

Backtracking

Triangular Approximation

Christofides

Nearest neighbour

Nearest insertion

## Routecraft

Routing Algorithms for Ocean Shipping and Urban Deliveries

Programming Project II

Desenho de Algortimos

2022/2023



**Developed by:** José Santos | Luís Du | Madalena Ye | G13\_4

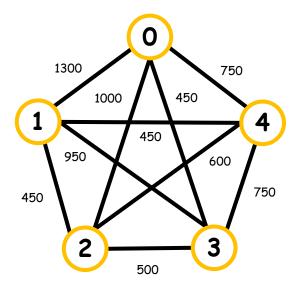








#### Backtracking using Brute Force



```
void Graph::tspBT(std::stack<Vertex*> &bestPath, double &minDist) {
   for(Vertex* v: vertexSet)
      v->setVisited(false);

   Vertex* startingNode = vertexSet[0];

   startingNode->setVisited(true);
   startingNode->setPathCost(0);
   startingNode->setPath(nullptr);

   minDist = INF;
   tspBTRec(0, 0, minDist, bestPath);
}
```



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#### Backtracking using Brute Force

```
void Graph::tspBTRec(int curVertex, int curIndex,
                     double &minDist, std::stack<Vertex*> &bestPath) {
   Vertex* v1 = findVertex(curVertex);
   if (curIndex == vertexSet.size() - 1){
        double cost = v1->getPathCost();
                                                           for(Edge* edge : v1->getAdj()) {
        bool hasCon = false;
                                                                Vertex* v2 = edge->getDest();
        for (auto e: v1->getAdj()) {
                                                                double distance = edge->getDistance();
            if (e->getDest()->getId() == 0) {
                                                                if(!v2->isVisited() && v1->getPathCost() + distance < minDist) {</pre>
                hasCon = true;
                                                                    v2->setPath(edge);
                cost += e->getDistance();
                                                                    v2->setPathCost(v1->getPathCost() + distance);
                break;
                                                                    v2->setVisited(true);
                                                                    tspBTRec(v2->getId(), curIndex+1, minDist, bestPath);
                                                                    v2->setVisited(false);
        if (hasCon && cost < minDist) {</pre>
            minDist = cost;
            bestPath = savePath(v1);
        return;
```





```
1300
                              750
           1000
                       450
                 450
      950
                            600
                                   750
450
                 500
```

```
void Graph::triangularApproximation
  (std::queue<Vertex*> &tour, double &dist) {
    prim();

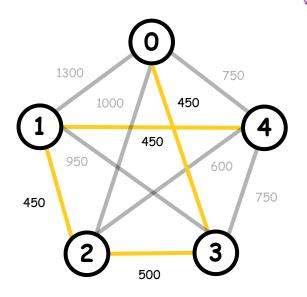
    tour = preOrderTraversal();
    tour.push(vertexSet[0]);

    std::queue<Vertex*> aux = tour;
    Vertex* cur = aux.front();
    aux.pop();
    Vertex* next = aux.front();
    while (!aux.empty()){
        dist += distance(cur, next);
        cur = next;
        aux.pop();
        next = aux.front();
}
```

- Compute MST at root (id = 0) using Prim's algorithm
- 2. Define a pre-order walk of the MST
- Tour the graph using the path induced by the pre-order walk.





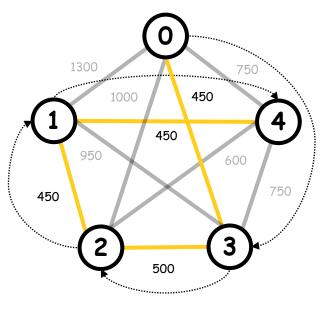


```
void Graph::prim() {
    if (vertexSet.empty()) return;
    for (auto v: vertexSet){
        v->setVisited(false);
        v->setPath(nullptr);
        v->setPathCost(INF);
        v->setMstAdj({});
        v->setDegree(0);
    PriorityQueue q;
    vertexSet[0]->setPathCost(0);
    q.insert(vertexSet[0]);
    while(!q.empty()){
        auto u = q.extractMin();
        u->setVisited(true);
        if (u->getPath() != nullptr) updateMst(u);
        for (auto e : u->getAdj()){
            auto w = e->getDest();
            if (!w->isVisited()){
                auto oldDist = e->getDest()->getPathCost();
                if (e->getDistance() < oldDist){</pre>
                    w->setPathCost(e->getDistance());
                    w->setPath(e);
                    if (oldDist == INF) q.insert(w);
                    else q.decreaseKey(w);
```

- Compute MST at root (id = 0) using Prim's algorithm
- Define a pre-order walk of the MST
- 3. Tour the graph using the path induced by the pre-order walk.



