Lista 5

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1- a) $\vec{BF} = \vec{BA} + \vec{AF} = -\vec{b} + \vec{f}$

b) $\vec{AG} = \vec{AC} + \vec{BA} + \vec{DG} = \vec{c} - \vec{b} + \vec{f}$

c) $\vec{AE} = \vec{AF} - \vec{FE} = \vec{f} - \vec{b}$

d) $\vec{BG} = \vec{BA} + \vec{AG} = -\vec{b} + \vec{c} - \vec{b} + \vec{f} = -2\vec{b} + \vec{c} + \vec{f}$

e) $\vec{HB} = -\vec{HG} + \vec{GD} + \vec{DC} + \vec{CA} + \vec{AB} = \vec{b} - \vec{f} + \vec{b} - \vec{c} + \vec{b} = 3\vec{b} - \vec{f} - \vec{c}$

f) $\vec{AB} + \vec{FG} = \vec{AB} + \vec{BA} + \vec{AC} = \vec{b} - \vec{b} + \vec{c} = \vec{c}$

g) $\vec{AD} + \vec{HG} = \vec{AC} + \vec{CD} + \vec{AB} = \vec{c} - \vec{b} + \vec{b} = \vec{c}$

h) $\vec{HF} + \vec{AG} - \vec{EF} = \vec{HG} + \vec{GA} + \vec{AF} + \vec{AG} - \vec{AB} = \vec{b} - \vec{b} + \vec{f} = \vec{f}$

i) $2\vec{AD} - \vec{FG} - \vec{BA} + \vec{GH} = 2(\vec{AC} + \vec{CD}) - (\vec{BA} + \vec{AC}) - (\vec{BA} + \vec{AC} + \vec{CD} + \vec{DG} +$
$\vec{GH}) + \vec{GH} = 2\vec{c} - 2\vec{b} + \vec{b} - \vec{c} + \vec{b} - \vec{c} + \vec{b} - \vec{f} + \vec{b} - \vec{b} = \vec{b} - \vec{f}$

2- a) $\vec{DF} = \vec{DC} + 2\vec{DE}$

b) $\vec{DA} = \vec{DC} + \vec{CO} + \vec{OB} + \vec{BA} = 2\vec{DC} + 2\vec{DE}$

c) $\vec{DB} = \vec{DC} + \vec{DE} + \vec{DC} = 2\vec{DC} + \vec{DE}$

d) $\vec{DO} = \vec{DC} + \vec{CO} = \vec{DC} + \vec{DE}$

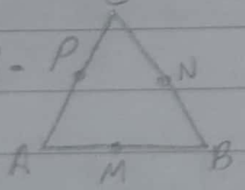
e) $\vec{EC} = \vec{ED} + \vec{DC} = -\vec{DE} + \vec{DC}$

f) $\vec{EB} = \vec{ED} + \vec{DC} + \vec{CO} + \vec{OB} = -\vec{DE} + \vec{DC} + \vec{DE} + \vec{DC} = 2\vec{DC}$

g) $\vec{OB} = \vec{DC}$

h) $\vec{AF} = -\vec{DC}$

3-

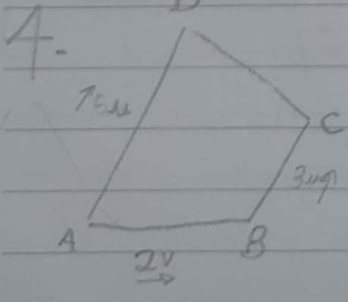


$\vec{BP} = \vec{BA} + \vec{AP} = -\vec{AB} + \frac{1}{2}AC$

$\vec{AN} = \vec{AB} + \vec{BC} = AB +$

$\vec{CM} = \vec{CA} + \vec{AM} = -\vec{AC} + \frac{1}{2}\vec{AB}$

4-



a) $\vec{CD} = \vec{CB} + \vec{BA} + \vec{AD} = -3\vec{u} - 2\vec{v} + 5\vec{u} = -2\vec{v} + 2$

$\vec{BD} = \vec{BA} + \vec{AD} = -2\vec{v} + 5\vec{u}$

$\vec{CA} = \vec{CB} + \vec{BA} = -3\vec{u} - 2\vec{v}$

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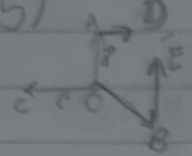
4- b) como $\vec{AD}$ e $\vec{BC}$ possuem mesma direção então eles são paralelos. Junto com $\vec{AB}$ e $\vec{CD}$ forma-se um conjunto de vetores linearmente dependentes, no qual, formam um trapézio

5) 

$$\vec{DE} = \vec{DA} + \vec{AO} + \vec{OB} + \vec{BE} = -\frac{1}{4}\vec{c} - \vec{a} + \vec{b} + \frac{5}{6}\vec{a} = -\frac{1}{4}\vec{c} - \frac{1}{6}\vec{a} + \vec{b}$$

6) $\vec{AC} = \vec{AO} + \vec{OC} = -\vec{a} - 2\vec{b} + 5\vec{a} + x\vec{b} = 4\vec{a} + (-2+x)\vec{b}$

