NORGES HANDELSHØYSKOLE

BAN402: DECISION MODELLING IN BUSINESS

PROFESSOR MARIO GUAJARDO

Project 3:
Non-Linear Programming

November 14^{th}, 2023

Contents

List of Tables

List of Figures

1 Part A: Time Series Forecasting

To forecast the price index for existing dwellings (PIFED) on a quarterly basis from 1993 to 2022, we formulate a modified version of the simple exponential smoothing method. This results in the following non-linear model:

Indexes and Sets:
$t \in T$: set of quarters.

Parameters:
D_t : Actual value of PIFED observed in quarter t.

Decision Variables:
$\gamma \in [0.01, 0.99]$: smoothing constant for the level.
$\beta \in [0.01, 0.99]$: smoothing constant for the trend.
F_t: forecast of PIFED for quarter t.
L_t: level of PIFED in quarter t.
Tr_t: trend of PIFED in quarter t.

The goal of this model is to minimize the forecast error, measured by the mean absolute percentage error (MAPE). The MAPE is given by the sum of forecast errors over all quarters t times 100, divided by the cardinality of T, in this case 120 quarters, which results in the following objective function:

$$minimize\ MAPE = \frac{\sum_{t \in T} \left| \frac{F_t - D_t}{D_t} \right| * 100}{|T|} \tag{1}$$

To model the forecasting method we define a set of constraints. In this modified version of the simple exponential smoothing method, the forecast for quarter t, F_t, is given by a level L_{t-1} and a trend component Tr_{t-1}, both for the previous quarter. Hence, the forecast is given by the following equation:

$$F_t = L_{t-1} + Tr_{t-1} \quad \forall\, t \in T : t > 1 \tag{2}$$

The level L_t is given by the actual value in quarter t plus the level and trend for the previous period, weighed by the smoothing constant γ:

$$L_t = \gamma * D_t + (1 - \gamma) * (L_{t-1} + Tr_{t-1}) \quad \forall\, t \in T : t > 1 \tag{3}$$

The trend Tr_t is calculated using a combination of the the difference in level between period t and period $t-1$ and the trend of period $t-1$, weighed by the smoothing constant β:

$$Tr_t = \beta * (L_t - L_{t-1}) + (1 - \beta) * Tr_{t-1} \quad \forall\, t \in T : t > 1 \tag{4}$$

Finally, we have to incorporate the initial values for the first forecast, level and trend:

$$F_1 = 18.8 + 0.5 \tag{5}$$

$$L_1 = \gamma * D_1 + (1 - \gamma) * (18.8 + 0.5) \tag{6}$$

$$Tr_1 = \beta * (L_1 - 18.8) + (1 - \beta) * 0.5 \tag{7}$$

We use the following code to implement the model in AMPL:

```
set T;
param D{T};
var gamma;
var beta;
var F{T};
var L{T};
var Tr{T};
```

```
minimize MAPE:
   (sum{t in T}abs((F[t]-D[t])/D[t])*100)/card(T);

subject to

Forecast{t in T:t>1}:
   F[t]= L[t-1] + Tr[t-1];
Level{t in T:t>1}:
   L[t]=gamma*D[t]+(1-gamma)*(L[t-1]+Tr[t-1]);
Trend{t in T:t>1}:
   Tr[t]=beta*(L[t]-L[t-1])+(1-beta)*Tr[t-1];

ForecastInitial:
   F[1]=18.8+0.5;
TrendInitial:
   Tr[1]=beta*(L[1]-18.8)+(1-beta)*0.5;
LevelInitial:
   L[1]=gamma*D[1]+(1-gamma)*19.3;

LowerGamma:
   0.01<=gamma;
UpperGamma:
   0.99>=gamma;
LowerBeta:
   0.01<=beta;
UpperBeta:
   0.99>=beta;
```

Implementing this model in AMPL and solving it using the minos solver yields an optimal MAPE of 2.189. Note that, because of the non-linear nature of the model, this is only a local and not a global optimum. In this local optimum, the optimal values for the smoothing constants are $\gamma = 0.719$ and $\beta = 0.0245$. The highest percentage error occurs in quarter 4 of 2008, where our forecast deviates from the reported data by 10.27%. The lowest deviation occurs in 2011 Q3 and 2018 Q1, where we predict the actual value and hence have an error of 0.00%.

