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VIRTUAL INSTRUMENT FOR SPACE NAVIGATION AND CONTROL IN FORMATION FLYING

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A virtual instrument for spacecraft formation flying navigation is presented. This instrument allows to simulate the flight up to eight space vehicles using different models for relative movements. Such models include perturbed circular reference orbits as well as elliptic ones. This paper shows comparative results for all different models and allows to visualize the trajectories of all spacecraft.

INTRODUCTION

Formation Flying (FF) is a subset of the denominated distributed space systems (DSS) which also comply with having a common control law for all spacecraft or satellites that constitutes the formation.[1,2]

Some advantages of a FF mission over a single ship mission are: launching flexibility since every single spacecraft can be sent independently in time and place, and ultimately they are failure tolerant because one faulty ship not necessarily compromises all mission. Furthermore, the several vehicles of FF allow to extend actual space capabilities. Thus, there are ongoing developing missions based on big scale distributed sensors to detect gravitational waves, interferometry, deep space telescopes, as well as Earth observations and spectroscopy.[3,4,5]

Space vehicles under FF missions establish different possible control architectures which possible required proper dynamic models for the different mission stages. Linear modeling is ample used because it is easy and versatile.[6,7] However, when highly eccentric orbits or perturbations are involved linear models can not apply due to the lack of precision. On the other hand, nonlinear models allow wide area of applications because of its exactness despite of more elaborated numerical methods and computer time than the linear models.[8,9]

The Virtual Instrument shown in this paper permits the simulation of relative movements among several spacecraft considering the most common parameters and variables of the FF research area. Thus, for example, the Graphical User Interface developed uses orbital elements in either its mean or osculatriz formats.

Results are available in both forms: graphical and in structured tables. Graphics can be chosen to show relative trajectories, or position and velocities in at least two frames of reference Hill or inertial. While data tables are exportable ready to be used by other analysis programs.

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Establishing and reconfiguring flight formation is part of capabilities for this instrument which in fact means changing relative orbits for each space vehicle.

For a given mission, to test the amount of spacecraft is important while developing just to see the true performance of the corresponding formation. Thus, this virtual instrument permits adding or removing vehicles as well to change formation configuration which in time can be also selected from a wide selection.

This instrument was thought to complement other simulations resulting of those like Satellite Constellation Visualisation[*] (SaVi), General Mission Analysis Tool[†] (GMAT) or the Satellite Tool Kit[‡] (STK) that exist nowadays. But what makes simulations under this instrument easy to use and learn is its GUI.

The Virtual instrument is a tool that allows to experiment with a specific formation previously selected from different existing models were the amount of ships can be as well selected and then apply to them a given control law.

This article is organized as follows, Theoretical frame section establishes fundamental concepts for different models of relative motion. Virtual instrument section presents the Guide User Interface developed while Results section shows some results using the instrument and finally, last section gives conclusions.

THEORETICAL FRAME

This section presents dynamic models for relative movements to be used by the virtual instrument. All models consider a leader spacecraft and its corresponding Hill frame of reference and how the follower spacecraft performing trajectories around such leader. The Hill frame has its origin specifically at the leader spacecraft and its corresponding axes are: axis 1 has the same direction of the leader position vector with respect to the inertial frame of reference, axis 3 has the same orientation than the angular momentum of the leader orbit, and axis 2 is chosen under the right hand convention.

Clohessy-Wiltshire Equations

The most used linear model to describe the relative movement of a deputy or secondary spacecraft with respect to a leader one which is using a Hill frame of reference and whose orbit is circular or near to ($e<0.01$) is based on the Clohessy-Wiltshire equations (CW) also known as Hill equations or Hill-Clohessy-Wiltshire (HCW).[10,11,12] This equations are a set of ordinary differential equations with constant coefficients given by:

[*] http://savi.sourceforge.net/

[†] http://gmatcentral.org/display/GW/GMAT+Wiki+Home

[‡] http://www.agi.com/

$$
\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{z}(t) \\ \dot{u}(t) \\ \dot{v}(t) \\ \dot{w}(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 3n^2 & 0 & 0 & 0 & 2n & 0 \\ 0 & 0 & 0 & -2n & 0 & 0 \\ 0 & 0 & -n^2 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \\ z(t) \\ u(t) \\ v(t) \\ w(t) \end{bmatrix} \tag{1}
$$

where $n = \sqrt{\mu / a^3}$ is the mean movement for the leader satellite.

