

Sista 4

Aluno: Luis Felipe Barbosa Leite

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$$1-a) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \quad \begin{cases} x=2 \\ y=-1 \end{cases}$$

$$b) \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ 2 \\ 1 \end{pmatrix} \quad \begin{cases} x=4 \\ y=3 \\ z=2 \\ w=1 \end{cases}$$

$$c) \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ & & & w \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \\ 2 \end{pmatrix} \quad \begin{cases} x=6 \\ y=3 \\ z+w=2 \end{cases}$$

$$d) \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & -1 \\ & & z \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad \begin{cases} x+z=1 \\ y-z=2 \end{cases} \quad \begin{cases} x=1-3z \\ y=2+\alpha \\ z=\alpha \end{cases}$$

Seja $z=\alpha; \alpha \in \mathbb{R}$

$$e) \begin{pmatrix} 1 & 0 & 0 & -7 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \\ & & & w \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 8 \\ 2 \\ -5 \end{pmatrix} \quad \begin{cases} x-7w=8 \\ y+3w=2 \\ z+w=-5 \end{cases} \quad \begin{cases} x=8+7\beta \\ y=2-3\beta \\ z=-5-\beta \\ w=\beta \end{cases}$$

Seja $w=\beta; \beta \in \mathbb{R}$

$$f) \begin{pmatrix} 1 & -6 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 \\ & & & & t \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} -2 \\ 7 \\ 8 \\ 0 \end{pmatrix} \quad \begin{cases} x-6y+3t=-2 \\ z+4t=7 \\ w+5t=8 \\ 0=0 \end{cases} \quad \begin{cases} x=-2+6\alpha-3\beta \\ z=7-4\beta \\ w=8-5\beta \\ y=\alpha \\ t=\beta \end{cases}$$

Seja $y=\alpha$ e $t=\beta; \alpha, \beta \in \mathbb{R}$



$$2-a) \begin{pmatrix} 3 & -4 & 1 \\ 1 & 3 & 9 \end{pmatrix} \xrightarrow{l_1 \leftrightarrow l_2} \begin{pmatrix} 1 & 3 & 9 \\ 3 & -4 & 1 \end{pmatrix} \xrightarrow{l_2 \leftarrow l_2 - 3l_1} \begin{pmatrix} 1 & 3 & 9 \\ 0 & 13 & 26 \end{pmatrix} \xrightarrow{l_2 \leftarrow l_2 / 13}$$

$$\begin{pmatrix} 1 & 3 & 9 \\ 0 & 1 & 2 \end{pmatrix} \xrightarrow{l_1 \leftarrow l_1 - 3l_2} \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \end{pmatrix} \xrightarrow{l_1 \leftarrow l_1 - 3l_2} \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \end{pmatrix} \Rightarrow \begin{cases} x=3 \\ y=2 \end{cases}$$

$$b) \begin{pmatrix} 5 & 8 & 34 \\ 10 & 16 & 50 \end{pmatrix} \xrightarrow{l_2 \leftarrow 2l_1} \begin{pmatrix} 5 & 8 & 34 \\ 10 & 16 & 50 \end{pmatrix} \text{ Sistema Impossível}$$

$$c) \begin{pmatrix} 1 & 2 & 5 \\ 2 & -3 & -4 \end{pmatrix} \xrightarrow{l_2 \leftarrow l_2 - 2l_1} \begin{pmatrix} 1 & 2 & 5 \\ 0 & -7 & -14 \end{pmatrix} \xrightarrow{l_2 \leftarrow l_2 / -7} \begin{pmatrix} 1 & 2 & 5 \\ 0 & 1 & 2 \end{pmatrix} \xrightarrow{l_1 \leftarrow l_1 - 2l_2}$$

$$\Rightarrow \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix} \Rightarrow \begin{cases} x=1 \\ y=2 \end{cases}$$

