



1-Q:  $|0\rangle =$   ;  $|1\rangle =$  

$|\tilde{0}\rangle =$   ;  $|\tilde{1}\rangle =$  

$$|\tilde{x}\rangle = \frac{1}{\sqrt{2}} \sum_{y=0}^{y=2-1} e^{i \frac{2\pi}{2} x \cdot y} |y\rangle \Rightarrow \frac{1}{2} \left[ e^{\frac{2\pi i}{2} x \cdot 0} |0\rangle + e^{\frac{2\pi i}{2} x \cdot 1} |1\rangle \right]$$

$$\Rightarrow \frac{1}{\sqrt{2}} [ |0\rangle + e^{i\pi x} |1\rangle ]$$

$$\text{QFT}|0\rangle = \frac{1}{\sqrt{2}} [ |0\rangle + e^{i\pi \cdot 0} |1\rangle ] = \frac{1}{\sqrt{2}} [ |0\rangle + |1\rangle ] = |+\rangle$$

$$\text{QFT}|1\rangle = |-\rangle$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

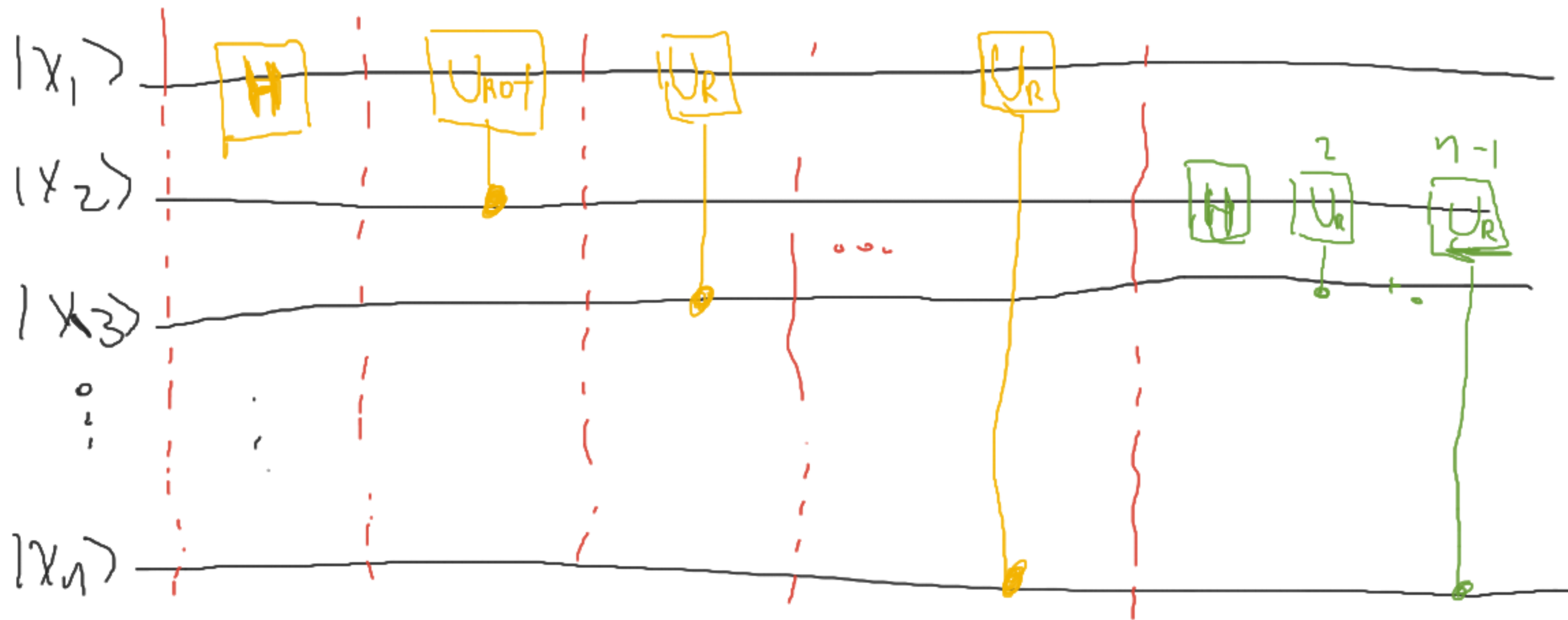
$$\theta = \pi \Rightarrow \boxed{e^{i\pi} + 1 = 0}$$

2. Ingredients

$$\begin{aligned} 1) \quad H |x_k\rangle &\Rightarrow |\tilde{0}\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \\ &|\tilde{1}\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}} \\ &= \left( |0\rangle + e^{i\pi x_k} |1\rangle \right) / \sqrt{2} \end{aligned}$$

$$2) \quad U_{\text{ROT}_k} |x_j\rangle = e^{\frac{2\pi i x_j}{2^k}} |x_j\rangle; \quad x_j = 0 \Rightarrow e^{\frac{2\pi i \cdot 0}{2^k}} |0\rangle = |0\rangle \quad \checkmark$$

$$\begin{aligned} &\downarrow \\ &|1\rangle \Rightarrow e^{\frac{2\pi i}{2^k}} |1\rangle \quad \checkmark \\ &\begin{pmatrix} 1 & 0 \\ 0 & e^{2\pi i / 2^k} \end{pmatrix} \end{aligned}$$



Step 0:  $|x_1, x_2, x_3, \dots, x_n\rangle$

Step 1:  $[|0\rangle + e^{\frac{2\pi i}{2^1} x_1} |1\rangle] \otimes |x_2, x_3, \dots, x_n\rangle$

Step 2:  $[|0\rangle + e^{\frac{2\pi i}{2^2} x_2} e^{\frac{2\pi i}{2^1} x_1}] \otimes |x_2, x_3, \dots, x_n\rangle$

Step N:

$$[|0\rangle + e^{\frac{2\pi i}{2^n} x_n} e^{\frac{2\pi i}{2^{n-1}} x_{n-1}} \dots e^{\frac{2\pi i}{2} x_1} |1\rangle] \otimes [x_2, x_3, \dots, x_n]$$

QFT  $\rightarrow$  Circuits

$$|x\rangle = \frac{1}{\sqrt{N}} \sum_{y=0}^{y=N-1} e^{\frac{2\pi i}{N} xy} |y\rangle$$