Tschauner-Hempel Equations

When the reference orbit is elliptic, Tschauner-Hempel equations (TH equations) are used. These equations are described using the true anomaly (*f*) as independent variable instead of time and are given by:[13,14,15]

$$
\begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix} + \begin{bmatrix} 0 & -2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} + \begin{bmatrix} -\dfrac{3}{1+e\cos(f)} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0 \tag{2}
$$

where x' is the derivative with respect to the true anomaly.

Non-linear Equations for Relative Movement

A general form for the equations that describe relative movements under a gravitational central force and without perturbations are:

$$
\begin{aligned}
\ddot{x} &= 2\dot{f}\dot{y} + \ddot{f}y + \dot{f}^2 x - \frac{\mu(r_0 + x)}{((r_0+x)^2 + y^2 + z^2)^{3/2}} + \frac{\mu}{r_0^2} \\
\ddot{y} &= -2\dot{f}\dot{x} - \ddot{f}x + \dot{f}^2 y - \frac{\mu y}{((r_0+x)^2 + y^2 + z^2)^{3/2}} \\
\ddot{z} &= -\frac{\mu z}{((r_0+x)^2 + y^2 + z^2)^{3/2}}
\end{aligned} \tag{3}
$$

where f and r_0 correspond to the true anomaly and the orbital radio of the leader, respectively.

The system of Equation (3) are commonly used as the basis to linear models to describe relative movements. Solutions to these system of equations are found by numerical integration. When perturbations are introduced to the system, care must be taken to avoid numerical problems due to different orders of magnitude involved.

Orbital elements

Orbital elements can be used to describe the relative movements between spacecraft. There are several representations with orbital elements so it can choose the most suitable for the task being performed. For example, the set of equations for position and velocity with respect to differential classical orbit elements in the Hill frame are:[7,16]

$$
\begin{aligned}
& x = \delta r \\
& y = r\left(\delta\theta + \cos(i)\delta\Omega\right) \\
& z = r\left(\sin(\theta)\delta i - \cos(\theta)\sin(i)\delta\Omega\right) \\
& u = -\frac{V_r}{2a}\delta a + \frac{p-r}{rp}h\delta\theta + \left(V_r a q_1 + h\sin(\theta)\right)\frac{\delta q_1}{p} + \left(V_r a q_2 - h\cos(\theta)\right)\frac{\delta q_2}{p} \\
& v = -\frac{3V_r}{2a}\delta a - V_r\delta\theta + \left(3V_t a q_1 + 2h\cos(\theta)\right)\frac{\delta q_1}{p} + \left(3V_t a q_2 + 2h\sin(\theta)\right)\frac{\delta q_2}{p} \\
& \quad + V_r\cos(i)\delta\Omega \\
& w = \left(V_t\cos(\theta) + V_r\sin(\theta)\right)\delta i + (V_t\sin(\theta) - V_r\cos(\theta))\sin(i)\delta\Omega
\end{aligned}
\tag{4}
$$

Where

$\text{eo} = [a, e, i, \Omega, \omega, f]^T$ are orbital elements of leader,

h is the magnitude of angular moment of leader orbit,

$\theta = \omega + f$,

$q_1 = e\cos(\omega)$,

$q_2 = e\sin(\omega)$,

$$r = \frac{a(1 - q_1^2 - q_2^2)}{1 + q_1\cos(\theta) + q_2\sin(\theta)} ,$$

$$\delta r = r\left(\frac{1}{a}\delta a + \frac{V_r}{V_t}\delta\theta - \frac{1}{p}\left(2aq_1 + r\cos(\theta)\right)\delta q_1 - \frac{1}{p}\left(2aq_2 + r\sin(\theta)\right)\delta q_2\right)$$

$$V_r = \frac{h}{p}\left(q_1\sin(\theta) - q_2\cos(\theta)\right)$$

$$V_t = \frac{h}{p}\left(1 + q_1\cos(\theta) + q_2\sin(\theta)\right)$$

If is used differential orbit elements vector $\delta\text{eo} = [\delta a, \delta e, \delta i, \delta\Omega, \delta\omega, \delta f]^T$, it can get a linear representation of Equation (4) for elliptical orbits:[17]