$$d) \begin{pmatrix} 3 & 2 & -5 & 8 \\ 2 & -4 & -2 & -4 \\ 1 & -2 & -3 & -4 \end{pmatrix} \xrightarrow{l_1 \leftarrow l_1 - l_2} \begin{pmatrix} 1 & 6 & -3 & 12 \\ 2 & -4 & -2 & -4 \\ 1 & -2 & -3 & -4 \end{pmatrix} \xrightarrow{l_3 \leftarrow l_3 - l_1} \begin{pmatrix} 1 & 6 & -3 & 12 \\ 2 & -4 & -2 & -4 \\ 0 & -8 & 0 & -16 \end{pmatrix}$$

$$\xrightarrow{l_2 \leftarrow l_2 - 2l_1} \begin{pmatrix} 1 & 6 & -3 & 12 \\ 0 & -16 & 4 & -28 \\ 0 & -8 & 0 & -16 \end{pmatrix} \xrightarrow{l_3 \leftarrow l_3 \cdot 2} \begin{pmatrix} 1 & 6 & -3 & 12 \\ 0 & -16 & 4 & -28 \\ 0 & -16 & 0 & -32 \end{pmatrix} \xrightarrow{l_3 \leftarrow l_3 - l_2} \begin{pmatrix} 1 & 6 & -3 & 12 \\ 0 & -16 & 4 & -28 \\ 0 & 0 & -4 & -4 \end{pmatrix}$$

$$\xrightarrow{l_3 \leftarrow l_3 / -4} \begin{pmatrix} 1 & 6 & -3 & 12 \\ 0 & -16 & 4 & -28 \\ 0 & 0 & 1 & 1 \end{pmatrix} \xrightarrow{l_2 \leftarrow l_2 / 4} \begin{pmatrix} 1 & 6 & -3 & 12 \\ 0 & -4 & 1 & -7 \\ 0 & 0 & 1 & 1 \end{pmatrix} \xrightarrow{l_2 \leftarrow l_2 + l_3} \begin{pmatrix} 1 & 6 & -3 & 12 \\ 0 & -4 & 0 & -6 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\xrightarrow{l_2 \leftarrow l_2 / -4} \begin{pmatrix} 1 & 6 & -3 & 12 \\ 0 & 1 & 0 & 3/2 \\ 0 & 0 & 1 & 1 \end{pmatrix} \xrightarrow{l_1 \leftarrow l_1 - 6l_2} \begin{pmatrix} 1 & 0 & -3 & 3 \\ 0 & 1 & 0 & 3/2 \\ 0 & 0 & 1 & 1 \end{pmatrix} \xrightarrow{l_1 \leftarrow l_1 + 3l_3} \begin{pmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 3/2 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

$$e) \begin{pmatrix} 2 & -6 & -4 \\ 1 & 3 & 1 \\ 4 & 12 & 2 \end{pmatrix} \xrightarrow{l_2 \leftarrow l_2 - 4l_1} \begin{pmatrix} 2 & -6 & -4 \\ 1 & 3 & 1 \\ 0 & 0 & -2 \end{pmatrix} \text{ Sistema Impossível}$$

$$f) \left(\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 2 & -1 & 3 & 9 \\ 3 & 3 & -2 & 3 \end{array} \right) \begin{array}{l} l_2 \leftarrow l_2 - 2l_1 \\ l_3 \leftarrow l_3 - 3l_1 \end{array} \Rightarrow \left(\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & -5 & 5 & 5 \\ 0 & -3 & 1 & -3 \end{array} \right) \begin{array}{l} l_2 \leftarrow -l_2 \\ l_2 \leftarrow l_2 / 5 \end{array} \Rightarrow \left(\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & -3 & 1 & -3 \end{array} \right) \begin{array}{l} l_1 \leftarrow l_1 - 2l_2 \end{array}$$

$$\Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 1 & -1 & -1 \\ 0 & -3 & 1 & -3 \end{array} \right) \begin{array}{l} l_3 \leftarrow l_3 + 3l_2 \\ l_3 \leftarrow l_3 / 2 \end{array} \Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & -2 & -6 \end{array} \right) \begin{array}{l} l_3 \leftarrow l_3 / -2 \\ l_1 \leftarrow l_1 - l_3 \end{array} \Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 3 \end{array} \right) \begin{array}{l} l_2 \leftarrow l_2 + l_3 \end{array}$$

