Lista 2

ALUNO Luis Felipe Barbosa Leite 2024.1.08.018

MATÉRIA: Geometria Analítica

1-a) $a_{11}\cdot\tilde{a}_{11} + a_{12}\cdot\tilde{a}_{12} = 2\cdot(-1)^2\cdot 3 + 1\cdot(-1)^3\cdot(-4) = 10$

b) $a_{11}\cdot\tilde{a}_{11}+a_{12}\cdot\tilde{a}_{12} = \sqrt{2}\cdot(-1)^2\cdot\sqrt{3} + 3\sqrt{6}\cdot(-1)^3\cdot 2 = -5\sqrt{6}$

c) $a_{31}\cdot\tilde{a}_{31}+a_{32}\cdot\tilde{a}_{32}+a_{33}\cdot\tilde{a}_{33} = 1\cdot(-1)^4\cdot\begin{vmatrix}0&2\\-1&1\end{vmatrix} +0+0 = 1\cdot b_{11}\tilde{b}_{11}\cdot b_{12}\cdot\tilde{b}_{12} =$

$=1\cdot(0+2\cdot(-1)^3\cdot -1) = 2$

d) $a_{11}\tilde{a}_{11}+a_{12}\cdot\tilde{a}_{12}+a_{13}\cdot\tilde{a}_{13} = -2\cdot(-1)^2\cdot\begin{vmatrix}5&4\\4&2\end{vmatrix} + 1\cdot(-1)^3\cdot\begin{vmatrix}1&4\\-3&2\end{vmatrix} + (-1)\cdot(-1)^4\cdot\begin{vmatrix}1&5\\-3&4\end{vmatrix} =$

$=-2(5\cdot(-1)^2\cdot2+4\cdot(-1)^3\cdot4)+(-1)\cdot(1\cdot(-1)^2\cdot2+4\cdot(-1)^3\cdot(-3))+(-1)(1\cdot(-1)^2\cdot4+5\cdot(-1)^3\cdot(-3)) =$

$=-2\cdot(-6)+(-1)14+(-1)19 = -21$

e) $a_{11}\cdot\tilde{a}_{11}+a_{12}\cdot\tilde{a}_{12}+a_{13}\cdot\tilde{a}_{13} = 0+2\cdot(-1)^3\cdot\begin{vmatrix}1&5\\2&2\end{vmatrix}+0 = -2(1\cdot(-1)^2\cdot2+5\cdot(-1)^3\cdot2) =$

$=-2\cdot(-8)=16$

f) $a_{11}\cdot\tilde{a}_{11}+a_{21}\cdot\tilde{a}_{21}+a_{31}\cdot\tilde{a}_{31}+a_{41}\cdot\tilde{a}_{41} = 3\cdot(-1)^2\begin{vmatrix}1&0&1\\-1&1&0\\0&1&-1\end{vmatrix}+0+0+0=$

$=3\left(1\cdot(-1)^2\cdot\begin{vmatrix}1&0\\1&-1\end{vmatrix}+0+1\cdot(-1)^4\cdot\begin{vmatrix}-1&1\\0&1\end{vmatrix}\right) = 3[1(1\cdot(-1)^2\cdot(-1)+0+1(-1\cdot(-1)^2\cdot1+0)] =$

$=3\cdot-2=-6$

g) $1\cdot(-1)^2\cdot\begin{vmatrix}1&2&5&3\\2&\sqrt{3}&0&0\\-3&6&1&0\\-3&0&0&0\end{vmatrix}+0+0+0+0 = 1\left(3\cdot(-1)^5\cdot\begin{vmatrix}2&\sqrt{3}&0\\-3&6&1\\-3&0&0\end{vmatrix}+0+0+0\right) =$

