Lista 8

Nome: [illegible]

1- a) $\vec{d_2} = (\frac{1}{3}, \frac{1}{2}, 1)$ $\quad \vec{d_1} = (\frac{1}{2}, 1, 1)$

$\vec{d_1} \cdot \vec{d_2} = (\frac{1}{2}, 1, 1) \cdot (\frac{1}{3}, \frac{1}{2}, 1) = \frac{1}{6} + \frac{1}{2} + 1 = \frac{5}{3}$

$\|d_1\| = \sqrt{(\frac{1}{2})^2 + 1 + 1} = \sqrt{\frac{9}{4}} = \frac{3}{2}$ $\quad \|d_2\| = \sqrt{(\frac{1}{3})^2 + (\frac{1}{2})^2 + 1} = \sqrt{\frac{1}{9} + \frac{1}{4} + 1} = \sqrt{\frac{49}{36}} = \frac{7}{6}$

$\cos\alpha = \frac{\frac{5}{3}}{\frac{3}{2} \cdot \frac{7}{6}} = \frac{\frac{5}{3}}{\frac{21}{12}} = 60/63$ $\quad \operatorname{sen}\alpha = \sqrt{1 - (\frac{20}{21})^2} = \sqrt{1 - \frac{400}{441}} = \sqrt{\frac{41}{441}} = \frac{\sqrt{41}}{21}$

b) $\vec{d_1} = (0, -1, 1)$ $\quad d_2 = (1, -1, 0)$

$d_1 \cdot d_2 = 1$

$\|d_1\| = \sqrt{2}$ $\quad \cos\alpha = \frac{1}{\sqrt{2} \cdot \sqrt{2}} = \frac{1}{2}$ $\quad \operatorname{sen}\alpha = \sqrt{1 - (\frac{1}{2})^2} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

$\|d_2\| = \sqrt{2}$

c) $d_1 = (-3, 0, 1)$ $\quad \cos\alpha = \frac{-5}{\sqrt{50}} = \frac{-5}{5\sqrt{2}} = \frac{-1 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{-\sqrt{2}}{2}$

$d_2 = (2, 0, 1)$

$d_1 \cdot d_2 = -6 + 1 = -5$ $\quad \operatorname{sen}\alpha = \sqrt{1 - (\frac{\sqrt{2}}{2})^2} = \sqrt{1 - \frac{2}{4}} = \sqrt{\frac{2}{4}} = \frac{\sqrt{2}}{2}$

$\|d_1\| = \sqrt{9 + 1} = \sqrt{10}$

$\|d_2\| = \sqrt{4 + 1} = \sqrt{5}$

d) $\vec{d_1} = (-7, -14, -7)$ $\quad d_1 \cdot d_2 = 7 + 14 - 7 = 14$ $\quad \|d_1\| = \sqrt{3}$

$d_2 = (-1, -1, 1)$ $\quad \|d_2\| = \sqrt{49 + 196 + 49} = \sqrt{294} = 7\sqrt{6}$

$\cos\alpha = \frac{14}{\sqrt{3} \cdot 7\sqrt{6}} = \frac{14}{21\sqrt{2}} = \frac{2}{3\sqrt{2}} = \frac{2\sqrt{2}}{6} = \frac{\sqrt{2}}{3}$

2- $P = (0, 2 + \lambda, 0) \Rightarrow \vec{PQ} = (1, -\lambda, \mu)$ $\quad \cos 45^\circ = \frac{\sqrt{2}}{2} = \frac{(1, -\lambda, \mu) \cdot (0, 1, 0)}{\sqrt{1 + \lambda^2 + \mu^2}}$

$Q = (1, 2, \mu)$

$\cos 45^\circ = \frac{2}{4} = \frac{\lambda^2}{1 + \lambda^2 + \mu^2} = \frac{1}{2}(\lambda^2 + 1^2 + \mu^2) = \lambda^2 \Rightarrow \frac{\lambda^2}{2} + \frac{1}{2} + \frac{\mu^2}{2} = \lambda^2 \Rightarrow \frac{\lambda^2}{2} = \frac{1}{2} + \frac{\mu^2}{2} \Rightarrow \lambda^2 = 1 + \mu^2$