$$\Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array} \right) \begin{array}{l} X=1 \\ Y=2 \\ Z=3 \end{array}$$

$$g) \left(\begin{array}{ccc|c} 1 & 0 & 3 & -8 \\ 2 & -4 & 0 & -4 \\ 3 & -2 & -5 & 26 \end{array} \right) \begin{array}{l} l_2 \leftarrow l_2 - 2l_1 \\ l_3 \leftarrow l_3 - 3l_1 \end{array} \Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 3 & -8 \\ 0 & -4 & -6 & 12 \\ 0 & -2 & -14 & 50 \end{array} \right) \begin{array}{l} l_3 \leftarrow l_3 / -2 \\ l_2 \leftarrow l_2 + 3l_3 \end{array} \Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 3 & -8 \\ 0 & -1 & 15 & -63 \\ 0 & -1 & 7 & -25 \end{array} \right)$$

$$\begin{array}{l} l_3 \leftarrow l_3 + l_2 \\ l_3 \leftarrow l_3 / 22 \end{array} \Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 3 & -8 \\ 0 & -1 & 15 & -63 \\ 0 & 0 & 22 & -88 \end{array} \right) \begin{array}{l} l_1 \leftarrow l_1 - 3l_3 \\ l_2 \leftarrow l_2 + 15l_3 \end{array} \Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & -1 & 15 & -63 \\ 0 & 0 & 1 & -4 \end{array} \right) \begin{array}{l} l_2 \leftarrow -l_2 \\ l_2 \leftarrow l_2 + 15l_3 \end{array} \Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -4 \end{array} \right)$$

$$X=4 \quad Y=3 \quad Z=-4$$

$$h) \left(\begin{array}{ccc|c} 1 & 2 & 3 & 10 \\ 3 & 4 & 6 & 23 \\ 2 & 2 & 3 & 13 \end{array} \right) \begin{array}{l} l_2 \leftarrow l_2 - 3l_1 \\ l_3 \leftarrow l_3 - 2l_1 \end{array} \Rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 3 & 10 \\ 0 & -2 & -3 & -7 \\ 0 & -2 & -3 & -7 \end{array} \right) \begin{array}{l} l_3 \leftarrow l_3 - l_2 \end{array} \Rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 3 & 10 \\ 0 & -2 & -3 & -7 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\begin{array}{l} l_1 \leftarrow l_1 + l_2 \\ l_2 \leftarrow l_2 / -2 \end{array} \Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & \frac{3}{2} & \frac{7}{2} \\ 0 & 0 & 0 & 0 \end{array} \right) \begin{array}{l} X=3 \\ Y=\frac{7}{2} - \frac{3}{2}X \\ Z=\alpha \end{array} \quad \alpha \in \mathbb{R}$$

$$i) \left(\begin{array}{ccc|c} 1 & -3 & 4 & -1 \\ 2 & -1 & 3 & 19 \end{array} \right) \begin{array}{l} l_2 \leftarrow l_2 - 2l_1 \end{array} \Rightarrow \left(\begin{array}{ccc|c} 1 & -3 & 4 & -1 \\ 0 & 5 & -5 & 15 \end{array} \right) \begin{array}{l} l_2 \leftarrow l_2 / 5 \end{array}$$

$$\Rightarrow \left(\begin{array}{ccc|c} 1 & -3 & 4 & -1 \\ 0 & 1 & -1 & 3 \end{array} \right) \begin{array}{l} l_1 \leftarrow l_1 + 3l_2 \end{array} \Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 1 & 11 \\ 0 & 1 & -1 & 3 \end{array} \right)$$