___/___/___

1-g) Continuação

$1[-3(-3\cdot(-1)^4\cdot\begin{vmatrix}\sqrt{3} & 0\\ 6 & 1\end{vmatrix})]$ $1\{-3[-3(\sqrt{3}\cdot(-1)^2\cdot 1+0)]\}=1\cdot 9\sqrt{3}=9\sqrt{3}$

h) $3\cdot(-1)^2\cdot\begin{vmatrix}0&0&0&-2\\0&2&0&0\\0&0&1&0\\-2&0&0&0\end{vmatrix}+0+0+0+0=3(-2\cdot(-1)^5\cdot\begin{vmatrix}0&2&0\\0&0&1\\-2&0&0\end{vmatrix}+0+0+0)=$

$=3[2(1\cdot(-1)^5\begin{vmatrix}0&2\\-2&0\end{vmatrix})]=3\{2[-1(2\cdot(-1)^3\cdot(-2)+0)]\}=3\cdot(-8)=-24$

OBS: b está antes da a

2- b) $\det(A)=\begin{vmatrix}3&-5&7\\4&2&8\\1&-9&6\end{vmatrix}\begin{matrix}3&-5\\4&2\\1&-9\end{matrix}=36-40-252-(14+(-216)+(-120))=$

$=256-(-322)=66$

+14 -216 -120 36 -40 -252

$\det(B)=\begin{vmatrix}4&3&7\\-1&0&2\\3&1&-4\end{vmatrix}\begin{matrix}4&3\\-1&0\\3&1\end{matrix}=0+18-7-(0+8+12)=11-20=-9$

0 8 12 0 18 -7

$\det(AB)=\det(A)\cdot\det(B)=66\cdot-9=-594$

a) $\det(A+B)=\begin{vmatrix}3+4&-5+3&7+7\\4+(-1)&2+0&8+2\\1+3&-9+1&6+(-4)\end{vmatrix}=\begin{vmatrix}7&-2&14\\3&2&10\\4&-8&2\end{vmatrix}\begin{matrix}7&-2\\3&2\\4&-8\end{matrix}=$

112 -560 -12 28 -80 -336

$=28-80-336-(112-560-12)=-388+460=72$

c) $\det(B^tA^t)=\det(B^t)\cdot\det(A^t)=\det(B)\cdot\det(A)=-9\cdot 66=-594$

d) $\det(2A-3C+B) =$

$$2A-3C+B = \begin{pmatrix} 6 & -10 & 14 \\ 8 & 4 & 16 \\ 2 & -18 & 12 \end{pmatrix} - \begin{pmatrix} 6 & 9 & -3 \\ 18 & 27 & -6 \\ 24 & 36 & -9 \end{pmatrix} + \begin{pmatrix} 4 & 3 & 7 \\ -1 & 0 & 2 \\ 3 & 1 & -4 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & -16 & 24 \\ -11 & -23 & 24 \\ -19 & -53 & 17 \end{pmatrix}$$

$$\det(2A-3C+B) = \left|\begin{array}{ccc|cc} 4 & -16 & 24 & 4 & -16 \\ -11 & -23 & 24 & -11 & -23 \\ -19 & -53 & 17 & -19 & -53 \end{array}\right| = -1564 + 7296 + 13992 - (10488 - 5083 + 2992) =$$

$-1564 \quad 7296 \quad 13992$

$$= 11332$$

e)

$$\det(C) = \left|\begin{array}{ccc|cc} 2 & 3 & -1 & 2 & 3 \\ 6 & 9 & -2 & 6 & 9 \\ 8 & 12 & -3 & 8 & 12 \end{array}\right| = -54 - 48 - 72 - (-54 - 48 - 72) = 0$$

$-72 \quad -48 \quad -54 \quad -54 \quad -48 \quad -72$

$\det(AC^t) = \det(A) \cdot \det(C^t) =$
$\det(A) \cdot \det(C) = 66 \cdot 0 = 0$

