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BAN402: Decision Modelling in Business

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Project 1:

Linear Programming

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1 Part A: Optimization of GHG abatement

1.1 Linear Programming Model

The aim of the first model is to find an optimal abatement plan for a factory that has to reduce its emissions of carbon dioxide (CO_2), chlorofluorocarbons (CFC), and nitrous oxide (N_2O) to be in line with newly implemented environmental regulations. The goal is to use the three different abatement methods selective catalytic reduction (SCR), lower-emission fuels (LEF), and electrostatic precipitators (EP) in a cost-effective way to fulfill the minimum abatement requirements defined by the government. The abatement methods differ regarding both costs and effectiveness in reducing the emissions of the three GHGs. Therefore, a linear programming model is set up to help decision makers identifying the least costly path for emission reduction:

Indexes and sets:
$g \in G$: set of GHGs.
$m \in M$: set of abatement methods.

Parameters:
C_m: Cost of abatement method m (in \$).
$R_{g,m}$: Annual reduction in gas g with one unit of intensity of method m (in tonnes).
R_g^{Min}: Minimum abatement goal for gas g (in tonnes).

Decision variables:
u_m: Units of intensity at which method m is used.

The objective function (Eq. 1) aims to minimize the sum of costs over all three abatement methods:

$$Min\ Costs = \sum_{m \in M} C_m * u_m \tag{1}$$

This cost minimization is subject to the following constraints:

$$\sum_{m \in M} R_{g,m} * u_m \geq R_g^{Min} \quad \forall g \in G \tag{2}$$

$$u_m \geq 0 \quad \forall m \in M \tag{3}$$

The first constraint (Eq. 2) ensures that all abatement methods together achieve the required minimum reduction for each gas, while the second constraint (Eq. 3) is a non-negativity constraint in place to avoid potential illogical results.

After solving the model in AMPL, we obtain the following results: To minimize costs, the factory should implement EP at 11.1111 units of intensity and SCR at 1.3333 units of intensity; LEF should not be used at all. This makes good sense, since method LEF is almost 6 times as expensive as SCR while beeing less than 3 times as effective. Adhering to this plan yields total costs of $13,000 to be in line with the emission reduction constraint.

1.2 Role of the Constraints in the optimal Solution

For CO_2 and N_2O the reduction constraint of 140 tonnes and 180 tonnes respectively, are active, i.e. the optimal reduction plan achieves exactly the required minimum. This is also referred to as a slack equal to 0. For CFC, the optimal result yields abatement of 121.7777 tonnes, i.e., more than the minimum of 100 tonnes, which means the constraint is not active and the slack is equal to 21.7777.

1.3 Sensitivity Analysis I

The discrepancies regarding the satisfaction of the constraints also explain differences regarding the sensitivity of optimal costs to the reduction targets defined by the government. We conduct a sensitivity analysis in AMPL by modifying the run-file:

```
[...]
# option needed for sensitivity analysis
option presolve 0;
# cplex option to do sensitivity analysis
option cplex_options 'sensitivity';
solve;
display [...],
reduction_constraint,
reduction_constraint.current, # sensitivity analysis I
reduction_constraint.down, # sensitivity analysis I
reduction_constraint.up, # sensitivity analysis I
```

Abating an extra tonne of CO_2 or N_2O would increase total costs by an additional $80 for the first and an extra $10 for the latter. This additional cost can also be called the shadow price. On the other hand, however, because the constraint for CFC is not binding, the optimal costs would not be affected should the government increase the reduction for CFC by one unit, i.e., the shadow price is 0. These statements are only valid for the following intervals:
$90 \leq \mathrm{R}_{CO2}^{Min} \leq 144$,
$175 \leq \mathrm{R}_{N2O}^{Min} \leq 280$,
$0 \leq \mathrm{R}_{CFC}^{Min} \leq 121,7778$.

Since a potential reduction goal of 155 tonnes for N_2O is outside the lower bound of 175, the effects of such a new reduction goal are unpredictable without modifying the model. If the emission target for this type of GHG increases to 220 tonnes, however, one can calculate the additional costs to be \$400.