$$3-a) \begin{pmatrix} 1 & 2 & 3 & 1 & 8 \\ 1 & 3 & 0 & 1 & 7 \\ 1 & 0 & 2 & 1 & 3 \end{pmatrix} \begin{array}{l} l_2 \leftarrow l_2 - l_1 \\ l_3 \leftarrow l_3 - l_1 \end{array} \Rightarrow \begin{pmatrix} 1 & 2 & 3 & 1 & 8 \\ 0 & 1 & -3 & 0 & 1 \\ 0 & -2 & -1 & 0 & -5 \end{pmatrix} \begin{array}{l} l_3 \leftarrow l_3 + 2l_2 \\ l_1 \leftarrow l_1 - 2l_2 \end{array} \Rightarrow \begin{pmatrix} 1 & 2 & 3 & 1 & 8 \\ 0 & 1 & -3 & 0 & 1 \\ 0 & 0 & -7 & 0 & -7 \end{pmatrix}$$

$$\begin{cases} X + 2Y + 3Z + W = 8 \\ Y - 3Z = -1 \\ Z = 1 \end{cases} \Rightarrow \begin{cases} X + 2Y + 3(1) + W = 8 \\ Y = -1 + 3(1) \\ Z = 1 \\ W = \alpha \end{cases} \Rightarrow \begin{cases} X = 1 - W \\ Y = 2 \\ Z = 1 \\ W = \alpha \end{cases}$$

$$W = \alpha; \alpha \in \mathbb{R}$$

$$b) \begin{pmatrix} 1 & 1 & 3 & -3 & 0 \\ 0 & 2 & 1 & -3 & 3 \\ 1 & 0 & 2 & -1 & -1 \end{pmatrix} \begin{array}{l} l_3 \leftarrow l_3 - l_1 \\ l_2 \leftarrow l_2 - l_3 \end{array} \Rightarrow \begin{pmatrix} 1 & 1 & 3 & -3 & 0 \\ 0 & 3 & 2 & -5 & 4 \\ 0 & -1 & -1 & 2 & -1 \end{pmatrix} \begin{array}{l} l_1 \leftarrow l_1 + l_3 \\ l_2 \leftarrow l_2 + 2l_3 \end{array} \Rightarrow \begin{pmatrix} 1 & 0 & 2 & -1 & -1 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & -1 & -1 & 2 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 2 & -1 & -1 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & -1 & 1 & 1 \end{pmatrix} \begin{array}{l} l_3 \leftarrow l_3 + l_2 \\ l_1 \leftarrow l_1 + 2l_3 \end{array} \Rightarrow \begin{pmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 1 & -1 & -1 \end{pmatrix}$$

$$\begin{cases} X + W = 1 \\ Y - W = 2 \\ Z - W = -1 \end{cases} \Rightarrow \begin{cases} X = 1 - \alpha \\ Y = 2 + \alpha \\ Z = -1 + \alpha \\ W = \alpha \end{cases}$$

$$W = \alpha; \alpha \in \mathbb{R}$$

$$c) \begin{pmatrix} 1 & 2 & 3 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 2 & 0 \\ 1 & 3 & 3 & 0 \end{pmatrix} \begin{array}{l} l_2 \leftarrow l_2 - l_1 \\ l_3 \leftarrow l_3 - l_1 \\ l_4 \leftarrow l_4 - l_1 \end{array} \Rightarrow \begin{pmatrix} 1 & 2 & 3 & 0 \\ 0 & -1 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{array}{l} l_1 \leftarrow l_1 - 2l_2 \\ l_2 \leftarrow l_2 + l_3 \\ l_4 \leftarrow l_4 - l_3 \end{array} \Rightarrow \begin{pmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{array}{l} l_1 \leftarrow l_1 - 3l_3 \\ l_2 \leftarrow l_2 - 2l_3 \\ l_4 \leftarrow l_4 - l_3 \end{array} \Rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow \begin{cases} X = 0 \\ Y = 0 \\ Z = 0 \end{cases}$$



$$4. \begin{pmatrix} 1 & -2 & 1 & 1 & 2 \\ 2 & -5 & 1 & -2 & -1 \\ 3 & -7 & 2 & -1 & 2 \end{pmatrix} \begin{array}{l} l_2 \leftarrow l_2 - 2l_1 \\ l_3 \leftarrow l_3 - 3l_1 \end{array} \Rightarrow \begin{pmatrix} 1 & -2 & 1 & 1 & 2 \\ 0 & -1 & -1 & -4 & -5 \\ 0 & -1 & -1 & -4 & -4 \end{pmatrix} \begin{array}{l} l_3 \leftarrow l_3 - l_2 \end{array}$$

