4ª EVALUACIÓN

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INSTRUCCIONES GENERALES: La presente evaluación consta de 5 reactivos, los cuales serán evaluados considerando el resultado y el procedimiento para llegar al mismo.

Lea cuidadosamente los reactivos antes de resolverlos.

A. Anote sus respuestas con letra clara y legible, sin borrones, enmendaduras y tachaduras, (se recomienda uso de lápiz)
B. Envíe sus respuestas en formato **PDF y en un solo archivo,** deberá subir este archivo a la plataforma de Classroom.

Resuelva la ecuación diferencial aplicando transformada de Laplace (2 puntos)

1. $t^2x'' + 4tx' + (t^2 + 2)x = sen\ t$

Utilice el teorema de convolución para resolver la siguiente ecuación diferencial (2 puntos)

2. $x' + x = f(t) \qquad x_0 = 1$

Resuelva el siguiente sistema de ecuaciones diferenciales usando el método de eliminación de variables (2 puntos)

3. $x' = 4x - 3y$
   $y' = 6x - 7y$

Resuelva el siguiente sistema de ecuaciones diferenciales usando el método de operadores (2 puntos)

4. $x' - 4x + 3y = 0$
   $-6x + y' + 7y = 0$

Resuelva el siguiente sistema de ecuaciones diferenciales usando transformada de Laplace (2 puntos)

5. $2x'' = -6x + 2y, \qquad x(0) = x'(0) = 0$
   $y'' = 2x - 2y + 40\ sen\ 3t \qquad y(0) = y'(0) = 0$

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Examen unidad 4

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1.- $t^2x'' + 4tx' + (t^2+2)x = \operatorname{sen} t$

$$\mathcal{L}[t^2x''] = \frac{d^2}{ds^2}\mathcal{L}[x''] = \frac{d^2}{ds^2}(s^2L - sx_0 - x_0')$$

$$= \frac{d}{ds}(2sL + s^2L' - x_0 - 0)$$

$$= 2L + 2sL' + 2sL' + s^2L''$$

$$\mathcal{L}[tx'] = -\frac{d}{ds}\mathcal{L}[x'] = -\frac{d}{ds}(sL - x_0)$$

$$= -L - sL'$$

$$\mathcal{L}[t^2x] = \frac{d^2}{ds^2}\mathcal{L}[x] = L'' \qquad \mathcal{L}[x] = L$$

$$\mathcal{L}[\operatorname{sen} t] = \frac{1}{s^2+1} \qquad \mathcal{L}[\operatorname{sen} at] = \frac{a}{s^2+a^2}$$

sustituyendo

$$2L + \cancel{4sL'} + s^2L'' - 4L - \cancel{4sL'} + L'' + 2L = \frac{1}{s^2+1}$$

$$s^2L'' + L''' + \cancel{4L} - \cancel{4L} = \frac{1}{s^2+1}$$

$$(s^2+1)L'' = \frac{1}{s^2+1}$$

$$L'' = \frac{1}{(s^2+1)^2}$$

$L'' = \mathcal{L}[t^2x]$

$$\mathcal{L}[t^2x] = \frac{1}{(s^2+1)^2}$$

$$t^2x = \mathcal{L}^{-1}\left[\frac{1}{(s^2+1)^2}\right]$$

$$t^2 x = \mathcal{L}^{-1}\left[\frac{1}{(s^2+1)} \cdot \frac{1}{s^2+1}\right]$$

usando teorema de convolución

$$= \mathcal{L}^{-1}\left[\frac{1}{s^2+1}\right] * \mathcal{L}^{-1}\left[\frac{1}{s^2+1}\right] *$$

$$\mathcal{L}^{-1}\left[\frac{a}{s^2+a^2}\right] = \operatorname{sen} at$$

$$t^2 x = \operatorname{sen} t * \operatorname{sen} t$$

$$t^2 x = \int_0^t \operatorname{sen} x (\operatorname{sen}(t-x))\, dx$$

usando identidades

$$\operatorname{sen} a \operatorname{sen} b = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right]$$

$$t^2 x = \frac{1}{2}\int_0^t \left[\cos(x-t+x) - \cos(x+t-x)\right] dx$$

$$= \frac{1}{2}\left[\int_0^t \cos(2x-t)\, dx - \int_0^t \cos t\, dx\right]$$

$$= \frac{1}{2}\left[\frac{1}{2}\operatorname{sen}(2x-t)\Big|_0^t - \cos t\; x\Big|_0^t\right]$$

