Lista 3

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1- $a_{11}=2-1=1$ $a_{12}=1+2=3$ $a_{13}=1+3=4$ $a_{21}=1-2=-1$ $a_{22}=4-2=2$ $a_{23}=2+3=5$

$a_{31}=1-3=-2$ $a_{32}=2-3=-1$ $a_{33}=6-3=3$

$$AX=B \iff \begin{bmatrix}1 & 3 & 4\\ -1 & 2 & 5\\ -2 & -1 & 3\end{bmatrix}\begin{bmatrix}X_1\\ X_2\\ X_3\end{bmatrix}=\begin{bmatrix}-1\\ 2\\ -3\end{bmatrix}$$

$$\begin{cases}1X_1+3X_2+4X_3=-1 & l_1 \leftarrow l_2+l_1\\ -1X_1+2X_2+5X_3=2 & l_1 \leftarrow 2l_1+l_3\\ -2X_1+(-1)X_2+3X_3=-3\end{cases}$$

$$\begin{cases}5X_2+9X_3=+1\\ 5X_2+11X_3=-5\end{cases} \quad l_2-l_1 \Rightarrow 2X_3=-6 \qquad 5X_2+(-27)=1$$

$$X_3=-3 \qquad X_2=\frac{28}{5}$$

$$1X_1+3X_2+4X_3=-1 \Rightarrow X_1+\frac{3\cdot 28}{5}+(-12)=-1 \Rightarrow X_1=11-\frac{84}{5}=-\frac{29}{5}$$

$$X=\begin{pmatrix}-3\\ \frac{28}{5}\\ -\frac{29}{5}\end{pmatrix}$$

2- a) $A^{-1}=\frac{1}{det(A)}\begin{pmatrix}d & -b\\ -c & a\end{pmatrix}$ $A^{-1}=\frac{1}{-5}\begin{pmatrix}3 & -4\\ -2 & 1\end{pmatrix}=\begin{pmatrix}-\frac{3}{5} & \frac{4}{5}\\ \frac{3}{5} & -\frac{1}{5}\end{pmatrix}$

$A=\begin{pmatrix}1 & 4\\ 2 & 3\end{pmatrix}$ $det(A)=-5$

$$X=A^{-1}B \Rightarrow \begin{pmatrix}-\frac{3}{5} & \frac{4}{5}\\ \frac{2}{5} & -\frac{1}{5}\end{pmatrix}\cdot\begin{pmatrix}-1\\ -1\end{pmatrix}=X \Rightarrow \begin{pmatrix}-\frac{3}{5}\times-1+\frac{4}{5}\times-1\\ \frac{2}{5}\times-1+-\frac{1}{5}\times-1\end{pmatrix}=\begin{pmatrix}-\frac{1}{5}\\ -\frac{1}{5}\end{pmatrix}=X$$

b) $C+AY=B$ $A^{-1}=-1\begin{pmatrix}5 & -2\\ -3 & 1\end{pmatrix}=\begin{pmatrix}-5 & 2\\ 3 & -1\end{pmatrix}$

$AY=B-C$

$$B-C=\begin{pmatrix}2 & 3\\ 5 & 5\end{pmatrix}+\begin{pmatrix}1 & 7\\ 2 & 7\end{pmatrix}=\begin{pmatrix}3 & 10\\ 7 & 12\end{pmatrix} \qquad Y=\begin{pmatrix}-5 & 2\\ 3 & -1\end{pmatrix}\begin{pmatrix}3 & 10\\ 7 & 12\end{pmatrix}$$

$$Y=\begin{pmatrix}-5\cdot3+2\cdot7 & -5\cdot10+2\cdot12\\ 3\cdot3+-1\cdot7 & 3\cdot10+-1\cdot12\end{pmatrix} \Rightarrow Y=\begin{pmatrix}-1 & -26\\ 2 & 18\end{pmatrix}$$

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c) $\det C = 1 \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix} = 1$ $\quad \tilde{C}_{11} = + \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix} = +1$ $\quad \tilde{C}_{12} = - \begin{vmatrix} 2 & 0 \\ 2 & 1 \end{vmatrix} = -2$ $\quad \tilde{C}_{13} = + \begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix} = 4$