$$\Rightarrow \begin{pmatrix} 1 & -2 & 1 & 1 & 2 \\ 0 & -1 & -1 & -4 & -5 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix} \begin{array}{l} l_2 \leftarrow -l_2 \\ l_1 \leftarrow l_1 + 2l_2 \end{array} \Rightarrow \begin{pmatrix} 1 & 0 & 3 & 9 & 12 \\ 0 & 1 & 1 & 4 & 5 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

a) $\begin{cases} x + 3z = 9 \\ y + z = 4 \\ z = \alpha; \alpha \in \mathbb{R} \end{cases} \Rightarrow \begin{cases} x = 9 + 3\alpha \\ y = 4 - \alpha \\ z = \alpha \end{cases}$ b) *biestima impossível*

$$5. a) \begin{pmatrix} 1 & 0 & 5 \\ 1 & 1 & 1 \\ 0 & 1 & -4 \end{pmatrix} + 4 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 0 & 5 \\ 1 & 5 & 1 \\ 0 & 1 & 0 \end{pmatrix} = A + 4I_3$$

$$(A + 4I_3)X = 0 \Rightarrow \begin{pmatrix} 5 & 0 & 5 \\ 1 & 5 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 5 & 0 & 5 & 0 \\ 1 & 5 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{array}{l} l_1 \leftarrow l_1/5 \end{array} \Rightarrow$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 1 & 5 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{array}{l} l_2 \leftarrow l_2 - l_1 \end{array} \Rightarrow \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{array}{l} l_2 \leftarrow l_2/5 \end{array} \Rightarrow \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{array}{l} l_3 \leftarrow l_3 - l_2 \end{array} \Rightarrow \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{cases} x + z = 0 \\ x + z = 0 \\ y = 0 \end{cases} \Rightarrow \begin{cases} x = -z \\ z = -x \\ y = 0 \end{cases} \quad S = \begin{pmatrix} 2 \\ 0 \\ -2 \end{pmatrix}$$

b) $AX = 2X \Leftrightarrow AX - 2X = 0 \Rightarrow (A - 2I_3)X = 0$

$$\begin{pmatrix} 1 & 0 & 5 \\ 1 & 1 & 1 \\ 0 & 1 & -4 \end{pmatrix} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 5 \\ 1 & -1 & 1 \\ 0 & 1 & -6 \end{pmatrix} = A - 2I_3$$

→ 2x

3x x



$$(A - 2I_3)X = 0 \quad \begin{pmatrix} -1 & 0 & 5 \\ 1 & -1 & 1 \\ 0 & 1 & -6 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} -1 & 0 & 5 & 0 \\ 1 & -1 & 1 & 0 \\ 0 & 1 & -6 & 0 \end{pmatrix} \begin{cases} -X + 5Z = 0 \\ X - Y + Z = 0 \\ Y - 6Z = 0 \end{cases}$$

$$\begin{cases} X = 5Z \\ X - Y + Z = 0 \\ Y = 6Z \end{cases} \quad \begin{cases} X = 5Z \\ 5Z - 6Z + Z = 0 \\ Y = 6Z \end{cases} \quad \begin{cases} X = 5Z \\ 0 = 0 \\ Y = 6Z \end{cases} \quad \begin{cases} X = 5\alpha \\ Y = 6\alpha \\ Z = \alpha \end{cases} \quad S = \begin{pmatrix} 5\alpha \\ 6\alpha \\ \alpha \end{pmatrix}$$

$$Z = \alpha \in \mathbb{R}$$

$$I - a) \begin{pmatrix} 2 & -2 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{pmatrix} \begin{matrix} l_1 \leftarrow l_1/2 \\ l_2 \leftarrow l_2 - 3l_1 \end{matrix} \Rightarrow \begin{pmatrix} 1 & -1 & \frac{1}{2} & 0 \\ 0 & 4 & -\frac{3}{2} & 1 \end{pmatrix} \begin{matrix} l_3 \leftarrow l_3/4 \\ \Rightarrow \end{matrix} \begin{pmatrix} 1 & -1 & \frac{1}{2} & 0 \\ 0 & 1 & -\frac{3}{8} & \frac{1}{4} \end{pmatrix}$$