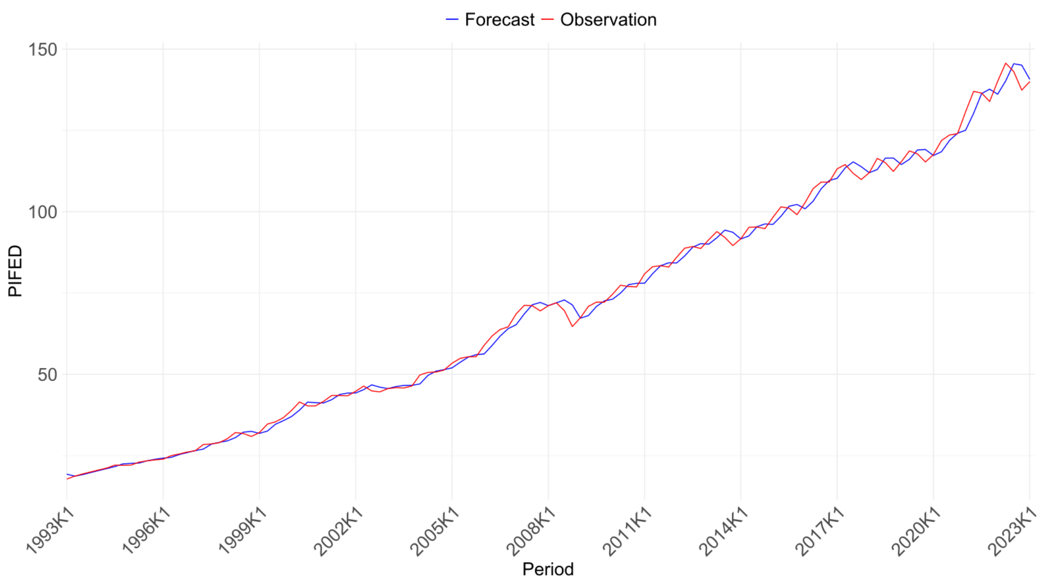

Figure 1: PIFED forecast vs. observation 1993-2023

The comparison between our forecast and the actual data is visualized in Figure 1. It is apparent that the model can predict the PIFED fairly accurately. Larger fluctuations however are depicted with a small time lag, which makes sense given the formulation of the model. For the first quarter of 2023, our model predicts a PIFED of \$140.719, which is quite accurate considering the observed value is \$140.0; we incur an error of 0.51%.

2 Part B: Gerrymandering for Justice: Redistricting U.S. Liver Allocation

In this part we will work with an integer model as introduced in Gentry et al., 2015. The model aims to redistrict the liver allocation in the U.S. by minimizing the sum of the absolute difference between the ideal and the actual number of donors in each districts. The decision to be taken is, which "mutually exclusive, geographically defined donor service area" (DSAs) i is assigned to which of the N districts. For further specifications of the model, please refer to Gentry et al., 2015. Note that the only modifications we conduct are described in the next two subsections. We assume that all other constraints as described in the paper remain in place.

2.1 Unfair Distribution of Transplant Centers

The first modification we will introduce to the model, takes care of an appropriate distribution of transplant-centers across the districts. The number of transplant centers in every district must not be deviate by more than six transplant centers from the average number of transplant centers across districts. In the model, h_i is defined as the number of transplant centers in DSA i. We now add a new parameter A to the model, containing the average number of transplant centers across districts. A is then calculated as follows, where N is the number of districts:

$$A = \frac{\sum_{i \in I} h_i}{N} \tag{8}$$

For all districts K (k is chosen as district center), the sum of transplant centers h_i in DSAs I assigned to district k must be within six transplant centers from the average:

$$(A) * y_k - 6 \leq \sum_{i \in I} h_i * w_{i,k} \leq (A) * y_k + 6 \quad \forall k \in K \tag{9}$$

