PROJECT 3

Submit your project through *WISEflow*. Please do not share nor reproduce questions or answers of this project in other internet websites or digital platforms. The submission deadline is Tuesday November 14th, at 14:00 hrs. The project can be done individually or in a group of at most 2 students. No cooperation between people who are not submitting this project as a group is allowed. It is possible to change groups throughout the semester and it is also possible to do some project(s) alone and other project(s) in a group. Provide **all your AMPL files** (model code, data, running commands, solution file, etc.) and other files you have used in your computations (e.g. Excel file) compressed in a single file (.zip or .rar). Include all files needed to calculate and run all parts of the project, even if from one to another task the changes are just marginal (we need all files to be able to run without modifying what you submit). In addition, provide a written report in **pdf** with your model formulations and the answers to the questions required in each part. The formulation of your models in the report can be typed using a text editor (e.g. Word, LaTeX), written by hand and scanned, or copied directly as text or screenshot from the AMPL code files when it applies (please just be careful the presentation must be clear enough for a reader). In the written report, it is fine that when there is just a marginal change from one task to another, in the latter you include just the modified part of the formulation (e.g., in task 2 you just defined a new variable or modified one constraint of the model you formulated in task 1, then it is fine that you included the full model formulation in task 1 and only the new variable definition and new constraint that you modified in task 2). Provide a short description (no more than two sentences, e.g. "#demand fulfillment") for every objective function and constraint in your formulations. Expected (not required) length of your report: Part A two pages, Part B one page, Part C two pages, Part D one page.

Part A

In the context of time series forecasting, a modified version of the simple exponential smoothing method that we studied in Lecture 13 incorporates an additional smoothing constant to capture a trend. The aim of this task is to apply this modified method to forecast the *Price index for existing dwellings* (PIFED), an index used in Norway to express the value of the stock of existing dwellings, based on current price information of existing dwellings sold on the free market. The index is updated quarterly.[1]

Standing on trimester t, we will calculate the forecast of the PIFED for the next trimester $(t+1)$ according to the following expression:

$$F_{t+1} = L_t + T_t, \tag{1}$$

where L_t is called the *level* (which tries to capture information on the current value of PIFED), and T_t is called the *trend* (which tries to capture the growth or decline in the value of PIFED). With initial values of the level and trend given, and after observing the actual value of PIFED in trimester $t+1$, the level and the trends are updated as follows:

$$L_{t+1} = \gamma \cdot PIFED_{t+1} + (1-\gamma) \cdot (L_t + T_t) \tag{2}$$

$$T_{t+1} = \beta \cdot (L_{t+1} - L_t) + (1-\beta) \cdot T_t \tag{3}$$

In expressions (2) and (3) above, γ and β are smoothing constants, playing a similar role as α in the original method we studied in the lecture. We will restrict both γ and β to take real numbers in the interval $[0.01, 0.99]$.

The file "DataA.txt" contains data on the value of PIFED for all the 120 quarters spanning from 1993 to 2022. Consider as initial level a value equal to 18.8, and as initial trend a value equal to 0.5. These values somehow capture data about estimates done before 1993, and are given to you as data to implement a model using expressions (1)-(3). Your task is to formulate a non-linear model to find the values of γ

[1] More information about the index and the source of the data for this question can be found on this webpage from *Statistics Norway*: *https://www.ssb.no/statbank/table/07221*

and β that minimize the mean absolute percentage error (MAPE) over the interval of 120 quarters in years 1993-2022. In your report, include a screenshot of your .mod file (or copy-paste its code lines). Implement and run the model in AMPL, using the solver **minos**, and answer the following tasks in your report.
a) What is the optimal value of MAPE?
b) What are the optimal values of γ and β?
c) Among the 120 quarters, when is the maximum absolute percentage error and how much is it?
d) Among the 120 quarters, when is the minimum absolute percentage error and how much is it?
e) What would be your forecast for the PIFED of the first quarter of 2023? Considering that the actual value published by *Statistics Norway* was 140.0, how far is your forecast?
f) Show in a same plot the forecast values and the actual values of PIFED, including the 121 quarters spanning from 1993 until the first quarter of 2023. You may generate the plot using a software of your choice (e.g. Excel, R) and then paste it in your report as a figure.

Part B

In the article "Gerrymandering for Justice: Redistricting U.S. Liver Allocation" (available on a link in the *Complementary readings* folder in *Canvas*), the authors describe the use of a decision model to partition USA's donor service areas into sharing districts that minimize the disparity in liver availability among districts. The Appendix of the article outlines a model formulation. As an expert in decision modelling, you have been approached by the health authorities to incorporate some modifications to the model. These modifications must be **linear** and may involve the definition of new variables, the formulation of new constraints, and/or the modification of constraints of the original formulation.