#### Triangular Approximation



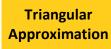
```
void Graph::preOrder
 (Vertex* vertex, std::queue<Vertex *> &1) {
    1.push(vertex);
    vertex->setVisited(true);
    for (Edge* edge : vertex->getMstAdj()) {
        Vertex* w = edge->getDest();
        if (!w->isVisited())
            preOrder(w,1);
std::queue<Vertex *> Graph::preOrderTraversal() {
    std::queue<Vertex *> 1;
    for (auto v : vertexSet)
        v->setVisited(false);
    Vertex* startingNode = vertexSet[0];
    preOrder(startingNode,1);
    return 1;
```

- . Compute MST at root (id = 0) using Prim's algorithm
- Define a pre-order walk of the MST
- 3. Tour the graph using the path induced by the pre-order walk.





Backtracking

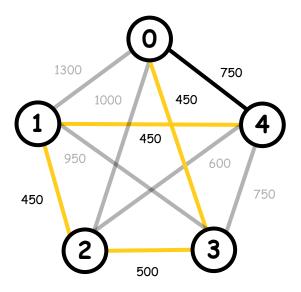


Christofides

Nearest neighbour

Nearest insertion





Path: 0 3 2 1 4 0

Distance: 2600 meters

Execution Time: 0.024375 milliseconds

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- 2. Define a pre-order walk of the MST
- Tour the graph using the path induced by the pre-order walk.





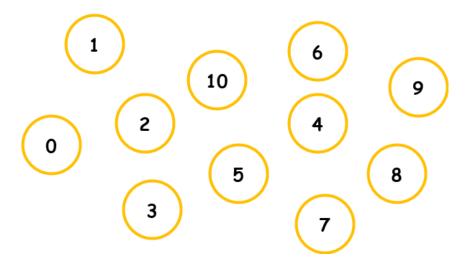
Backtracking

Triangular Approximation

Christofides

Nearest neighbour





- 1. Compute MST
- 2. Add a minimum-weight perfect matching M of the odd vertices in T
- 3. Find an Eulerian Circuit
- 4. Transform the Circuit into a Hamiltonian Cycle





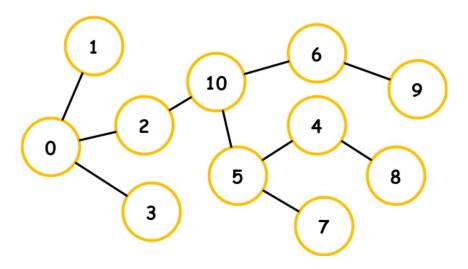
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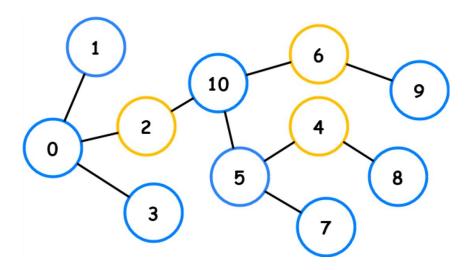
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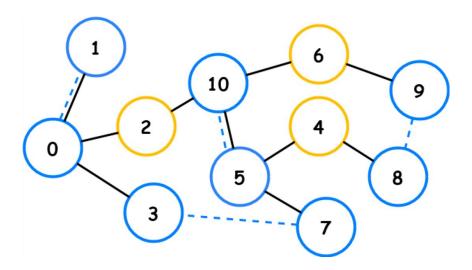
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Backtracking

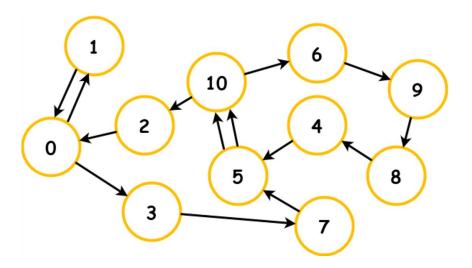
Triangular Approximation

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Backtracking

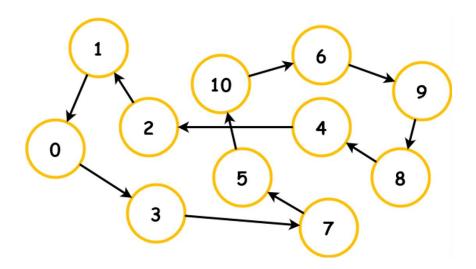
Triangular Approximation

Christofides

Nearest neighbour

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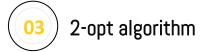


