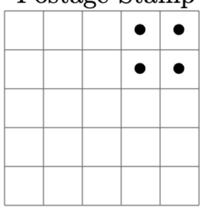
A game of bingo is played on a 5x5 bingo card. The card is populated randomly with 25 values in the range 1-75. Next, 30 numbered balls are drawn in the range 1-75. A card position with number matching a numbered ball is called *daubed*.

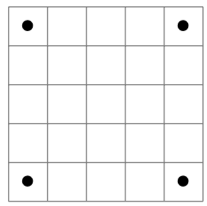
A bingo pattern is a collection of bingo card spots. A pattern is hit if, after the 30 balls are drawn, the bingo card is daubed with at least the spots contained on the pattern.

Consider 2 bingo patterns: Postage Stamp and 4 Corners.

Postage Stamp



4 Corners



1. Since both patterns have 4 daubs, hitting either pattern in 30 balls drawn has the same probability. What is this probability?

We're looking for the probability that a specific 4-daub pattern is selected given that there are **m** daubs on the bingo card.

Let A be the event of getting a specific 4-daub pattern, given **m** daubs, and let B be the event of selecting **m** daubts from 30 number balls, where **m** is a interger between 0 and 25, inclusive.

To start, the probability of A can be described as follows: Reserve 4 numbers out of the 25 available. Thus, you'll have (25 - 4) available numbers on a Bingo card that can be anything. If any additional daubs are found, then the (**m**-4)-daubs can be rearrange in the (25 - 4) available spaces.

The probability of B can be described as follows: From 25 numbers in the bingo card, select **m** numbers to reserve. In addition, we must generate the remaining (30 – **m**) numbers out of the 50 remaining numbers from 75 sample space that were not found in the bingo card.

Thus, we have the following formula

$$P(A) = f(m) = \begin{cases} 0 \quad m < 4 \\ \frac{nCr(21, m-4)}{nCr(25, m)} \quad 4 \le m \le 21 \\ 0 \quad m > 21 \end{cases}$$

$$P(B) = h(m) = \begin{cases} 0 \quad m < 0 \\ \dfrac{nCr(25, m) \cdot nCr(50, 30 - m)}{nCr(75, 30)} \quad 0 \le m \le 25 \\ 0 \quad m > 25 \end{cases}$$

With these two formulas, we can compute the probability that a specific 4-daubs pattern is selected given **m-**daubs.

$$P(4 - daubs\ pattern) = \sum_{m=0}^{25} h(m) \cdot f(m) = 0.022547$$

2. What is the probability of hitting both patterns in 30 balls drawn?

Like question 1, Let A be the event of getting a specific 7-daub pattern, given **m** daubs, and let B be the event of selecting **m** daubts from 30 number balls, where **m** is a interger between 0 and 25, inclusive.

To start, the probability of A can be described as follows: Reserve 7 numbers out of the 25 available. Thus, you'll have (25 - **7**) available numbers on a Bingo card that can be anything. If any additional daubs are found, then the (**m - 7**)-daubs can be rearrange in the (25 - 7) available spaces.

The probability of B can be described as follows: From 25 numbers in the bingo card, select **m** numbers to reserve. In addition, we must generate the remaining (30 – **m**) numbers out of the 50 remaining numbers from 75 sample space that were not found in the bingo card.

Thus, we have the following formula

$$P(A) = f(m) = \begin{cases} 0 \quad m < 7 \\ \frac{nCr(18, m-7)}{nCr(25, m)} \quad 7 \leq m \leq 18 \\ 0 \quad m > 18 \end{cases}$$

$$P(B) = \ h(m) = \begin{cases} 0 \quad m < 0 \\ \dfrac{nCr(25, m) \cdot nCr(50,30-m)}{nCr(75,30)} \quad 0 \leq m \leq 25 \\ 0 \quad m > 25 \end{cases}$$

With these two formulas, we can compute the probability that a specific 7-daubs pattern is selected given **m-**daubs.

$$P(7 - daub\ pattern) = \sum_{m=0}^{25} h(m) \cdot f(m) = 0.001025$$

3. What is the probability of hitting the 4-corner pattern given the postage stamp pattern is not hit in 30 balls drawn?

We're looking for the probability of selecting the 4-corner pattern **and** not selecting the stamp pattern and there being **m** daubs.