1.4 Sensitivity Analysis II

To analyse how sensitive our model is to changes in costs for the abatment methods, we run a sensitivity analysis for the decision variable u_m in AMPL:

```
[...]
# Sensitivity analysis 2:
# Robustness of the optimal solution for u
display u, u.current, u.down, u.up; [...]
```

We obtain the following lower and upper bounds of cost per unit of intensity for the respective methods:
$\$462{,}5 \leq C_{SCR} \leq \$740$,
$\$750 \leq C_{EP} \leq \$1200$,
$\$1290 \leq C_{LEF} \leq \$\infty$.
This means that our optimal solution will remain the same if the costs for the methods remain within the specified ranges.

Looking at these intervals, a positive factor regarding the robustness of our model is that it is relatively insensitive to changes in costs of reduction methods and is therefore not reliant on the exact accuracy of the cost data provided. Especially when comparing the current cost of LEF with the lower bound for the method (\$1290), we observe a big difference. To cause a change in our optimal solution, we must decrease the cost of LEF to below \$1290. Therefore, both the optimal decisions as well as the optimal costs stay unaffected, should the costs of LEF drop from \$3300 to \$2950.

2 Part B: Optimal Production Plan for Recycling Company

2.1 Linear Programming Model

Indexes and sets:

$p \in P$: set of products.

$c \in C$: set of cardboards.

$s \in S$: set of suppliers.

$q \in Q$: set of quality features.

$a \in A$: set of products in category A.

$b \in B$: set of products in category B.

Parameters:

$AVAIL_{c,s}$: Maximum availability of cardboard c from supplier s (in tonnes).

M_s^{PURCH}: Minimum purchase amount from supplier s (in \$).

$C_{c,s}{}^{PURCH}$: Cost of one tonne of cardboard c from supplier s (in \$).

C_p^{PROD}: Cost of producing one tonne of product p (in \$).

P_p^{SALE}: Sale price for one tonne of product p (in \$).

D_p^{MIN}: Minimum demand for product p (in tonnes)

K^{Amax}: Production capacity for products in category A (in tonnes)

K^{Bmax}: Production capacity for products in category B (in tonnes)

$QUAL_{p,q}$: Minimum quality for feature q in product p.

$IQUAL_{c,q}$: Quality for feature q in cardboard c.

Decision variables:

$j_{c,s}$: Amount of cardboard c purchased from supplier s.

$u_{c,p}$: Amount of cardboard c used in blending for product p.

x_p: Amount produced of product p.

We are maximizing the profit, which is the sale price of a product times the quantity of the product sold over all products in set P, subtracting the purchasing costs and the production costs:

$$Max\ Profit = \sum_{p \in P} P_p^{SALE} * x_p - \sum_{c \in C} \sum_{s \in S} C_{c,s}^{\ PURCH} * j_{c,s} - \sum_{p \in P} C_p^{PROD} * x_p \quad (4)$$

The objective function is subject to the following constraints. The amount of cardboard purchased from each of the suppliers cannot exceed the maximum supply offered by these suppliers:

$$j_{c,s} \leq AVAIL_{c,s} \quad \forall c \in C, s \in S \quad (5)$$

In blending the cardboards to produce final products, we can not exceed the amount purchased of the respective cardboards:

$$\sum_{p \in P} u_{c,p} \leq \sum_{s \in S} j_{c,s} \quad \forall c \in C \quad (6)$$

There is a minimum purchase agreement in place with both suppliers:

$$\sum_{c \in C} C_{c,s}^{\ PURCH} * j_{c,s} \geq M_s^{PURCH} \quad \forall s \in S \quad (7)$$