$\vec{BC} = \vec{BO} + \vec{OC} = -3\vec{a} - 2\vec{b} + 5\vec{a} + x\vec{b} \quad 2\vec{a} + (-2+x)\vec{b}$

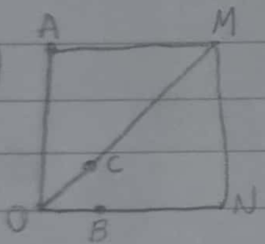
$\vec{AC} = k \cdot \vec{BC}$

$4\vec{a} + (-2+x)\vec{b} = 2k\vec{a} + k(-2+x)\vec{b} \Rightarrow 2\vec{a} + ((-2+x) - k(-2+x))\vec{b} = 0$

$\{\vec{a}, \vec{b}\}$ é LI $\rightarrow \begin{cases} 4 - 2k = 0 \quad k = 2 \\ (-2+x) - k(-2+x) = 0 \end{cases}$

$(-2+x) - 2(-2+x) = 0 \Rightarrow -2 + x + 4 - 2x \Rightarrow -x + 2 = 0$

$\underline{x = 2}$

7) 

$\vec{AB} = \vec{AO} + \vec{OB} \Leftrightarrow -\vec{OM} + \vec{ON} + \frac{1}{n}\vec{ON} \Leftrightarrow -\vec{OM} + (1 + \frac{1}{n})\vec{ON}$

$\vec{AC} = \vec{AO} + \vec{OC} \Leftrightarrow -\vec{OM} + \vec{ON} + \frac{1}{1+n}\vec{OM} \Rightarrow \vec{ON} + \frac{-n}{1+n}\vec{OM}$

$\vec{AB} = k\vec{AC}$

$\Leftrightarrow -\vec{OM} + \frac{n+1}{n}\vec{ON} = k\left(\frac{-n}{1+n}\vec{OM} + \vec{ON}\right) = \vec{0}$

$= \left(\frac{n+1}{n} - k\right)\vec{ON} + \left(-1 + k\frac{n}{1+n}\right)\vec{OM} = \vec{0}$ $\qquad \{\vec{OM}, \vec{ON}\}$ é LI.

$\begin{cases} \frac{n+1}{n} - k = 0 \quad k = \frac{n+1}{n} \\ -1 + k\frac{n}{1+n} = 0 \quad k = \frac{1+n}{n} \end{cases}$

8) $\{\vec{u}, \vec{v}\}$ e $\{2\vec{u} + \vec{v}, \vec{u} - 2\vec{v}\}$ representam a mesma base

$f = a\vec{u} + b\vec{v} \qquad f = c(2\vec{u} + \vec{v}) + d(\vec{u} - 2\vec{v})$

8) continuação

$a\vec{u} + b\vec{v} = 2c\vec{u} + c\vec{v} + d\vec{u} - 2d\vec{v} =$

$= -a\vec{u} + 2c\vec{u} + d\vec{u} - 2d\vec{v} - b\vec{v} + c\vec{v} = 0$

$\vec{u}(-a + 2c + d) + \vec{v}(-2d - b + c) = 0$

para ser ambos bom, $(-a+2c+d)$ e $(-2d-b+c)$ tem que ser 0.

9-a) $\alpha(\vec{u}+\vec{v}) + \beta(\vec{u} - \vec{v} + \vec{w}) + \lambda(\vec{u} + \vec{v} + \vec{w})$

$$\begin{cases} \alpha + \beta + \lambda \\ \alpha - \beta + \lambda \\ \beta + \lambda \end{cases} \qquad det = \left(\begin{array}{ccc|cc} 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 & -1 \\ 0 & 1 & 1 & 0 & 1 \end{array}\right) = -1+1-(1+1) = -2$$

com $det \neq 0$ os vetores são L.I.

10) a) $\vec{AB} = B - A = (0,-3,-3)$ $\quad \vec{BC} = C - B = (0,1,1)$

$\vec{CA} = A - C = (0,2,2)$

b) $\vec{AB} + \frac{2}{3}\vec{BC} = (0,-3,-3) + (0,\frac{2}{3},\frac{2}{3}) = (0,-\frac{7}{3},-\frac{7}{3})$

c) $C + 1/2\,\vec{AB} = (1,1,0) + (0,-\frac{3}{2},-\frac{3}{2}) = (1,-\frac{1}{2},-\frac{3}{2})$

d) $A - 2\vec{BC} = (1,3,2) - (0,2,2) = (1,1,0)$

11-a) $\{(2,3),(0,2)\}$ L.I

$$\begin{cases} 2\alpha = 0 \\ 3\alpha + 2\beta = 0 \end{cases} \qquad det \begin{vmatrix} 2 & 0 \\ 3 & 2 \end{vmatrix} = 4$$

b) $\{(3,0),(-2,0)\}$ L.D.

$$\begin{cases} 3\alpha + (-2)\beta = 0 \\ \end{cases} \qquad det = \begin{vmatrix} 3 & -2 \\ 0 & 0 \end{vmatrix} = 0$$

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11) Continuação

c) $\{(2,3,4),(0,3,3)\}$ L.I

$\begin{cases} 2\alpha + 0 = 0 \\ 3\alpha + 3\beta = 0 \\ 4\alpha + 3\beta = 0 \end{cases}$ $\alpha = 0$ $\beta = \alpha$ $\beta = \frac{4}{3}\alpha$

d) $\{(1,-1,2),(1,1,0),1,-1,1)\}$ L.I.