2.2 Limiting Strong Districts

As another independent modification, we will limit the number of strong districts to two. A strong district is defined as a district, which contains three or more DSAs from the Top 10 DSAs as decreasingly ordered by number of livers recovered for transplantation. Let $J \in \{1, 2, ..., 10\}$ denote a new set containing the top 10 DSAs as indexed with 1 through 10. Obviously, J is a subset of I. We define a new variable $s_k \in \{0, 1\}$, which we set to 1 if a district with chosen center k contains at least three DSAs from set J. Otherwise, s_k will be 0:

$$\sum_{i \in J} w_{i,k} \geq 3 * s_k \quad \forall k \in K \tag{10}$$

$$\sum_{i \in J} w_{i,k} \leq M * s_k + 2 \quad \forall k \in K \tag{11}$$

Where M is a sufficiently large number, for example greater than the number of DSAs. The sum over s_k is then restricted by the upper limit of two:

$$\sum_{k \in K} s_k \leq 2 \quad \forall k \in K \tag{12}$$

3 Part C: Nord-Pool Day-Ahead Market

In this section we will look at the principles for market clearing on the northern European electricity market. The procedure in footnote 2 of the task definition implies that we are dealing with period / data file 3 in the following.

3.1 Plotting the Day-Ahead Market

The demand and supply bids can be visualized by a step function as done in figure 2 for hour 3. While this step function is the most accurate representation of supply and demand, it has some negative features. A major problem with these piecewise linear curves is that they do not necessarily yield a unique equilibrium. Since step functions do not always have a single intersection, there are cases where an ambiguous price or quantity is possible.

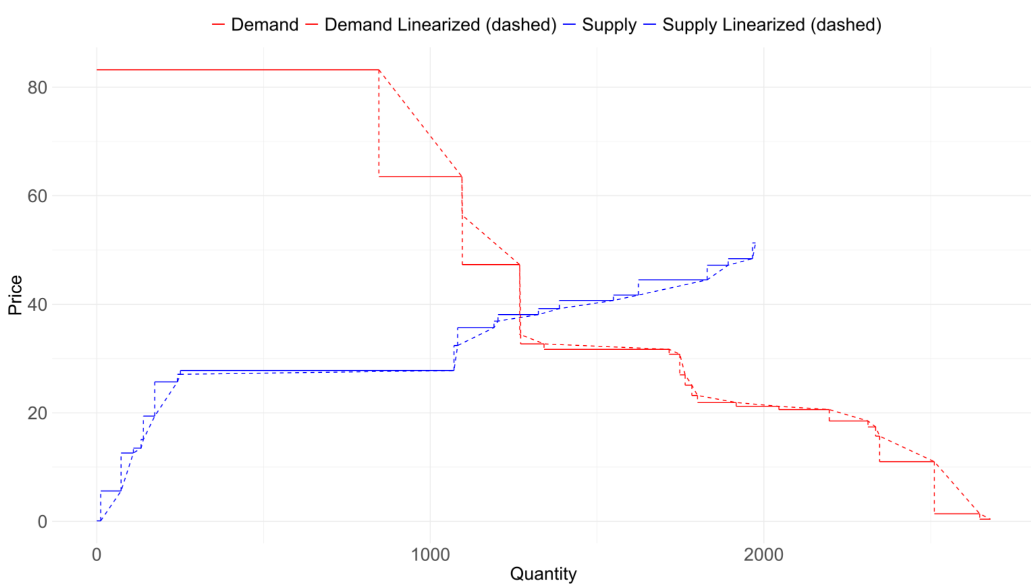

Figure 2: Supply and demand curves in period 3 including linearized curves

This is why in practice Nord Pool linearizes the curves as represented by the dashed lines in figure 2. On the one hand, this ensures that there is a clear intersection of supply and demand, i.e. a unique equilibrium. On the other hand, this is also not a perfect solution, since the linearized functions overestimate willingness to pay on the demand side and underestimate the price on the supply side. However, this property is accepted to ensure efficient and fair market clearing. Moreover, market participants know how the market clearing works and can incorporate the framework into their bids.

