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Dron Simulation for the Control of a Satellite Trajectory with Fixed Attitude

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Abstract

This work focuses on the modelling, control, optimization and actual flight of a quadrotor helicopter to emulate the control of a satellite working under a given fix attitude while in orbit. The modelling uses the unit Quaternion to represent attitude and the control study the performance in presence of parametric uncertainty considering optimal energy consumption for two different controllers: the first strategy is the Linear Quadratic Regulator Control that uses Bryson's rule as algorithm for determining the weight matrices, while the second strategy is the use of Fuzzy Control for the dynamic´s attitude. Results show that the first controller present a steady state error quite large on trajectory tracking, whereby second process decreases the phase error while remaining within the optimal energy consumption. Finally, a human computer interface is developed to introduce the proper satellite parameters and make them similar to the actual quadrotors.

Keywords: (Control, Quaternion, Attitude, Quadrotor, Satellite

Nomenclature

Unity quaternion

$$\mathbf{q} = \begin{pmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{pmatrix} = q_0 + \mathbf{q}_{1:3} = q_0 + q_1\mathbf{i} + q_2\mathbf{j} + q_3\mathbf{k} \,.$$

in terms of the 3D Cartesian orthonormal basis vectors **i**, **j** and **k** while q_o describes the rotation magnitude.
* represents the conjugate of a quaternion.
$\otimes$ is the quaterninon product.

Acronyms/Abbreviations

Linear Quadratic Regulator (LQR)
Unmanned Aerial Vehicle (UAV).

1. Introduction

Modelling and simulation have become important tools to previously test some of the actual major mathematical approaches in Control Theory. For the particular research area of Astronautics -embracing its counterpart the Aeronautics- efforts in developing and improving actual simulators have considerably taken both ingenious and technical amounts of work. Fortunately, nowadays virtual instrumentation gives the opportunity to diminish and yet to effectively improve those efforts.

Here, the modelling and control of an unmanned aerial vehicle (UAV) is presented. A quadrotor is used as an example of a particular UAV and to show the control of a 3D navigational flight as a feasible emulation of a 3 degrees of freedom space ship orbiter.

2. UAV (quadrotor) Model

A Quadrotor is chosen to model and represent a given UAV under the quaternion approach [1]

2.1 State Variables

The linear acceleration of a quadrotor is given by [2][3]:

$$\dot{\mathbf{v}} = -g\mathbf{z}_i + \frac{F_{U_1}}{m} R_{M^i}\mathbf{z}_i - \boldsymbol{\omega}\times\mathbf{v} + \boldsymbol{\Gamma}_{\boldsymbol{p}} \qquad (1)$$

where $\mathbf{z}_i = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}^T$ is the position vector in the direction of the gravitational force g, R_{M^i} is the rotation matrix in terms of the Cardan angles (Tait-Bryan) with respect to the body frame of reference (Fig. 1) and F_{U_1} is the control input affecting height.

The angular acceleration is [4]

$$\dot{\boldsymbol{\omega}} = I^{-1}\left(-\boldsymbol{\omega}\times I\boldsymbol{\omega} + \boldsymbol{\tau}_G + \boldsymbol{\tau} + \boldsymbol{\Gamma}_{\omega}\right) \qquad (2)$$

where $\boldsymbol{\tau} = (\tau_{\phi,U_2} \quad \tau_{\theta,U_3} \quad \tau_{\psi,U_4})^T$ is the input control vector on the heading, roll and pitch rotation and $\boldsymbol{\tau}_G$ is the complete torque produced by all four engines given by [5]:

$$\boldsymbol{\tau}_G = -\sum_{i=1}^{4} J_R \Omega_i \begin{pmatrix} Q\, I_{xx}^{-1} \\ -P I_{yy}^{-1} \\ 0 \end{pmatrix} \quad (3)$$

Where Ω_i is the angular velocity J_R and the moment of inertia of each helix and Q, P are the angular velocities with respect to the UAV's frame of reference M^c.