We assume linearity of yield. The amount of cardboard used to produce a final product equals the output amount for the respective product:

$$\sum_{c \in C} u_{c,p} = x_p \quad \forall p \in P \quad (8)$$

The production of category A products (P1, P2) is limited to a certain capacity:

$$\sum_{a \in A} x_a \leq K^{Amax} \quad (9)$$

The production of category B products (P3, P4) is limited to a certain capacity:

$$\sum_{b \in B} x_b \leq K^{Bmax} \tag{10}$$

Demand for each product is constrained by a lower bound only:

$$x_p \geq D_p^{MIN} \quad \forall p \in P \tag{11}$$

Each product must fulfill certain quality features defined in set Q. The average feature quality of all cardboard used in a product equals the feature quality of the respective product:

$$\sum_{c \in C} IQUAL_{c,q} * u_{c,p} \geq QUAL_{p,q} * \sum_{c \in C} u_{c,p} \quad \forall p \in P, q \in Q \tag{12}$$

Please note that we multiplied both sides of Eq. 12 by the amount of cardboard used in a product, to make this constraint linear. Lastly, we need the non-negativity constraint:

$$j_{c,s},\ u_{c,s},\ x_{c,p} \geq 0 \quad \forall c \in C, s \in S, p \in P \tag{13}$$

The model was solved in AMPL for which one important remark must be made. As mentioned above, we define set Q, containing the quality features "strength" and "transparency", strength being favorable and transparency being unfavorable. For the quality constraint to be formulated in one equation for all $q \in Q$ (Eq. 12), we must modify the transparency input data for $QUAL_{p,q}$ and $IQUAL_{c,q}$ in the AMPL data file by multiplying with a factor of -1, as less transparency is seen as preferable.

Solving the model in AMPL with the display precision option $= 6$ yields a maximum profit of \$2,915,522. The optimal purchasing and blending plan for the recycling company is displayed in Table 1.

Table 1: Optimal purchasing and blending plan for recycling company

	Purchasing		Blending			
	S1	S2	P1	P2	P3	P4
C1	0	20000	0	0	5000	15000
C2	25956	0	0	5956	5000	15000
C3	0	23133	0	23133	0	0
C4	911	20000	9000	11911	0	0

Amount of every cardboard purchased from suppliers and used in blending for products, in tons per week

We purchase everything available from cardboard C1, which is an intuitive result, because C1 is the cardboard with the highest quality. Hence we can produce more higher quality products that sell for a higher price (the difference in cost for high- and low-quality cardboard is smaller than the difference in sale price for high- and low-quality products). Furthermore, cardboards C3 and C4 are almost entirely purchased from supplier S2, which makes sense because S2 has the better price for both.

For the maximized profit, we produce at the minimum demand limit of product P1 (9000) and P3 (10000), since P1 is the product with the lowest return (considering the difference in price of cardboard suitable to fulfill quality and sales price of P1), and P3 is only slightly more difficult to produce within the quality constraint than P4, but yields less profit. P2 is the product with be largest quantity (41000) because a lot of C3 can be used, which is comparably cheap. Product P4 is produced at 30000 tonnes per week.

Which supplier is the most profitable one?

To answer this question without explicit use of data on costs and prices, we evaluate how our optimal purchase amount varies for the cardboard between the two suppliers. Since we are maximizing profit, we expect to purchase more from the more profitable supplier than from the less profitable one. Hence, we need to examine to what extent we exploit the supply constraint (Eq. 5). To do this, we first calculate the slack variables in AMPL:

```
# AMPL code added to runfile (slack variables)
display supply_constraint.body, supply_constraint.ub,
   supply_constraint.slack;
```

Table 2 shows the results from the code above. If we want to make a statement about the profitability of the suppliers, it is not helpful to look at the slack variables from purchasing product C1 and C2 because C1 is offered by S2 only, and C2 is offered by S1 only. Hence, we do not have a choice where we buy these cardboards from. However, C3 and C4 are offered by both suppliers and can thus provide insight into which supplier we prefer to maximize profit.

