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Lista 1 2024.1.08.018

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1- a) $A+2B = \begin{pmatrix} 1 & 0 \\ 3 & 7 \end{pmatrix} + 2\begin{pmatrix} 0 & 5 \\ 2 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 3 & 7 \end{pmatrix} + \begin{pmatrix} 0 & 10 \\ 4 & -4 \end{pmatrix} = \begin{pmatrix} 1 & 11 \\ 7 & 3 \end{pmatrix}$

b) $AB-BA = \begin{pmatrix} 1 & 0 \\ 3 & 7 \end{pmatrix} \cdot \begin{pmatrix} 0 & 5 \\ 2 & -2 \end{pmatrix} - \begin{pmatrix} 0 & 5 \\ 2 & -2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 3 & 7 \end{pmatrix} = \begin{pmatrix} 1\cdot0+0\cdot2 & 1\cdot5+0\cdot(-2) \\ 3\cdot0+7\cdot2 & 3\cdot5+7(-2) \end{pmatrix} - \begin{pmatrix} 0\cdot1+5\cdot3 & 0\cdot0+5\cdot7 \\ 2\cdot1+(-2)3 & 2\cdot0+(-2)\cdot7 \end{pmatrix} =$

$= \begin{pmatrix} 0 & 5 \\ 14 & 1 \end{pmatrix} - \begin{pmatrix} 15 & 35 \\ -4 & -14 \end{pmatrix} = \begin{pmatrix} -15 & -30 \\ 18 & 15 \end{pmatrix}$

c) $2C-D = 2\begin{pmatrix} -2 & 3 & -7 \\ 7 & -3 & -2 \end{pmatrix} - \begin{pmatrix} -3 & 2 & 0 \\ 1 & 1 & 4 \\ -2 & 0 & 2 \end{pmatrix}$ = Não é definida, pois 2C e D possuem ordem distintas.

d) $2D^t - 3E^t = 2\begin{pmatrix} -3 & 1 & 0 \\ 2 & 1 & 4 \\ 0 & 0 & 2 \end{pmatrix} - 3\begin{pmatrix} 2 & -1 & -6 \\ 4 & 0 & 0 \\ -3 & -4 & -1 \end{pmatrix} = \begin{pmatrix} 2\cdot(-3) & 2\cdot1 & 2\cdot0 \\ 2\cdot2 & 2\cdot1 & 2\cdot4 \\ 2\cdot0 & 2\cdot0 & 2\cdot2 \end{pmatrix} - \begin{pmatrix} 3\cdot2 & 3(-1) & 3(-6) \\ 3\cdot4 & 3\cdot0 & 3\cdot0 \\ 3(-3) & 3(-4) & 3(-1) \end{pmatrix}$

$= \begin{pmatrix} -6 & 2 & 0 \\ 4 & 2 & 8 \\ 0 & 0 & 4 \end{pmatrix} - \begin{pmatrix} 6 & -3 & -18 \\ 12 & 0 & 0 \\ -9 & -12 & -3 \end{pmatrix} = \begin{pmatrix} -12 & 5 & 18 \\ -8 & 2 & 8 \\ 9 & 12 & 7 \end{pmatrix}$

e) $D^2 + DE = \begin{pmatrix} -3 & 2 & 0 \\ 1 & 1 & 4 \\ -2 & 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} -3 & 2 & 0 \\ 1 & 1 & 4 \\ -2 & 0 & 2 \end{pmatrix} + \begin{pmatrix} -3 & 2 & 0 \\ 1 & 1 & 4 \\ -2 & 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} 2 & 4 & -3 \\ -1 & 0 & -4 \\ -6 & 0 & -1 \end{pmatrix} =$

