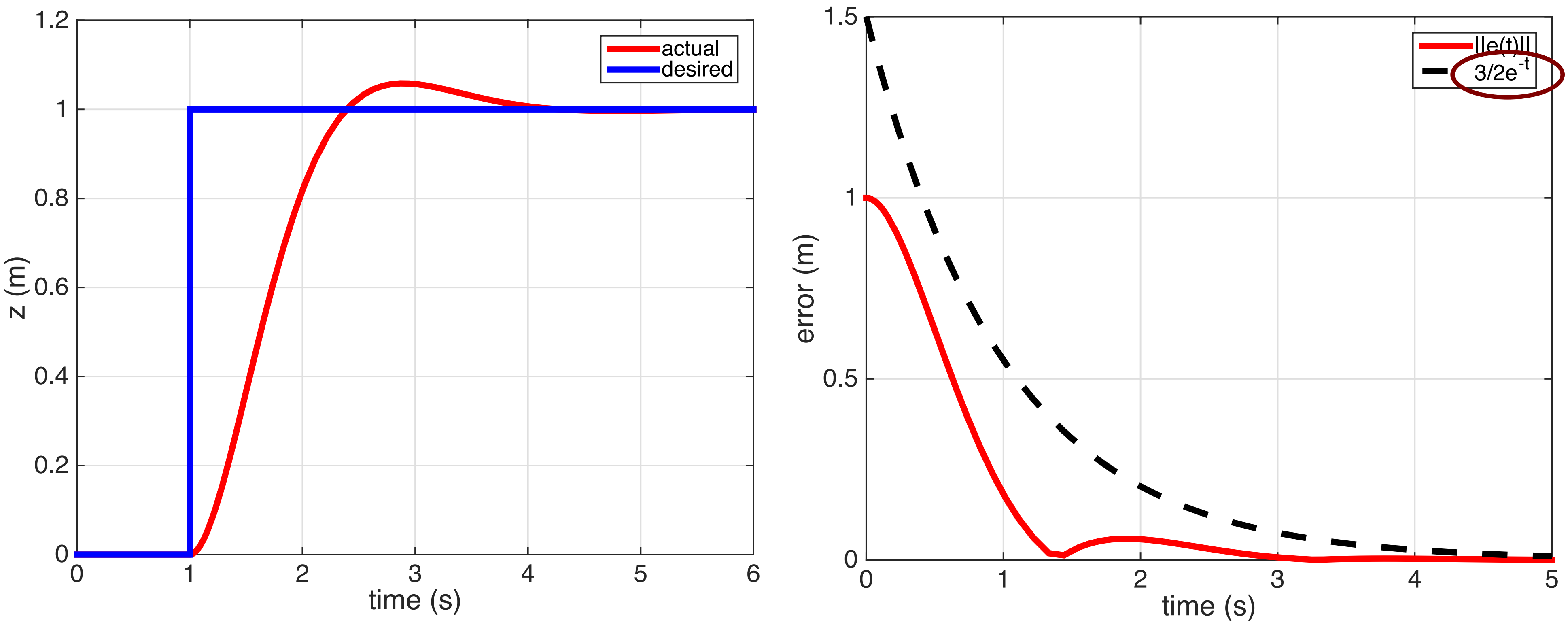
$$u(t) = \ddot{x}^{\mathrm{des}}(t) + K_v \dot{e}(t) + K_p e(t)$$

Consider the controllers we used before to control the height of a quadrotor.