Table 2: Slack variables of supply constraint (Eq. 5)

	supply_constraint.body	supply_constraint.ub	supply_constraint.slack
C1 S1	0	0	0
C1 S2	20000	20000	0
C2 S1	25956	40000	14044
C2 S2	0	0	0
C3 S1	0	25000	25000
C3 S2	23133	30000	6867
C4 S1	911	25000	24089
C4 S2	20000	20000	0

Amount of body, upper bound and slack for supply constraint (in tonnes)

Table 2 shows that the slack variable for purchasing C4 from S2 is 0, so we purchase all C4 that S2 has available. The slack variable for purchasing C4 from S1 on the other hand is relatively high (24089), because we only purchase a small amount of 911 out of the total 25000 that S1 has on offer. The purchase of C3 follows a similar pattern. Although we do not purchase all C3 that S2 has on offer (slack 6867), S2 is the only supplier we purchase it from (upper bound = slack for C3 S1). It is thus obvious that we prefer supplier S2 for both C3 and C4. Concluding, our optimal purchasing plan for maximizing profit tells us that supplier S2 must be the most profitable.

2.2 New Supplier S3 enters the Market

a) S3 offers a fixed amount of 1000 tonnes C1

A new supplier S3 enters the market offering a fixed amount of 1000 tonnes C1 and we want to determine the maximum price we are ready to accept to gain at least 2.5% in profit. We thus conduct the following changes to our model from section B.1:

New set of suppliers:

$n \in SN$: Set of suppliers, including S3.

Changes to Parameters:

$Avail_{c,n}$: Maximum availability now indexed to all suppliers, not just S1, S2.
$AvailS3$: New parameter, fixed amount of C1 available from S3.
$Profmin$: New parameter, minimum profit that must be reached (=1.025*$2,915,522).

Changes to Variables:

$j_{c,n}$: Now indexed to n instead of s.
$CostS3$: New variable, cost for one unit of C1 from S3 we are willing to accept.

We are now maximizing the cost of C1 from S3 instead of the profit. Hence, our objective function changes:

$$Max\ CostS3 \tag{14}$$

Both the supply constraint (Eq. 5) and the blending constraint (Eq. 6) must be changed to sum over the set of all suppliers (SN):

$$j_{c,n} \leq AVAIL_{c,n} \quad \forall c \in C, n \in SN \tag{15}$$

$$\sum_{p \in P} u_{c,p} \leq \sum_{n \in SN} j_{c,n} \quad \forall c \in C \tag{16}$$

Furthermore, we add the constraint of fixed purchase amount from S3:

$$j_{C1,S3} \; = \; AvailS3 \tag{17}$$

Finally, we have to make sure that the profit is at least 2.5% higher than the profit obtained in task 1:

$$\sum_{p \in P} P_p^{SALE} * x_p - \sum_{c \in C} \sum_{s \in S} C_{c,s}{}^{PURCH} * j_{c,s} - \sum_{p \in P} C_p^{PROD} * x_p - AvailS3 * CostS3 \; \geq \; Profmin \tag{18}$$

Note that in Eq. 17, we use the old set of suppliers S, because we are separately subtracting the purchasing cost of S3 in the last term of the LHS. All other definitions of the LPM in Section 2.1 remain the same.

Solving the model in AMPL yields a maximum acceptable price of $61.3 per tonne of C1 when purchased from S3 to gain a 2.5% increase in profit. While this price may seem very high at the beginning compared to the prices of the other suppliers, it actually makes good sense. C1 is the cardboard with the highest quality and is therefore necessary to produce the most qualitative product P4 that sells for the highest price. However, we already fully exploit the supply of C1 from the other suppliers, and thus are now willing to pay a higher price for additional C1 from supplier 3. The additional 1000 tonnes of C1 result in a production increase of 2000 tonnes of our most expensive product P4, hence more possible profit.

b) S3 offers a fixed amount of 1000 tonnes of new cardboard C5

Now supplier S3 offers 1000 tonnes of a new cardboard C5 instead of C1. With respect to Part B.2.a we introduce only minor changes to the model. We add C5 to the set of cardboards C with the respective values of 97 for strength and 0.3% of transparency.

Consequently, Eq. 17 must be changed to:

$$j_{C5,S3} = AvailS3 \tag{19}$$

Furthermore, variable $CostS3$ will not denote the cost of one tonne of C1 anymore, but for the new cardboard C5. Everything else remains the same as defined in part 2.2.a or the original model in 2.1.