Thinking about this in single terms, let A be the event of selecting the 4-corner pattern, let B be the event of not selecting the stamp pattern, and let C be event of selecting **m** daubs, then :

$$P\big((A\ and\ B)\ and\ C\big) = P(A\ and\ B) \cdot P(C) = P(A) \cdot P(B|A) \cdot P(C)$$

From question 1, we know that

$$P(A) = D(m) = \begin{cases} 0 & m < 0 \\ \dfrac{nCr(21, m-4)}{nCr(25, m)} & 4 \leq m \leq 21 \\ 0 & m > 21 \end{cases}$$

$$P(C) = V(m) = \begin{cases} 0 & m < 0 \\ \dfrac{nCr(25, m) \cdot nCr(50, 30-m)}{nCr(75, 30)} & 0 \leq m \leq 25 \\ 0 & m > 25 \end{cases}$$

Let's think about what P(B|A) means. Suppose you have four daubs, then four daubs will be used for the 4-corner pattern, leaving zero for the stamp pattern. Thus, P(B|A) = 1 when the number of daubs equals four, five and six. Given that event A happens, there is only 21 numbers left on the bingo card. One of the numbers from the stamp pattern is taken, so only 3 more numbers remain to complete the pattern.

We can take the complement of the probability of selecting a stamp pattern given the 4-corner pattern is selected to get the probability of not selecting the stamp pattern given the 4-corner is selected.

Thus, using the same methology from question 1, we'll reserve 3 numbers from 21 and let the other 18 values be any number. Any additional daubs will be rearranged in the 18 available numbers.

$$P(B|A): E(m) = \begin{cases} 0 & m < 4 \\ 1 & 4 \leq m \leq 6 \\ 1 - \dfrac{nCr(18, m-7)}{nCr(21, m-4)} & 7 \leq m \leq 25 \\ 0 & m > 25 \end{cases}$$

Thus, probability of selecting the 4-corner pattern **and** not selecting the stamp pattern **and** there being **m-** daubs on the bingo card can be expressed as follows:

$$P(selecting\ 4 - corner\ pattern\ and\ no\ stamp\ pattern)$$

$$= \sum_{m=0}^{m=25} D(m) \cdot E(m) \cdot V(m) = 0.021522$$

4. Write (Code) a Monte Carlo bingo simulator to validate part (c) running at least 100,000 bingo game simulations. Create a 90% confidence interval to validate your results.

Simulation:

a. For every game played
    i. Generate a bingo card
    ii. Generate number balls
    iii. Determine if the pattern(s) is found
b. Return a nGames by nPatterns boolean matrix, where each row represents a game and each column identifying a pattern(s), where 1 means pattern was found, 0 otherwise

Using 30 simulations, with each simulation having 100,000 bingo games, we're 90% confidence that the population probability means for obtaining a 4-corner pattern and not obtaining a Postage Stamp pattern is within (0.0214, 0.0217) interval.

The Image below shows the sample mean for each pattern after 30 simulations

```
               Patterns  Probability
0             4 Corners     0.022614
1  4 Corners & Not Stamp     0.021559
2         Both patterns     0.001054
3         Postage Stamp     0.022509

With 90% Confidence, the population probability mean of the pattern '4 Corners & Not Stamp' is within this confidence interval
Confidence Interval:(0.0214,0.0217)
```

5. Time part (d). How long does it take to run? Make some optimizations to decrease the program's runtime.

Initial:

Each simulation runs 100,000 bingo games on an average of 6.4 seconds.

Changes:

- Instead of doing a "+" operation to set my boolean variable from 0 to 1, I used "=" to set my boolean varibale to 1
- Set array data types from int to bool
- This decreased the avg. runtime for a simulation to 6.11 seconds.

Possible Changes:

- Instead of using a boolean matrix of NxN to present as my pattern, I could use a $1\ x\ N$^2 vector and potetntially decrease the slicing time by a few milliseconds.
- I could also check the value that the two patterns have in common (upper right corner). If I see that the value is not in my number ball list, then I know the two patterns are not complete and I save myself 6 value checks.