$$
\begin{aligned}
& x(f) \approx \frac{r}{a}\delta a + \frac{ae\sin(f)}{\eta}\delta M - a\cos(f)\delta e \\
& y(f) \approx r\left(\frac{(1 + e\cos(f))^2\,\delta M}{\eta^3} + \delta\omega + \frac{\sin(f)(2 + e\cos(f))\delta e}{\eta^2} + \cos(i)\delta\Omega\right) \\
& z(f) \approx r\left(\sin(\theta)\delta i - \cos(\theta)\sin(i)\delta\Omega\right)
\end{aligned}
\tag{5}
$$

Also, in Equation 5 if the eccentricity is small, it can be performed a simplification more:

$$
\begin{aligned}
x(f) &\approx (1-e\cos(f))\delta a + \frac{ae\sin(f)}{\eta}\delta M - a\cos(f)\delta e \\
y(f) &\approx \frac{a}{\eta}(1+e\cos(f))\delta M + a(1-e\cos(f))\delta\omega + a\sin(f)(2-e\cos(f))\delta e \\
&\quad + a(1-e\cos(f))\cos(i)\delta\Omega \\
z(f) &\approx a(1-e\cos(f))\left(\sin(\theta)\delta i - \cos(\theta)\sin(i)\delta\Omega\right)
\end{aligned}
\tag{6}
$$

Perturbations

The two-body problem more realist have to consider perturbations, and then the equation to motion can be written like:[18]

$$\ddot{\mathbf{r}} = -\frac{\mu}{r^3}\mathbf{r} + \mathbf{a}_p \tag{7}$$

where $\boldsymbol{a}_p$ is the acceleration vector because perturbations.

For spacecraft or satellites orbiting the Earth, gravitational disturbances are always present; the non-sphericity of the Earth cause a gravity force different a central force, in special, the oblateness of the Earth.

Other gravitational perturbations are due to the attraction of the moon and the sun, known as the third-body effect. The gravitational pull of the moon and the sun causes other effects on the gravitational potential of the Earth called tidal effect. In addition, the forces exerted by the rest of planets of the solar system, being Venus and Jupiter the most important planets, can influenced in the orbits of spacecraft. Furthermore, considering the gravity model from general relativity theory, it is also necessary to introduce relativistic effects.[19,20,21]

Furthermore, obit of an object may be altered due to the solar radiation pressure, which is considered a direct radiation pressure. There are reflected radiation like the radiation pressure of albedo, the pressure of infrared and the visible light, all produced by the Earth.

Solar radiation also causes the earth absorbs energy and transform part of it into heat, producing a non-uniform temperature at the surface of the Earth. The surface re-emits thermal radiation is also a source of disturbance.

Another non-gravitational perturbation is atmospheric resistance, which becomes important for objects with low or high eccentricity orbits since the magnitude of this force may become dominant over the rest of the disturbances. One can speak of a resistance due to the electrically neutral atmosphere and a resistance due to the electrical charges on the surface of the satellite or spacecraft. The size and shape of the satellite or spacecraft become important for this disturbance.[22]

Now will delve into the disturbance caused by the non-sphericity of the earth. Regardless of the model, the perturbation due to the flattening of the earth is the predominant disturbance in many problems. This disturbance is reflected in the term C20 = J2 of geopotential model, and cannot be neglected in most cases with spacecraft orbiting the Earth. It has three major effects: nodal regression, drift in perigee, and drift in the mean anomaly. Expressions in terms of mean orbit elements for these effects are:[4,23]

$$\frac{d\Omega}{dt} = -\frac{3}{2} J_2 n \left(\frac{R_e}{p}\right)^2 \cos(i)$$

$$\frac{d\omega}{dt} = \frac{3}{4} J_2 n \left(\frac{R_e}{p}\right)^2 (5\cos^2(i) - 1) \tag{8}$$

$$\frac{dM}{dt} = n + \frac{3}{4} J_2 n \left(\frac{R_e}{p}\right)^2 \sqrt{1-e^2}\,(3\cos^2(i) - 1)$$

Due to disturbances is common choose to work with mean orbit elements. The reason to choose mean elements and not osculating elements is the prolonged time of missions.