Looking at figure 2, we can conclude that the system price for period 3 should be between 37€ and 38€.

3.2 Bids and Profit

a) One bid per period with the same volume and varying prices

For this task, we will break down our strategy in to eight steps. Since we assume perfect knowledge about the other supply and demand bids, we include that data in our reasoning. For an overview of the data please refer to the appendix.

Step 1: The first step of our strategy is to analyze the existing equilibrium before entering the market ourselves. We observe that both system price and volume increase consistently from hour 1 to hour 5, with a system price of 22.75€ at volume 820 during hour 1 and a system price of 40.35€ at volume 1596 during hour 5. We thus conclude that hours 4 and 5 have a bigger impact on our overall profit than hour 1. Additionally, the overall higher volumes and bigger price ranges on the demand side of hour 5 offer us more flexibility to bid higher volumes and leave more room for price adjustments, i.e., using our market power as a big producer to influence the equilibrium in a beneficial way. Thus, not only will the biggest part of our profit come from hours 4 and 5, but it will also be easier to drive the system price.

Step 2: We take a bottom-up approach to analyze how our bids drive prices and volumes. That means that in this step we are only focusing on a single hour with our efforts to maximize profit. Since we have to bid the same volume in all periods, it seems sensible to start our analysis in hour 5 for the reasons described in the previous paragraph.

Step 3: Our underlying reasoning is that, in a perfect world, we would maximize our profit by selling as much as possible for as high of a price as possible. That is why we start by bidding the highest volume possible (1000 MWh) and try to see how high we can push the system price selling this volume.

Step 4: We take a look at the bids of the other suppliers to reason where we can enter the market with such a high volume without being "kicked out of the market" for a too aggressive price. For that, we order the bids in hour 5 by increasing bid prices on the supply side and decreasing bid prices on the demand side. If we were to go in the market with a price of 39 just below the last supply price accepted PS (39.7€), we know that no additional demand will arise, because the next bid after the last accepted demand price PD is demand bid 14 with a price of 34€. We can also observe that there is not much demand coming after the demand bid for 42.5€. Consequently, we conclude that it is not a good strategy to try lower the price to get more demand in to the market. We turn to the supply side and try to underbid suppliers with high volume bids in price. Because the system price is determined by the linearized function, we cannot definitely say how low we have to bid below a certain bid to kick out this supplier. Therefore, starting from 36€ we try lowering our price. Bidding 32€ lets us sell all 1000 MWh at a system price of 32.24€. Looking at the data, we see that we kick out high-volume bids 1 and 2 at this price, which allows us to sell our whole production capacity. Further lowering our bid price would decrease the system price, whereas increasing it would result in either less volume no volume sold.

Step 5: While in step 4 we were primarily concerned about finding the optimal price, in this step we start analyzing different volumes. From the previous step we arrived at a profit of 22240€ for hour 5. We calculate the which prices would compensate for which volume loss and conclude that to reach at least the same profit when bidding lower volumes, we must reach substantially higher system prices. For a volume of 800 for example, we would need a system price of at least 37.8€. However, this system price would not allow any other buyers ton enter the market compared to the initial equilibrium (next after 42.5€ is 34€), and on the supply side, there are enough suppliers willing to sell below this, so we could not push those out of the market. Trying slightly lower volumes like 990 and 980 to make sure we do not miss out on any local optimums close to the 1000 optimum, we can still not improve the profit of 22240€. We conclude that for hour 5, bidding 1000 MWh for 32€ results in the optimal profit.

Step 6: We set the volume for all periods to 1000, because we have to bid the same volume every period. We lower the bid price for every period until we sell all 1000 units in every period. This way we achieve the highest possible system price to sell 1000 units in each period.

Step 7: We carry out a basic sensitivity analysis of our solution by lowering the volumes and raise our bid prices across all periods to get more expensive bids accepted in the market and hence a higher system price, that can make up for the lower volume we sell.