$\tilde{C}_{21} = - \begin{vmatrix} 0 & 0 \\ 3 & 1 \end{vmatrix} = 0$ $\quad \tilde{C}_{22} = + \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} = 1$ $\quad \tilde{C}_{23} = - \begin{vmatrix} 1 & 0 \\ 2 & 3 \end{vmatrix} = -3$ $\quad \tilde{C}_{31} = + \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0$

$\tilde{C}_{32} = - \begin{vmatrix} 1 & 0 \\ 2 & 0 \end{vmatrix} = 0$ $\quad \tilde{C}_{33} = + \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} = 1$ $\quad Cof(C) = \begin{pmatrix} 1 & -2 & 4 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix}$

$adj(C) = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 4 & -3 & 1 \end{pmatrix}$ $\quad C^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 4 & -3 & 1 \end{pmatrix}$

$CW = B \Rightarrow W = C^{-1}B \Rightarrow W = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 4 & -3 & 1 \end{pmatrix} \begin{pmatrix} 5 \\ 7 \\ 2 \end{pmatrix} = \begin{pmatrix} 1\cdot 5 + 0 + 0 \\ -2\cdot 5 + 1\cdot 7 + 0 \\ 4\cdot 5 + -3\cdot 7 + 1\cdot 2 \end{pmatrix} \Rightarrow W = \begin{pmatrix} 5 \\ -3 \\ -3 \end{pmatrix}$

3- a) $AXB = C$
$X = A^{-}CB^{-}$

b) $A(B+X) = A$
$AB + AX = A$
$AX = A - AB$
$X = A^{-}(A - AB)$
$X = I - B$

c) $ACXB = C$
$X = (AC)^{-}CB^{-}$

d) $(AB)^{-}(AX) = CC^{-}$
$AX = AB$
$X = A^{-}(AB)$
$X = B$

e) $AB^{t}XB^{-} = A^{t}$
$AB^{t}X = A^{t}B$
$X = (AB^{t})^{-}(A^{t}B)$

4- a) $\begin{pmatrix} 3 & -4 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 9 \end{pmatrix}$ $\quad \det(A) = 9 - (-4) = 13$

$\det(A_x) = \begin{vmatrix} 1 & -4 \\ 9 & 3 \end{vmatrix} = 3 - (-36) = 39$

$\det(A_y) = \begin{vmatrix} 3 & 1 \\ 1 & 9 \end{vmatrix} = 27 - 1 = 26$

$X = \dfrac{\det(A_x)}{\det(A)} = \dfrac{39}{13} = 3$ $\quad Y = \dfrac{\det(A_y)}{\det(A)} = \dfrac{26}{13} = 2$ $\quad S = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

b) $\begin{pmatrix} 5 & 8 \\ 10 & 16 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 34 \\ 50 \end{pmatrix}$ $det(A) = 80 - 80 = 0$

$det(A_x) = \begin{vmatrix} 34 & 8 \\ 50 & 16 \end{vmatrix} = 544 - 400 = -144$

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c) $\begin{pmatrix} 1 & 2 \\ 2 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ -4 \end{pmatrix}$ $det(A) = -3 - 4 = -7$

$det(A_x) = \begin{vmatrix} 5 & 2 \\ -4 & -3 \end{vmatrix} = -15 - (-8) = -7$

$det(A_y) = \begin{vmatrix} 1 & 5 \\ 2 & -4 \end{vmatrix} = -4 - 10 = -14$

$x = \frac{-7}{-7} = 1$ $y = \frac{-14}{-7} = 2$

d) $\begin{pmatrix} 3 & 2 & -5 \\ 2 & -4 & -2 \\ 1 & -2 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 8 \\ -4 \\ -4 \end{pmatrix}$ $det(A) \left|\begin{array}{ccc|cc} 3 & 2 & -5 & 3 & 2 \\ 2 & -4 & -2 & 2 & -4 \\ 1 & -2 & -3 & 1 & -2 \end{array}\right| = 36 + (-4) + 20 - (12 + 20 + (-12)) = 32$