Solving the model in AMPL, compared to the previous section we now obtain a much smaller maximum price of $22.1 per tonne C5. While this price may sound comparably low at first, we can conclude the reason when looking at the quality features of C5 compared to C1. C1 has a higher quality than the required quality for the most qualitative product P4 and can thus be used together with another less qualitative cardboard to blend P4. C5 on the other hand has the exact same quality as the requirement for P4. It follows that we can only produce 1000 tonnes more of P4 with 1000 tonnes of C5, which accounts for much less profit increase than the 2000 tonnes that we were able to increase P4 production by in the previous section. Concluding, we are willing to pay only a low price for the new cardboard C5.

3 Part C: Transport Cost Minimization

3.1 Linear Programming Model

In Part C, we try to minimize transport costs of salmon for a seafood company. The company sources its salmon from two different regions and has two processing facilities who supply fifteen different markets. To ensure full satisfaction of demand at the fifteen markets with minimal transport costs, the following linear programming model is set up:

Indexes and sets:
$r \in R$: set of sourcing regions for salmon.
$f \in F$: set of processing facilities.
$k \in K$: set of markets.

Parameters:
$C_{r,f}^{PURCH}$: Cost of transporting 1 ton of salmon from region r to facility f (in \$).
$C_{f,k}^{SHIP}$: Cost of shipping 1 ton of salmon from facility f to market k (in \$).
M_r^{PURCH}: Maximum purchase limit from region r (in tonnes).
M_f^{PROC}: Maximum processing limit at facility f (in tonnes).
D_k: Demand at market k (in tonnes).

Decision variables:
$x_{r,f}$: Tonnes of salmon transported from region r to facility f.
$y_{f,k}$: Tonnes of salmon shipped from facility f to market k.

The goal of the objective function (Eq. 20) is to minimize the transport costs both from the regions to the production sites as well as from the production sites to the markets.

$$Min\ Costs = \sum_{r\in R}\sum_{f\in F} C_{r,f}^{PURCH} * x_{r,f} + \sum_{f\in F}\sum_{k\in K} C_{f,k}^{SHIP} * y_{f,k} \tag{20}$$

This cost minimization takes place within several constraints. The fact that the amount transported from the regions to the facilities cannot exceed the purchasing limit from region r and the amount shipped to the markets from processing facility f cannot be greater than its production capacity, is reflected in Eq. 21 - 22.

$$\sum_{f\in F} x_{r,f} \leq M_r^{PURCH} \quad \forall r \in R \tag{21}$$

$$\sum_{k\in K} y_{f,k} \leq M_f^{PROC} \quad \forall f \in F \tag{22}$$

Furthermore, outgoing shipments from facility f to markets K cannot be greater then what facility f has previously received from regions R.

$$\sum_{k\in K} y_{f,k} \leq \sum_{r\in R} x_{r,f} \quad \forall f \in F \tag{23}$$

Additionally, the amount of salmon market k receives from facilities F has to satisfy the demand on said market.

$$\sum_{f\in F} y_{f,k} = D_k \quad \forall k \in K \tag{24}$$

Finally, we need the non-negativity constraint:

$$x_{r,f},\ y_{f,k} \geq 0 \quad \forall r \in R, f \in F, k \in K \tag{25}$$

Solving this model in AMPL yields the following optimal transport plan: Region R1 provides Facility F1 with 150 tons of salmon, while transporting nothing to Facility F2. F2, in turn, receives 137 tons of salmon from Region R2. No transport takes place between R2 and F1. This ensures that the 287 tons of salmon needed to satisfy demand at the markets is transported to the processing facilities at minimum costs.

Table 3: Optimal Shipping Plan from Facilities to Markets

	K1	K2	K3	K4	K5	K6	K7	K8	K9	K10	K11	K12	K13	K14	K15
F1	0	21	15	15	0	0	0	0	15	28	0	18	0	15	23
F2	17	0	0	0	22	15	20	12	0	0	19	0	25	7	0

Amount of salmon (in tons) shipped from facility f to market k to minimize costs

The optimal shipping plan from Facilities F to Markets K can be found in Table 3. Note that, with the exception of K14, all of the markets are supplied from only one of the two facilities. Implementing this plan results in total monthly transport costs of $10480.