3- a) $\det(A^t) = \det(A) = -2$

b) $\det(5A) = 5^4 \cdot \det(A) = 625 \cdot -2 = -1250$

c) $\det(A^6) = [\det(A)]^6 = -2^6 = 64$

d) $\det(A^{-1}) = 1/\det(A) = -\frac{1}{2}$

4- a)

$$\begin{vmatrix} a & b & c \\ d & e & f \\ 5g & 5h & 5i \end{vmatrix} = 5 \cdot \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 5 \cdot (-3) = -15$$

b)

$$\begin{vmatrix} a & b & -2c \\ 3d & 3e & -6f \\ g & h & -2i \end{vmatrix} = (-2) \cdot 3 \cdot \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = (-2) \cdot 3 \cdot (3) = 18$$

c) $\begin{vmatrix} -a & -b & -c \\ g & h & i \\ -d & -e & -f \end{vmatrix} \overset{l_2 \leftrightarrow l_3}{=} (-1)\cdot \begin{vmatrix} -a & -b & -c \\ -d & -e & -f \\ g & h & i \end{vmatrix} \cdot(-1) = (-1)\cdot(-1)\cdot(-1)\cdot \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = (-1)\cdot(-1)\cdot(-1)\cdot(-3) = +3$

d) $\begin{vmatrix} g & h & i \\ a & b & c \\ d & e & f \end{vmatrix} \overset{l_1 \leftrightarrow l_2}{=} (-1) \begin{vmatrix} a & b & c \\ g & h & i \\ d & e & f \end{vmatrix} \overset{l_2 \leftrightarrow l_3}{=} (-1)(-1) \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = (-1)\cdot(-1)\cdot(-3) = -3$

e) $\begin{vmatrix} a & b & c \\ 2d+a & 2e+b & 2f+c \\ g & h & i \end{vmatrix} \overset{l_2 \leftarrow l_2 - l_1}{=} \begin{vmatrix} a & b & c \\ 2d & 2e & 2f \\ g & h & i \end{vmatrix} \div 2 = 2\cdot \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 2\cdot(-3) = -6$

f) $\begin{vmatrix} ka+a & kb+b & kc+c \\ d & e & f \\ g & h & i \end{vmatrix} = \begin{vmatrix} ka & kb & kc \\ d & e & f \\ g & h & i \end{vmatrix} + \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = k \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} + \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$

$= k\cdot(-3) + (-3) = -3k - 3$

5) $\det(A) = \begin{vmatrix} 5 & 4 & 20 & 1 \\ 4 & 6 & 20 & -4 \\ -5 & -7 & -30 & 9 \\ 3 & -6 & -30 & 12 \end{vmatrix} = 10 \begin{vmatrix} 5 & 4 & 2 & 1 \\ 4 & 6 & 2 & -4 \\ -5 & -7 & -3 & 9 \\ 3 & -6 & -3 & 12 \end{vmatrix} \begin{matrix} \\ \div 2 \\ \\ \div 3 \end{matrix} = 3\cdot 2\cdot 10 \begin{vmatrix} 5 & 4 & 2 & 1 \\ 2 & 3 & 1 & -2 \\ -5 & -7 & -3 & 9 \\ 1 & -2 & -1 & 4 \end{vmatrix} \begin{matrix} l_1 \leftarrow l_1 - 5l_4 \\ l_2 \leftarrow l_2 - 2l_4 \\ l_3 \leftarrow l_3 + 5l_4 \end{matrix} =$

$= 60 \begin{vmatrix} 0 & 14 & 7 & -19 \\ 0 & 7 & 3 & -10 \\ 0 & -17 & -8 & 29 \\ 1 & -2 & -1 & 4 \end{vmatrix} = 60\cdot 1\cdot(-1)^5 \cdot \begin{vmatrix} 14 & 7 & -19 \\ 7 & 3 & -10 \\ -17 & -8 & 29 \end{vmatrix} \begin{matrix} 14 & 7 \\ 7 & 3 \\ -17 & -8 \end{matrix} = -60\cdot[1218+1190+1064-[969+1120+1421]]$

$= -60\cdot(-38) = +2.280$

969 1120 1421 1218 1190 1064

6 - a) $\begin{vmatrix} 4 & 6 & x \\ 7 & 4 & 2x \\ 5 & 2 & x \end{vmatrix}\begin{matrix} 4 & 6 \\ 7 & 4 \\ 5 & 2 \end{matrix} = -128 \iff -16x + 60x + 14x - (-42x + 16x + 20x) = -128$