$$= \frac{1}{2}\left[\frac{1}{2}\operatorname{sen} t - \frac{1}{2}\operatorname{sen}(-t) - t\cos t + \cos t(0)\right]$$

$$\operatorname{sen}(-t) = -\operatorname{sen} t$$

$$t^2 x = \frac{1}{2}\left[\frac{1}{2}\operatorname{sen} t + \frac{1}{2}\operatorname{sen} t - t\cos t\right]$$

$$t^2 x = \frac{1}{2}\left[\operatorname{sen} t - t\cos t\right]$$

$$\boxed{x = \frac{1}{2} t^{-2} \operatorname{sen} t - \frac{1}{2t}\cos t}$$

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Examen Unidad 4 21/09/2021

2.- $x' + x = f(t) \qquad x_0 = 1$

$\mathcal{L}[x'] = sL - x_0 = sL - 1$

$\mathcal{L}[x] = L$

$\mathcal{L}[f(t)] = F(s)$

sustituyendo

$sL - 1 + L = F(s)$

$L(s+1) = F(s) + 1$

$L = \dfrac{F(s)+1}{(s+1)}$

$L = \mathcal{L}[x]$

$\mathcal{L}[x] = \dfrac{F(s)+1}{(s+1)}$

$x = \mathcal{L}^{-1}\left[\dfrac{F(s)}{(s+1)}\right] + \mathcal{L}^{-1}\left[\dfrac{1}{s+1}\right] \qquad \mathcal{L}^{-1}\left[\dfrac{1}{s-a}\right] = e^{at}$

$x = \mathcal{L}^{-1}[F(s)] * \mathcal{L}^{-1}\left[\dfrac{1}{s+1}\right] + e^{-t}$

$= f(t) * e^{-t} + e^{-t} \qquad f(t) * g(t) = \int_0^t f(x)\, g(t-x)\, dx$

$x = \int_0^t f(x)\, e^{-(t-x)}\, dx + e^{-t}$

$$\boxed{x = e^{-t} \int_0^t f(x)\, e^{x}\, dx + e^{-t}}$$

3.- $\begin{cases} x' = 4x - 3y & (1) \\ y' = 6x - 7y & (2) \end{cases}$

Despejamos y de (1)

$$3y = 4x - x'$$
$$y = \frac{4}{3}x - \frac{1}{3}x' \quad (3) \quad \therefore \quad y' = \frac{4}{3}x' - \frac{1}{3}x''$$

Sustituyendo (3) en (2)

$$\frac{4}{3}x' - \frac{1}{3}x'' = 6x - 7\left(\frac{4}{3}x - \frac{1}{3}x'\right)$$

$$\frac{4}{3}x' - \frac{1}{3}x'' = 6x - \frac{28}{3}x + \frac{7}{3}x'$$

$$-\frac{1}{3}x'' - x' + \frac{10}{3}x = 0$$

$$x'' + 3x' - 10x = 0$$

Escribimos la ecuación característica

$$x = e^{rt}$$

$$r^2 + 3r - 10 = 0$$
$$(r+5)(r-2) = 0 \qquad r = -5 \quad r = 2$$

$$x = C_1 e^{-5t} + C_2 e^{2t}$$

sustituimos en (3)

$$y = \frac{4}{3}(C_1 e^{-5t} + C_2 e^{2t}) - \frac{1}{3}(-5C_1 e^{-5t} + 2C_2 e^{2t})$$
$$= \frac{4}{3}C_1 e^{-5t} + \frac{4}{3}C_2 e^{2t} + \frac{5}{3}C_1 e^{-5t} - \frac{2}{3}C_2 e^{2t}$$
$$y = \frac{9}{3}C_1 e^{-5t} + \frac{2}{3}C_2 e^{2t} = 3C_1 e^{-5t} + \frac{2}{3}C_2 e^{2t}$$

Solución:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} c_1 e^{-5t} + \begin{pmatrix} 1 \\ 2/3 \end{pmatrix} c_2 e^{2t}$$

