**PROJECT 1**
**Linear Programming**

Submit your project through *WISEflow*. Please do not share nor reproduce questions or answers of this project in other internet websites or digital platforms. The submission deadline is Tuesday September 26th, at 14:00 hrs. The project can be done individually or in a group of at most 2 students. No cooperation between people who are not submitting this project as a group is allowed. It is possible to change groups throughout the semester and it is also possible to do some project(s) alone and other project(s) in a group. Provide **all your AMPL files** (model code, data, running commands, solution file, etc.) compressed in a single file (.zip). Include all files needed to run all parts of the project, even if from one to another task the changes are just marginal (we need all files to be able to run without modifying what you submitted). In addition, provide a written report as a pdf file with your model formulations and the answers to the questions required in each part. The formulation of your models can be typed in a text editor (e.g. Word, LaTeX), written by hand and scanned, or copied directly as text or screenshot from the AMPL code files when it applies (please just be careful the presentation must be clear enough for a reader). In the written report, it is fine that when there is just a marginal change from one task to another, in the latter you include just the modified part of the formulation (e.g., in task 2 you just defined a new variable or modified one constraint of the model you formulated in task 1, then it is fine that you included the full model formulation in task 1 and only the new variable definition and new constraint that you modified in task 2). Recall using the solver *cplex* to solve the models. Provide a short description (no more than two sentences, e.g. "demand fulfillment") for every objective function and constraint in your formulations. Expected (not required) length of your report: Parts A and D between one and two pages each; Parts B and C between 2 and 3 pages each. All model formulations in this project must be linear.

## Part A

A factory emits three types of greenhouse gases into the air, all of which cause global warming. The three greenhouse gases are: carbon dioxide (CO2), chlorofluorocarbons (CFC), and nitrous oxide (N2O). The government has just passed a new environmental law designed to slow the growth of greenhouse gases in the atmosphere. Under the new law, this factory must reduce its annual emission of CO2 by at least 140 thousand kg, CFC by at least 100 thousand kg, and N2O by at least 180 thousand kg. Assume the factory has available three abatement methods to achieve these targets: selective catalytic reduction (SCR), lower-emission fuels (LEF), and electrostatic precipitators (EP). The effectiveness of each method depends on how *intensively* is used. We will measure the usage of each method in *units of intensity.* Table 1 shows the reduction of emission achieved per each unit of intensity and the corresponding cost for each method. For example, if the SCR method is used at 3.5 *units of intensity*, it results in simultaneous reductions of $3.5 \times 5 = 17.5$ thousand kg of CO2, $3.5 \times 8 = 28.0$ thousand kg of CFC, and $3.5 \times 10 = 35.0$ thousand kg of N2O at a cost of $3.5 \times 500 = \$1750$.

| Method | CO2 | CFC | N2O | Cost |
|---|---|---|---|---|
| SCR | 5 | 8 | 10 | 500 |
| LEF | 13 | 9 | 25 | 3300 |
| EP | 12 | 10 | 15 | 1110 |

Table 1: Annual emission reduction (in thousand kg) and annual cost (in \$) per each unit of intensity.

1. Formulate a linear programming model for deciding the units of intensity at which each abatement method must be used, such that the optimal solution minimizes the cost of reducing emissions. Implement the model in AMPL and solve it. What is the optimal solution? What is the optimal objective value?

2. Briefly (no more than a couple of paragraphs) discuss how the constraints are satisfied in the optimal solution.

3. How sensitive is the optimal cost to the targets set by the government? In particular, if the target of N2O emissions decreases to 155 thousand kg, can you conclude what is the effect in the optimal cost

without running the model again? And if the target of N2O emissions increases to 220 thousand kg, can you conclude what is the effect in the optimal cost without running the model again?

4. Investigate the effect of changes of the cost coefficients on the optimal solution. How sensitive are the decisions to the accuracy in these coefficients? In particular, if the cost of the LEF method would be reduced from 3300 to 2950 because of a cheaper source of fuels, would your optimal decisions remain the same? Would your optimal cost remain the same?

## Part B

Consider a recycling company producer of four types of paper-based products which are then used for packaging in industrial contexts. We refer to these paper-based products as *final products.* There are four types of final products (P1, P2, P3, and P4). Each final product is produced by blending four types of cardboard (C1, C2, C3 and C4). The cardboard is obtained from two different suppliers ($S1$ and $S2$).

The weekly purchases from $S1$ of cardboard C2 is limited to a maximum of 40,000 tonnes per week, while the weekly purchases of C3 and C4 is limited to a maximum of 25,000 tonnes each. Supplier $S1$ has no offer of cardboard C1.

The weekly purchases from $S2$ of cardboard C1 and C4 is limited to a maximum of 20,000 tonnes each, while the purchases of cardboard C3 is limited to a maximum of 30,000 tonnes per week. Supplier $S2$ has no offer of cardboard C2.