Step 8: In Step 7 we realize, that we can achieve higher profits with lower volume bids in hour 1. We now keep the volume at 1000 in these periods but increase our price, knowing well that we will sell less of what we bid. In this we try to be the last supplier that is accepted with a volume that is just shortly below our bid of 1000. We are able to do this strategy and achieve a higher profit only in period 1. In the other periods, we observe a high-volume bid just above our bid, which means if we raised our bid price, we would loose a lot of volume. In these cases, Step 8 does not yield a higher profit.

Table 1 displays our bids, system prices, volumes accepted and our profit for each period. We reach an overall profit of 63,286 € by applying our high-volume low-price strategy.

Table 1: Profit Scenario 1

Bid Quantity (MWh)	Bid Price (€)	Realized Supply (MWh)	System Price (€)	Profit (€)
1000	14	972	14.08	3966
1000	9	1000	18.31	8310
1000	27	1000	27.54	17540
1000	21	1000	21.23	11230
1000	32	1000	32.24	22240
			Total Profit:	**63286**

Concluding, we can infer from our strategy that volume is the much bigger leverage than price in bidding as a supplier and in general it pays off to place a bid in a way that allows you to sell the maximum amount, at least when the volume cannot vary across periods. Furthermore, it seems to be a good strategy to change the variables *ceteris paribus*, i.e., while keeping the other variables constant. Obviously, we cannot be certain that our optimum is a global optimum or just a local one. Especially marginal effects around the volumes that we were not able to test all, could potentially yield higher profits.

b) At most one bid per period with varying volume

In the second scenario we have the flexibility to not submit a bid in every period. However, not submitting a bid would only make sense in an equilibrium where the system price drops below our production costs of 10€/MWh, which does not occur with the given data.

To maximize our profit in scenario 2, we start from the status quo displayed in table 1, and reevaluate the volume of our bids hour by hour as described above in step 5. Our goal is to find a lower volume such that the potential increase in system price overcompensates for the volume loss.

Looking at our competitors in hour 1, we see that the last supplier is the powerplant bidding in at 7.50€. After that there is a range of suppliers with higher prices but relatively low volumes. Getting these suppliers into the market could be a good strategy for us to drive up the system price without loosing to much volume. When doing that, we arrive at a bid of 790 MWh. If we decrease volume beyond that point, we drive the large demand at 16.50€ out of the market and loose too much volume, if we bid more than 790 MWh we do not exploit the full increase potential in system price. After finding this volume we look into how we can modify our bid price to further increase our profit. Again, referring to the demand data, we have to ensure to keep the large demand at 16.50€ in the market, which is why we conclude that a bidding price of 16€ is maximizing our profits.

When applying the same reasoning to the other time periods, we find that, for hour 2, lowering our volume to 760 looks like a suitable solution. The price at which we bid in cannot be higher than 13€, because we have to avoid driving the supplier at 13.50€ out of the auction. If that happens, because of the linearized nature of the demand and supply curves, the system price would be too high for the next demand point to enter the auction at 20.4€ and we cannot sell all the volume we offered anymore.

We analyze all other time periods in the same way, but do not find room for improvement there, as any deviation from the 1000 MWh bid would yield a lower profit. Hence, the bids we submit in scenario two can be obtained from table 2:

Table 2: Profit Scenario 2

Bid Quantity (MWh)	Bid Price (€)	Realized Supply (MWh)	System Price (€)	Profit (€)
790	16	790	16.50	5135
760	13	760	21.38	8648.8
1000	27	1000	27.54	17540
1000	21	1000	21.23	11230
1000	32	1000	32.24	22240
			Total Profit:	**64793.8**

3.3 Block Bids

In scenario 3, we are submitting a block bid for four consecutive periods. In analogy to Step 1 described above, we first analyze the equilibrium before entering the market ourselves. Since prices and volumes are higher in periods 1 to 4, we consider these first four hours for our block bid. Moreover, we stick to our strategy of maximizing our profit through high volume and use our full capacity as a starting point. The benchmark price for our block bid is 33.7€. If we bid higher than this price, our bid exceeds the average price and is not accepted.