4.- $\begin{cases} x' - 4x + 3y = 0 & (1) \\ -6x + y' + 7y = 0 & (2) \end{cases}$

$$D[x] - 4[x] + 3[y] = 0$$
$$-6[x] + D[y] + 7[y] = 0$$

$$(D-4)[x] + 3[y] = 0$$
$$-6[x] + (D+7)[y] = 0$$

$$\Delta = \begin{vmatrix} D-4 & 3 \\ -6 & D+7 \end{vmatrix} = (D-4)(D+7) - 3(-6)$$

$$\Delta = D^2 + 3D - 28 + 18 = D^2 + 3D - 10$$

$$(D^2 + 3D - 10)[x] = 0$$
$$(D^2 + 3D - 10)[y] = 0$$

$$x'' + 3x' - 10x = 0$$
$$y'' + 3y' - 10y = 0$$

Ecuación característica para x

$$r^2 + 3r - 10 = 0$$
$$(r+5)(r-2) = 0 \qquad r = -5 \qquad r = 2$$

$$x = C_1 e^{-5t} + C_2 e^{2t}$$
$$y = D_1 e^{-5t} + D_2 e^{2t}$$

Sustituyendo en ①

$$-5C_1e^{-5t}+2C_2e^{2t}-4C_1e^{-5t}-4C_2e^{2t}+3D_1e^{-5t}+3D_2e^{2t}=0$$

$$-9C_1e^{-5t}-2C_2e^{2t}+3D_1e^{-5t}+3D_2e^{2t}=0$$

$$-9C_1e^{-5t}+3D_1e^{-5t}=0$$

$$3D_1\cancel{e^{-5t}}=9C_1\cancel{e^{-5t}}$$

$$3D_1=9C_1$$

$$D_1=\frac{9}{3}C_1$$

$$D_1=3C_1$$

$$-2C_2e^{2t}+3D_2e^{2t}=0$$

$$3D_2\cancel{e^{2t}}=2C_2e^{2t}$$

$$3D_2=2C_2$$

$$D_2=\frac{2}{3}C_2$$

$$y=3C_1e^{-5t}+\frac{2}{3}C_2e^{2t}$$

Solución

$$\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}1\\3\end{pmatrix}C_1e^{-5t}+\begin{pmatrix}1\\2/3\end{pmatrix}C_2e^{2t}$$

5.- $\begin{cases} 2x'' = -6x + 2y \\ y'' = 2x - 2y + 40\,\text{sen}\,3t \end{cases}$ $\quad x(0) = x'(0) = 0$, $\quad y(0) = y'(0) = 0$

$$\mathcal{L}[x''] = s^2L - sx_0 - x'_0 = s^2L$$
$$\mathcal{L}[x] = L$$
$$\mathcal{L}[y] = M$$
$$\mathcal{L}[y''] = s^2M - sy_0 - y'_0 = s^2M$$
$$\mathcal{L}[\text{sen}3t] = \frac{3}{s^2+9} \qquad \mathcal{L}[\text{sen}at] = \frac{a}{s^2+a^2}$$

sustituyendo

$$2s^2L = -6L + 2M$$
$$s^2M = 2L - 2M + \frac{120}{s^2+9}$$

$$2s^2L + 6L - 2M = 0 \qquad\qquad (2s^2+6)L - 2M = 0$$
$$-2L + s^2M + 2M = \frac{120}{s^2+9} \quad = \quad -2L + (s^2+2)M = \frac{120}{s^2+9}$$

$$L = \frac{\begin{vmatrix} 0 & -2 \\ \frac{120}{s^2+9} & s^2+2 \end{vmatrix}}{\begin{vmatrix} 2s^2+6 & -2 \\ -2 & s^2+2 \end{vmatrix}} = \frac{2\left(\frac{120}{s^2+9}\right)}{(2s^2+6)(s^2+2) - (-2)(-2)}$$

$$= \frac{\frac{60}{(s^2+9)}}{(2s^4+4s^2+6s^2+12-4} = \frac{\frac{60}{s^2+9}}{2s^4+10s^2+8}$$

$$L = \frac{60}{2(s^4+5s^2+4)(s^2+9)} = \frac{30}{(s^2+4)(s^2+1)(s^2+9)}$$

$$M = \frac{\begin{vmatrix} 2s^2+6 & 0 \\ -2 & \frac{120}{s^2+9} \end{vmatrix}}{\begin{vmatrix} 2s^2+6 & -2 \\ -2 & s^2+2 \end{vmatrix}} = \frac{(2s^2+6)\left(\frac{120}{s^2+9}\right)}{(s^2+4)(s^2+1)(s^2+9)}$$

$$M = \frac{240s^2+720}{(s^2+4)(s^2+1)(s^2+9)^2}$$

$$L = \frac{30}{(s^2+4)(s^2+1)(s^2+9)} \qquad \mathcal{L}^{-1}\left[\frac{a}{s^2+a^2}\right] = \operatorname{sen} at$$