$$l_1 + l_1 + l_2 \Rightarrow \begin{pmatrix} 1 & 0 & \frac{11}{8} & \frac{1}{4} \\ 0 & 1 & -\frac{3}{8} & \frac{1}{4} \end{pmatrix} \quad A^{-1} = \begin{pmatrix} \frac{11}{8} & \frac{1}{4} \\ -\frac{3}{8} & \frac{1}{4} \end{pmatrix}$$

$$b) \begin{pmatrix} 2 & -2 & 0 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{pmatrix} \begin{matrix} l_1 \leftarrow l_1/2 \\ l_2 \leftarrow l_2 - l_1 \end{matrix} \Rightarrow \begin{pmatrix} 1 & -1 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 3 & 1 & -\frac{1}{2} & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{pmatrix} \begin{matrix} l_3 \leftarrow l_3 - l_2 \end{matrix}$$

$$\Rightarrow \begin{pmatrix} 1 & -1 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 2 & -\frac{1}{2} & 1 & -2 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{pmatrix} \begin{matrix} l_3 \leftarrow l_3 - l_2 \\ l_2 \leftarrow l_2/2 \end{matrix} \Rightarrow \begin{pmatrix} 1 & -1 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & -1 & 0 & -\frac{1}{2} & 1 & -4 \\ 0 & 1 & -1 & 0 & 0 & -1 \end{pmatrix} \begin{matrix} l_3 \leftarrow l_3 - l_2 \\ l_2 \leftarrow -l_2 \end{matrix}$$

$$\begin{pmatrix} 1 & -1 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} & -1 & 4 \\ 0 & 0 & 1 & -\frac{1}{2} & 1 & -4 \end{pmatrix} \begin{matrix} l_1 \leftarrow l_1 + l_2 \\ \Rightarrow \end{matrix} \begin{pmatrix} 1 & 0 & 0 & 1 & -1 & 4 \\ 0 & 1 & 0 & \frac{1}{2} & -1 & 4 \\ 0 & 0 & 1 & -\frac{1}{2} & 1 & -4 \end{pmatrix}$$

$$B^{-1} = \begin{pmatrix} 1 & -1 & 4 \\ \frac{1}{2} & -1 & 4 \\ -\frac{1}{2} & 1 & -4 \end{pmatrix}$$

$$c) \begin{pmatrix} 3 & 5 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{pmatrix} \begin{matrix} l_1 \leftarrow l_1 - 2l_2 \\ \Rightarrow \end{matrix} \begin{pmatrix} 1 & 1 & 1 & -2 \\ 1 & 2 & 0 & 1 \end{pmatrix} \begin{matrix} l_2 \leftarrow l_2 - l_1 \\ \Rightarrow \end{matrix} \begin{pmatrix} 1 & 1 & 1 & -2 \\ 0 & 1 & -1 & -3 \end{pmatrix}$$



$$L_1 \leftarrow L_1 - L_2 \quad \left(\begin{array}{cc|cc} 1 & 0 & 2 & 1 \\ 0 & 1 & -1 & -3 \end{array} \right) \quad C^{-1} = \begin{pmatrix} 2 & 1 \\ -1 & -3 \end{pmatrix}$$

$$D = \left(\begin{array}{ccc|ccc} 0 & -1 & 1 & 1 & 0 & 0 \\ 2 & 0 & -1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{array} \right) \quad \begin{array}{l} L_1 \leftarrow L_1/2 \\ L_2 \leftarrow L_2 - L_1 \\ L_3 \leftarrow L_3 - L_1 \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & 0 & -\frac{1}{2} & 1 \end{array} \right) \quad \begin{array}{l} L_3 \leftarrow L_3 - L_2 \\ L_1 \leftarrow L_1 + \frac{1}{2}L_3 \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \end{array} \right) \quad \begin{array}{l} L_2 \leftarrow L_2 + L_3 \\ L_1 \leftarrow L_1 + L_3 \end{array} \quad D^{-1} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

