CS/MATH111 ASSIGNMENT 4

Problem 1: Give an asymptotic estimate, using the Θ-notation, of the number of letters printed by the algorithms given below. Give a complete justification for your answer, by providing an appropriate recurrence equation and its solution.

(a)
```
algorithm PrintAs(n)
    if n ≤ 1 then
        print("A")
    else
        for j ← 1 to 10
            do print("A")
        for i ← 1 to 5 do
            PrintAs( ⌊n/2⌋ )
```

(b)
```
algorithm PrintBs(n)
    if n ≥ 4 then
        for j ← 1 to n^2
            do print("B")
        for i ← 1 to 6 do
            PrintBs( ⌊n/4⌋ )
        for i ← 1 to 10 do
            PrintBs( ⌈n/4⌉ )
```

(c)
```
algorithm PrintCs(n)
    if n ≤ 2 then
        print("C")
    else
        for j ← 1 to n^3
            do print("C")
        PrintCs( ⌊n/3⌋ )
        PrintCs( ⌊n/3⌋ )
        PrintCs( ⌊n/3⌋ )
        PrintCs( ⌊n/3⌋ )
```

(d)
```
algorithm PrintDs(n)
    if n ≥ 5 then
        print("D")
        print("D")
        print("D")
        if (x ≡ 0 (mod 2)) then
            PrintDs( ⌊n/5⌋ )
            PrintDs( ⌈n/5⌉ )
            x ← x + 3
        else
            PrintDs( ⌈n/5⌉ )
            PrintDs( ⌊n/5⌋ )
            x ← 5x + 3
```

In part (d), variable x is a global variable initialized to 1.

Solution 1:

a) $a \geq 1, b > 1, c > 0, d \geq 0$. If T(n) satisfies the recurrence, $T(n) = aT(\frac{n}{b}) + cn^d$,

$$T(n) = \theta(n^{log_b a}), a > b^d$$

$$T(n) = \theta(n^d logn), a = b^d$$

$$T(n) = \theta(n^d), a < b^d$$

So for part a print("A") runs 10 times and the recursive function runs 5 times with $(\frac{n}{2})$,
The recurrence: $5T(\frac{n}{2}) + 10$.

$$a = 5, b = 2, c = 10, d = 0; 2^0 = 1; 5 > 1$$

With this, we see that the recurrence is $\theta(n^{log_2 5})$.

b) Print ("B") runs n^2 times. The recursive function run 6 times in one loop and 10 times in another loop, so 16 times total (with $\frac{n}{4}$). The recurrence:

$$16T(\frac{n}{4}) + n^2$$

$$a = 16, b = 4, c = 1, d = 2; 4^2 = 16$$

$$16 = 16$$

With this we can see that the recurrence is $\theta(n^2 log(n))$

c) Print ("c") runs n^3 times. The recursive function runs simply 4 times despite not being in a loop with $\frac{n}{3}$. The recurrence:

$$4T(\frac{n}{3}) + n^3$$

$$a = 4, b = 3, c = 1, d = 3; 4^2 = 16$$

$$4 < 27$$

With this, we have that the recurrence is $\theta(n^3)$

d) Since $n \geq 5$, we can't simply disregard the contents like the past problems, as they only applied if n was small, which would not change much. So, print ("D") 3 times, For the if statement, regarding modulo, it is an if else statement, both with $2T(\frac{n}{5})$ recurrence so either way it will run the reccurence 2T(n/5 times). That being said,

$$2T(\frac{n}{5}) + 3$$

$$a = 2, b = 5, c = 3, d = 0; 5^0 = 1$$

$$2 > 1$$

With this, we get $\theta(n^{log_5 2})$

Problem 2: We have three sets A, B, C with the following properties:

(a) $|B| = 2|A|$, $|C| = 3|A|$,

(b) $|A \cap B| = 18$, $|A \cap C| = 20$, $|B \cap C| = 24$,

(c) $|A \cap B \cap C| = 11$,

(d) $|A \cup B \cup C| = 129$.