$20x \quad 16x \quad -42x \quad -16x \quad 60x \quad 14x$

$58x + 6x = -128$

$x = -128/64$

$= x = -2$

b) $\begin{vmatrix} 3 & 5 & 7 \\ 2x & x & 3x \\ 4 & 6 & 7 \end{vmatrix}\begin{matrix} 3 & 5 \\ 2x & x \\ 4 & 6 \end{matrix} = 39 \iff 21x + 60x + 84x - (28x + 54x + 70x) = 39$

$28x \quad 54x \quad 70x \quad 21x \quad 60x \quad 84x$

$165x - 152x = 39$

$x = 39/13$

$x = 3$

c) $\begin{vmatrix} x+3 & x+1 & x+4 \\ 4 & 5 & 3 \\ 9 & 10 & 7 \end{vmatrix}\begin{matrix} x+3 & x+1 \\ 4 & 5 \\ 9 & 10 \end{matrix} = -7 \iff 35x + 105 + 27x + 27 + 40x + 160 - (+45x + 180 + 30x + 90 + 28x + 28) = -7$

$45x+180 \quad 30x+90 \quad 28x+28 \quad 35x+105 \quad 27x+27 \quad 40x+160$

$102x - 103x + 292 - 298 = -7$

$-x = -7 + 6$

$x = 1$

d) $\begin{vmatrix} x & x+2 \\ 1 & x \end{vmatrix} = 0 \iff x^2 - x - 2 = 0 \quad \Delta = 1 - (-8) = 9$

$x+2 \quad x^2$

$x_1 = \frac{+1 + \sqrt{9}}{2} \quad x_2 = \frac{+1 - \sqrt{9}}{2} \quad \{x \in \mathbb{R} / x = 2 \text{ ou } x = -1\}$

$x_1 = 2 \quad x_2 = -1$

e) $\begin{vmatrix} x-4 & 0 & 3 \\ 2 & 0 & x-9 \\ 0 & 3 & 0 \end{vmatrix}\begin{matrix} x-4 & 0 \\ 2 & 0 \\ 0 & 3 \end{matrix} \neq 0 \iff 18 - (3x^2 - 39x + 108) \neq 0$

$-3x^2 + 39x - 90 \neq 0 \quad \Delta = 1521 - 1080 = 441$

$x_1 = \frac{-39 - \sqrt{441}}{-6} \quad x_2 = \frac{-39 + \sqrt{441}}{-6}$

$0 \quad (3x-12)(x-9) \quad 0 \quad 0 \quad 0 \quad 18$

$x_1 \neq 10 \quad x_2 \neq 3$

$\{x \in \mathbb{R} / x \neq 10 \text{ e } x \neq 3\}$

7 - a) A matriz inversa só irá existir se $\det(A) \neq 0$.

$A^{-1} = \frac{1}{ad - cb}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

___/___/___

7b) $\begin{vmatrix} 3 & 1 \\ 5 & 2 \end{vmatrix} = 6-5=1$ $\quad A^{-1} = 1\begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}$

$\begin{vmatrix} 4 & 7 \\ 1 & 2 \end{vmatrix} = 8-7=1$ $\quad B^{-1} = \begin{pmatrix} 2 & -7 \\ -1 & 4 \end{pmatrix}$

$AB = \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}\begin{pmatrix} 4 & 7 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 3\cdot4+1\cdot1 & 3\cdot7+1\cdot2 \\ 5\cdot4+2\cdot1 & 5\cdot7+2\cdot2 \end{pmatrix} = \begin{pmatrix} 13 & 23 \\ 22 & 39 \end{pmatrix}$

$\begin{vmatrix} 13 & 23 \\ 22 & 39 \end{vmatrix} = 507-506=1$ $\quad (AB)^{-1} = \begin{pmatrix} 39 & -23 \\ -22 & 13 \end{pmatrix}$