$= \begin{pmatrix} -3\cdot-3+2\cdot1+0\cdot-2 & -3\cdot2+2\cdot1+0\cdot0 & -3\cdot0+2\cdot4+0\cdot2 \\ 1\cdot-3+1\cdot1+4\cdot-2 & 1\cdot2+1\cdot1+4\cdot0 & 1\cdot0+1\cdot4+4\cdot2 \\ -2\cdot-3+0\cdot1+2\cdot-2 & -2\cdot2+0\cdot1+2\cdot0 & -2\cdot0+0\cdot4+2\cdot2 \end{pmatrix} + \begin{pmatrix} -3\cdot2+2\cdot-1+0\cdot-6 & -3\cdot4+2\cdot0+0\cdot0 & -3\cdot-3+2\cdot-4+0\cdot-1 \\ 1\cdot2+1\cdot-1+4\cdot-6 & 1\cdot4+1\cdot0+4\cdot0 & 1\cdot-3+1\cdot-4+4\cdot-1 \\ -2\cdot2+0\cdot-1+2\cdot-6 & -2\cdot4+0\cdot0+2\cdot0 & -2\cdot-3+0\cdot-4+2\cdot-1 \end{pmatrix}$

$= \begin{pmatrix} 11 & -4 & 8 \\ -10 & 3 & 12 \\ 2 & -4 & 4 \end{pmatrix} + \begin{pmatrix} -8 & -12 & 1 \\ -23 & 4 & -11 \\ -16 & -8 & 4 \end{pmatrix} = \begin{pmatrix} 3 & -16 & 9 \\ -33 & 7 & 1 \\ -14 & -12 & 8 \end{pmatrix}$

f) $C^t A = \begin{pmatrix}-2 & 7\\ 3 & -3\\ -7 & -2\end{pmatrix}_{3\times2} \cdot \begin{pmatrix}1 & 3\\ 0 & 7\end{pmatrix}_{2\times2} = \begin{pmatrix}-2\cdot1+7\cdot0 & -2\cdot3+7\cdot7\\ 3\cdot1+-3\cdot0 & 3\cdot3+-3\cdot7\\ -7\cdot1+-2\cdot0 & -7\cdot3+-2\cdot7\end{pmatrix} = \begin{pmatrix}-2 & 43\\ 3 & -12\\ -7 & -35\end{pmatrix}$

g) $E - AC = \begin{pmatrix}2 & 4 & -3\\ -1 & 0 & -4\\ -6 & 0 & -1\end{pmatrix} - \begin{pmatrix}1 & 0\\ 3 & 7\end{pmatrix}\begin{pmatrix}-2 & 3 & 7\\ 7 & -3 & -2\end{pmatrix} = \begin{pmatrix}2 & 4 & -3\\ -1 & 0 & -4\\ -6 & 0 & -1\end{pmatrix} - \begin{pmatrix}1\cdot-2+0\cdot7 & 1\cdot3+0\cdot-3 & 1\cdot7+0\cdot-2\\ 3\cdot-2+7\cdot7 & 3\cdot3+7\cdot-3 & 3\cdot7+7\cdot-2\end{pmatrix} =$

$= \begin{pmatrix}2 & 4 & -3\\ -1 & 0 & -4\\ -6 & 0 & -1\end{pmatrix} - \begin{pmatrix}-2 & 3 & 7\\ 43 & -12 & 7\end{pmatrix}$. não é definida, pois AC e E possuem ordem distintas.

h) $F^t E = (1\ \ -2\ \ 0) \cdot \begin{pmatrix}2 & 4 & -3\\ -1 & 0 & -4\\ -6 & 0 & -1\end{pmatrix} = (1\cdot2+-2\cdot-1+0\cdot-6 \quad 1\cdot4+-2\cdot0+0\cdot0 \quad 1\cdot-3+-2\cdot-4+0\cdot-1) =$

$= (4\ \ 4\ \ 5)$

i) $BCF = \begin{pmatrix}0 & 5\\ 2 & -2\end{pmatrix} \cdot \begin{pmatrix}-2 & 3 & 7\\ 7 & -3 & -2\end{pmatrix} \cdot \begin{pmatrix}1\\ -2\\ 0\end{pmatrix} = \begin{pmatrix}0\cdot-2+5\cdot7 & 0\cdot3+5\cdot-3 & 0\cdot7+5\cdot-2\\ 2\cdot-2+-2\cdot7 & 2\cdot3+-2\cdot-3 & 2\cdot7+-2\cdot-2\end{pmatrix}\begin{pmatrix}1\\ -2\\ 0\end{pmatrix} =$

