Lista 7

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1- a) $\vec{AB} = (-5, -2, 1)$

$\{\vec{AX}, \vec{AB}\}$ é L.D.

parâmetrica

$r: \begin{cases} X = 3 + 5\lambda \\ Y = 5 - 2\lambda \\ Z = 1 + 1\lambda \end{cases}$ $\Leftrightarrow$ $r: \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -5 \\ -2 \\ 1 \end{pmatrix}$

simétrica

$r: \begin{cases} \lambda = \frac{X-3}{-5} \\ \lambda = \frac{Y-5}{-2} \\ \lambda = \frac{Z-1}{1} \end{cases}$ $\Leftrightarrow$ $r: \frac{X-3}{-5} = \frac{Y-5}{-2} = \frac{Z-1}{1}$

b) $\vec{AB} = (1, -1, 0)$

$\{\vec{AX}, \vec{AB}\}$ é LD

parâmetrica

$r: \begin{cases} X = \lambda \\ Y = 1 - \lambda \\ Z = 0 \end{cases}$ $\Rightarrow$ $\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$

Não existe a forma simétrica, pois $Z_{AB} = 0$

c) $\vec{AB} = (0, -1, -1)$

$\{\vec{AX}, \vec{AB}\}$ é LD

parâmetrica

$r: \begin{cases} X = 0 \\ Y = 1 - \lambda \\ Z = 1 - \lambda \end{cases}$ $\Rightarrow$ $\begin{pmatrix} X \\ Z \\ Y \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix}$

Não existe a forma simétrica, pois $X_{AB} = 0$

d) $\vec{AB} = (3, -1, 3)$

$\{\vec{AX}, \vec{AB}\}$ é LD

parâmetrica

$r: \begin{cases} X = 3 + 3\lambda \\ Y = 2 - \lambda \\ Z = 1 + 3\lambda \end{cases}$ $\Rightarrow$ $\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -1 \\ 3 \end{pmatrix}$

simétrica

$\Leftrightarrow$ $r: \begin{cases} \lambda = \frac{X-3}{3} \\ \lambda = \frac{Y-2}{-1} \\ \lambda = \frac{Z-1}{3} \end{cases}$ $\Rightarrow$ $r: \frac{X-3}{3} = \frac{Y-2}{-1} = \frac{Z-1}{3}$

2 - a) $\overset{\lambda=1}{B=(0,1,6)}$, $\overset{\lambda=0}{A=(1,0,4)}$, $\vec{u}=(-1,1,2)$

b) $P=(1,3,-3)$ $\qquad\qquad$ $Q=(-3,4,12)$

$$\begin{cases} 1 = 1-\lambda & \lambda=0 \\ 3=\lambda & \lambda=3 \\ -3=4+2\lambda & \lambda=\frac{-7}{2} \end{cases} \qquad \begin{cases} -3=1-\lambda & \lambda=4 \\ 4=\lambda & \lambda=4 \\ 12=4+2\lambda & \lambda=4 \end{cases} \qquad \begin{array}{l} P\notin r \\ Q\in r \end{array}$$

c)

$r \quad \begin{cases} x = 1-\lambda \\ y = 4+\lambda \\ z = -7+2\lambda \end{cases} \; ; \; \lambda\in\mathbb{R}$

3 - a) $\{\vec{AB},\vec{BC},\vec{AC}\}$ é LD $\vec{AB}\cdot(\vec{BC}\cdot\vec{AC})$

$\vec{AB}=(-8,-4,10)$
$\vec{BC}=(9,-9,-9)$
$\vec{AC}=(1,-13,1)$

$\begin{array}{ccc|cc} -8 & -4 & 10 & -8 & -4 \\ 9 & -9 & -9 & 9 & -3 \\ 1 & -13 & 1 & 1 & -13 \end{array} = 72+36-1170+36+90+936=0$

Como o det. é 0, A,B,C são
colineares e formam um triângulo

b)