$det(A_x) = \left|\begin{array}{ccc|cc} 8 & 2 & -5 & 8 & 2 \\ -4 & -4 & -2 & -4 & -4 \\ -4 & -2 & -3 & -4 & -2 \end{array}\right| = 96 + 16 - 40 - (24 + 32 - 80) = 96$

$det(A_y) = \left|\begin{array}{ccc|cc} 3 & 8 & -5 & 3 & 8 \\ 2 & -4 & -2 & 2 & -4 \\ 1 & -4 & -3 & 1 & -4 \end{array}\right| = 36 - 16 + 40 - (-48 + 24 + 20) = 64$

$det(A_z) = \left|\begin{array}{ccc|cc} 3 & 2 & 8 & 3 & 2 \\ 2 & -4 & -4 & 2 & -4 \\ 1 & -2 & -4 & 1 & -2 \end{array}\right| = 48 - 8 - 32 - (-16 + 24 - 32) = 32$

$x = \frac{96}{32} = 3$ $y = \frac{64}{32} = 2$ $z = \frac{32}{32} = 1$

e) $\begin{pmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \\ 3 & 3 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 9 \\ 3 \end{pmatrix}$ $det(A) = \left|\begin{array}{ccc|cc} 1 & 2 & -1 & 1 & 2 \\ 2 & -1 & 3 & 2 & -1 \\ 3 & 3 & -2 & 3 & 3 \end{array}\right| = 2+18-6-(+3+9-8) = 10$

$det(A_x) = \left|\begin{array}{ccc|cc} 2 & 2 & -1 & 2 & 2 \\ 9 & -1 & 3 & 9 & -1 \\ 3 & 3 & -2 & 3 & 3 \end{array}\right| = 4+18-27-(-36+18+3) = 10$

$det(A_y) = \left|\begin{array}{ccc|cc} 1 & 2 & -1 & 1 & 2 \\ 2 & 9 & 3 & 2 & 9 \\ 3 & 3 & -2 & 3 & 3 \end{array}\right| = -18+18-6-(-27+9-8) = 20$

$det(A_z) = \left|\begin{array}{ccc|cc} 1 & 2 & 2 & 1 & 2 \\ 2 & -1 & 9 & 2 & -1 \\ 3 & 3 & 3 & 3 & 3 \end{array}\right| = -3+54+12-(12+27-6) = 30$

$X = \frac{10}{10} = 1$ $Y = \frac{20}{10} = 2$ $Z = \frac{30}{10} = 3$

f) $\begin{pmatrix} 1 & 0 & 3 \\ 2 & -4 & 0 \\ 3 & -2 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -8 \\ -4 \\ 26 \end{pmatrix}$ $det(A) = \left|\begin{array}{ccc|cc} 1 & 0 & 3 & 1 & 0 \\ 2 & -4 & 0 & 2 & -4 \\ 3 & -2 & -5 & 3 & -2 \end{array}\right| = 20+0-12-(0+0-36) = 44$

$det(A_x) = \left|\begin{array}{ccc|cc} -8 & 0 & 3 & -8 & 0 \\ -4 & -4 & 0 & -4 & -4 \\ 26 & -2 & -5 & 26 & -2 \end{array}\right| = -160+0+24-(-312+0+0) = 176$

$det(A_y) = \left|\begin{array}{ccc|cc} 1 & -8 & 3 & 1 & -8 \\ 2 & -4 & 0 & 2 & -4 \\ 3 & 26 & -5 & 3 & 26 \end{array}\right| = -20+0+156-(-80+0-36) = 252$

$$\det(A_x) = \left|\begin{array}{ccc|cc} 1 & 0 & -8 & 1 & 0 \\ 2 & -4 & -4 & 2 & -4 \\ 3 & -2 & 26 & 3 & -2 \end{array}\right| = -104+0+32-(96+8+0) = -176$$

$$X = \frac{176}{44} = 4 \quad Y = \frac{252}{44} \quad Z = \frac{-176}{44} = -4$$

g) $$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 4 & 6 \\ 3 & 2 & 3 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 10 \\ 23 \\ 10 \end{pmatrix} \quad \det(A) = \left|\begin{array}{ccc|cc} 1 & 2 & 3 & 1 & 2 \\ 3 & 4 & 6 & 3 & 4 \\ 3 & 2 & 3 & 3 & 2 \end{array}\right| = 12+36+18-(18+12+36) = \underline{0}$$