3.2 Allowing for Unsatisfied Demand

If demand on all markets was to rise by 25%, it becomes apparent, that, considering the processing limit at the facilities, not all demand can be met. That is why, when running the model in AMPL, we obtain an infeasibility message.

Allowing for unsatisfied demand at a penalty cost that varies from market to market requires adjustments to the model set up in 3.1. A new parameter C_k^{PEN} is defined that depicts the penalty costs per ton of unsatisfied demand at market k. The amount of unsatisfied demand at market k is given by $D_k - \sum_{f \in F} y_{f,k}$, i.e., the difference between demand at market k and shipment to the same market from all facilities F. Including the resulting mathematical expression into the minimization problem yields the following modified objective function (Eq. 18):

$$Min \sum_{r \in R} \sum_{f \in F} C_{r,f}^{PURCH} * x_{r,f} + \sum_{f \in F} \sum_{k \in K} C_{f,k}^{SHIP} * y_{f,k} + \sum_{k \in K} C_k^{PEN} * (D_k - \sum_{f \in F} y_{f,k}) \tag{26}$$

Furthermore, the demand constraint also has to be matched to the new situation, such that it is now possible to ship less than the demanded quantity (Eq. 19). All other constraints remain as described in section 3.1.

$$\sum_{f \in F} y_{f,k} \leq D_k \quad \forall k \in K \tag{27}$$

Finally, the demand increase of 25% is included in the data file of our AMPL model.

When solving this adjusted model, the results change in several aspects with respect to section 3.1. The total amount of salmon has decreased, with Region R1 supplying 117.5 tons to Facility F1 and Region R2 supplying 107.5 tons to facility F2. Like in 3.1, there is no transport between R1 and F2 and R2 and F1, respectively. That implies that the processing limit of 150 tons is not fully used at either of the facilities; F1 operates at 78.333% and F2 at 71.666% capacity.

The optimal shipping plan from facilities F to markets K is displayed in Table 4. It is noteworthy that unsatisfied demand only occurs in the markets that have been identified as less important for business and therefore associated with lower or no penalty costs. This is intuitive, since for markets K11 - K15 the shipping costs are always above the penalty regardless of the production facility. The largest amount of unsatisfied demand occurs in market K13 where 31.25 tons, i.e. all of the demand is left unsatisfied.

Sticking to the shipping plan outlined above yields costs of \$9122.5, with $\approx$ 77% (\$7022.5) of the costs coming from transport and the remaining $\approx$ 23% (\$2100) consisting of the penalties. Despite the penalty costs, these total costs are substantially lower than what we have obtained in 3.1.

Table 4: Optimal Shipping Plan allowing for Unsatisfied Demand

	K1	K2	K3	K4	K5	K6	K7	K8	K9	K10	K11	K12	K13	K14	K15
F1	0	26.25	18.75	18.75	0	0	0	0	18.75	35	0	0	0	0	0
F2	21.25	0	0	0	27.5	18.75	25	15	0	0	0	0	0	0	0

Amount of salmon (in tons) shipped from facility f to market k to minimize costs when allowing for unsatisfied demand

That both costs and production are decreasing when faced with higher demand and penalty costs may seem like a counter-intuitive result at first, which is however put into perspective by the fact that we are not maximizing profit, but running a sole cost minimization model, where less sales relate to lower transportation costs.

4 Part D: Bjørndal et. al., 2023

For the last part of this report, we looked at the work of Bjørndal et al., 2023, who developed a model on the optimal charging of electric vehicles (EV). Note that for all statements made in this section, we assume exogeneity of p_t, i.e. there is no causality between p_t and $\sum_{k \in K} x_{kt}$; demand in the EV fleet does not influence the power price.