VIRTUAL INSTRUMENT

The virtual instrument presented here allows the user creates relative trajectories of deputy satellites or spacecraft with respect to one leader using different dynamic models of formation flying presented in the previous section.

Figure 1 provides an overview of the main interface of virtual instrument whose operation is simple and convenient for many applications. It is noted that there are only two tabs; in the first one, "Settings", the necessary data is entered for the program to operate, and the second tab, "Graphics", the results are displayed.

To operate the virtual instrument, we first must define the number of secondary spacecraft and the type of orbital elements. The interface allows to work up to nine spacecraft, this is, one leader and eight deputy vehicles.

The interface allows working with classical mean orbital elements or their osculating counterparts. The virtual instrument internally always operate with mean orbital elements, then it includes an algorithm for mapping between mean and osculating elements, which works best for orbits with small eccentricities and inclinations.[24] Mapping takes into account the disturbance due to Earth oblateness, this is, the J2 effect.

Therefore, it is advisable to introduce mean orbital elements instead of osculating elements if the inertial reference orbit is highly elliptical. Note that when select the type of orbital elements to be introduced, the interface automatically calculates the other set of orbital elements and displayed them in the interface.

Subsequently, the parameters of reference orbit (inertial orbit of the leader around of Earth) are entered: semimajor axis (a), eccentricity (e), inclination (i), right ascension of the ascending node (Ω), argument of perigee (ω), and mean anomaly (M).

In the case of deputy spacecraft, rather than work with their orbital elements, differential orbit elements are used, which is convenient because of the relatively short distance between the spacecraft.

Finally, it must set the test interval and an integration step. The test interval is the time that training will be simulated from the initial conditions introduced, and can be specified in seconds or as multiples of periods of the reference orbit (for example 2T0, 5.5T0).

The integration step is the main step to be used in numerical methods for solving equation systems; although the virtual instrument could occupy another size if step of user is not convenient.

The step size affects the computation time of the virtual instrument, so a small integration step is more appropriate when the test interval is short.

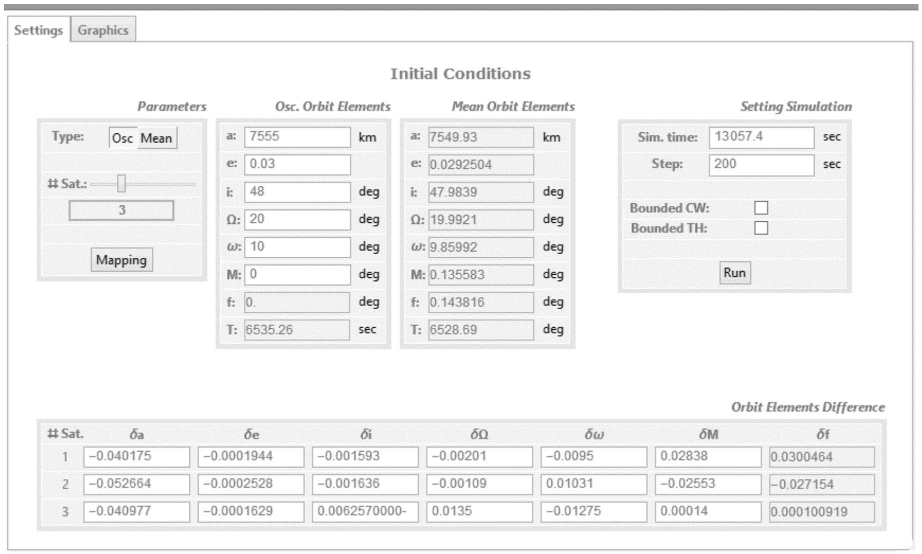

Figure 1. Main interface of virtual instrument.

RESULTS

The results obtained using this virtual instrument are on the tab "Graphics". It can be displayed three types of graphs:

- Graphs of inertial orbits
- Graphs of relative orbits
- Position and velocity plots

Figure 2 shows a screen capture from the orbit of the spacecraft leader at the inertial frame around of Earth. The inertial frame used is the framework ECI (Earth Centered Inertial). If required, it can also show the orbits around the Earth of the others spacecraft, although due to the closeness of them, it may not be noticeable differences between paths.