Use the inclusion-exclusion principle to determine the number of elements in A. Show your work.

Solution 2: By inclusion exclusion principle, $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$. Given the values for all but A, we know:

$$129 = |A| + 2|A| + 3|A| - 18 - 20 - 24 + 11$$

$$129 = 6|A| - 51$$

$$180 = 6|A|$$

$$|A| = 30$$

Problem 3: A company, Nice Inc., will award 45 fellowships to high-achieving UCR students from four different majors: computer science, biology, political science and history. They decided to give fellowship awards to at least 10 students majoring in computer science and at most 7 biology majors. The number of political science and history majors should be between 5 and 8 students each. How many possible lists of awardees are there? You need to give a complete derivation for the final answer, using the method developed in class. (Brute force listing of all lists will not be accepted.)

Solution 3: We have 45 fellowships spread out to 4 majors(Computer Science(c), Biology(b), Political Science(p), and History(h). We need to give c at least 10 fellowships, b at most 7 fellowships, and give p and h 5-8 fellowships each. This means we can make the equations

$$10 \leq c \leq 45$$
$$0 \leq b \leq 7$$
$$5 \leq p \leq 8$$
$$5 \leq h \leq 8$$

By making the left side of the inequality 0 we have:

$$0 \leq c' \leq 35$$
$$0 \leq b \leq 7$$
$$0 \leq p' \leq 3$$
$$0 \leq h' \leq 3$$

Now we should find M. We do this by finding the total number of objects, which will be the total amount of fellowships subtracted by the lower bounds of c,p,and h

$$c' + b + p' + h' = 35 = M$$

Since 36 is greater than 25 by a margin, we know that this cannot be true; calculating the combination of something greater than produce 0.

$$S_{tot} = \begin{matrix} m+k-1 \\ k-1 \end{matrix}$$

$$S_{tot} = \begin{matrix} 28 \\ 3 \end{matrix}$$

$$S_{tot} = 3276$$

The total amount of possible lists of awardees are given by the equation:

$$S(c' \leq 35 \wedge b \leq 7 \wedge p' \leq 3 \wedge h' \leq 3) = S_{tot} - S(c' \geq 36 \vee b \geq 8 \vee p' \geq 4 \vee h' \geq 4)$$

Now we can use inclusion-exclusion:

$$3276 - (S(b \geq 8) + S(p' \geq 4) + S(h' \geq 4) - S(b \geq 8 \wedge p' \geq 4) - S(b \geq 8 \wedge h' \geq 4) - s(p' \geq 4 \wedge h' \geq 4) + (b \geq 8 \wedge p' \geq 4 \wedge h' \geq 4))$$

We can find a numerical value now (all of the terms excluding the 3276 being combinations):

$$3276 - (\begin{matrix} 20 \\ 3 \end{matrix} + \begin{matrix} 24 \\ 3 \end{matrix} + \begin{matrix} 24 \\ 3 \end{matrix} - \begin{matrix} 16 \\ 3 \end{matrix} - \begin{matrix} 16 \\ 3 \end{matrix} - \begin{matrix} 16 \\ 3 \end{matrix} - \begin{matrix} 20 \\ 3 \end{matrix} + \begin{matrix} 12 \\ 3 \end{matrix})$$

$$= 3276 - (2024 + 2024 - 560 - 560 + 220)$$

Now to find the total amount of possible lists:

$$= 3276 - (2024 + 2024 - 560 - 560 + 220)$$
$$= 128$$

This means we can make a total of 128 lists of awardees with our conditions

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Academic integrity declaration. This homework was a collaboration between Luis Barrios and Andy Payan. For this homework specifically, the only sources we used to aid us were the lecture/discussion notes as well as messaging Biqian for further clarification on proceeding with the questions.

Submission. To submit the homework, you need to upload the pdf file to Gradescope. If you submit with a partner, you need to put two names on the assignment and submit it as a group assignment. Remember that only LaTeX papers are accepted.