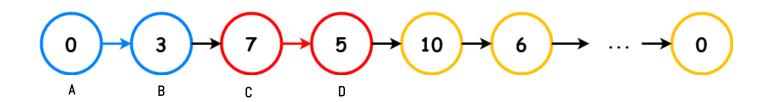


0 -> 3 -> 7 -> 5 -> 10 -> 6 -> 9 -> 8 -> 4 -> 5 -> 10 -> 0

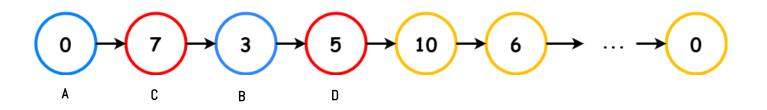
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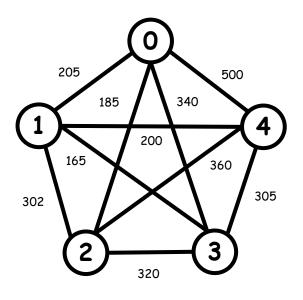




If distance(A,C) + distance(B,D) < distance(A,B) + distance(C,D)



# 04 Nearest neighbour

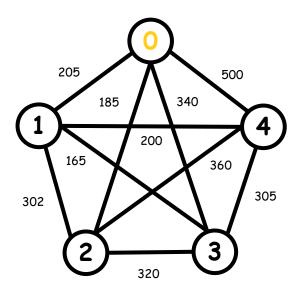


```
void Graph::nearestNeighborTSP
 (std::vector<Vertex *> &tour, double &distance) {
    for (auto v : vertexSet)
        v->setVisited(false);
    Vertex* currentVertex = vertexSet[0];
    currentVertex->setVisited(true);
    while(true){
        tour.push_back(currentVertex);
        double minDist = INT MAX;
        Vertex* nextVertex = nullptr;
        for (auto e : currentVertex->getAdj()){
            Vertex* neighbor = e->getDest();
           if (!neighbor->isVisited()){
                double dist = e->getDistance();
                if (dist < minDist){
                    minDist = dist;
                    nextVertex = neighbor;
        if (nextVertex == nullptr) break;
        nextVertex->setVisited(true):
        distance += minDist;
        currentVertex = nextVertex;
    distance += currentVertex->getAdj()[0]->getDistance();
    tour.push_back(vertexSet[0]);
```

- Start at the root
- Find out the shortest edge connecting the current vertex and an unvisited neighbour
- 3. Repeat the process until all the vertices have been visited
- 4. Return to starting point to obtain the tour