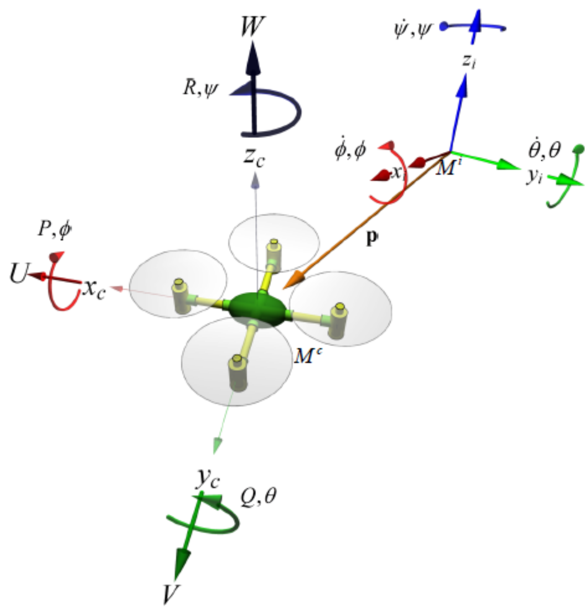

Fig. 1 Frames of reference and state variables.

Thus, the translational velocity with respect to the M I frame of reference and the angular velocity vectors are given by [6]

$$\dot{\mathbf{p}} = R_{M^i}\mathbf{v} \quad (4)$$

$$\dot{\boldsymbol{\alpha}} = \mathrm{T}\boldsymbol{\omega} \quad (5)$$

respectively and where T is

$$\mathrm{T} = \begin{pmatrix} 1 & sen(\phi)\ tan(\theta) & cos(\phi)\ tan(\theta) \\ 0 & cos(\phi) & -sen(\phi) \\ 0 & sen(\phi)/cos(\theta) & cos(\phi)/cos(\theta) \end{pmatrix} \quad (6)$$

If a unity quaternion is used to describe the kinematics of the UAV, then the angular velocity becomes [7][8]

$$\dot{\mathbf{q}} = \frac{1}{2}\mathbf{q}\otimes\begin{pmatrix} 0 \\ \boldsymbol{\omega} \end{pmatrix} \quad (7)$$

Finally, in terms of the quaternion, the full kinematics is described by the translational acceleration

$$\dot{\mathbf{v}} = -g\mathbf{z}_i + \frac{1}{m}\left(\mathbf{q}\otimes\begin{pmatrix} 0 \\ 0 \\ U_1 \end{pmatrix}\otimes\mathbf{q}^* + \boldsymbol{\Gamma}_p\right) - \boldsymbol{\omega}\times\mathbf{v} \quad (8)$$

and the angular acceleration given by the Equation (2).

3. Linear Quadratic Regulator Control

In this section, the operation points for the quadrotor in stationary flight are calculated through Jacobian linearization and the control law is stablished using the Bryson rule.

The UAV closed control loop is described by Figure (2) [7][8][9] where

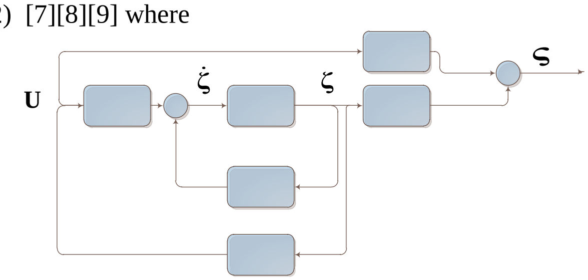

Fig. 2 Feedback Control System

A is the $n\times n$ state matrix, B a $n\times m$ constant matrix, C is the $n\times n$ coupling output matriz, D is the $n\times m$ perturbation matrix and K is a $m\times n$ control input, and $\mathbf{U} = -K\zeta$ with the linear and time invariant state variables

$$\dot{\zeta} = (A - BK)\zeta \quad (9)$$

$$\varsigma = C\zeta + D\mathbf{U} \quad (10)$$

Furthermore, for optimal performance the system ought to satisfy the cost function [10]

$$J(\mathbf{U}) = \int_0^\infty (\varsigma^T\varsigma)\,dt = \int_0^\infty (\zeta^T Q\zeta + \mathbf{U}^T R\mathbf{U})\,dt \quad (11)$$

where Q is the state control matrix and R is the performance matrix both being positive definite and calculated from [11]

$$\begin{aligned} C^T C &= Q \\ D^T D &= R \end{aligned} \quad (12)$$

which in turn can be satisfied if the Riccati equation [12]