$$\det(A_x) = \left|\begin{array}{ccc|cc} 10 & 2 & 3 & 10 & 2 \\ 23 & 4 & 6 & 23 & 4 \\ 10 & 2 & 3 & 10 & 2 \end{array}\right| = 120+120+138-(138+120+120) = 0$$

$$\det(A_y) = \left|\begin{array}{ccc|cc} 1 & 10 & 3 & 1 & 10 \\ 3 & 23 & 6 & 3 & 23 \\ 3 & 10 & 3 & 3 & 10 \end{array}\right| = 69+180+90-(90+60+207) = \underline{-18}$$

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5 - a) $$\begin{pmatrix} 3 & -4 \\ -6 & +8 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \det(A) = 24-24 = 0$$

$$\det(A_x) = \begin{vmatrix} 0 & -4 \\ 0 & 8 \end{vmatrix} = 0 \quad \det(A_y) = \begin{vmatrix} 3 & 0 \\ -6 & 0 \end{vmatrix} = 0$$

$X = 0 \quad Y = 0 \quad S = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$ Soluções únicas

b) $$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 4 \\ 1 & 1 & 3 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \det(A) = \left|\begin{array}{ccc|cc} 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 4 & 2 & 2 \\ 1 & 1 & 3 & 1 & 1 \end{array}\right| = 6+4+2-(6+4+2) = \underline{0}$$

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$$\det(A_x) = \left|\begin{array}{ccc|cc} 0 & 1 & 1 & 0 & 1 \\ 0 & 2 & 4 & 0 & 2 \\ 0 & 1 & 3 & 0 & 1 \end{array}\right| = 0 \quad \det(A_y) = \left|\begin{array}{ccc|cc} 1 & 0 & 1 & 1 & 0 \\ 2 & 0 & 4 & 2 & 0 \\ 1 & 0 & 3 & 1 & 0 \end{array}\right| = 0 \quad \det(A_z) = \left|\begin{array}{ccc|cc} 1 & 1 & 0 & 1 & 1 \\ 2 & 2 & 0 & 2 & 2 \\ 1 & 1 & 0 & 1 & 1 \end{array}\right| = 0$$

$X=0$ $y=0$ $z=0$ $S=(0\ 0\ 0)$ Soluções infinitas

c) $\begin{pmatrix} 1 & 1 & 2 \\ 1 & -1 & 3 \\ 1 & 4 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ $\det(A) = \left|\begin{array}{ccc|cc} 1 & 1 & 2 & 1 & 1 \\ 1 & -1 & 3 & 1 & -1 \\ 1 & 4 & 0 & 1 & 4 \end{array}\right| = 0+3+8-(0+12+(-2)) = 1$

$$\det(A_x) = \left|\begin{array}{ccc|cc} 0 & 1 & 2 & 0 & 1 \\ 0 & -1 & 3 & 0 & -1 \\ 0 & 4 & 4 & 0 & 4 \end{array}\right| = 0 \quad \det(A_y) = \begin{vmatrix} 1 & 0 & 2 \\ 1 & 0 & 3 \\ 1 & 0 & 4 \end{vmatrix} = 0 \quad \det(A_z) = \begin{vmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 1 & 4 & 0 \end{vmatrix} = 0$$

$X=0$ $y=0$ $z=0$ Solução Única
$S=(0\ 0\ 0)$

6 - a) $\begin{pmatrix} 3 & m \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $\det(A) = -3-m \neq 0$
$m \neq -3$

b) $\begin{pmatrix} 3 & 2(m-1) \\ m & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\det(A) = -12-(2m^2-2m) \neq 0$
$-2m^2+2m-12 \neq 0$
$\Delta = 4-96 = -95$ delta negativo
$m \in \mathbb{R}$

c) $\begin{pmatrix} 1 & -1 & 0 \\ 1 & m & 1 \\ -1 & 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}$ $\det(A) = \left|\begin{array}{ccc|cc} 1 & -1 & 0 & 1 & -1 \\ 1 & m & 1 & 1 & m \\ -1 & 1 & -1 & -1 & 1 \end{array}\right| = -m+1+0-(1+1+0) =$
$= -m-1 \neq 0$
$m \neq -1$