8-a) $\tilde{a}_{11} = (-1)^2 \cdot 1 = 1$ $\quad \tilde{a}_{21} = (-1)^3 \cdot -2 = 2$ $\quad Cof(A) = \begin{pmatrix} 1 & -3 \\ 2 & 2 \end{pmatrix}$

$\tilde{a}_{12} = (-1)^3 \cdot 3 = -3$ $\quad \tilde{a}_{22} = (-1)^4 \cdot 2 = 2$

b) $\tilde{b}_{11} = (-1)^2 \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = 1\cdot(-2-1) = -3$ $\quad \tilde{b}_{12} = (-1)^3 \cdot \begin{vmatrix} 1 & 1 \\ 0 & -1 \end{vmatrix} = -1\cdot(-1-0) = 1$

$\tilde{b}_{13} = (-1)^4 \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = 1\cdot(1-0) = 1$ $\quad \tilde{b}_{21} = (-1)^3 \begin{vmatrix} -2 & 0 \\ 1 & -1 \end{vmatrix} = -1\cdot(2-0) = -2$

$\tilde{b}_{22} = (-1)^4 \begin{vmatrix} 2 & 0 \\ 0 & -1 \end{vmatrix} = 1\cdot(-2-0) = -2$ $\quad \tilde{b}_{23} = (-1)^5 \begin{vmatrix} 2 & -2 \\ 0 & 1 \end{vmatrix} = -1\cdot(2-0) = -2$

$\tilde{b}_{31} = (-1)^4 \begin{vmatrix} -2 & 0 \\ 2 & 1 \end{vmatrix} = 1\cdot(-2-0) = -2$ $\quad \tilde{b}_{32} = (-1)^5 \begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} = -1(2-0) = -2$

$\tilde{b}_{33} = (-1)^6 \begin{vmatrix} 2 & -2 \\ 1 & 2 \end{vmatrix} = 1\cdot(4-(-2)) = 6$ $\quad Cof B = \begin{pmatrix} -3 & 1 & 1 \\ -2 & -2 & -2 \\ -2 & -2 & 6 \end{pmatrix}$

___/___/___

9 a) $\begin{vmatrix} 2 & -2 \\ 3 & 1 \end{vmatrix} = 2-(-6) = 8$ $\quad adj(A) = \begin{pmatrix} 1 & 2 \\ -3 & 2 \end{pmatrix}$ $\quad A^{-1} = \frac{1}{8}\begin{pmatrix} 1 & 2 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} \frac{1}{8} & \frac{1}{4} \\ -\frac{3}{8} & \frac{1}{4} \end{pmatrix}$

b) $\left|\begin{matrix} 2 & -2 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & -1 \end{matrix}\,\middle|\,\begin{matrix} 2 & -2 \\ 1 & 2 \\ 0 & 1 \end{matrix}\right| = -4-(2+2) = -8$

$0 \quad 2 \quad 2 \quad -4 \quad 0 \quad 0$

$\tilde{b}_{11} = (-1)^2 \cdot \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = 1\cdot(-2-1) = -3$ $\quad \tilde{b}_{12} = (-1)^3 \begin{vmatrix} 1 & 1 \\ 0 & -1 \end{vmatrix} = -1\cdot(-1-0) = 1$

$\tilde{b}_{13} = (-1)^4 \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = 1\cdot(1-0) = 1$ $\quad \tilde{b}_{21} = (-1)^3 \begin{vmatrix} -2 & 0 \\ 1 & -1 \end{vmatrix} = -1\cdot(2-0) = -2$

$\tilde{b}_{22} = (-1)^4 \begin{vmatrix} 2 & 0 \\ 0 & -1 \end{vmatrix} = 1\cdot(-2-0) = -2$ $\quad \tilde{b}_{23} = (-1)^5 \begin{vmatrix} 2 & -2 \\ 0 & 1 \end{vmatrix} = -1\cdot(2-0) = -2$

$\tilde{b}_{31} = (-1)^4 \begin{vmatrix} -2 & 0 \\ 2 & 1 \end{vmatrix} = 1\cdot(-2-0) = -2$ $\quad \tilde{b}_{32} = (-1)^5 \begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} = -1\cdot(2-0) = -2$