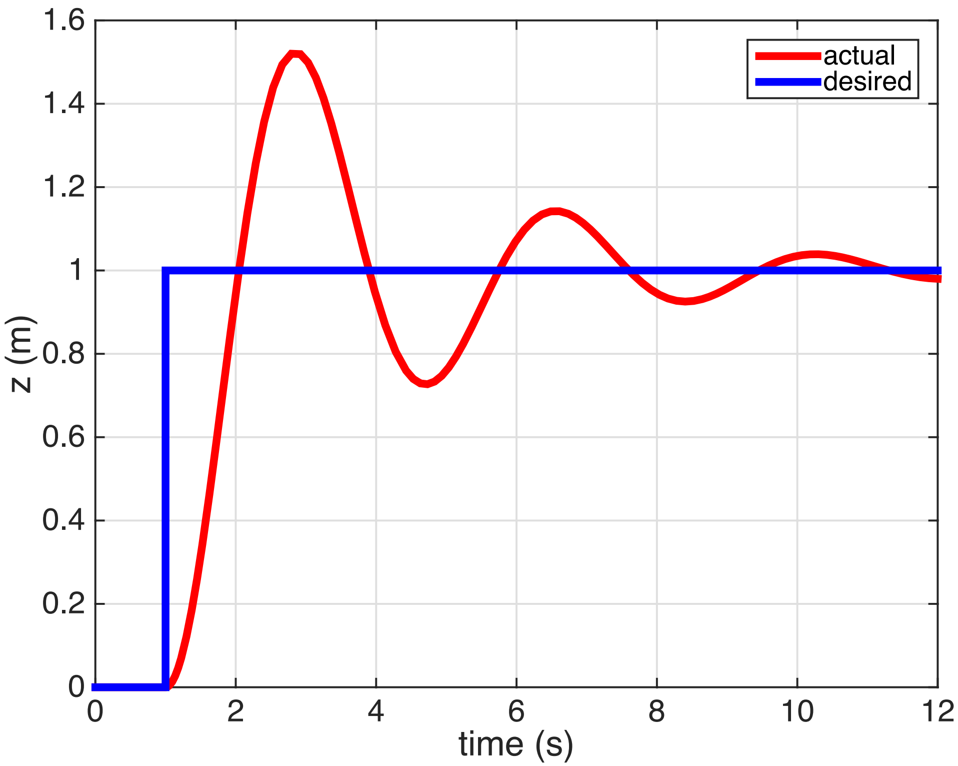
Example 1: PD Controller

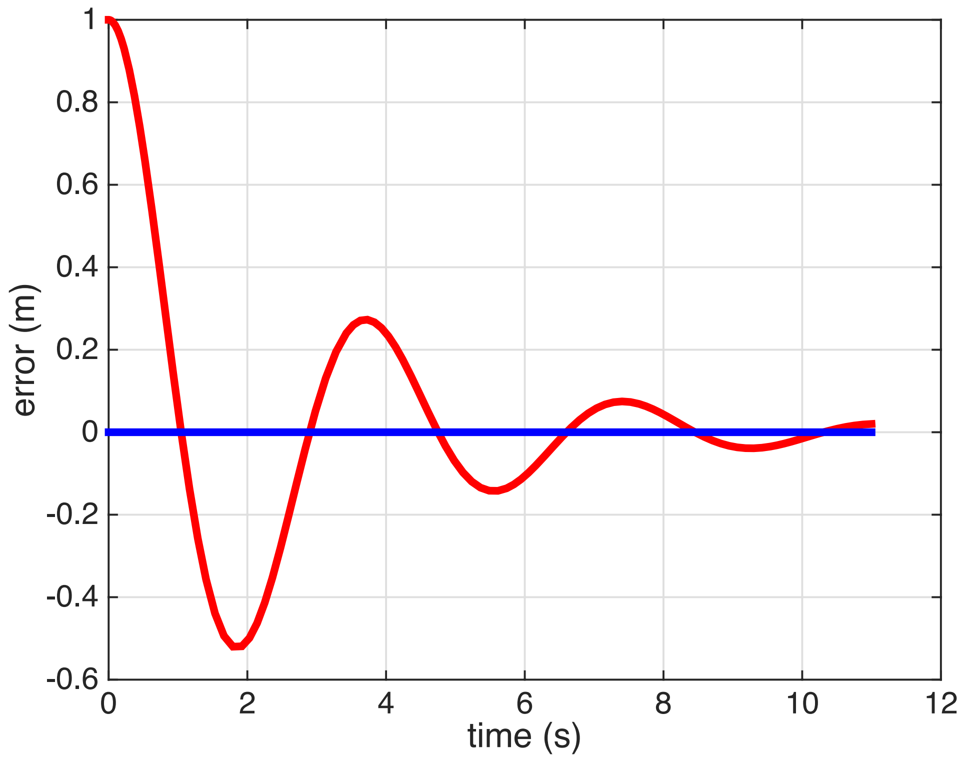


Example 1: PD Controller