$\begin{pmatrix}35 & -15 & -10\\ -18 & 12 & -10\end{pmatrix} \cdot \begin{pmatrix}1\\ -2\\ 0\end{pmatrix} = \begin{pmatrix}35\cdot1+-15\cdot-2+-10\cdot0\\ -18\cdot1+12\cdot-2+-10\cdot0\end{pmatrix} = \begin{pmatrix}65\\ -42\end{pmatrix}$

2 - a) $A_{2\times3}\, B_{3\times4} = C_{2\times4}$

b) $A_{4\times1}\, B_{1\times2} = C_{4\times2}$

c) $A_{1\times2}\, B_{3\times1}$ = não está definido

d) $A_{5\times2}\, B_{2\times3} = C_{5\times3}$

e) $A_{4\times4}\, B_{3\times3}$ = não está definido

f) $A_{2\times1}\, B_{1\times3} = C_{2\times3}$

h) $A_{2\times2}\, B_{2\times2} = C_{2\times2}$

3- a) $a_{11}=3-2=1$ $\quad a_{21}=6-2=4$ $\quad A=\begin{pmatrix}1 & -1 & -3\\ 4 & 2 & 0\end{pmatrix}$

$a_{12}=3-4=-1$ $\quad a_{22}=6-4=2$

$a_{13}=3-6=-3$ $\quad a_{23}\ 6-6=0$

b) $a_{11}=2+1=3$ $\quad a_{21}=4-1=3$ $\quad a_{31}=9-1=8$ $\quad B=\begin{pmatrix}3 & -1 & -2\\ 3 & 6 & 1\\ 8 & 7 & 9\end{pmatrix}$

$a_{12}=1-2=-1$ $\quad a_{22}=4+2=6$ $\quad a_{32}=9-2=7$

$a_{13}=1-3=-2$ $\quad a_{23}=4-3=1$ $\quad a_{33}=6+3=9$

c) $a_{11}=1$ $\quad a_{13}=3$ $\quad c=\begin{pmatrix}1 & 2 & 3 & 4\end{pmatrix}$

$a_{12}=2$ $\quad a_{14}=4$

d) $a_{11}=1+1=1$ $\quad a_{21}=2\cdot2\cdot1=4$ $\quad a_{31}=2\cdot3\cdot1=6$ $\quad a_{41}=2\cdot4\cdot1=8$

$a_{12}=2\cdot1\cdot2=4$ $\quad a_{22}=4+4=8$ $\quad a_{32}=2\cdot3\cdot2=12$ $\quad a_{42}=2\cdot4\cdot2=16$

$a_{13}=2\cdot1\cdot3=6$ $\quad a_{23}=2\cdot2\cdot3=12$ $\quad a_{33}=9+9=18$ $\quad a_{43}=2\cdot4\cdot3=24$

$a_{14}=2\cdot1\cdot4=8$ $\quad a_{24}=2\cdot2\cdot4=16$ $\quad a_{34}=2\cdot3\cdot4=24$ $\quad a_{44}=16+16=32$

$$D=\begin{pmatrix}1 & 4 & 6 & 8\\ 4 & 8 & 12 & 16\\ 6 & 12 & 18 & 24\\ 8 & 16 & 24 & 32\end{pmatrix}$$