The purchase prices per tonne obtained from supplier $S1$ are \$56 for C2, \$53.5 for C3 and \$51 for C4. The purchase prices per tonne obtained from supplier $S2$ are \$57.5 for C1, \$52.5 for C3, and \$50 for C4. The company has an agreement of minimum purchases with both suppliers. The agreement states that the total weekly purchases from each of these two suppliers must have a minimum value of \$1,500,000.

The final products have been classified in two categories: P1 and P2 are of *category A*, while P3 and P4 are of *category B.* The production cost per tonne of final product is \$4 for final products of category A and \$5 for final products of category B. The sales prices per tonne are \$80 for P1, \$85 for P2, \$97.5 for P3 and \$100 for P4.

The production capacity for final products of category A in total is limited to 50,000 tonnes per week, while the capacity for final products of category B in total is limited to 45,000 tonnes per week.

There are minimum demand quantities that must be satisfied for each final product. These quantities are: 9000 tonnes of P1; 7000 tonnes of P2; 10,000 tonnes of P3; and 12,000 tonnes of P4 per week. Any production above these minimum quantities is also sold in the market of final products.

The cardboard types differ in quality. The quality of the final products is dependent on the proportion of each cardboard type in blending (assume linearity of yield). The quality of cardboard types and final products is measured by two characteristics: *strength* and *transparency.* We will assume that the strength is measured in a scale of absolute numbers from 0 to 100, and the transparency in percentage. In general, strength is a desirable feature in the packaging industry, while transparency is an undesirable feature. Table 2 shows the quality of the cardboard types and Table 3 shows the minimum strength and maximum transparency required for each type of final product.

| Cardboard | Strength | Transparency |
|---|---|---|
| C1 | 98 | 0.2% |
| C2 | 96 | 0.4% |
| C3 | 92 | 1.5% |
| C4 | 90 | 2.0% |

Table 2: Quality of cardboard types in strength and transparency.

| Final product | Strength | Transparency |
|---|---|---|
| P1 | 88 | 2.0% |
| P2 | 92 | 1.5% |
| P3 | 96 | 0.3% |
| P4 | 97 | 0.3% |

Table 3: Minimum strength and maximum transparency content of the different types of final products.

1. Formulate a linear programming model to determine a weekly blending plan such that the company maximizes profit. Implement the model in AMPL and solve it. What is the optimal blending plan and how much profit it provides? Without explicit use of data on costs and prices, but only the rest of the

data and the optimal solution, could you conclude which supplier is overall the most profitable for the company?

2. Suppose there is a new supplier $S3$ in the market of recyclable cardboard. This supplier can only provide a fixed quantity of 1000 tonnes of cardboard C1 weekly to the company. The company is willing to purchase these 1000 tonnes, but only if the weekly profit increases by at least 2.5% with respect to the optimal profit obtained in task 1. Assume that all the other parameter values and conditions remain the same.
a) Find the maximum price the company should accept to pay per tonne from this new supplier such that by buying the 1000 tonnes it achieves the increase of at least 2.5% with respect to the optimal profit obtained in task 1.
b) If instead of cardboard C1, the offer of supplier $S3$ is 1000 tonnes of another cardboard type C5 characterized by strength 97 and transparency 0.3%, what is the maximum price that the company should accept to pay per each tonne of C5 such that by buying the 1000 tonnes the profit increases by at least 2.5% with respect to the optimal profit obtained in task 1?

## Part C

A seafood company obtains salmon from two regions. In Region R1 the company can obtain as many as 200 tonnes per month, and in Region R2 it can also obtain as many as 200 tonnes per month. From these regions, the salmon is sent to any of the two production facilities of the company, denoted by F1 and F2. The cost of transporting one tonne from R1 to F1 and F2 is $13 and $16, respectively. The cost of transporting one tonne from R2 to F1 and F2 is $14 and $11, respectively.

The production capacity at each facility is 150 tonnes per month. After the salmon is processed at the facilities, the final product is ready to be shipped to the markets (assume the total production in tonnes equals the number of tonnes processed). The company perceives demand from 15 markets, denoted as K1,K2,...,K15. The monthly demands of the markets are shown in Table 4 and the costs of shipping one tonne between facilities and markets are shown in Table 5.

| K1 | K2 | K3 | K4 | K5 | K6 | K7 | K8 | K9 | K10 | K11 | K12 | K13 | K14 | K15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 17 | 21 | 15 | 15 | 22 | 15 | 20 | 12 | 15 | 28 | 19 | 18 | 25 | 22 | 23 |

Table 4: Demand from each market expressed in tonnes.

| $ | K1 | K2 | K3 | K4 | K5 | K6 | K7 | K8 | K9 | K10 | K11 | K12 | K13 | K14 | K15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| F1 | 21 | 9 | 9 | 21 | 42 | 42 | 90 | 87 | 18 | 24 | 48 | 30 | 75 | 24 | 33 |
| F2 | 15 | 51 | 84 | 60 | 21 | 24 | 27 | 21 | 96 | 84 | 51 | 84 | 21 | 60 | 114 |

Table 5: Costs of shipping one tonne between facilities and markets.