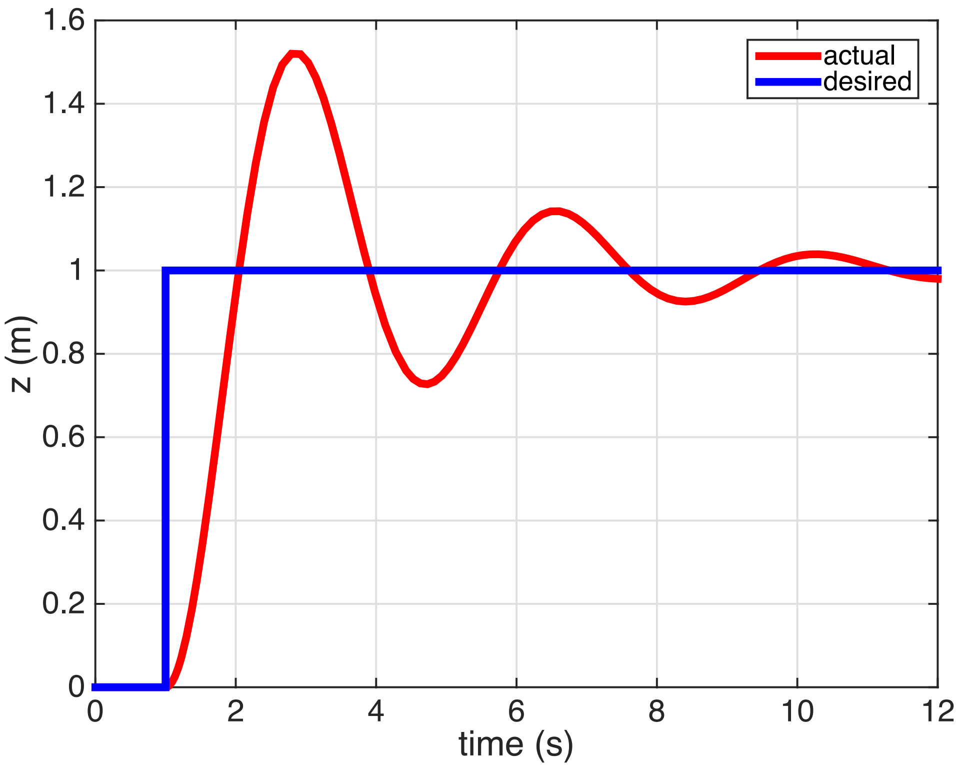
Example 2: High K_p

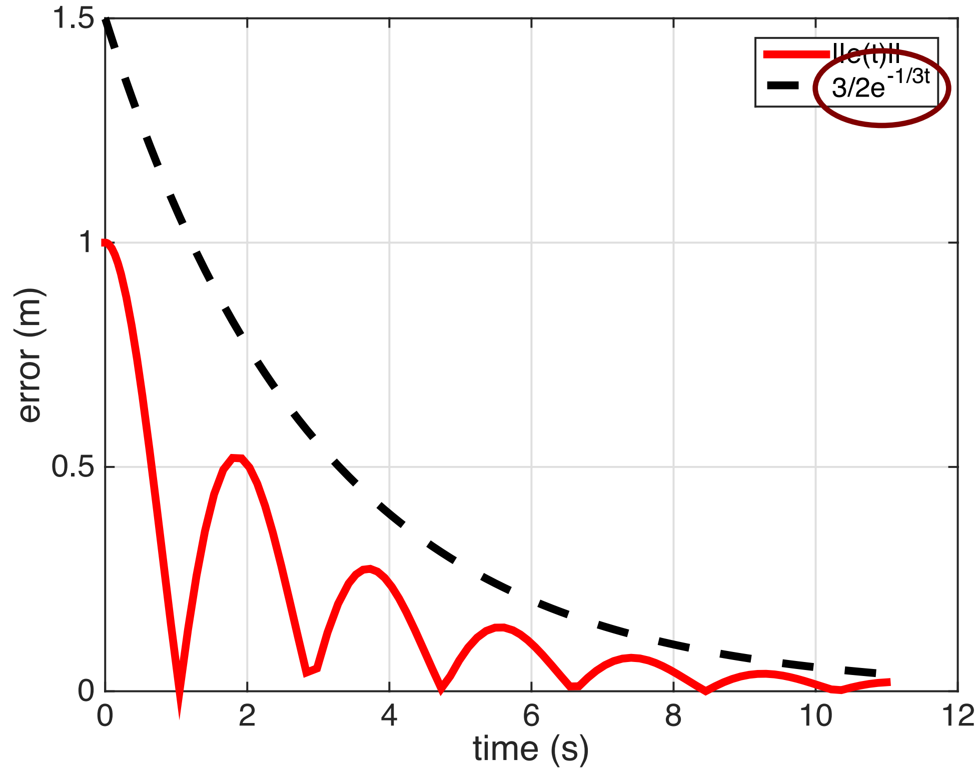




Penn Engineering

Example 2: High K_p





Penn Engineering