For formation flying are more interesting graphs about the relative trajectories of deputy spacecraft with respect to the leader at the Hill frame. An example with three relative orbits is presented in Figure 3. Although three paths are displayed, remember that there are four spacecraft, since one of them would be at the origin of the coordinate system.

With help of the controls on the top of the interface can select which dynamic model used to calculate the relative orbits and plot them; can be chose more than one at a time. Also, it is possible choose to display all secondary spacecraft or only some of them. This is convenient when you want to compare different models without to saturate the graph.

It can also view the position of the spacecraft on their orbits at any instant of time thanks to a slider of controls. It should be noted that depending on the characteristics of the reference orbit, some models not exhibit appropriate behavior. An example of this is given in orbits whose eccentricity is nonzero and want using the CW equations, which as mentioned before, are not valid for this type of orbits, so that the relative orbits obtained will be incorrect.

The conditions of the secondary spacecraft can also produce erroneous results, for example, if the separation between buildings is very large, all models except the nonlinear equations of relative motion will present trajectories with unacceptable errors.

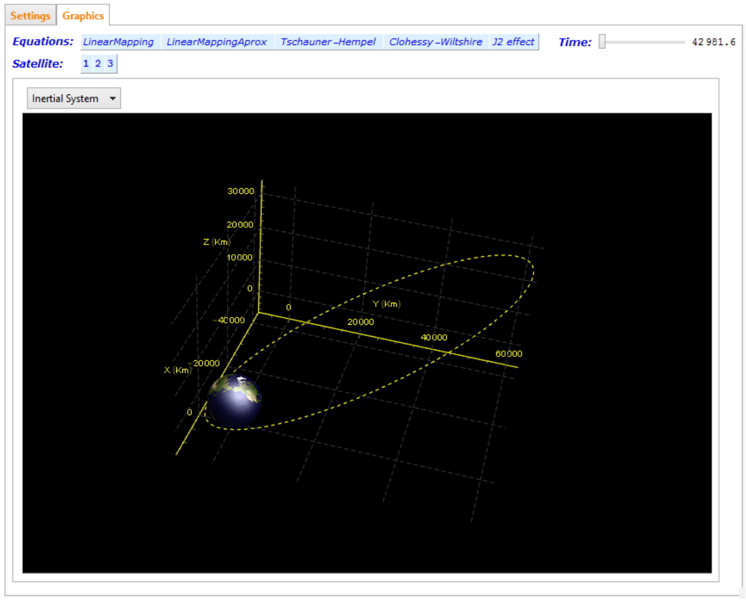

Figure 2. Graphics tab, inertial frame.