$$x = 30\mathcal{L}^{-1}\left[\frac{1}{(s^2+4)} \cdot \frac{1}{s^2+1} \cdot \frac{1}{s^2+9}\right] = 30\mathcal{L}^{-1}\frac{1}{s^2+4} * \mathcal{L}^{-1}\frac{1}{s^2+1} * \mathcal{L}^{-1}\frac{1}{s^2+9}$$

$$= 30\left[\frac{1}{2}\operatorname{sen}2t * \operatorname{sen}t * \frac{1}{3}\operatorname{sen}3t\right]$$

$$= 30\left[\frac{1}{2}\operatorname{sen}2t * \left[\frac{1}{3}\int_0^t \operatorname{sen}x \operatorname{sen}(3t-3x)\,dx\right]\right]$$

$$= 30\left[\frac{1}{2}\operatorname{sen}2t * \left[\frac{1}{6}\left[\int_0^t \cos(4x-3t) - \int_0^t \cos(3t-2x)\,dx\right]\right]\right]$$

$$= 30\left[\frac{1}{2}\operatorname{sen}2t * \left[\frac{1}{6}\left[\frac{1}{4}\operatorname{sen}(4x-3t)\Big|_0^t + \frac{1}{2}\operatorname{sen}(3t-2x)\Big|_0^t\right]\right]\right]$$

$$x = 30\left[\frac{1}{2}\operatorname{sen}2t * \left[\frac{1}{6}\left[\frac{1}{4}\operatorname{sen}t - \frac{1}{4}\operatorname{sen}(-3t) + \frac{1}{2}\operatorname{sen}t - \frac{1}{2}\operatorname{sen}3t\right]\right]\right]$$

$$= 30\left[\frac{1}{2}\operatorname{sen}2t * \left[\frac{1}{6}\left[\frac{3}{4}\operatorname{sen}t - \frac{1}{4}\operatorname{sen}3t\right]\right]\right]$$

$$= 30\left[\frac{1}{2}\operatorname{sen}2t * \left[\frac{1}{8}\operatorname{sen}t - \frac{1}{24}\operatorname{sen}3t\right]\right]$$

$$x = 15\int_0^t \operatorname{sen}2x\left[\frac{1}{8}\operatorname{sen}(t-x) - \frac{1}{24}\operatorname{sen}(3t-3x)\right]dx$$

$$= 15\left[\frac{1}{8}\int_0^t \operatorname{sen}2x\,\operatorname{sen}(t-x)\,dx - \frac{1}{24}\int_0^t \operatorname{sen}2x\,\operatorname{sen}(3t-3x)\,dx\right]$$

$*$ $\frac{1}{8}\left[\frac{1}{2}\int_0^t \cos(3x-t)\,dx - \int_0^t \cos(x+t)\,dx\right]$

$$= \frac{1}{16}\left[\frac{1}{3}\operatorname{sen}(3x-t)\Big|_0^t - \operatorname{sen}(x+t)\Big|_0^t\right]$$

$$= \frac{1}{16}\left[\frac{1}{3}\operatorname{sen}2t - \frac{1}{3}\operatorname{sen}(-t) - \operatorname{sen}2t + \operatorname{sen}t\right]$$

$$= \frac{1}{16}\left[-\frac{2}{3}\operatorname{sen}2t + \frac{4}{3}\operatorname{sen}t\right] = -\frac{1}{24}\operatorname{sen}2t + \frac{1}{12}\operatorname{sen}t$$

$**$ $-\frac{1}{24}\left[\frac{1}{2}\int_0^t \cos(5x-3t)\,dx - \int_0^t \cos(3t-x)\,dx\right]$

$$= -\frac{1}{48}\left[\frac{1}{5}\operatorname{sen}(5x-3t)\Big|_0^t + \operatorname{sen}(3t-x)\Big|_0^t\right]$$

$$= -\frac{1}{48}\left[\frac{1}{5}\operatorname{sen}2t - \frac{1}{5}\operatorname{sen}(-3t) + \operatorname{sen}2t - \operatorname{sen}3t\right]$$

$$= -\frac{1}{48}\left[\frac{6}{5}\operatorname{sen}2t - \frac{4}{5}\operatorname{sen}3t\right] = -\frac{1}{40}\operatorname{sen}2t + \frac{1}{60}\operatorname{sen}3t$$

sustituyendo en x

$$x = 15\left[-\frac{1}{24}\operatorname{sen}2t + \frac{1}{12}\operatorname{sen}t - \frac{1}{40}\operatorname{sen}2t + \frac{1}{60}\operatorname{sen}3t\right.$$

$$= 15\left[-\frac{1}{15}\operatorname{sen}2t + \frac{1}{12}\operatorname{sen}t + \frac{1}{60}\operatorname{sen}3t\right]$$

$$\boxed{x = -\operatorname{sen}2t + \frac{5}{4}\operatorname{sen}t + \frac{1}{4}\operatorname{sen}3t}$$