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$\cos 60^\circ = \frac{1}{2} = \frac{(1,-\lambda,\mu)(0,0,1)}{\sqrt{1+\lambda^2+\mu^2}} = \frac{1}{2} = \frac{\mu}{\sqrt{1+\lambda^2+\mu^2}} \Rightarrow \frac{1}{4} = \frac{\mu^2}{1+\lambda^2+\mu^2} = \frac{1}{4}+\frac{\lambda^2}{4}+\frac{\mu^2}{4} = 4\mu^2$

$3\mu^2 = \lambda^2+1$

$\begin{cases} \lambda^2 = 1+\mu^2 \Rightarrow 3\mu^2 = 2+\mu^2 \Rightarrow 2\mu^2 = 2 \Rightarrow \mu^2 = 1 \\ 3\mu^2 = 1+\lambda^2 \Rightarrow 3 = 1+\lambda^2 \Rightarrow \lambda^2 = 2 \Rightarrow \lambda = \sqrt{2} \end{cases}$

3- a) $d_r(1,1,1)$ $d_\pi = (0,0,1)$ $d_r d_\pi = 1$

$\|d_r\| = \sqrt{3}$

$\|d_\pi\| = \sqrt{1}$

$\operatorname{sen}\alpha = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

b) $d_r = (-1,1,1)$ $d_\pi = (2,-1,0)$

$\|d_r\| = \sqrt{3}$

$\|d_\pi\| = \sqrt{5}$

$\operatorname{sen}\alpha = \frac{-3}{\sqrt{15}} = \frac{-3\sqrt{15}}{15} = \frac{-\sqrt{15}}{5}$

c) $d_r = (1,1,-2)$ $\|d_r\| = \sqrt{1+1+4} = \sqrt{6}$

$d_\pi = (1,1,-1)$ $\|d_\pi\| = \sqrt{3}$

$\operatorname{sen}\alpha = \frac{4}{\sqrt{18}} = \frac{4}{3\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{6} = \frac{2\sqrt{2}}{3}$

4- $\pi_1 = (1,1,1)$ $\|d_1\| = \sqrt{2}$ $\frac{\vec{u}\cdot d}{\|d\|} = \frac{1}{2}(1,0,-1) = \left(\frac{1}{2}, 0, -\frac{1}{2}\right)$

$\pi_2 = (1,-1,0)$ $\|d_2\| = \sqrt{2}$

5- a) $u_1 \cdot u_2 = 2-1-3 = -2$

$\|u_1\| = \sqrt{4+1+1} = \sqrt{6}$ $\cos\alpha = \frac{-2}{\sqrt{66}}$

$\|u_2\| = \sqrt{9+1+1} = \sqrt{11}$

b) $v_1 = (1,0,1)$ $\Rightarrow v_1 \cdot v_2 = -1$

$v_2 = (-1,0,0)$

$\|v_1\| = \sqrt{1}$ $\cos\alpha = \frac{-1}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$

$\|v_2\| = \sqrt{3}$

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c)

$V_1 = (1,0,0)$, $V_2 = (1,1,1)$

$$\begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{vmatrix} = (0,-1,1)$$

$\cdot \quad = -1$

$V_3 = (-1,2,0)$, $V_2 = (0,1,0)$

$$\begin{vmatrix} i & j & k \\ -1 & 2 & 0 \\ 0 & 1 & 0 \end{vmatrix} = (0,0,-1)$$

$\|N_1\| = \sqrt{2}$ $\cos\alpha = \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$

$\|N_2\| = \sqrt{1}$

7- b) $P_\lambda = (4\lambda, 2\lambda, -3\lambda+4)$

$P_A = \sqrt{(4\lambda-3)^2+(2\lambda-2)^2+(4-3\lambda-5)^2}$ $\quad P_B = \sqrt{4\lambda^2+(2\lambda)^2+(3-3\lambda)^2}$

$P_A = P_B$ $\quad$ λ é equidistante

c) $P_\lambda = (8,7,-1)$

$P_A = \sqrt{(2+\lambda-1)^2+(3+\lambda-1)^2-(\lambda-3)^2}$ $\quad P_B = \sqrt{\lambda^2+(1+\lambda)^2+(\lambda-7)^2}$