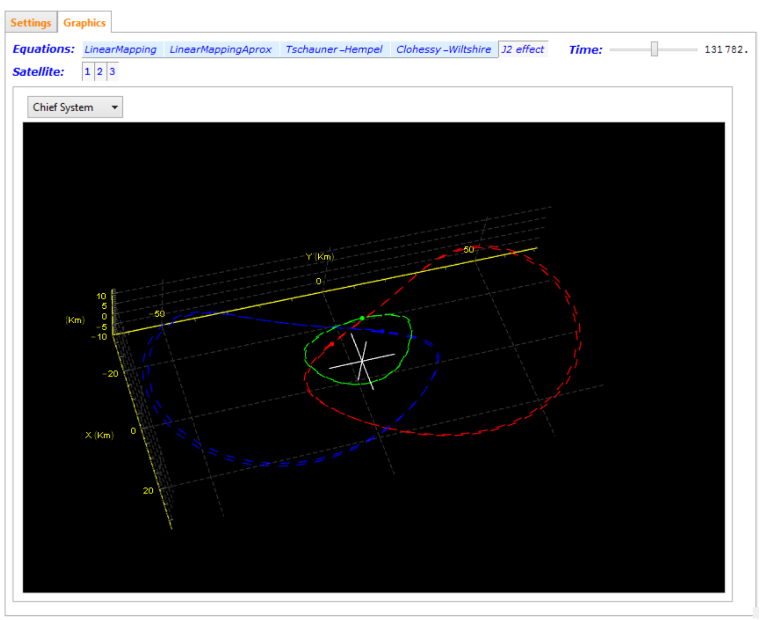

Figure 3. Graphics tab, Hill frame

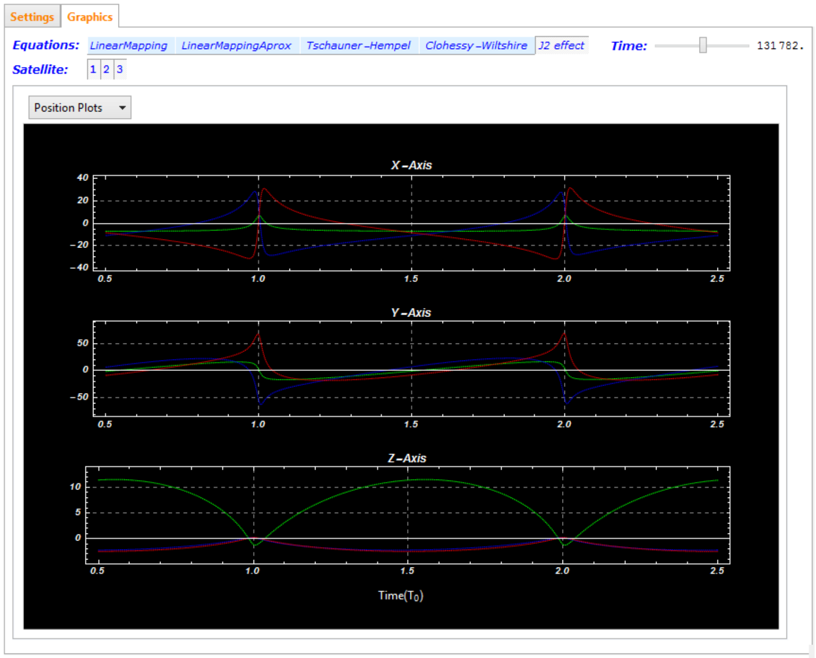

Figure 4. Graphics tab, position plot

On the other hand, it is possible to obtain position and velocity relative of all secondary spacecraft at the Hill frame. Figure 4 shows the three secondary vehicles position plot. Again, the user can select the spacecraft, the axis, and variables to plot.

The corresponding mean orbital elements for the reference orbit are organized on Table 1. Meanwhile, on Table 2 differential orbit elements corresponding to trajectories of the secondary vehicles of Figure 3 and 4.

Table 1. Mean orbital elements.

Element	Value	Unit
a	42095	km
e	0.81818	
i	28.5	deg
Ω	357.857	deg
ω	298.2253	deg
M	180	deg

Table 2. Differential orbit elements.

Num. Satellite	δa (km)	δe	δi (deg)	δΩ (deg)	δω (deg)	δM (deg)
1	-0.04018	-0.00019	-0.00159	-0.00201	-0.0095	0.02838
2	-0.05266	-0.00025	-0.00164	-0.00109	0.01031	-0.02553
3	-0.04098	-0.00016	0.006257	0.0135	-0.01275	0.00014

CONCLUSION

A virtual instrument to simulate relative trajectories of several space ships around Earth considering J4 perturbations was presented. Theoretical basis for formation flying coupled movements as well as J4 (perturbation due to true pole deformation) were discussed.

This instrument helps to show and select among several formation flying dynamic models which would be most convenient under specific circumstances. A GUI allows to introduce FF parameters such as amount of ships, individual initial conditions and selection of dynamic models which help to compare performances.

Orbits for all space vehicles are described using mean and difference orbital elements.

Future development for this virtual instrument considers other distinct perturbations beside J2 and to include other models for relative movements. Versatility is thought to increase by introducing orbital elements to secondary ships instead of just element differences.

REFERENCES

[1] D. P. Scharf, F. Y. Hadaegh y S. R. Ploen, “A Survey of Spacecraft Formation Flying Guidance and Control (Part I): Guidance”, American Control Conference, Denver, Colorado, 2003.

[2] D. P. Scharf, F. Y. Hadaegh y S. R. Ploen, "A Survey of Spacecraft Formation Flying Guidance and Control (Part II): Control", American Control Conference, Boston, Massachusetts, 2004.

[3] R. W. Beard, J. Lawton y F. Y. Hadaegh, "A Coordination Architecture for Spacecraft Formation control", IEEE Transactions on Control system Technology, vol. 9, nº 6, pp. 777-789, 2001.