When we bid the highest possible price with which we are still accepted, our block bid is the fourth block bid accepted. Block bids 2, 3, and 6 are accepted before ours. The volume of accepted block bids is fixed in the market, and there is only supply block bids. Since demand does not increase, accepted block bids drive regular bids out of the market. The suppliers that are pushed out of the market are the ones with the highest prices, resulting in a lower system price. From this, it follows, that the more block bid volume is accepted, the more normal bids are outside the equilibrium, and hence the lower the system price is. This logic is critical for the next step of our strategy.

We now place a block bid of 1000 MWh for the lowest possible price, 1€. By this we ensure, that our block bid is accepted first and with the low bid price, we hope to lower the benchmark average price in the next iteration that determines whether the next higher block bid is accepted. We observe that with this bid we are able to drive block bid 3 and 6 out of the accepted block bids. And indeed, the system price increases and we obtain a higher profit. However, it seems that even with the lowest bid price, and a volume of 1000 MWh, we are not able to get block bid 2 rejected.

We test our strategy by increasing the price and find that increasing it up to 25 € gives the same system prices, so no additional block bids apart from 2 and 11 are accepted. If we go above 25 €, new bids are accepted and system prices decreases again.

In the last step of our strategy, similarly to task 2, we try to lower the volume, keeping the system price at 1 €. Even though the system prices increase, this increase can not make up for the loss in volume, and our profit declines. Thus, we reason that bidding 1000 MWh for 1 € gives the optimal profit for us and we stick with this block bid. We obtain an optimal profit 58,420 €, and hourly profits are displayed in table 3.

Table 3: Profit Block Bids

Period	Bid Quantity (MWh)	Bid Price (€)	Realized Supply (MWh)	System Price (€)	Profit (€)
1	1000	1	1000	32.03	22030
2	1000	1	1000	18.06	8060
3	1000	1	1000	27.52	17530
4	1000	1	1000	20.80	10800
				Total Profit:	**58420**

4 Part D: Optimal Public Auction

We define the following integer linear model to minimize the cost of procurement from companies that provide their service to municipalities in our country.

Indexes and Sets:

$k \in K$: set of counties.
$j \in J$: set of municipalities.
$j \in \bar{J}_k, \bar{J}_k \subset J \; \forall k \in K$: set of municipalities in county k.
$i \in I$: set of companies.
$b \in B$: set of bids.

Parameters:

$a_{i,b} \in \{0, 1\}$: 1 if bid b is placed by company i, 0 otherwise.
$m_{j,b} \in \{0, 1\}$: 1 if municipality j is included in bid b, 0 otherwise.
P_b: procurement price for bid b.
D_i: discount offered by company i when purchase amount is at least T_i.
T_i: minimum purchase amount from company i to get discount D_i.
$maxN$: maximum number of companies that can be accepted in the whole country.

Decision Variables:

$x_b \in \{0, 1\}$: 1, if bid b is accepted, 0 otherwise.
$y_i \in \{0, 1\}$: 1, if threshold of company i is reached, 0 otherwise.
$z_i \in \{0, 1\}$: 1, if company i has at least one bid accepted, 0 otherwise.
$w_{i,k} \in \{0, 1\}$: 1, if company i has a bid accepted in county k, 0 otherwise.
$u_i \in \{0, 1\}$: auxiliary variable to capture if threshold is reached or not.

The total procurement cost can be described by the sum of the prices of all accepted bids minus granted discounts. Thus, our objective follows as:

$$minimize\ cost = \sum_{b \in B} x_b * P_b - \sum_{i \in I} y_i * D_i \tag{13}$$

The objective is subject to the following constraints. First, we need to ensure service coverage to all municipalities J. It is important to emphasize that we can accept more than one bid containing municipality j, and thus this case becomes a set covering problem:

$$\sum_{b \in B} x_b * m_{j,b} \geq 1 \quad \forall j \in J \tag{14}$$

Furthermore, every county k must be covered by at least 2 different companies. To cover this, we set up a separate variable $w_{i,k}$ as described above. The following constraints ensure that $w_{i,k} = 1$ when company i has one or more bids accepted in county k (Eq. 15 and 16) and that at least 2 companies have one or more bids accepted in every county (Eq. 17):