$$A^T P + PA - (PB + S)R^{-1}(B^T P + S^T) + Q = 0 \quad (13)$$

is solved to obtain the matrix P. Thus, the K matrix is obtained on the form of

$$K = -R^{-1}(B^T P + S^T) \quad (14)$$

Now, in agreement with Figure (2), the output state vector can be written in terms of the control vector U as

$$\varsigma = g\left(\boldsymbol{\zeta},\mathbf{U}\right) \tag{15}$$

while Eq. 9 can be written as a non-linear differential equation

$$\dot{\boldsymbol{\zeta}} = f(\boldsymbol{\zeta})+\sum_{i=1}^{4} g\left(\boldsymbol{\zeta}\right)\mathbf{U}_i \tag{16}$$

Then, when the input state vector associated to the UAV is

$$\boldsymbol{\zeta}=\begin{pmatrix} q_1 & P & q_2 & Q & q_3 & R & x & U & y & V & z & W\end{pmatrix}^T$$

and the components of the tensor of inertia are

$$g\left(\boldsymbol{\zeta}\right)=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & g_{1,8} & 0 & g_{1,10} & 0 & g_{1,12} \\ 0 & 1/I_{xx} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/I_{yy} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/I_{zz} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}^T$$

with

$$g_{1,8} = \frac{1}{m}\left(2\left(q_1q_3 - q_2q_0\right)\right)$$

$$g_{1,10} = \frac{1}{m}\left(2\left(q_2q_3 + q_1q_0\right)\right)$$

$$g_{1,12} = \frac{1}{m}\left(q_0^2 - q_1^2 - q_2^2 + q_3^2\right)$$

the function $f(\boldsymbol{\zeta})$ is

$$f\left(\boldsymbol{\zeta}\right)=\begin{pmatrix} \frac{1}{2}\left(q_3Q - q_0P - q_2R\right) \\ \left(\frac{I_{yy}-I_{zz}}{I_{xx}}\right)QR + \frac{\Gamma_P}{I_{xx}} \\ \frac{1}{2}\left(q_1R - q_3P - q_0Q\right) \\ \left(\frac{I_{zz}-I_{xx}}{I_{yy}}\right)PR + \frac{\Gamma_Q}{I_{yy}} \\ \frac{1}{2}\left(q_2P - q_1Q - q_0R\right) \\ \left(\frac{I_{xx}-I_{yy}}{I_{zz}}\right)PQ + \frac{\Gamma_R}{I_{zz}} \\ U \\ \frac{\Gamma_x}{m} \\ V \\ \frac{\Gamma_y}{m} \\ W \\ -g+\frac{\Gamma_z}{m} \end{pmatrix} \tag{17}$$

The Bryson rule

$$Q_{ii} = \frac{1}{\zeta_{max}^2}$$

$$R_{ii} = \frac{1}{U_{max}^2}$$

will give the maximum allowed variations for each matrix named ζ_{max} and U_{max}.

Using Equations (15) and (16), in order to simplify and associate the model to a space ship, some considerations can be done [13]: i) the Coriolis force would be small for a given modern quadrotor, ii) all torques produced by the rotor motors can be insignificant and iii) the operation point is chosen while the quadrotor is at stationary flight.

Thus, and under the iii) consideration, a given UAV using the quaternion approach [14]

$$\begin{aligned} \mathbf{q}_E &= \begin{pmatrix}1 & 0 & 0 & 0\end{pmatrix}^T \\ \mathbf{v}_E &= \begin{pmatrix}0 & 0 & 0\end{pmatrix}^T \\ \boldsymbol{\omega}_E &= \begin{pmatrix}0 & 0 & 0\end{pmatrix}^T \end{aligned} \tag{18}$$

$\boldsymbol{\omega}\times\mathbf{v}$ is negligible as well as the external forces $\boldsymbol{\Gamma}_{\mathbf{p}}$. Therefore, the translation velocity can be simplified to

$$\dot{\mathbf{v}} = -g\mathbf{z}_i + \frac{1}{m}\mathbf{q}\otimes\begin{pmatrix}0\\0\\U_1\end{pmatrix}\otimes\mathbf{q}^* \tag{19}$$

and the cinematics can also be simplified as

$$\dot{\mathbf{q}} = \frac{1}{2}\mathbf{q}_{\omega} \tag{19}$$

Finally, the controllability of this system assured by doing the analysis for the matriz

$$S = \begin{pmatrix}B & AB & A^2B & \cdots & A^{n-1}B\end{pmatrix}$$

system using two different popular computer software

Thus, the full LQR feedback control system becomes as shown in Figure (3)