$\tilde{b}_{33} = (-1)^6 \begin{vmatrix} 2 & -2 \\ 1 & 2 \end{vmatrix} = 1\cdot(4-(-2)) = 6$ $\quad cof = \begin{pmatrix} -3 & 1 & 1 \\ -2 & -2 & -2 \\ -2 & -2 & 6 \end{pmatrix}$

$adj(B) = [cof(B)]^t$

$adj(B) = \begin{pmatrix} -3 & -2 & -2 \\ 1 & -2 & -2 \\ 1 & -2 & 6 \end{pmatrix}$ $\quad B^{-1} = \frac{1}{-8}\begin{pmatrix} -3 & -2 & -2 \\ 1 & -2 & -2 \\ 1 & -2 & 6 \end{pmatrix} = \begin{pmatrix} \frac{3}{8} & \frac{1}{4} & \frac{1}{4} \\ -\frac{1}{8} & \frac{1}{4} & \frac{1}{4} \\ -\frac{1}{8} & \frac{1}{4} & -\frac{3}{4} \end{pmatrix}$

c) $\left|\begin{matrix} 0 & -1 & 1 \\ 2 & 0 & -1 \\ 1 & 1 & 0 \end{matrix}\,\middle|\,\begin{matrix} 0 & -1 \\ 0 & 0 \\ 1 & 1 \end{matrix}\right| = 2+1-0 = 3$

$0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 2$

c) continuação

$\tilde{C}_{11} = (-1)^2 \begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix} = 1\cdot(0-(-1)) = 1$ $\quad \tilde{C}_{12} = (-1)^3 \begin{vmatrix} 2 & -1 \\ 1 & 0 \end{vmatrix} = -1\cdot(0-(-1)) = -1$

$\tilde{C}_{13} = (-1)^4 \begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} = 1\cdot(2-0) = 2$ $\quad \tilde{C}_{21} = (-1)^3 \begin{vmatrix} -1 & 1 \\ 1 & 0 \end{vmatrix} = -1\cdot(0-1) = 1$

$\tilde{C}_{22} = (-1)^4 \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = +1\cdot(0-1) = -1$ $\quad \tilde{C}_{23} = (-1)^5 \begin{vmatrix} 0 & -1 \\ 1 & 1 \end{vmatrix} = -1\cdot(0-(-1)) = -1$

$\tilde{C}_{31} = (-1)^4 \begin{vmatrix} -1 & 1 \\ 0 & -1 \end{vmatrix} = 1\cdot(1-0) = 1$ $\quad \tilde{C}_{32} = (-1)^5 \begin{vmatrix} 0 & 1 \\ 2 & -1 \end{vmatrix} = -1\cdot(0-2) = 2$

$\tilde{C}_{33} = (-1)^6 \begin{vmatrix} 0 & -1 \\ 2 & 0 \end{vmatrix} = 1\cdot(0-(-2)) = 2$ $\quad cof(C) = \begin{pmatrix} 1 & -1 & 2 \\ 1 & -1 & -1 \\ 1 & 2 & 2 \end{pmatrix}$

$adj(C) = \begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & 2 \\ 2 & -1 & 2 \end{pmatrix}$ $\quad C^{-1} = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & 2 \\ 2 & -1 & 2 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \end{pmatrix}$

d) $\det D = -1\cdot(-1)^7 \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & 0 & 3 \end{vmatrix}\begin{matrix} 1 & 0 \\ 0 & 1 \\ 2 & 0 \end{matrix} = 1\cdot(3-2) = 1$

$2\ 0\ 0 \quad 3\ 0\ 0$

$\tilde{d}_{11} = (-1)^2 \cdot \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 3 \\ 0 & -1 & 0 \end{vmatrix}\begin{matrix} 1 & 0 \\ 0 & 2 \\ 0 & -1 \end{matrix} = 1\cdot 3 = 3$ $\quad \tilde{d}_{12} = (-1)^3 \cdot \begin{vmatrix} 0 & 0 & 0 \\ 2 & 2 & 3 \\ 0 & -1 & 0 \end{vmatrix} = -1\cdot 0 = 0$