Para M

$$M = \frac{240s^2 + 720}{(s^2+4)(s^2+1)(s^2+9)^2}$$

$$y = 240\mathcal{L}^{-1}\underbrace{\frac{s^2}{(s^2+4)(s^2+1)(s^2+9)^2}}_{1} + 720\mathcal{L}^{-1}\underbrace{\frac{1}{(s^2+4)(s^2+1)(s^2+}}_{2}$$

$$1 = 240\mathcal{L}^{-1}\left[\frac{s}{s^2+4} \cdot \frac{s}{s^2+9} \cdot \frac{1}{s^2+1} \cdot \frac{1}{s^2+9}\right]$$

$$= 240\left[\cos 2t * \cos 3t * \underbrace{\operatorname{sen}t * \frac{1}{3}\operatorname{sen}3t}_{\text{ya lo calcule en x}}\right]$$

$$= 240\left[\cos 2t * \cos 3t * \left[\frac{1}{8}\operatorname{sen}t - \frac{1}{24}\operatorname{sen}3t\right]\right.$$

$$\cos a \, \text{sen}\, b = \frac{1}{2}\left(\text{sen}\,[a+b) - \text{sen}(a-s)\right]$$

$$\cos 3t * \left[\frac{1}{8}\text{sen}\,t - \frac{1}{24}\text{sen}\,3t\right]$$

$$= \int_0^t \cos 3x \left[\frac{1}{8}\text{sen}(t-x) - \frac{1}{24}\text{sen}(3t-3x)\right] dx$$

$$= \frac{1}{8}\int_0^t \cos 3x \, \text{sen}(t-x)\,dx - \frac{1}{24}\int_0^t \cos 3x\, \text{sen}(3t-3x)\,dx$$

① ②

parte ①

$$\frac{1}{8}\cdot\frac{1}{2}\left[\int_0^t \text{sen}(2x+t)\,dx - \int_0^t \text{sen}(4x-t)\,dx\right]$$

$$= \frac{1}{16}\left[-\frac{1}{2}\cos(2x+t)\Big|_0^t + \frac{1}{4}\cos(4x-t)\Big|_0^t\right]$$

$$= \frac{1}{16}\left[-\frac{1}{2}\cos 3t + \frac{1}{2}\cos t + \frac{1}{4}\cos 3t - \frac{1}{4}\cos(-t)\right]$$

$$= \frac{1}{16}\left[-\frac{1}{4}\cos 3t + \frac{3}{4}\cos t\right] = -\frac{1}{64}\cos 3t + \frac{3}{64}\cos t$$

parte ②

$$-\frac{1}{24}\cdot\frac{1}{2}\left[\int_0^t \text{sen}\,3t\,dx - \int_0^t \text{sen}(6x-3t)\,dx\right]$$

$$= -\frac{1}{48}\left[\text{sen}\,3t\, x\Big|_0^t + \frac{1}{6}\cos(6x-3t)\Big|_0^t\right]$$

$$= -\frac{1}{48}\left[t\,sen3t + \frac{1}{6}\cos 3t - \frac{1}{6}\cos(-3t) \right]$$

$$= -\frac{1}{48}\,t\,sen3t - \frac{1}{144}\cos 3t$$

$$\cos 3t * \left[\frac{1}{8}sent - \frac{1}{24}sen3t\right]$$

$$L = -\frac{1}{64}\cos 3t + \frac{3}{64}\cos t - \frac{1}{48}t\,sen3t - \frac{1}{144}\cos 3t$$

$$= -\frac{13}{576}\cos 3t + \frac{3}{64}\cos t - \frac{1}{48}t\,sen3t$$

$$y = \cos 2t * \left[-\frac{13}{576}\cos 3t + \frac{3}{64}\cos t - \frac{1}{48}t\,sen3t\right]$$