$$w_{i,k} * M \geq \sum_{j \in \bar{J}_k} \sum_{b \in B} (a_{i,b} * m_{j,b}) * x_b \quad \forall i \in I, k \in K \tag{15}$$

$$w_{i,k} \leq \sum_{j \in \bar{J}_k} \sum_{b \in B} (a_{i,b} * m_{j,b}) * x_b \quad \forall i \in I, k \in K \tag{16}$$

$$\sum_{i \in I} w_{i,k} \geq 2 \quad \forall k \in K \tag{17}$$

However, we need to make sure that the total number of companies accepted in the whole country does not exceed $maxN$. Similar to constraints 15-17, we introduce another variable z_i. $z_i = 1$ when company i has one or more bids accepted (Eq. 18 and 19) the sum over z is then limited by $maxN$ (Eq. 20):

$$z_i * M \geq \sum_{b \in B} a_{i,b} * x_b \quad \forall i \in I \tag{18}$$

$$z_i \leq \sum_{b \in B} a_{i,b} * x_b \quad \forall i \in I \tag{19}$$

$$\sum_{i \in I} z_i \leq maxN \tag{20}$$

Lastly, we must make sure that the company discount D_i is only subtracted when the purchase amount from company i is at least as high as threshold T_i. To capture this logic, we introduced variable y_i and auxiliary variable u_i. The following three constraints restrict y_i to 1 only when $\sum_{b \in B} x_b * (a_{i,b} * P_b) - T_i \geq 0$ for i, meaning the purchase amount is bigger than the threshold. Otherwise, y_i will be 0:

$$y_i * M \geq \sum_{b \in B} (a_{i,b} * P_b) * x_b - T_i + 1 \quad \forall i \in I \tag{21}$$

$$u_i * (-M) \leq \sum_{b \in B} (a_{i,b} * P_b) * x_b - T_i \quad \forall i \in I \tag{22}$$

$$u_i + y_i = 1 \quad \forall i \in I \tag{23}$$

References

Gentry, S., Chow, E., Massie, A., & Segev, D. (2015). Gerrymandering for justice: Redistricting u.s. liver allocation. *Interfaces*, *45*(5), 462–480. https://doi.org/10.1287/inte.2015.0810

Appendix

Table A1: Supply and Demand Period 1

DEMAND				SUPPLY			
Period	**Consumer No.**	**Quantity**	**Price**	**Period**	**Consumer No.**	**Quantity**	**Price**
1	2	1	76.6	1	15	71	0.1
1	13	21	49.2	1	3	119	4
1	5	222	41.6	1	10	38	7.5
1	8	3	35.3	1	4	13	11.4
1	14	21	33	1	6	76	14.7
1	3	179	32.5	1	17	61	15.5
1	11	13	31.1	1	9	101	17.7
1	7	2	29.7	1	18	203	18.8
1	18	114	29.1	1	11	138	21.5
1	4	18	29	1	19	98	22.9
1	20	247	26.5	1	1	378	28.5
1	15	1	25	1	5	8	33.6
1	16	31	23.7	1	12	56	37.2
1	1	265	16.5	1	16	8	40.3
1	9	73	16.5	1	2	35	41.4
1	10	2	14.9	1	14	54	43.7
1	17	114	12.3	1	13	64	46.2
1	12	2	6.9	1	7	26	47.5
1	19	328	1.3	1	20	33	48.5
1	6	20	0.3	1	8	99	51.9

Table A2: Supply and Demand Period 2

DEMAND				SUPPLY			
Period	**Consumer No.**	**Quantity**	**Price**	**Period**	**Consumer No.**	**Quantity**	**Price**
2	2	1	63.2	2	11	24	0.1
2	1	815	52.7	2	16	6	6.6
2	5	249	51	2	1	878	9.1
2	3	190	50.3	2	18	64	11.3
2	6	21	35.2	2	13	39	13.5
2	20	22	34.1	2	4	12	15.8
2	11	122	30.4	2	2	122	25.8
2	14	26	29.5	2	5	6	31.2
2	13	33	29	2	19	8	33.2
2	9	11	27.9	2	7	70	34.3
2	17	134	27.4	2	9	79	35.7
2	8	74	27.3	2	3	9	36.5
2	12	37	24.8	2	6	155	38.9
2	4	18	23.2	2	14	30	40.1
2	15	151	20.4	2	10	157	41.5
2	16	165	18.1	2	17	63	42.2
2	18	114	15.5	2	8	6	44.2
2	7	3	12.3	2	15	102	45
2	10	16	3.3	2	12	265	45.3
2	19	30	0.5	2	20	63	49.8