$0\ {-3}\ 0 \quad 0$

$\tilde{d}_{13} = (-1)^4 \begin{vmatrix} 0 & 1 & 0 \\ 2 & 0 & 3 \\ 0 & 0 & 0 \end{vmatrix} = 1\cdot 0 = 0$ $\quad \tilde{d}_{14} = (-1)^5 \begin{vmatrix} 0 & 1 & 0 \\ 2 & 0 & 2 \\ 0 & 0 & -1 \end{vmatrix}\begin{matrix} 0 & 1 \\ 2 & 0 \\ 0 & 0 \end{matrix} = -1\cdot 2 = -2$

$0\ 0\ {-2} \quad 0\ 0\ 0$

$\tilde{d}_{21} = (-1)^3 \begin{vmatrix} 0 & 0 & 1 \\ 0 & 2 & 3 \\ 0 & -1 & 0 \end{vmatrix} = -1\cdot 0 = 0$ $\quad \tilde{d}_{22} = (-1)^4 \begin{vmatrix} 1 & 0 & 1 \\ 2 & 2 & 3 \\ 0 & -1 & 0 \end{vmatrix}\begin{matrix} 1 & 0 \\ 2 & 2 \\ 0 & -1 \end{matrix} = 1\cdot 1 = 1$

$0\ {-3}\ 0 \quad 0\ 0\ {-2}$

d) Continuação

$\tilde{d}_{23} = (-1)^5 \begin{vmatrix} 1 & 0 & 1 \\ 2 & 0 & 3 \\ 0 & 0 & 0 \end{vmatrix} = -1 \cdot 0 = 0$ $\qquad$ $\tilde{d}_{24} = (-1)^6 \begin{vmatrix} 1 & 0 & 0 \\ 2 & 0 & 2 \\ 0 & 0 & -1 \end{vmatrix} = 1 \cdot 0 = 0$

$\tilde{d}_{31} = (-1)^4 \left|\begin{matrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{matrix}\right|\begin{matrix} 0 & 0 \\ 1 & 0 \\ 0 & -1 \end{matrix}\right| = 1 \cdot -1 = -1$ $\qquad$ $\tilde{d}_{32} = (-1)^5 \begin{vmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} = -1 \cdot 0 = 0$

0 0 0 0 -1

$\tilde{d}_{33} = (-1)^6 \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix} = 1 \cdot 0 = 0$ $\qquad$ $\tilde{d}_{34} = (-1)^7 \left|\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{matrix}\right|\begin{matrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{matrix}\right| = -1 \cdot -1 = 1$

0 0 0 -1 0 0

$\tilde{d}_{41} = (-1)^5 \left|\begin{matrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 2 & 3 \end{matrix}\right|\begin{matrix} 0 & 0 \\ 1 & 0 \\ 0 & 2 \end{matrix}\right| = -1 \cdot 2 = -2$ $\qquad$ $\tilde{d}_{42} = (-1)^6 \begin{vmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 2 & 2 & 3 \end{vmatrix} = 1 \cdot 0 = 0$

0 0 0 0 0 2

$\tilde{d}_{43} = (-1)^7 \left|\begin{matrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & 0 & 3 \end{matrix}\right|\begin{matrix} 1 & 0 \\ 0 & 1 \\ 2 & 0 \end{matrix}\right| = -1 \cdot 1 = -1$ $\qquad$ $\tilde{d}_{44} = (-1)^8 \left|\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 2 \end{matrix}\right|\begin{matrix} 1 & 0 \\ 0 & 1 \\ 2 & 0 \end{matrix}\right| = 1 \cdot 2 = 2$