$3\lambda^2+\lambda+18 = 3\lambda^2 \; 12\lambda+50$

$18\lambda = 36$

$\lambda = 2$

8- a)

$A = (1,-2,0)$, $d = (3,2,1)$, $AP = (-3,2,1)$

$$\begin{vmatrix} i & j & k \\ -3 & 2 & 1 \\ 3 & 2 & 1 \end{vmatrix} = (0,-6,-12) = AP \times d$$

$\|d\| = \sqrt{9+4+1} = \sqrt{14}$

$\|AP \times d\| = \sqrt{36+144} = \sqrt{180} = 3\sqrt{20}$

dist $\frac{3\sqrt{20}}{\sqrt{14}}$

b) $A = (2,3,1)$, $\vec{d} = (4,1,-2)$, $\overrightarrow{AP} = (-1,-4,3)$

$$\begin{vmatrix} i & j & k \\ -1 & -4 & 3 \\ 4 & 1 & -2 \end{vmatrix} = (5,-10,15)$$

$\|AP \times d\| = \sqrt{25+100+225} = \sqrt{350}$

$\|d\| = \sqrt{16+1+4} = \sqrt{21}$

dist $= \sqrt{\frac{350}{21}}$

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c) $A = (-3, 0, -\frac{1}{2})$

$\vec{d} = (2, 1, 1)$

$AP = (3, -1, \frac{1}{2})$

$\begin{vmatrix} i & j & k \\ 3 & -1 & \frac{1}{2} \\ 2 & 1 & 1 \end{vmatrix} = (\frac{-3}{2}, -2, 5)$

$\|AP \times d\| = \sqrt{\frac{9}{4} + 4 + 25} = \frac{\sqrt{125}}{2} = \frac{5\sqrt{5}}{2}$

$\|d\| = \sqrt{4+1+1} = \sqrt{6}$

$dist = \dfrac{\frac{5\sqrt{5}}{2}}{\sqrt{6}} = \dfrac{5\sqrt{30}}{2}$

10) a)

$v_1 = (1, 0, 0)$

$v_2 = (-1, 0, 3)$

$P_1 = (0, 3, 4)$

$\begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ -1 & 0 & 3 \end{vmatrix} = (0, -3, 0) = N$

$d = \dfrac{|(0,3,4)(0,-3,0)|}{\|(0,-3,0)\|} = \dfrac{-9}{3} = 3$

b)

$P = (0, 0, -6)$

$a = 1$

$b = 2$

$c = -2$

$d = -6$

$d = \dfrac{|12 \cdot 6|}{\sqrt{1+4+4}} = \dfrac{6}{\sqrt{9}} = \dfrac{6}{3} = 2$

c)

$P = (1, 1, 1)$

$a = 2 \quad b = -1$

$c = 2 \quad d = -3$

$d = \dfrac{|2 - 1 + 2 - 3|}{\sqrt{4+1+4}} = 0$

11) $\begin{cases} x = 2 - \lambda \\ y = \lambda \\ z = 2\lambda - 2 \end{cases} \Rightarrow$

$\sqrt{6} = \dfrac{|(2-\lambda) \cdot 2\lambda - (2\lambda - 2) \cdot 1|}{\sqrt{6}} = \dfrac{3 - 5\lambda}{\sqrt{6}} = \sqrt{6} \Rightarrow 6 = 3 - 5\lambda$

$\lambda = \frac{-3}{5}$

$P = (\frac{13}{5}, \frac{-3}{5}, -\frac{16}{5})$

12 - a) $\begin{cases} x+y+z=0 \\ 3x-y-1=0 \end{cases} \Rightarrow 3x-y-1=x+y+z$ $\quad -2x+y+1=0$