Table A3: Supply and Demand Period 3

DEMAND				SUPPLY			
Period	**Consumer No.**	**Quantity**	**Price**	**Period**	**Consumer No.**	**Quantity**	**Price**
3	1	846	83.2	3	6	12	0.1
3	5	249	63.5	3	17	61	5.6
3	2	1	56.4	3	13	37	12.6
3	3	172	47.3	3	11	23	13.5
3	6	3	34.4	3	8	7	15.1
3	7	70	32.7	3	18	34	19.4
3	20	375	31.7	3	15	68	25.7
3	10	32	30.8	3	5	9	27.1
3	4	16	27	3	1	820	27.8
3	9	20	25.1	3	4	11	32.4
3	8	17	23.2	3	12	109	35.7
3	13	116	21.9	3	3	12	36.9
3	16	128	21.2	3	10	121	38.1
3	14	151	20.6	3	19	63	39.2
3	12	116	18.5	3	7	162	40.7
3	17	23	17.4	3	2	75	41.7
3	19	12	15.7	3	16	206	44.5
3	15	164	11	3	20	63	47.2
3	18	136	1.4	3	9	73	48.4
3	11	32	0.4	3	14	9	51.3

Table A4: Supply and Demand Period 4

DEMAND				SUPPLY			
Period	**Consumer No.**	**Quantity**	**Price**	**Period**	**Consumer No.**	**Quantity**	**Price**
4	1	873	67.5	4	15	61	0.1
4	2	1	66.2	4	5	9	6.1
4	5	212	60	4	14	67	8.7
4	3	172	53	4	10	37	10.4
4	6	34	36.5	4	8	111	12.3
4	7	3	34.1	4	17	65	16.4
4	20	33	27.6	4	18	230	17.5
4	9	13	27	4	9	118	21.3
4	8	70	26.4	4	1	182	22.4
4	13	32	25.9	4	2	36	29.7
4	15	33	24.9	4	11	70	31.9
4	12	120	23.6	4	12	64	33.1
4	17	11	22.1	4	19	109	33.7
4	14	132	20.3	4	3	9	34.1
4	16	232	16	4	20	29	37.9
4	19	162	11.2	4	7	98	40.1
4	10	20	9.9	4	13	66	41.2
4	11	24	2.4	4	16	8	42.1
4	18	162	1.5	4	4	13	45.1
4	4	17	0.3	4	6	449	49.5

Table A5: Supply and Demand Period 5

DEMAND				SUPPLY			
Period	**Consumer No.**	**Quantity**	**Price**	**Period**	**Consumer No.**	**Quantity**	**Price**
5	1	821	75.7	5	5	38	0.1
5	2	1	58.8	5	14	38	1
5	6	345	53.3	5	3	152	4.4
5	3	188	44.5	5	4	35	6.6
5	5	243	42.5	5	16	66	8.7
5	14	28	34	5	12	48	11.8
5	11	12	32.2	5	6	99	16.1
5	16	19	31.1	5	18	97	22.3
5	12	28	30	5	8	27	31.7
5	17	181	27.7	5	2	159	33
5	10	32	26.1	5	20	34	34.9
5	15	31	25.3	5	1	674	36.5
5	18	204	22.4	5	10	51	37.5
5	19	167	20.8	5	11	78	39.7
5	9	68	20.2	5	9	28	41.7
5	7	40	11.9	5	17	66	45.1
5	20	242	11.7	5	7	140	48.8
5	8	3	7.5	5	13	152	51.7
5	4	17	4.1	5	19	40	58.7
5	13	120	0.3	5	15	41	59.5