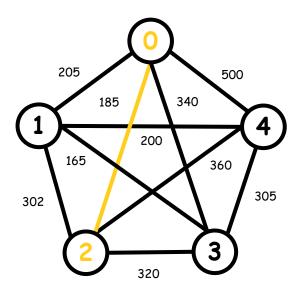


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    while(true){
        tour.push_back(currentVertex);
        double minDist = INT MAX;
        Vertex* nextVertex = nullptr;
        for (auto e : currentVertex->getAdj()){
            Vertex* neighbor = e->getDest();
           if (!neighbor->isVisited()){
                double dist = e->getDistance();
                if (dist < minDist){
                    minDist = dist;
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        if (nextVertex == nullptr) break;
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        currentVertex = nextVertex;
    distance += currentVertex->getAdj()[0]->getDistance();
    tour.push back(vertexSet[0]);
```

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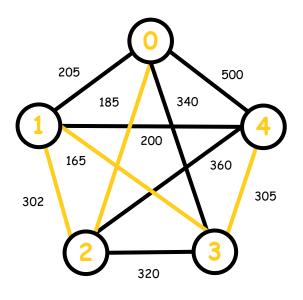


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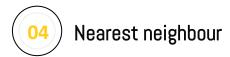


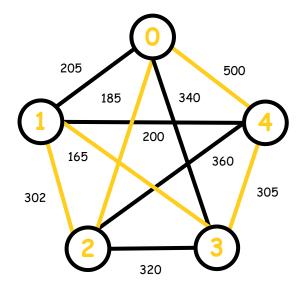


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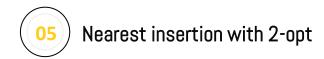


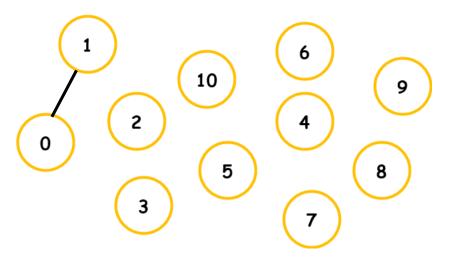
Backtracking

Triangular Approximation

Christofides

Nearest neighbour





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- 3. Apply the 2-opt algorithm



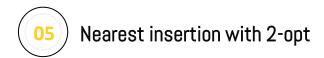


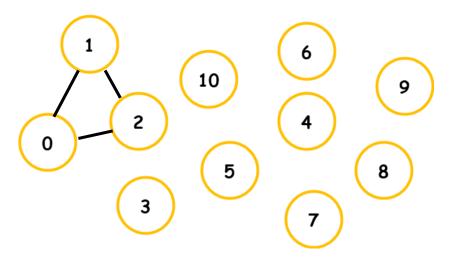
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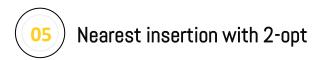


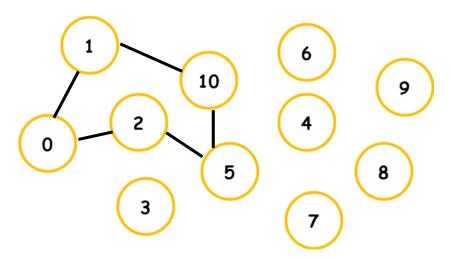
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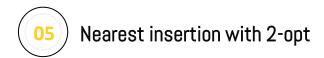


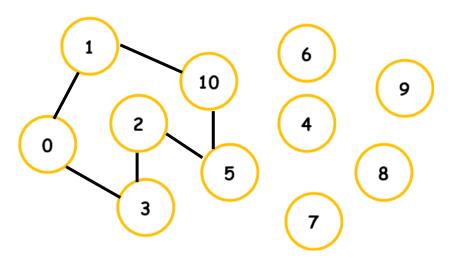
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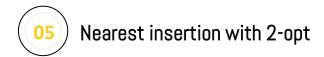


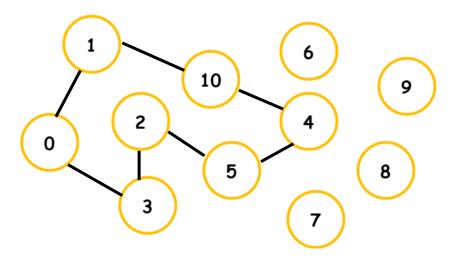
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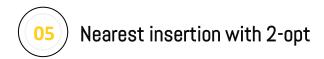


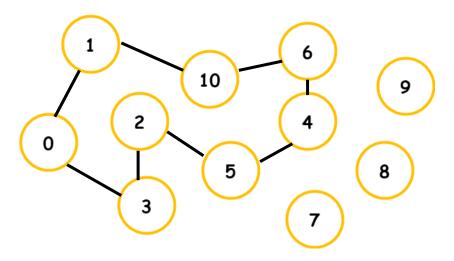
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Backtracking

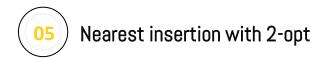
Triangular Approximation

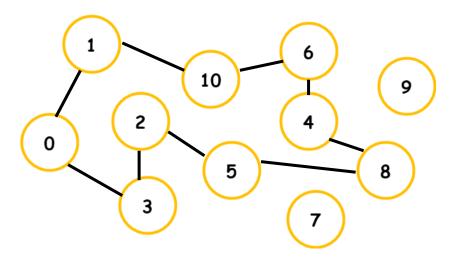
Christofides

Nearest neighbour

Nearest insertion

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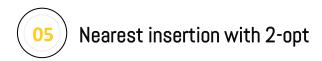


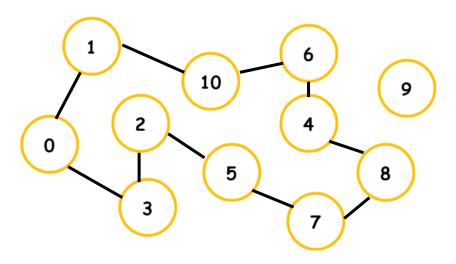
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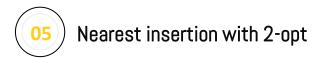


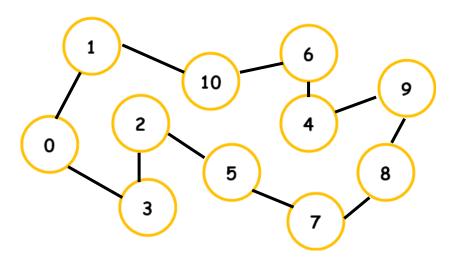
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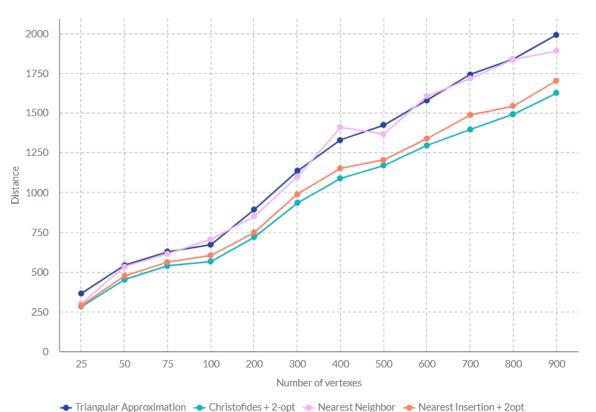
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## **Distance comparison**





### Time comparison