A $\quad$ M $\quad$ B

$\tilde{\pi}: M = C + \alpha\,\vec{CA} + \beta\,\vec{CB}, \; \alpha,\beta\in\mathbb{R} \qquad \vec{CA}=(-1,13,-1)$
$\gamma: M = C + \alpha(-1,13,-1)+\beta(-9,9,9); \; \alpha,\beta\in\mathbb{R} \quad \vec{CB}=(-9,9,9)$

4 - a)

$\begin{array}{ccc|cc} 1 & 0 & 8 & 1 & 0 \\ -3 & 0 & 9 & -3 & 0 \\ 1+\lambda & 2+\lambda & -3\lambda & 1+\lambda & 2+\lambda \end{array} = -48-24\lambda-18--9\lambda=0 \quad \{A,B,C\}: LD$

$\Rightarrow -66-33\lambda=0 \Rightarrow \lambda=-2.5$

$C = (1+\lambda, 2+\lambda, -3\lambda)$

$C = (-1, 0, 6)$

b) $d(C,A)=\sqrt{(1+\lambda-1)^2+(2+\lambda-1)^2+(2-\lambda-1)^2}=\sqrt{3\lambda^2-4\lambda+2} \qquad d(C,B)=\sqrt{(1+\lambda-5)^2+(\lambda-0)^2+(\lambda-0)^2}=\sqrt{3\lambda^2+2}$

$d(C,A)=d(C,B)$

$\sqrt{3\lambda^2-4\lambda+2}=\sqrt{3\lambda^2+2} \qquad\qquad C=(1,0,0)$

$-4\lambda=0$

$\lambda=0$

5. a) $\pi: x(1,2,0) + \lambda(1,1,0) + \beta(2,3,-1) =$

$\begin{cases} x = 1 + \lambda + 2\beta \\ y = 2 + \lambda + 3\beta \\ z = -\beta \end{cases}$

b) $AB = (0,-2,-1)$

$\pi: x = (1,1,0) + \lambda(0,-2,-1) + \beta(2,1,0)$

$\begin{cases} x = 1 + 2\beta \\ y = 1 - 2\lambda + \beta \\ z = -\lambda \end{cases}$

c) $AB = (1,1,-3)$

$Ac = (0,-1,-1)$

$\pi: x = (1,0,1) + \lambda(1,1,-3) + \beta(0,-1,-1)$

$\begin{cases} x = 1 + \lambda \\ y = \lambda - \beta \\ z = 1 - 3\lambda - \beta \end{cases}$

6 - a) $\pi: x = (9,-1,0) + \lambda(0,1,0) + \beta(1,1,1)$

$\vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix} = (1,0,-1)$

$1(x-9) + 0(y+1) - 1(z-0) \Rightarrow x - 9 - z = \boxed{x + z = 9}$

b) $AB = (-2,0,0)$

$AC = (1,1,1)$

$\begin{vmatrix} i & j & k \\ -2 & 0 & 0 \\ 1 & 1 & 1 \end{vmatrix} = (0,-2,-2)$

$-2(y-0) - 2(z-1) = 0 \Rightarrow -2y - 2z + 2 = 0$

$\boxed{-y - z = -2}$

c) $\vec{AB} = (0, -2, -1)$

$\begin{vmatrix} i & j & k \\ 0 & -2 & -1 \\ 2 & 1 & 0 \end{vmatrix} = (-1, -2, +4)$

$-1(x-1) -2(y-1) +4(z-1) = 0 \Rightarrow X-1-2y+2+4z-4 \Rightarrow X-2y+4z = 3$

d) $\begin{cases} x = 1 + \lambda \\ y = -1 + \lambda \\ z = 1 - \lambda \end{cases}$ $x+1 = y+1 = z-1$