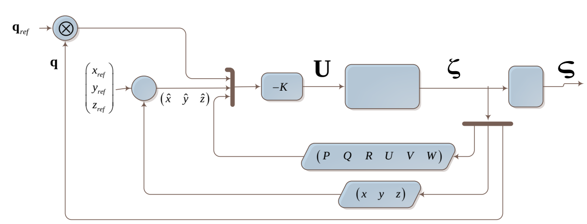

Fig 3. LQR Control system

4. Fuzzy Attitude Control

Here, theoretical basis [15].

5. Simulation Results

For the simulation of a particular quadrotor all specific data were taken from [1X] to accomplish the following:

i) Translational position: the initial ground effect over the UAV was eliminated since the flight is considered already stationary.
ii) Rotational position: All angles were restricted within $\pm 30°$ from the equilibrium point.
iii) Angular velocity: is limited to an operation range of $\pm\pi\ rad/s$
iv) Control Forces: From equation all control inputs are limited to

$$0.000N \le U_1 \le 9.67N$$
$$-0.57Nm \le U_2 \le 0.57Nm$$
$$-0.57Nm \le U_3 \le 0.57Nm$$
$$-0.11Nm \le U_4 \le 0.11Nm$$

The response positions for the LQR controller are presented in Figure (4) where the error in each axis is easily appreciated giving the biggest error of 25% for the z axis [16].

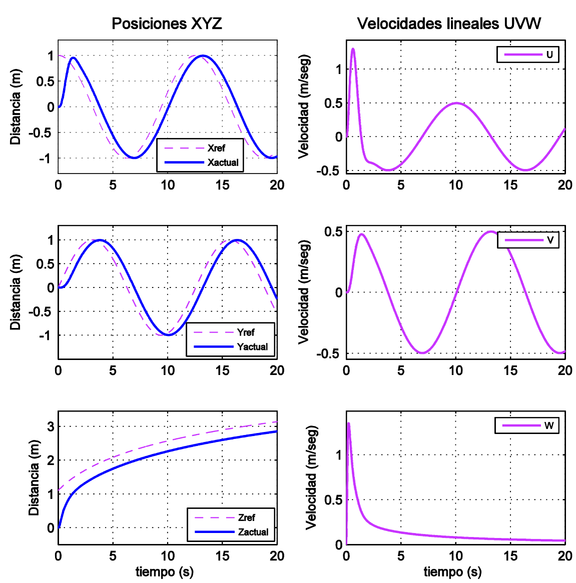

Fig. 4 Position and velocity tracking

Fig. 5 shows the response to a step quaternion input as well as its corresponding control signals.

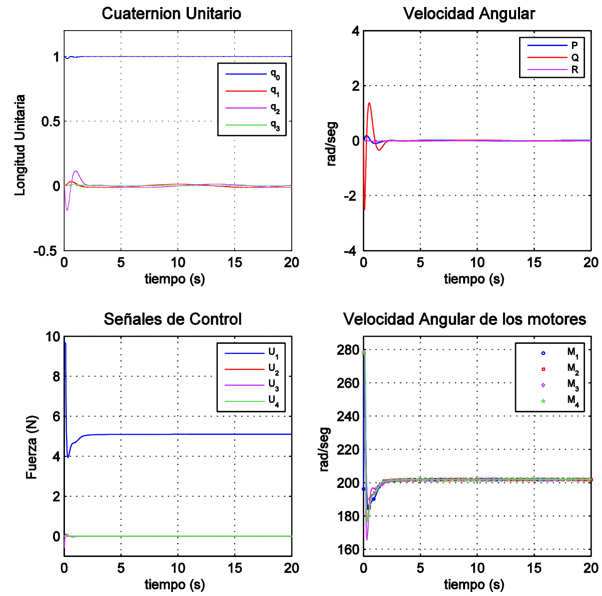

Fig. 5 $\mathbf{q}, \boldsymbol{\omega}, \mathbf{U}, \boldsymbol{\Omega}$ responses at trajectory tracking

It is important to note that all state variables are bounded as expected despite the phase error presented in Figure (4). However, the control force U_1 requires its maximum value and so making that the rotors go to its maximum speed $278\, rad/s$. This rotor reaction implies a very high energy consumption for the UAV and yet, the error remains high as well.