d) $\begin{pmatrix} m & 1 & -1 \\ 1 & m & 1 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix}$ $\quad 0 \neq \det(A) = \left| \begin{matrix} m & 1 & -1 \\ 1 & m & 1 \\ 1 & -1 & 0 \end{matrix} \middle| \begin{matrix} m & 1 \\ 1 & m \\ 1 & -1 \end{matrix} \right| = \rightarrow$

$= 1 + 1 - (-m - m) = 2m + 2 \neq 0$

$m \neq -1$

7- $\begin{cases} 6x - 2y = 750 \\ x + y = 225 \end{cases} \Rightarrow \begin{pmatrix} 6 & -2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 750 \\ 225 \end{pmatrix}$ $\quad \det(A) = 6 - (-2) = 8$

$\det A_x = \begin{vmatrix} 750 & -2 \\ 225 & 1 \end{vmatrix} = 750 + 450 = 1200$ $\quad \det(A_y) = \begin{vmatrix} 6 & 750 \\ 1 & 225 \end{vmatrix} = 1350 - 750 = 600$

$x = \frac{1200}{8} = 150 \qquad y = \frac{600}{80} = 75$

8- $\begin{cases} 0{,}6x + 0{,}2y = 300 \\ x + y = 540 \end{cases} \Rightarrow \begin{pmatrix} 0{,}6 & 0{,}2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 300 \\ 540 \end{pmatrix}$ $\quad \det(A) = 0{,}6 - 0{,}2 = 0{,}4$

$\det(A_x) = \begin{vmatrix} 300 & 0{,}2 \\ 540 & 1 \end{vmatrix} = 192$ $\quad \det(A_y) = \begin{vmatrix} 0{,}6 & 300 \\ 1 & 540 \end{vmatrix} = 324 - 300 = 24$

$x = \frac{192}{0{,}4} = 480 \qquad y = \frac{24}{0{,}4} = 60$

9- $\begin{matrix} 2x + 5y + 10x = 500 \\ 2x + y = 92 \end{matrix} \Rightarrow \begin{pmatrix} 12 & 5 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 500 \\ 92 \end{pmatrix}$ $\quad \det(A) = 12 - 10 = 2$

$\det(A_x) = \begin{vmatrix} 500 & 5 \\ 92 & 1 \end{vmatrix} \; 500 - 460 = 40$ $\quad \det(A_y) \cdot \begin{vmatrix} 12 & 500 \\ 2 & 92 \end{vmatrix} = 1104 - 1000 = 104$

$x = \frac{40}{2} = 20 \qquad y = \frac{104}{2} = 52$

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10- $\begin{cases} K+A=109 \\ K+T=142 \\ T+A=97 \end{cases} \Rightarrow \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} K \\ A \\ T \end{pmatrix} = \begin{pmatrix} 109 \\ 142 \\ 97 \end{pmatrix}$ $\det(A) = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix} \begin{matrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{matrix} = -1-1=-2$

$\det(A_K) = \begin{vmatrix} 109 & 1 & 0 \\ 142 & 0 & 1 \\ 97 & 1 & 1 \end{vmatrix} \begin{matrix} 109 & 1 \\ 142 & 0 \\ 97 & 1 \end{matrix} = 97-(142+109)=-154$

$\det(A_A) = \begin{vmatrix} 1 & 109 & 0 \\ 1 & 142 & 1 \\ 0 & 97 & 1 \end{vmatrix} \begin{matrix} 1 & 109 \\ 1 & 142 \\ 0 & 97 \end{matrix} = 142-(109+97)=-64$

$\det(A_T) = \begin{vmatrix} 1 & 1 & 109 \\ 1 & 0 & 142 \\ 0 & 1 & 97 \end{vmatrix} \begin{matrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{matrix} = 109-(97+142)=-130$

$K = \frac{-154}{-2} = 77$ $A = \frac{-64}{-2} = 32$ $T = \frac{-130}{-2} = 65$