[4] N. Dadkhah, L. Rodrigues y A. G. Aghdam, "Satellite Formation Flying Controller Design Using an Optimal Decentralized Approach", de American Control Conference, New York, USA, 2007.

[5] D. Lee, S. P. Viswanathan, L. Holguin, A. K. Sanyal y E. A. Butcher, "Decentralized Guidance and Control for Spacecraft Formation Flying Using Virtual Leader Configuration", de American Control Conference, Washington, 2013.

[6] M. Mesbahil y F. Y. Hadaegh, "Mode and Logic-Based Switching for the Formation Flying Control of Multiple Spacecraft", de American Control Conference, Arlington, Virginia, 2001.

[7] H. Cho, P. Sang-Young, P. Han-Earl y C. Kyu-Hong, "Analytical Solution to Optimal Reconfigurations of Satellite Formation Flying in Circular Orbit under J2 Perturbation", IEEE Transactions on Aerospace and Electronic System, vol. 48, nº 3, pp. 2180-2196, 2012.

[8] W. Ren y R. W. Beard, "Decentralized Scheme for Spacecraft Formation Flying via the Virtual Structure Approach", Journal of Guidance, Control, and Dynamics, vol. 27, nº 1, pp. 73-82, 2004.

[9] R. S. Smith, M. V. Subbotin y F. Y. Hadaegh, "Decentralized Estimation and Control in High Precision Spacecraft Formations: Comparision Studies", de IEEE Aerospace conference, Big Sky, Montana, 2007.

[10] T. Reid y A. K. Misra, "Formation Flying of Satellites in the Presence of Atmospheric Drag", Journal of Aerospace Engineering, Sciences and Applications, vol. III, nº 1, pp. 64-91, 2011.

[11] C. Sabol, R. Burns y C. A. McLaughlin, "Satellite Formation Flying Design and Evolution", Journal of Spacecraft and Rockets, vol. 38, nº 2, pp. 270-278, 2001.

[12] D. Xue, X. Cao y Y. Wu, "Decentralized Determination of Relative Orbit for Formation Flying Satellite", de 1st International Symposium on Systems and Control in Aerospace and Astronautics, Harbin, China, 2006.

[13] R. E. Sherrill, "Dynamics and Control of Satellite Relative Motion in Elliptic Orbits using Lyapunov-Floquet Theory", Auburn University, Alabama, 2013.

[14] P. Sengupta, "Dynamics and Control of Satellite Relative Motion in a Central Gravitational Field", Texas A&M, Texas, 2006.

[15] R. G. Melton, "Time-Explicit Representation of Relative Motion Between Elliptical Orbits", Journal of Guidance, Control, and Dynamics, vol. 23, nº 4, pp. 604-610, 2000.

[16] C. W. T. Roscoe, S. R. Vadali y K. T. Alfriend, "Design of Satellite Formation in Orbits of High Eccentricity with Performance Constraints Specified Over a Region of Interest", Advances in the Astronautical Sciences, vol. 139, pp. 145-162, 2011.

[17] H. Schaub y J. L. Junkins, Analytical Mechanics of Space Systems, AIAA Education Series, 2003.

[18] V. A. Chobotov, Orbital Mechanics, AIAA Education Series, 2002.

[19] M. Eshagh y M. N. Alamdari, "Perturbations in Orbital Elements of a Low Earth Orbiting Satellite", Journal of the Earth & Space Physics, vol. 33, nº 1, pp. 1-12, 2007.

[20] O. Montenbruck y E. Gill, Satellite Orbits, Models, Methods and Applications, Springer, 2001.

[21] G. Petit y B. Luzum, "IERS Conventions (2010)", Frankfurt, 2010.

[22] G. Beutler, Methods of Celestial Mechanics, Vol. 1, Springer, 2005.

[23] S. R. Vadali, S. S. Vaddi y K. T. Alfriend, "An Intelligent Control Concept for Formation Flying Satellite Constellations", International Journal of Robust and Nonlinear Control, vol. 12, pp. 97-115, 2002.

[24] R. H. Lyddane, "Small Eccentricities or Inclinations in the Brouwer Theory of the Artificial Satellite", Astronomical Journal, vol. 68, nº 8, pp. 555-558, 1963.