The modifications requested by the authorities are described in the two tasks below (each task is independent from each other). Your answer must be formulated in mathematical terms (not in AMPL code).

1. Concerned about solutions that may look unfair regarding large differences in the number of transplant centers of the districts, the health authorities require that no district can have more than six neither less than six transplant centers than the average number of transplant centers across districts. Which modification(s) would you introduce in the model to capture this condition?

2. The health authorities have created a *Top 10* list with the ten DSAs which most livers have recovered for transplantation (during the time window under study). For the ease of notation, we will say that $i = 1, 2, \ldots, 10$ correspond to the indexes of these DSAs in the set $\mathcal{J}$. We will refer as a *strong district* to a district containing three or more DSAs from the Top 10 list. The health authorities require that your solution can contain at most two strong districts. Which modification(s) would you introduce in the model to capture this condition?

Part C

In this part we will study some of the key aspects of the Nord Pool day-ahead market. These are the basis behind system prices and two bid types: hourly bids and block bids. As a basis for the questions, the file model.mod is provided. This file is ready to run (so you don't have to create your own .run file). Since some Linux and Mac operative systems use the syntax *./minos* to call the solver, you might need to edit line 36 in file model.mod by adding the characters "./". The models implemented in this file attempt to maximize the social surplus on the electricity market. It uses two different descriptions of the supply and demand curves. The first description is based on the step functions obtained from the total volume demand (supply) given a price below (above) each distinct price of any bid. The other resembles the approach at Nord Pool by using linear approximations of the step functions to guarantee a unique intersection between the demand and supply, which defines the equilibrium price. The code also implements a procedure for block bids, which works as follows. First, the problem without block bids is solved. Then, the block bids are studied iteratively, where the price of each block bid is compared against the average system prices over the time periods included in each block bid. If there is more than one block bid that should be accepted (better block price than the average price) the one with lowest price is selected. Then, the volume of the selected block bid is assumed to be fixed and the model (without block bids) is run again. This procedure is repeated until no more block bids are accepted.

Day-ahead market without block bids

1. In the data file hour.dat there is data for five time periods or hours. This instance includes 20 supply bids and 20 demand bids for each period. Study the file and draw a figure of the supply and demand curves for one of these five periods[2]. Draw also the linearized curves. Based on your figure, which price (approximately) would you propose as the system price for this period and which bids should be accepted? Briefly (no more than 200 words) discuss the advantages and disadvantages with the step function curves and with the linearized curves in this example.
   Note: You can run model.mod and use the report file plot.txt to draw the figures in Excel (follow instructions in lines 15-30 of model.mod).

2. Run the model using the file hour.dat and study the results. Suppose you are a new supplier with a power plant with a capacity of 1000 MW per time period. Your own production cost is 10 euro per MWh. Decide the best bid you can come up with in order to maximize your profit in each of these two scenarios:

   a) You must place exactly one bid per period, the volume must be the same for all periods and can be any non-negative integer number, and the price may vary from one to another period but it always has to be an integer number greater than 1 euro.
   b) You can place at most one bid per period, the price may vary from one to another period but it always has to be an integer number greater than 1 euro, and the volume may also vary from one to another period but it always must be a multiple of 10.
   Calculate the profit you obtain in each scenario and explain the reasoning behind your strategy.

Day-ahead market with block bids

3. In the files blockbids_$\{1, 2, 3, 4, 5\}$.dat there is data for hourly and block bids. The data file you have to use is defined in analogous way as for task 1. Run the model and study the results (you don't have to write about this in the report; it is just for you to get familiar with this data instance and the equilibrium). Suppose you are a new supplier with a power plant with a capacity of 1000 MW per time period. Your own production cost is 10 euro per MWh. Suppose you only can place one bid. This bid must be a block bid that covers four consecutive time periods (note there are five periods in the data file). The price of your block bid cannot be equal to the price of any other block bid in the file and must be greater than 1.0 euro (you may use up to one decimal in your price). The volume must be a multiple of 10. Decide the best block bid you can come up with in order to maximize your profit. Explain the reasoning behind your strategy. How much is your profit?