$$e) \quad L_1 \leftarrow L_1/2 \quad \left(\begin{array}{cccc|ccc} 1 & -\frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & \frac{1}{2} & 2 & \frac{5}{2} & -\frac{1}{2} & 0 & 1 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 1 \end{array} \right) \quad \begin{array}{l} L_2 \leftarrow L_2/2 \\ L_1 \leftarrow L_1 + \frac{1}{2}L_2 \end{array}$$

$$\left(\begin{array}{cccc|ccc} 1 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 1 & 0 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 2 & \frac{5}{2} & -\frac{1}{2} & 0 & 1 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 1 \end{array} \right) \quad \begin{array}{l} L_3 \leftarrow L_3 - \frac{1}{2}L_2 \\ L_4 \leftarrow L_4 + \frac{1}{2}L_2 \end{array}$$

$$\left(\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & \frac{3}{11} & \frac{3}{11} & -\frac{1}{11} & -\frac{1}{11} \\ 0 & 1 & 0 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & \frac{11}{8} & -\frac{1}{4} & -\frac{1}{8} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 & -\frac{2}{11} & -\frac{1}{11} & \frac{4}{11} & \frac{4}{11} \end{array} \right) \quad \begin{array}{l} L_2 \leftarrow L_2 + \frac{1}{2}L_4 \\ L_3 \leftarrow L_3 - \frac{11}{8}L_4 \end{array}$$

$$E^{-1} = \begin{pmatrix} \frac{6}{11} & \frac{3}{11} & -\frac{1}{11} & -\frac{1}{11} \\ -\frac{1}{11} & \frac{5}{11} & \frac{3}{11} & \frac{2}{11} \\ 0 & 0 & 0 & -\frac{1}{2} \\ -\frac{2}{11} & -\frac{1}{11} & \frac{4}{11} & \frac{4}{11} \end{pmatrix}$$

$$8. \begin{cases} 1C + 2S + 3B = 26 \\ 2C + 5S + 6B = 60 \\ 3C + 3S + 4B = 40 \end{cases} \quad \det(A) = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 5 & 6 \\ 3 & 3 & 4 \end{vmatrix} = 20 + 18 - (16 + 30) = -2$$

$$\det(A_c) = \begin{vmatrix} 26 & 2 & 3 \\ 60 & 5 & 6 \\ 40 & 3 & 4 \end{vmatrix} = 520 + 480 + 540 - (480 + 468 + 600) = -8$$

$$\det(A_s) = \begin{vmatrix} 1 & 26 & 3 \\ 2 & 60 & 6 \\ 3 & 40 & 4 \end{vmatrix} = 240 + 312 + 240 - (208 + 240 + 360) = -16$$

$$\det(A_B) = \begin{vmatrix} 1 & 2 & 26 \\ 2 & 5 & 60 \\ 3 & 3 & 40 \end{vmatrix} = 200 + 240 + 156 - (160 + 180 + 260) = -4$$

$$Blusa = \frac{-4}{-2} = 2 \quad Calça = \frac{-8}{-2} = 4 \quad Short = \frac{-16}{-2} = 8$$

$$9. \begin{cases} 2C + 5S + 6B = 2200 \\ C = 3b \\ C = b + 2 \end{cases} \rightarrow \begin{cases} 2C + 5S + 6B = 2200 \\ C - 3b = 0 \\ C - b - 2 = 0 \end{cases}$$

$$\det(A) = \begin{vmatrix} 2 & 5 & 6 \\ 1 & 0 & -3 \\ 1 & -1 & -1 \end{vmatrix} = -15 - 6 - (-5 + 6) = -22$$

$$\det(A_c) = \begin{vmatrix} 2200 & 5 & 6 \\ 0 & 0 & -3 \\ 0 & -1 & -1 \end{vmatrix} = -6600 \quad \det(A_s) = \begin{vmatrix} 2 & 2200 & 6 \\ 1 & 0 & -3 \\ 1 & 0 & -1 \end{vmatrix} = -6600$$