4- a) $[BA]_{23}=2\cdot1+(-1)\cdot2+4\cdot5=20$

b) $[AB]_{23}=(-2)\cdot3+(-3)\cdot4+2(-17)=-52$

c) $[B^2]=1\cdot1+4\cdot2+5\cdot(-3)=-6$

d) $tr(A)=1+(-3)+4=2$

e) $tr(B^t)=1+(-1)+(-17)=-17$

f) $tr(A-B)=[1-1]+[(-3)-(-1)]+[5-(-17)]=0+(-2)+22=20$

g) $tr(AB)=$

$$A\cdot B=\begin{pmatrix}1 & 2 & 1\\ -2 & -3 & 2\\ 1 & 4 & 5\end{pmatrix}\cdot\begin{pmatrix}1 & 0 & 3\\ 2 & -1 & 4\\ -3 & -1 & -17\end{pmatrix}=\begin{pmatrix}1\cdot1+2\cdot2+1\cdot-3 & 1\cdot0+2\cdot-1+1\cdot-1 & 1\cdot3+2\cdot4+1\cdot-17\\ -2\cdot1+-3\cdot2+2\cdot-3 & -2\cdot0+3\cdot-1+2\cdot-1 & -2\cdot3+-3\cdot4+2\cdot-17\\ 1\cdot1+4\cdot2+5\cdot-3 & 1\cdot0+4\cdot-1+5\cdot-1 & 1\cdot3+4\cdot4+5\cdot-17\end{pmatrix}$$

g) Continuação $AB = \begin{pmatrix} 2 & -3 & -6 \\ -14 & 1 & -52 \\ -6 & -9 & -66 \end{pmatrix}$ $tr(AB) = 2 + 1 + (-66) = -63$

5) a) $2X + A - A = 3B + C - A = 2X = \begin{pmatrix} 6 & 3 \\ 12 & 9 \end{pmatrix} + \begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} -1 & 7 \\ 2 & 6 \end{pmatrix} =$

$= 2X = \begin{pmatrix} 7 & -2 \\ 11 & 3 \end{pmatrix} = X = 2 \div \begin{pmatrix} 7 & -2 \\ 11 & 3 \end{pmatrix} = X = \begin{pmatrix} \frac{7}{2} & -1 \\ \frac{11}{2} & \frac{3}{2} \end{pmatrix}$

b) $Y + \begin{pmatrix} -1 & 7 \\ 2 & 6 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 2-0 & 4-1 \\ 1-2 & 3-0 \end{pmatrix} \Leftrightarrow Y = \begin{pmatrix} 1 & \frac{3}{2} \\ -\frac{1}{2} & \frac{3}{2} \end{pmatrix} - \begin{pmatrix} -1 & 7 \\ 2 & 6 \end{pmatrix} \Leftrightarrow Y = \begin{pmatrix} 2 & -\frac{11}{2} \\ -\frac{5}{2} & -\frac{9}{2} \end{pmatrix}$

c) $3X + X = B - A \Leftrightarrow 4X = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix} - \begin{pmatrix} -1 & 7 \\ 2 & 6 \end{pmatrix} \Leftrightarrow X = 4 \div \begin{pmatrix} 3 & -6 \\ 2 & -3 \end{pmatrix} =$

$= X = \begin{pmatrix} \frac{3}{4} & -\frac{6}{4} \\ \frac{1}{2} & -\frac{3}{4} \end{pmatrix}$

d) (I) $-2Y + 3A = 2B + C \Leftrightarrow -2Y = \begin{pmatrix} 4 & 2 \\ 8 & 6 \end{pmatrix} + \begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} -3 & 21 \\ 6 & 18 \end{pmatrix} \Leftrightarrow$

$\Leftrightarrow Y = -\frac{1}{2} \cdot \begin{pmatrix} 7 & -17 \\ 3 & -12 \end{pmatrix} = Y = \begin{pmatrix} -\frac{7}{2} & \frac{17}{2} \\ -\frac{3}{2} & 6 \end{pmatrix}$

(II) $X + \begin{pmatrix} -\frac{7}{2} & \frac{17}{2} \\ -\frac{3}{2} & 6 \end{pmatrix} = \begin{pmatrix} -3 & 21 \\ 6 & 18 \end{pmatrix} \Leftrightarrow X = \begin{pmatrix} -3 & 21 \\ 6 & 18 \end{pmatrix} - \begin{pmatrix} -\frac{3}{2} & \frac{17}{2} \\ -\frac{3}{2} & 6 \end{pmatrix} = X = \begin{pmatrix} \frac{1}{2} & \frac{25}{2} \\ \frac{15}{2} & 12 \end{pmatrix}$