$3x+z-1=0 \quad y=2x-1$

$z=1-3x$

$d_r=(1,-1,1)$; $d_s=(0,-1,1)$

$\begin{vmatrix} i & j & k \\ 1 & -1 & 1 \\ 0 & -1 & 1 \end{vmatrix} = (0,1,1)$

$P_1=(2,1,0)$

$P_2=(0,-1,-1)$

$P_1-P_2=(2,2,-1)$

$v\cdot(P_1-P_2)=(1,1,3)\cdot(2,2,-1)=7 \quad \|v\|=\sqrt{1+1+9}=\sqrt{26}$

$d=\frac{7}{\sqrt{26}}$

b) $d_r=(3,4,-2)$; $d_s=(6,-8,-1)$

$\begin{vmatrix} i & j & k \\ 3 & 4 & -2 \\ 6 & -4 & -1 \end{vmatrix} = (-4,-3,-12)$

$P_1=(-4,0,5)$

$P_2=(21,-5,2)$

$P_1-P_2=(-25,5,-7)$

$v\cdot(P_1-P_2)=(-4,-3,-12)\cdot(-25,5,-7)=169$

$\|v\|=\sqrt{16+144+9}=\sqrt{169}=13$

$d=\frac{169}{13}=13$

c) $d_r=(-2,1,2)$; $d_s=(-1,1,2)$

$\begin{vmatrix} i & j & k \\ -2 & 1 & 2 \\ -1 & 1 & 2 \end{vmatrix} = (0,-1,2)$

$P_1=(1,0,0)$

$P_2=(0,0,2)$

$P_1-P_2=(1,0,-2)$

$v\cdot(P_1-P_2)=-4$

$\|v\|=\sqrt{10}$

$d=\frac{4}{\sqrt{10}}$

13 - a)

$v_1=(1,0,0)$; $v_2=(0,1,0)$

$\begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = (0,0,1) = \sqrt{1}$

$d=9$

$d=\frac{|4-9|}{\sqrt{1}}=\frac{5}{\sqrt{1}}=5$

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b) $\begin{cases} x - y + z = 0 \Rightarrow x - z - 4 + \cancel{z} \Rightarrow x = 4 \\ 2x + y - z = 3 \\ y = z + 4 \end{cases}$

$d = \frac{|8+4|}{\sqrt{4+1+1}} = \frac{12}{\sqrt{6}} \cdot \frac{12\sqrt{6}}{6} = 2\sqrt{6}$

c) $\begin{cases} x = \lambda \\ y = \lambda - 1 \\ z = x - 3 = \lambda - 3 \end{cases}$

$d = \frac{|1+9-10|}{\sqrt{4+1+9}} = 0$

14- b)

$\vec{N_1} = (-1, 0, 3)$
$\vec{N_2} = (1, 1, 0)$

$\begin{vmatrix} i & j & k \\ -1 & 0 & 3 \\ 1 & 1 & 0 \end{vmatrix} = (-3, 3, -1)$

$-3(x-2) + 3(y-1) - 1(z-2) = 0$

$-3x + 6 + 3y - 3 - z + 2 = 3x - 3y + z + 5 = 0$

$d = \frac{|-5-5|}{\sqrt{4+4+4}} = \frac{10}{\sqrt{12}} \cdot \frac{10\sqrt{12}}{12} = \frac{5\sqrt{12}}{6}$

c)

$N_1 = (1, 1, 1)$
$N_2 = (2, 1, 1)$

$\begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 2 & 1 & 1 \end{vmatrix} = (0, -1, 1)$

$\|N\| = \sqrt{1+1} = \sqrt{2}$

$\|N_1\| = \sqrt{3}$

$\|N_2\| = \sqrt{6}$

$d = \frac{\sqrt{3}}{\sqrt{3}} = 1$

15)

$P_1 = (1, 1, 4)$
$P_2 \ (4, 1, 1)$
$v = (4, 2, -3)$

$P_1 - P_2 = (-3, 0, 3)$

$\begin{vmatrix} i & j & k \\ -3 & 0 & 3 \\ 4 & 2 & -3 \end{vmatrix} = (6, -3, -6)$

15- continuação

$6(x-1) - 3(y-1) - 6(z-4) = 0$

$6x - 6 - 3y + 3 - 6z + 24 = 0$

$6x - 3y - 6z + 21$

$d = \frac{|6 - 3 - 6 + 21|}{\sqrt{36 + 9 + 36}} = \frac{18}{9} = 2u$