$$\det(A_b) = \begin{vmatrix} 2 & 5 & 2200 \\ 1 & 0 & 0 \\ 1 & -1 & 0 \end{vmatrix} = -2200 \quad C = \frac{-6600}{-22} = 300 \quad b = \frac{-2200}{22} = 100$$

$$S = \frac{-4400}{-22} = 200$$



$$10 - \begin{cases} 40C + 30A + 10P = 7000 \\ 20C + 40A + 30P = 6000 \\ 10C + 20A + 40P = 5000 \end{cases} \rightarrow \begin{cases} 4C + 3A + 1P = 700 \\ 2C + 4A + 3P = 600 \\ 1C + 2A + 4P = 500 \end{cases}$$

$$\det(A) = \begin{vmatrix} 4 & 3 & 1 & | & 4 & 3 \\ 2 & 4 & 3 & | & 2 & 4 \\ 1 & 2 & 4 & | & 1 & 2 \end{vmatrix} = 64 + 9 + 4 - (24 + 24 + 4) = 25$$

$$\det(A_C) = \begin{vmatrix} 700 & 3 & 1 & | & 700 & 3 \\ 600 & 4 & 3 & | & 600 & 4 \\ 500 & 2 & 4 & | & 500 & 2 \end{vmatrix} = 11200 + 4500 + 1200 - (7200 + 4200 + 2000) = 3500$$

$$\det(A_A) = \begin{vmatrix} 4 & 700 & 1 & | & 4 & 700 \\ 2 & 600 & 3 & | & 2 & 600 \\ 1 & 500 & 4 & | & 1 & 500 \end{vmatrix} = 9600 + 2100 + 1000 - (600 + 6000 + 500) = 500$$

$$\det(A_P) = \begin{vmatrix} 4 & 3 & 700 & | & 4 & 3 \\ 2 & 4 & 600 & | & 2 & 4 \\ 1 & 2 & 500 & | & 1 & 2 \end{vmatrix} = 8000 + 1800 + 2800 - (3000 + 4800 + 2800) = 2000$$

$$\text{Torra de Carne} = \frac{3500}{25} = 140 \quad \text{Salada} = \frac{500}{25} = 20 \quad \text{Pizzão} = \frac{2000}{25} = 80$$

$$11 - \begin{cases} 2A + 3B + 1C = 8420 \\ 1A + 2B + 2C = 7940 \\ 4A + 3B = 8110 \end{cases} \quad \det(A) = \begin{vmatrix} 2 & 3 & 1 & | & 2 & 3 \\ 1 & 2 & 2 & | & 1 & 2 \\ 4 & 3 & 0 & | & 4 & 3 \end{vmatrix} = 24 + 3 - (12 + 8) = 7$$

$$\det(A_C) = \begin{vmatrix} 8420 & 3 & 1 & | & 8420 & 3 \\ 7940 & 2 & 2 & | & 7940 & 2 \\ 8110 & 3 & 0 & | & 8110 & 3 \end{vmatrix} = 46660 + 23820 - (16270 + 50520) = 5740$$

$$\text{det}(x_0) = \begin{vmatrix} 2 & 8420 & 1 \\ 1 & 7940 & 2 \\ 4 & 8110 & 0 \end{vmatrix} \begin{vmatrix} 2 & 8420 \\ 1 & 7940 \end{vmatrix} = 67360 + 8110 - (32440 + 31760) = 11270$$

$$\text{det}(x_1) = \begin{vmatrix} 2 & 3 & 8420 \\ 1 & 2 & 7940 \\ 4 & 3 & 8110 \end{vmatrix} \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 32440 + 95280 + 25260 - (6730 + 47640 + 24330) = 13650$$

$$A = \frac{5740}{7} = 820 \quad B = \frac{11270}{7} = 1610 \quad C = \frac{13650}{7} = 1950$$

$$\text{Excode} = 1950 - 820 = 1130$$



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