2 0 0 3 0 0 $\qquad$ 0 0 0 2 0 0

$$\text{Cof}(D) = \begin{pmatrix} 3 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ -2 & 0 & -1 & 2 \end{pmatrix} \qquad \text{adj}(D) = \begin{pmatrix} 3 & 0 & -1 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ -2 & 0 & 1 & 2 \end{pmatrix}$$

$$D^{-1} = \frac{1}{1} \begin{pmatrix} 3 & 0 & -1 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ -2 & 0 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 0 & -1 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ -2 & 0 & 1 & 2 \end{pmatrix}$$

10 - a) $A^t - BX = 2C$

$BX = A^t - 2C$

$X = B^{-1}(A^t - 2C)$

B não pode possuir determinante igual à 0.

b) $\begin{vmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{vmatrix} \begin{matrix} 1 & 0 \\ 2 & -1 \\ 4 & 1 \end{matrix} = -8+4-(-8+3) = 1$

$-8 \quad 3 \quad 0 \quad -8 \quad 0 \quad 4$

$\tilde{b}_{11} = (-1)^2 \cdot \begin{vmatrix} -1 & 3 \\ 1 & 8 \end{vmatrix} = 1 \cdot -11 = -11$ $\qquad \tilde{b}_{12} = (-1)^3 \cdot \begin{vmatrix} 2 & 3 \\ 4 & 8 \end{vmatrix} = -1 \cdot 4 = -4$

$\tilde{b}_{13} = (-1)^4 \begin{vmatrix} 2 & -1 \\ 4 & 1 \end{vmatrix} = 1 \cdot 6 = 6$ $\qquad \tilde{b}_{21} = (-1)^3 \begin{vmatrix} 0 & 2 \\ 1 & 8 \end{vmatrix} = -1 \cdot -2 = -2$

$\tilde{b}_{22} = (-1)^4 \begin{vmatrix} 1 & 2 \\ 4 & 8 \end{vmatrix} = 1 \cdot 0 = 0$ $\qquad \tilde{b}_{23} = (-1)^5 \begin{vmatrix} 1 & 0 \\ 4 & 1 \end{vmatrix} = -1 \cdot -3 = 3$

$\tilde{b}_{31} = (-1)^4 \begin{vmatrix} 0 & 2 \\ -1 & 3 \end{vmatrix} = 1 \cdot 2 = 2$ $\qquad \tilde{b}_{32} = (-1)^5 \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = -1 \cdot -1 = 1$

$\tilde{b}_{33} = (-1)^6 \begin{vmatrix} 1 & 0 \\ 2 & -1 \end{vmatrix} = 1 \cdot -1 = -1$ $\qquad cof(B) = \begin{pmatrix} -11 & -4 & 6 \\ -2 & 0 & 3 \\ 2 & 1 & -1 \end{pmatrix}$ $\qquad adj(B) = \begin{pmatrix} -11 & -2 & 2 \\ -4 & 0 & 1 \\ 6 & 3 & -1 \end{pmatrix}$

$B^{-1} = \begin{pmatrix} -11 & -2 & 2 \\ -4 & 0 & 1 \\ 6 & 3 & -1 \end{pmatrix}$

$A^t - BX = 2C \Leftrightarrow X = B^{-1}(A^t - 2C) : X = \begin{pmatrix} -11 & -2 & 2 \\ -4 & 0 & 1 \\ 6 & 3 & -1 \end{pmatrix} \cdot \begin{pmatrix} -1-0 & 2-2 & 6-6 \\ 2-2 & -1-0 & 4-4 \\ 6-6 & 4-4 & -1-0 \end{pmatrix} =$

$X = \begin{pmatrix} -11 & -2 & 2 \\ -4 & 0 & 1 \\ 6 & 3 & -1 \end{pmatrix} \cdot \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} -11 \cdot -1 & -2 \cdot -1 & 2 \cdot -1 \\ -4 \cdot -1 & 0 & 1 \cdot -1 \\ 6 \cdot -1 & 3 \cdot -1 & -1 \cdot -1 \end{pmatrix} = X = \begin{pmatrix} 11 & 2 & -2 \\ 4 & 0 & -1 \\ -6 & -3 & 1 \end{pmatrix}$