Despite the acceptable behavior of the Bryson rule, the analysis [16] of the error of state variable ζ_{11} at the stability point show that it can be diminished if an integral filter is used. Therefore, a modified LQR controller was tested using an integrator: ILQR. The corresponding results after this improvement are shown in Figures (6) and (7).

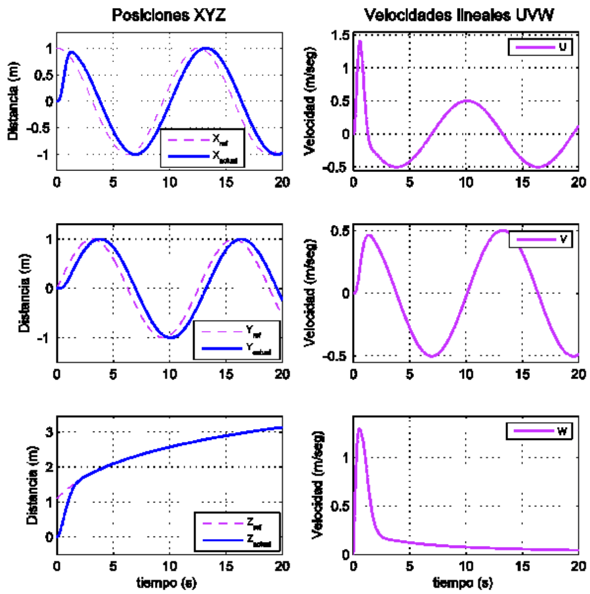

Fig. 6 ILQR $\mathbf{p}, \mathbf{v}$ trajectory tracking

The error is lower for the ILQR than the LQR as well as the energy consumption

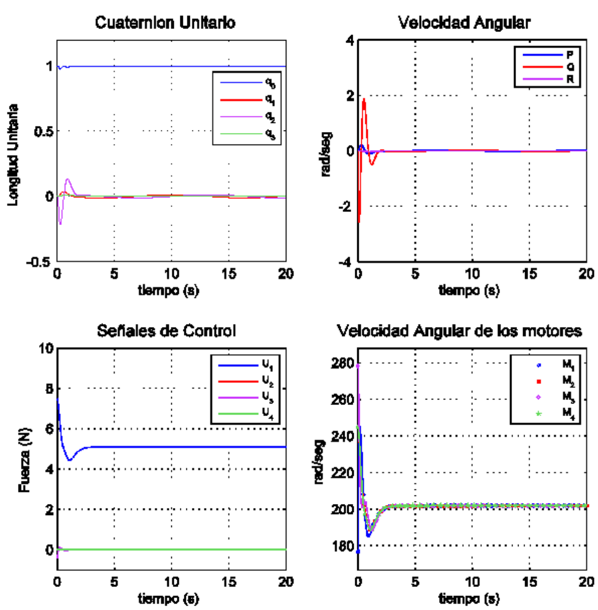

Fig. 7 ILQR $\mathbf{q}, \boldsymbol{\omega}, \mathbf{U}, \boldsymbol{\Omega}$ at trajectory tracking

For this case, all state variables remain bounded and furthermore the rotors do not reach their maximum.

Now, with respect to the Fuzzy Control the following figures show that

6. Space Ship Emulation

On the one hand, a quadrotor has the hoovering capability that produces the very same degrees of freedom as those of a spaceship while in a specific orbit, and on the other hand, because the UAV and an orbiter ship are both rigid bodies it is only needed, from the Control Theory point of view, to impose restrictions on the roll and pitch attitude for the quadrotor, then the two vehicles can be considered equivalent.

Figure (8) describes how this UAV controllers emulates a space vehicle while in a given orbit.