1. Formulate a linear programming model to minimize the monthly transport costs in meeting demands of all markets. Implement the model in AMPL and solve it. What is the optimal shipping plan and how much does it cost?

2. Suppose for the incoming month the company anticipates an increase of 25% in the demand from each market. Run your model for this new scenario. What do you obtain? Assume the marketing and operation managers have agreed on allowing for unsatisfied demand, but at a certain penalty cost (due to the negative impact in reputation for the company). The penalty cost has been defined according to priorities the company has on the markets. Per each tonne of unsatisfied demand in markets K1,K2,...,K4, the penalty cost is $100; for markets K5,K6,...,K10, it is $50 per tonne; for markets K11,K12,...,K14 it is $20 per tonne; for market K15 there is no penalty cost. Formulate a linear programming model to minimize the monthly costs (including transportation and penalty costs) in this new situation. Implement the model in AMPL and solve it. How much is the optimal cost? How much of this cost is due to transportation and how much due to penalties? What is the optimal shipping plan? Which market is the one with the largest amount of unsatisfied demand? How much is the unsatisfied demand for this market? Is the total capacity of the facilities used at its maximum?

## Part D

The article "Smart home charging of electric vehicles using a digital platform" (available in the module *Complementary readings* in *Canvas*), addresses a scheduling problem by a linear programming model. The description of the model is presented in Section 3.2 of the article, including a mathematical formulation outlined in expressions (1)–(5). In the following tasks we study this model and some modifications. Justify your answers clearly.

1. a) Could the model be infeasible? If yes, what is (are) the possible reason(s) for that? If not, why?
   b) Assume that all the parameters of the problem are non-negative. Could the model be unbounded? If yes, what is (are) the possible reason(s) for that? If not, why?
   c) Assume that all the parameters of the problem are non-negative, except for the price per kWh for some few times (this is an uncommon phenomenon, but it happens from time to time in reality[1]). Let us say there are two time periods in set $T$ for which $p_t$ is negative. Denote by $t_1$ and $t_2$ such time periods. Could the model be unbounded? If yes, what is (are) the possible reason(s) for that? If not, why?

2. Suppose that a smoothing condition requires the energy aggregator to keep some balance in the hourly power consumption (in total, accounting for all cars) during a day. Suppose the time periods represent the hours contained in this day (i.e., set $T = \{1, 2, \dots, 24\}$). This smoothing condition states that the difference between the power consumption in every pair of time periods cannot be greater than certain parameter $\delta$. How would you modify the original linear programming model to capture this new condition while keeping its linearity? What are the consequences that this modification could imply on the results of the new model with respect to the original one?

3. Suppose that you have solved the original model for a data set covering a time horizon of one day (say $T = \{1, 2, \dots, 24\}$), and obtained an optimal objective value equal to 37,500 NOK.
   a) In a new data set, you notice that all the data remain the same, except for the price at time period 12 which increased by 10 NOK, and the price at time period 20 which decreased by 10 NOK. Suppose that you solve the model for this new data set, and call the optimal objective value $v^*$. Can you conclude how $v^*$ compares with respect to 37,500?
   b) In a third data set, you notice that all the data remain the same as the first data set, except that the final time of the interval available for charging a specific car is three hours later. This car belongs to a customer called Helon Muzk, who in the first data set drove from home to work at 8:00, while in the third data set he did home office some hours and then drove at 11:00. His car is identified in the data set as "TeslaX". In other words, the tuple $(\text{“}TeslaX\text{”}, 1, 8)$ belongs to $U$ in the first data set, where there exists a corresponding parameter $s^{end}_{\text{“}TeslaX\text{”},8}$ expressing the state-of-charge desired by Helon. In contrast, set $U$ in the third data set contains the tuple $(\text{“}TeslaX\text{”}, 1, 11)$ instead of tuple $(\text{“}TeslaX\text{”}, 1, 8)$, and there exists a corresponding parameter $s^{end}_{\text{“}TeslaX\text{”},11}$ expressing the state-of-charge desired by Helon (whose value equals the value of $s^{end}_{\text{“}TeslaX\text{”},8}$ from the previous data set). Suppose that you solve the model for this third data set, and call the optimal objective value $w^*$. How does $w^*$ compare with 37,500?

[1] See e.g. a recent note in a local newspaper here:
`https://www.bt.no/nyheter/okonomi/i/rlJmAA/negativ-stroempris-i-bergen-hele-mandag`