   Hints: You are not expected to create a model to properly "optimize" your bids nor to run thousand different combinations. Rather, try to explore the equilibrium and design a strategy which allows you to progressively arrive at a favourable result. In order to explore the performance of your bids, you can incorporate your bids in the data files: for hourly bids, write the volume and price of your bids as values for parameters *InputQS* and *InputPS* at the end of the file hour.dat; for block bids, do it after line 29 in the file blockbids_$\{1, 2, 3, 4, 5\}$.dat. Note you will have to update the value of parameter *no_supply_bids* and *no_block_bids* in tasks 2 and 3, respectively (yours will be the hourly bid 21 for each period in task 2 and the block bid 11 in task 3). The data file name must be given in lines 44 or 45 of the file model.mod (do not modify other lines in model.mod). If the price of your hourly bid coincides with the highest price of an accepted supply bid in the corresponding period (which can be seen under the column header "PS" in results.txt), assume that the volume accepted of your bid was either all the volume that you bid or the volume of the last supply bid accepted (the value under the column header "s" in results.txt) in case the latter is less than the former. When calculating your revenue during a period, you must consider the equilibrium price for the corresponding period (which can be seen under the column header "Price" in results.txt).

[2] The period you have to study is given as follows. Consider the ordered set of numbers $N = \{1, 2, 3, 4, 5\}$. Divide the last digit of your candidate number by 2 and approximate it to the closest greater or equal integer in N. If you work in teams of two persons, there might be two different last digits in your candidate numbers. If so, choose the lowest one. For example, if you work alone and your number is 83 or 84 you have to study period 2 (because $3/2 = 1.5$ and $4/2 = 2$ are approximated to 2); if your number is 31 and you work in a team with someone whose number is 77 you have to study period 1 (because 1 is lower than 7, and $1/2 = 0.5$ is approximated to 1).

Part D

Auctions are a useful mechanism not only in electricity markets but also in the procurement of goods and services with public funding. Suppose you work for a public organization and the government has asked you to secure the annual coverage of an important service for the population of the country. This service can be provided by a number of private companies. Instead of choosing one or more companies in arbitrary manner, you decide to carry out a public auction, where each of those companies can place bids. To add transparency to the process, a number of rules have been previously stipulated by a public committee, as explained below.

First of all, the country is divided in counties and the counties are divided in municipalities. Denote by K the set of counties and by J the set of municipalities. For all $k \in K$, denote by $\bar{J}_k$ the set of municipalities belonging to county k (of course, $\bar{J}_k$ is a subset of set J).

The set of companies participating in the auction is denoted by I. Each company may present several bids. Denote by B the set of all bids. Each bid contains a price and one or more municipalities. If bid b is accepted, the company which presents this bid commits to provide the service in all the municipalities contained in the bid during a whole year. As companies may present several bids, they eventually might get more than one bid accepted. Let us define the binary parameter a_{ib} equal to 1 if bid b is placed by company i and zero otherwise. Also, let us define the binary parameter m_{jb} equal to 1 if municipality j is contained in bid b and zero otherwise. Note the idea of allowing companies to place bids involving several municipalities helps to capture economies of scales, which may turn beneficial for the auctioneer (for example, because a company can reduce costs by deploying operations in a bundle of municipalities nearby). This principle is similar to the one behind the block bids in the Nord Pool market.

Your task as an auctioneer is to decide which bids must be accepted. Taking care of the public funds, your goal is to minimize the total cost of procuring services from the companies. In the computation of this cost, you must consider the prices of the accepted bids and potential discounts offered by the companies. The price of accepting bid b is equal to P_b, data given to you by the companies when they place their bids. In addition, each company $i \in I$ participating in the auction provides you other two numbers: a discount parameter D_i and a threshold parameter T_i. Here, D_i is a discount offered by company i which becomes effective only if the sum of the prices of the accepted bids of company i is at least T_i.

In meeting the goal of minimizing total cost, your decision must satisfy several conditions. First, you must assure service coverage to all the municipalities, that is, for all $j \in J$ at least one bid containing municipality j must be accepted. Based on bad experiences from the past, it is important to reduce the risk of having only one company serving a whole county, which would become critical if such a single company would go bankrupt. Therefore, the total number of companies with accepted bids including municipalities of county k must be at least two for all $k \in K$. On the other hand, due to the administrative issues in dealing with a large number of service providers at national level, the stipulations also specify a maximum number $maxN$ of companies which can get bids accepted (in total over the whole country).

Formulate an integer **linear** model for this problem. Briefly explain all constraints in your model. AMPL code is not required in this part. In addition to the sets and parameters defined above, you may introduce new notation in your formulation.