6- $A^2 = \begin{pmatrix} 1 & \frac{1}{x} \\ x & 1 \end{pmatrix} \begin{pmatrix} 1 & \frac{1}{x} \\ x & 1 \end{pmatrix} = \begin{pmatrix} 1 \cdot 1 + \frac{1}{x} \cdot x & 1 \cdot \frac{1}{x} + \frac{1}{x} \cdot 1 \\ x \cdot 1 + 1 \cdot x & x \cdot \frac{1}{x} + 1 \cdot 1 \end{pmatrix} = \begin{pmatrix} 2 & x \\ 2x & 2 \end{pmatrix} = 2 \begin{pmatrix} 1 & \frac{1}{x} \\ \frac{1}{x} & 1 \end{pmatrix} = 2A$

$\therefore A^2 = 2A$

$A^3 = A^2 \cdot A = 2A \cdot A = 2A^2 = 2^2 A$

$A^4 = A^3 \cdot A = 2A \cdot A^2 = 2^3 A$

$A^n = 2^{n-1} A$

7- a) $A(B+C) = AB + AC = \underline{X+Y}$

b) $B^t A^t = (AB)^t = X^t$

c) $C^t A^t = (AC)^t = Y^t$

d) $(ABA)C = (AB) \cdot (AC) = X \cdot Y$

8- a) $\begin{pmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{pmatrix} = \begin{pmatrix} 4 & 2x-3 \\ x+2 & x+1 \end{pmatrix}$ $\quad x+2 = 2x-3 \quad x = 5 \quad A = \begin{pmatrix} 4 & 7 \\ 7 & 6 \end{pmatrix}$

b) $\begin{pmatrix} 0 & x & y \\ -4 & 0 & 2z \\ 2 & 1-z & 0 \end{pmatrix} = \begin{pmatrix} 0 & 4 & -2 \\ -x & 0 & -1+z \\ -y & -2z & 0 \end{pmatrix}$ $\quad x=4 \quad y=2 \quad 2z = -1+z \quad z=-1 \quad B = \begin{pmatrix} 0 & -4 & 2 \\ 4 & 0 & +2 \\ 2 & -2 & 0 \end{pmatrix}$

9- $\begin{pmatrix} 3x & 3y \\ 3z & 3t \end{pmatrix} = \begin{pmatrix} x+4 & 6+x+y \\ -1+z+t & 2t+3 \end{pmatrix}$ $\quad 3x = x+4 \quad x=2 \quad 3y = 6+2+y \quad y=4 \quad 3t = 2t+3 \quad t=3$

$3z = -1+z+3 \Leftrightarrow z = 1$

10- a) se considerar $\theta = 30°$

$R(30°) \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{1}{2} & \frac{\sqrt{3}}{2} \cdot -\frac{1}{2} + \frac{1}{2} \cdot \frac{\sqrt{3}}{2} \\ -\frac{1}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{1}{2} & -\frac{1}{2} \cdot -\frac{1}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

b) $\begin{pmatrix} 1 & 0 & x \\ 0 & \frac{1}{\sqrt{2}} & y \\ 0 & \frac{1}{\sqrt{2}} & z \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ x & y & z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $\quad 1 \cdot 1 + 0 \cdot 0 + x \cdot x = 1 \quad x^2 = 1-1 \quad x = 0$

$0 \cdot 0 + \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + y \cdot y = 1$ $\quad 0 \cdot 0 + \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \cdot z = 0$

$y^2 = 1 - \frac{1}{2}$ $\quad 0 + \frac{1}{2} + \frac{1}{\sqrt{2}} z = 0$

$y = \frac{1}{\sqrt{2}}$ $\quad z = -\frac{1}{\sqrt{2}}$