4.1 Unboundedness and Infeasibility

First, we check the model for potential infeasibility. A model is said to be infeasible if there is no solution satisfying all constraints. In the case of the model set up by Bjørndal et al., 2023, this could be the case if there is not enough capacity c_t over all time periods T, to be able to fulfill the first constraint (starting load plus the aggregate power charged has to be equal to the desired end load) for all tuples (k,i,f). In other words, the fleets charging demand could exceed what is feasible to be in line with the capacity constraint. On an individual level, the amount of power needed in every point in time t between i and f to reach the desired end load $s^{end}_{k,f}$ could be larger than what is technically possible (m_k). The feasibility of the model therefore depends on the data it is running on.

Secondly, we assess whether the model could be unbounded. Unbounded models are models where the objective function value can be improved without any limit, i.e., the objective value strives towards plus (maximization) or minus (minimization) infinity. When assuming positive parameters, this is not possible with this specific model. With positive prices $x_{k,t}$ would need to become negative for the objective value to strive towards minus infinity. However, this is ruled out by the non-negativity constraint (Eq. 5 in the paper). This does not change, even if prices occasionally become negative. In that case, the capacity constraint $\sum_{k \in K} x_{k,t} \leq c_t$ keeps $x_{k,t}$ from limitless growth, assuming that capacity c_t can not be infinitive and that there is always a capacity limit. In conclusion, while there might be feasibility issues in certain situations and for certain data sets, the model can not become unbounded.

4.2 Smoothing Condition

Introducing a smoothing condition as described in task 2 of part D could be done by adding the following constraint to the model:

$$\left| \sum_{k \in K} x_{k,t} - \sum_{k \in K} x_{k,t-1} \right| \leq \delta \quad \forall t \in T : t > 1 \tag{28}$$

To ensure linearity, we can convert this into two separate linear constraints:

$$\sum_{k \in K} x_{k,t} - \sum_{k \in K} x_{k,t-1} \leq \delta \quad \forall t \in T : t > 1 \tag{29}$$

$$\sum_{k \in K} x_{k,t-1} - \sum_{k \in K} x_{k,t} \leq \delta \quad \forall t \in T : t > 1 \tag{30}$$

This constraint ensures that the difference in overall power consumption between to points in time (in absolute value) cannot exceed the parameter δ. This constraint further decreases the possibilities to exploit the full potential of times of low prices, which is why the objective value is going to be at least the same, likely higher.

4.3 Mr. Helon Muzk

Without additional information or assumptions it is not possible to assess exactly how the price change as described in task 3a will affect the objective value. If, for example, the capacity limit c_t is already reached for all points in time, there is no flexibility in the system and the direction of effect of the price change would depend on whether more power is charged at t12 or t20:

$$\sum_{k\in K} x_{k12} \leq \sum_{k\in K} x_{k20} \quad \rightarrow \quad v^* < 37,500 \ NOK \tag{31}$$

$$\sum_{k\in K} x_{k12} \geq \sum_{k\in K} x_{k20} \quad \rightarrow \quad v^* > 37,500 \ NOK \tag{32}$$

On the other hand, if there is still free capacity and demand could react to the price change, the direction of the effect would depend on the initial values of p_{12} and p_{20} as well as on how many people can switch from a now higher price to a now lower price and how many can exploit the lower price versus how many are still stuck with the higher price. In a nutshell, in order to draw reliable conclusions, you would need information on c_t, $\sum_{k\in K} x_{k,12}$, $\sum_{k\in K} x_{k,20}$ and p_t to determine the effect this price has on the objective value.

Considering the change in charging preferences of a certain Mr. Helon Muzk allows for more concrete statements. We conclude that:

$$w^* \leq 37,500 \tag{33}$$

The original situation where Helon charged his car from t=1 to t=8 is still possible in case prices p_9, p_{10}, p_{11} are higher then from t=1 to t=8. If however, prices drop, and there is still free capacity between t=9 and t=11, he will have lower overall charging costs, which will then also lower the objective value. In reality, this would actually not be an unlikely scenario, as electricity prices tend drop at around 9 or 10 am after peaking in the morning.

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