7- a) $z = 4x + 2y + 5$

$\begin{cases} x = \lambda \\ y = \beta \\ z = 4\lambda + 2\beta + 5 \end{cases}$

d) $z = y - 2$

$\begin{cases} x = \beta \\ y = z+2 \\ z = y-2 \end{cases}$

b) $y = 5x + 1$

$\begin{cases} x = \lambda \\ y = 5\lambda - 1 \\ z = \beta \end{cases}$

c) $z = 3$

$\begin{cases} x = \lambda \\ y = \beta \\ z = 3 \end{cases}$

8- a) $\begin{cases} x = 1 + \lambda - \beta \\ y = 2\lambda + \beta \\ z = 3 - \beta \end{cases}$ $\begin{vmatrix} i & j & k \\ 1 & 2 & 0 \\ -1 & 1 & -1 \end{vmatrix} = (-2, 1, 3)$

$-2(x+1 + y - 0 + 3(z-3) = 0 \Rightarrow -2x + y + 3z = 11$

b) $\begin{cases} x = 1+\lambda \\ y = 2 \\ z = 3 - \lambda + \beta \end{cases}$ $\quad \begin{vmatrix} i & j & k \\ 0 & 0 & 1 \\ 1 & 0 & -1 \end{vmatrix} = (0, -1, 0)$

$-1(y-2) = 0 \Rightarrow -y + 2 = 0$

$y = 2$

c) $\begin{cases} x = -2 + \lambda - \beta \\ y = 2\lambda + 2\beta \\ z = \lambda + \beta \end{cases}$ $\quad \begin{vmatrix} i & j & k \\ 1 & 2 & 1 \\ -1 & 2 & 1 \end{vmatrix} = (0, 2, 4)$

$2(y-0) + 1(z-0) = 0$

$2y + z = 0$

10-a) $r : \begin{cases} x + 2y + 3z - 1 = 0 \\ x - y + 2z = 0 \end{cases} \Rightarrow 3y + z - 1 = 0$

$z = 1 - 3y$

$\begin{cases} x = 7\lambda - 2 \\ y = \lambda \\ z = 1, 3\lambda \end{cases}$ $\quad r : \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 7 \\ 1 \\ 3 \end{pmatrix}$

b) $\begin{cases} x + y + z - 1 = 0 \\ x + y - z = 0 \end{cases}$ $\quad x + y - z = 0$ $\quad \begin{cases} x = \frac{1}{2} - \lambda \\ y = \lambda \\ z = \frac{1}{2} \end{cases}$

$2x + 2y - 1 = 0$ $\quad z = \frac{1}{2} - y + y$

$x + y = \frac{1}{2}$ $\quad z = \frac{1}{2}$

$x = \frac{1}{2} - y$ $\quad r : \begin{pmatrix} \frac{1}{2} \\ 0 \\ \frac{1}{2} \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

c) $2x - z + 1 = 0$ $\quad \begin{cases} x = 3 \\ y = \lambda \\ z = 7 \end{cases}$ $\quad r : \begin{pmatrix} 3 \\ 0 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$6 - z + 1 = 0 \Rightarrow$

$z = 7$

d) $\begin{cases} x=\lambda \\ y=2 \\ z=0 \end{cases} \Rightarrow \pi: \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

11-a)

$\vec{v_r} = (-2, 0, -1)$

$\begin{cases} x = 9 - 2t \\ y = \lambda \\ z = 3 - \lambda \end{cases}$

$z = 3 - y$ $x + 2y - 3 = 6$

$x + 2y = 9$

$x = 9 - 2y$

$X = (9, 0, 3) + \lambda(-2, 0, -1)$

$\begin{cases} 9 = 1 - 2\lambda \Rightarrow \lambda = 4 \\ 0 = -1 + \lambda \Rightarrow \lambda = 1 \\ 3 = 1 - \lambda \Rightarrow \lambda = -2 \end{cases}$ • são paralelas e não coincidentes

b) $r: \dfrac{x+1}{2} = \dfrac{y}{3} = \dfrac{z+1}{2}$

$v_r = (2, 3, 2)$ $v_s (1, 2, 0)$ $X = (-1, 0, -1) + (2, 3, 2)\lambda \neq B(1, 2, 0)$