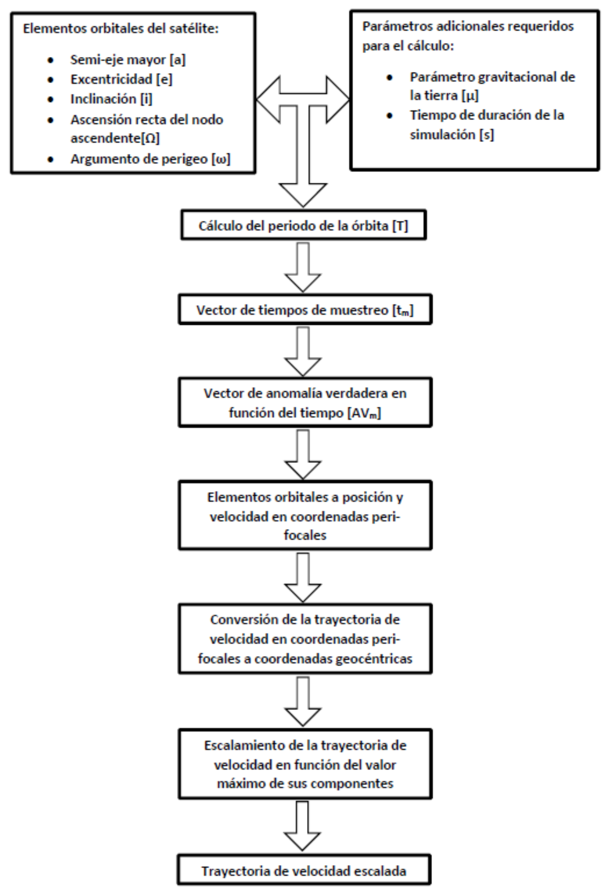

Fig. 8 UAV emulation of a spaceship

7. Computer Interface

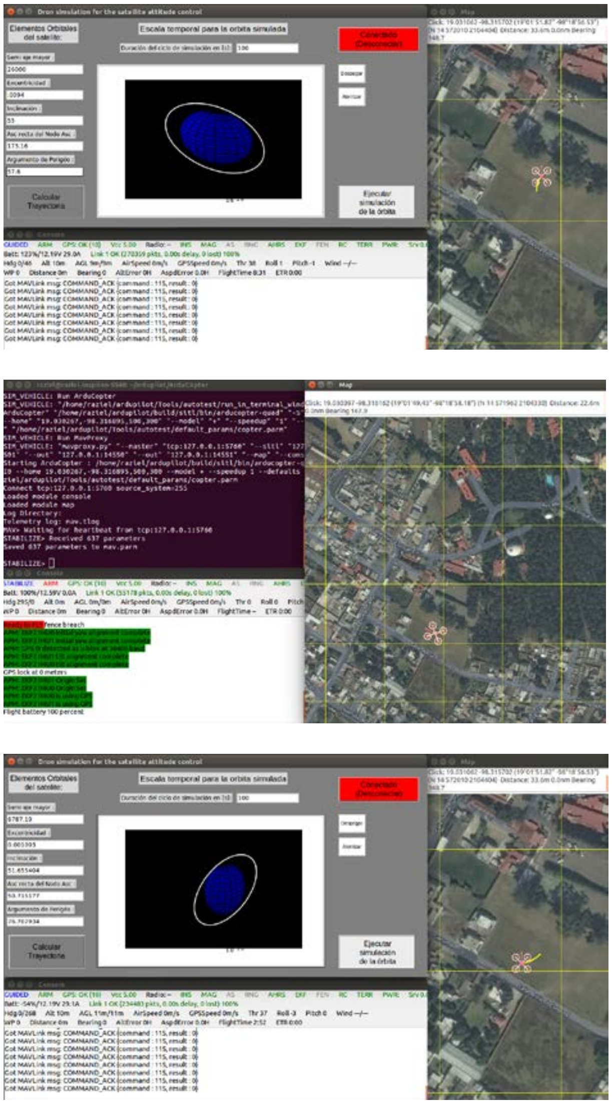

Figure 9. Emulator interface.

8. Conclusions

The mathematical model of a quadrotor is given in terms of a unit quaternion for both, static and dynamics. Simulated results for LQR controller and its improvement, an ILQR were, presented. Fuzzy controller was discussed as a way to actually implement a controller into a based Arduino UAV.

Simulations were oriented to prove that and actual UAV can be taken as an emulator of a space vehicle in a given orbit considering some characteristics as:

1. Good overshot Quaternion Control
2. Energy consumption monitoring and control
3. Stable state with a small error in trajectory tracking
4. Good stability control under perturbations.

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