$\begin{cases} \alpha + \beta + \lambda = x \\ -\alpha + \beta - \lambda = y \\ 2\alpha + \lambda = z \end{cases}$

$det = \begin{vmatrix} 1 & 1 & 1 \\ -1 & 1 & -1 \\ 2 & 0 & 1 \end{vmatrix} \begin{matrix} 1 & 1 \\ -1 & 1 \\ 2 & 0 \end{matrix} = 1-2-(-1+2) = -2$

e) $\{1,-1,1),(-1,2,1),(-1,2,2)\}$ L.I

$det \begin{vmatrix} 1 & -1 & -1 \\ -1 & 2 & 2 \\ 1 & 1 & 2 \end{vmatrix} \begin{matrix} 1 & -1 \\ -1 & 2 \\ 1 & 1 \end{matrix} = 4-2+1-(2+2-2) = 3-2 = 1$

f) $\{(1,0,1),(0,0,1),(2,0,5)\}$ LD

$det = \begin{vmatrix} 1 & 0 & 2 \\ 0 & 0 & 0 \\ 1 & 1 & 5 \end{vmatrix} = 0$

12) $w = \alpha\vec{u} + \beta\vec{v}$

$\begin{cases} 2\alpha + \beta = 1 \\ -\alpha - \beta = 1 \end{cases}$ $2\alpha - 1 - \alpha = 1$ $\beta = -1 - \alpha$ $\alpha = 2$ $-\beta - 2 = 1$ $\beta = -3$

$\vec{W} = 2\vec{u} - 3\vec{v}$

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12) b) $\vec{z} = \alpha\vec{a} + \beta\vec{b} + \lambda\vec{c}$

$$\begin{cases} \alpha + \lambda = 1 \\ \alpha + \beta + \lambda = 2 \\ \alpha + \beta = 3 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 2 \\ 1 & 1 & 0 & 3 \end{array}\right) \begin{array}{l} l_2 - l_1 \\ l_3 \cdot l_2 \end{array} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 2 \end{array}\right) l_3 - l_1 \Rightarrow$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{array}\right) \xrightarrow{l_1 + l_3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{array}\right) \xrightarrow{l_3 \cdot -1} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

$\vec{z} = 2\vec{a} + \vec{b} - \vec{c}$

13) b) $K\vec{u} = \vec{v}$

$(K, K(m-1), Km) = (m, 2n, 4)$

$$\begin{cases} K = m \\ Km - K = 2n \\ Km = 4 \end{cases} \quad K = m = \pm 2$$

$Km - K = 2n$

$m = -2$: $4 + 2 = 2n \Rightarrow n = 3$

$m = +2$: $4 - 2 = 2n \Rightarrow n = 1$

$$\begin{cases} m = -2 \text{ e } n = 3 \\ m = +2 \text{ e } n = 1 \end{cases}$$

b) $K\vec{u} = \vec{v}$

$(K, Km, Kn + K) = (m, n+1, 8)$

$$\begin{cases} K = m \\ Km = n + 1 \\ Kn + K = 8 \end{cases}$$

14) $\det \left|\begin{array}{ccc} m & m^2+1 & m \\ -1 & m & 1 \\ m^2+1 & 0 & 1 \end{array}\right| \begin{array}{cc} m & m^2+1 \\ -1 & m \\ m^2+1 & 0 \end{array}$

$= m^2 + (m^2+1)^2 + m^2 + 1 - m^4 - m^2$

$= m^2 + m^4 + 2m^2 + 1 + m^2 + 1 - m^4 - m^2$

$= 3m^2 + 2$

$\{\vec{u}, \vec{v}, \vec{w}\}$ L.D. $m^2 = -\frac{2}{3}$ $\quad m = \sqrt{-\frac{2}{3}}$ $\quad m \notin \mathbb{R}$

Com isso para todo $m \in \mathbb{R}$ $\{\vec{u}, \vec{v}, \vec{w}\}$ é L.I.

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15 a) $\begin{cases} f_1 = e_1 + e_2 \\ f_2 = e_1 + e_3 \\ f_3 = e_1 + e_2 - e_3 \end{cases}$ $\begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix}^t = \left(\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & -1 \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} \right)^t$

$(f_1 \ f_2 \ f_3) = (e_1 \ e_2 \ e_3) \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix}$

b) $(2,3,7)_c = (f_1 \ f_2 \ f_3) \begin{pmatrix} 2 \\ 3 \\ 7 \end{pmatrix} = (e_1 \ e_2 \ e_3) \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ 7 \end{pmatrix} =$

$= (e_1 \ e_2 \ e_3) \begin{pmatrix} 12 \\ 9 \\ -4 \end{pmatrix} = (12 \ 9 \ -4)_B$