$P_r = (-1, 0, -1)$

$\begin{cases} -1 + 2\lambda = \beta \\ 3\lambda = 2\beta \\ -1 + 2\lambda = 0 \Rightarrow \lambda = \frac{1}{2} \end{cases}$ $\begin{aligned} \beta &= -1 + 2 \cdot \tfrac{1}{2} = 0 \\ 3 \cdot \tfrac{1}{2} &= 2\beta \\ \beta &= \tfrac{3}{4} \end{aligned}$ • são reversas

c) $v_r = (2, -1, 3)$ $X = (8, 1, 9) + \lambda(2, -1, 3) = (3, -4, 4) + \beta(1, -2, 2)$

$v_s = (1, -2, 2)$ $\begin{cases} 2\lambda - \beta = -5 \\ \lambda + 2\beta = -5 \\ 3\lambda + 2\beta = -5 \end{cases}$ • são concorrentes

12-a) $\begin{cases} x=1 \\ y=1+\lambda \\ z=\lambda \end{cases}$ $1-(1+\lambda)-(-\lambda)=2$

$\lambda+\lambda=2$

$2\lambda=2$

$\lambda=1$

$\begin{cases} x=1 \\ y=2 \\ z=1 \end{cases}$

$\hookrightarrow (1,2,1)$ · transversais

b) $\begin{cases} x=1+2\lambda \\ y=\lambda \\ z=\lambda \end{cases}$ $\pi: x=(3,0,1)+\alpha(1,0,1)+\beta(2,2,0)$ $\begin{cases} 1+2\lambda=3+\alpha+2\beta \\ \lambda=2\beta \\ \lambda=1+\alpha \end{cases}$

$1+2\lambda=3+2\beta-1+2\beta$

$1+2\lambda=2+4\beta$

$1+4\beta=2+4\beta$ · são paralelas

c) $\begin{cases} x=y+1 \\ x=2y \end{cases}$ $2y=y+1$ · O ponto $(2,1)$ não está no plano

$y=1$

$x=y+1 \Rightarrow x=2$

13-a)

$v_1=(1,2,0)$ $v_2=(1,0,1)$ $\begin{vmatrix} i & j & k \\ 1 & 2 & 0 \\ 1 & 0 & 1 \end{vmatrix} = (2,1,-2)=\vec{n}$

$d=(2,m,1)$ $d \cdot n = (2,m,1)(2,1,-2)$

$4+m-2=0$

$m=-2$

b) $\vec{n}=(2,m,m)$ $d \cdot v = 2-3m+m=0$

$\vec{d}=(1,-3,1)$ $m=2$

$N-3 \cdot 2+0=1$

$N-6=1$

$N=7$

c) $\vec{d} = (m, 2, m)$ $d \cdot V = 1m + 2m + m \neq 0$ • transversal ao

$\vec{v} = (1, m, 1)$ $4m \neq 0$ plano

15 — $V_1 = (-1, m, 1)$ $\Big\}$ $\begin{vmatrix} i & j & k \\ -1 & m & 1 \\ 2 & 0 & 1 \end{vmatrix} = (m, 3, -2m) = N_1$

$V_2 = (2, 0, 1)$

$V_3 = (m, 1, 0)$ $\Big\}$ $\begin{vmatrix} i & j & k \\ m & 1 & 0 \\ 1 & 0 & m \end{vmatrix} = (m, -m^2, -1) = N_2$

$V_4 = (1, 0, m)$

$N_1 \times N_2 = \begin{vmatrix} i & j & k \\ m & 3 & -2m \\ m & -m^2 & -1 \end{vmatrix} = (3 - 2m^3, -m - 2m^3, -3m - m^3)$ • não

transversais