PIB – Inverno 2021/2022 – Partial Exam 1

1º Teste Parcial – 24/11/2021

							Σ			
1	a	b		1,25	1,25		2,5	6,25	13	20
2	a	b	c	1,25	1,25	1,25	3,75			
3	a	b		1,5	1,5		3	6,75		
4	a	b	c	1,25	1,25	1,25	3,75			
5	a	b	c	1,5	1,5	1,5	4,5	7	7	
6	a	b		1,25	1,25		2,5			

1 a) $I_1 = \begin{bmatrix} 10 & 42 & 17 \\ 17 & 17 & 17 \end{bmatrix}$ (2) (3) 50

• one possible solution

$m_{I_1} = \frac{1}{2\times3}\left(10+42+\underbrace{a+b+c+d}_{a=b=c=d,\ \text{simplification}}\right)=20$

$= \frac{1}{6}(52+4a) = 20$

$= \frac{1}{6}\cdot(52+4a) = 20 \Leftrightarrow 52+4a = 120$

$\Leftrightarrow 4a = 68 \Leftrightarrow a = 17$ //

$I_2 = \begin{bmatrix} 0 & 255 & 70 & 80 \\ 90 & 100 & 110 & 95 \end{bmatrix}$ (2) (4) 50

$m_{I_2} = \frac{1}{2\times4}\sum_{m=0}^{1}\sum_{n=0}^{3} I_2[m,n] = 100$

$\Leftrightarrow m_{I_2} = \frac{1}{8}\left(0+255+a+b+c+d+e+f\right) = 100$

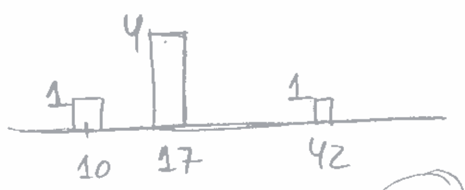
$\Leftrightarrow 255+a+b+c+d+e+f = 800$

$\Leftrightarrow a+b+c+d+e+f = 545$

70 80 90 100 110 95

b) Histogram

I_1



pmf = $\{\frac{1}{6}, \frac{4}{6}, \frac{1}{6}\}$ 12,5

Entropy

$H(I_1) = -2\frac{1}{6}\log_2\left(\frac{1}{6}\right) - \frac{4}{6}\log_2\left(\frac{4}{6}\right)$

$= 1{,}25$ bit/pixel 12,5

Total bits

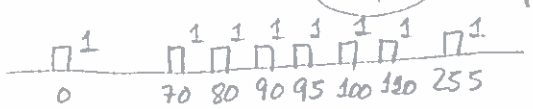
#bits = 2×3×6 = 36 12,5

minimum n

LSB

$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ 12,5

I_2 12,5



pmf = $\{\frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}\}$

$H(I_2) = \log_2(8)$

$= 3$ bit/pixel 12,5

#bits = 2×4×8 = 64 12,5

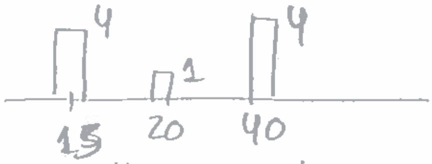
$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ 12,5

b) Alternative Version

$$I_1 = \begin{bmatrix} 15 & 40 & 40 \\ 40 & 40 & 20 \\ 15 & 15 & 15 \end{bmatrix} \qquad I_2 = \begin{bmatrix} 10 & 12 & 14 \\ 9 & 11 & 13 \end{bmatrix}$$

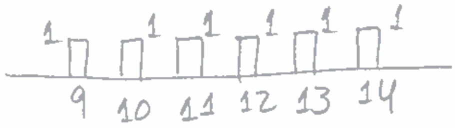
Histogram

I_1



$pmf = \{ \frac{4}{9}, \frac{1}{9}, \frac{4}{9} \}$

I_2



$pmf = \{ \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6} \}$

Entropy

$H(I_1) = -2 \times \frac{4}{9} \log_2\left(\frac{4}{9}\right) - \frac{1}{9} \log_2\left(\frac{1}{9}\right) = 1{,}39$ bit/pixel

$H(I_2) = \log_2(6) = 2{,}58$ bit/pixel

Total Bits (minimum 'm')

$\#bits = 3 \times 3 \times 6 = 54$

$\#bits = 2 \times 3 \times 4 = 24$

LSB

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

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② a) Figure (a) → (i) Authentication / Verification action on a biometric system (a one to one comparison). (30)

(ii) Feature Extractor – From the input image, it computes a vector with features / attributes [- - - - -] (35)

(iii) Template – is the vector (or set of vectors) that represents an individual on the database (35)

b) Figure (b) → (i) (25) YY axis measures the error rate (% of misclassifications)

(ii) (25) EER = Equal Error Rate

(iii) (25) A → False match Rate (FMR) (False Positives)
B → False Non-match Rate (FNMR) (False Negatives)

(iv) (25) P1 → Point of Zero FNMR
P2 → Point of Zero FMR

c) Fingerprint recognition. (50) The picture shows a fingerprint sensor. (25)
Use of fingerprint recognition on smartphones and personal computers, for instance. Access control / labor control.

It is widespread with its use on many scena[illegible] (25) Very acceptable. Easy to acquire.

3 a) $T_1 \rightarrow$ For pixels from $\emptyset \ldots 100$, the intensity is kept. (identity)
For pixels above 100, the intensity is saturated at 100 (saturation) 25

T_1

0	1	2	3	4	...	98	99	100	100	100	100	...	100	100
0	1	2	3	4	...	98	99	100	101	102	103	...	254	255

25

256 position lookup table

$T_2 \rightarrow$ contrast stretching function for pixels in the range of 100 to 200 25

$\{100 \ldots 200\} \longrightarrow \{0 \ldots 255\}$
input output

- Pixels in the range 0 to 100 are set to zero ($\emptyset$) – minimum
- Pixels in the range 200 to 255 are set to 255 – maximum

T_2

0	0	...	0	0	3	5	...	?	...	252	255	255	255	...	255
0	1	...	99	100	101	102	...		...	199	200	201	202	...	255

25

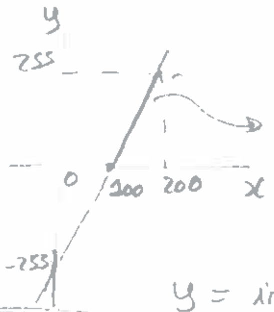
256 position lookup table

b) $I_A = \begin{bmatrix} 0 & 10 & 50 \\ 140 & 230 & 200 \\ 40 & 20 & 150 \end{bmatrix} \longrightarrow I_1 = \text{NOT}[I_A] = 255 - I_A$

50 $= \begin{bmatrix} 255 & 245 & 205 \\ 115 & 25 & 55 \\ 215 & 235 & 105 \end{bmatrix}$

$\rightarrow I_2 = T_2[I_A]$

50 $= \begin{bmatrix} 0 & 0 & 0 \\ 102 & 255 & 255 \\ 0 & 0 & 128 \end{bmatrix}$



$y = \frac{255}{100} x + (-255)$

$= 2{,}55x - 255$

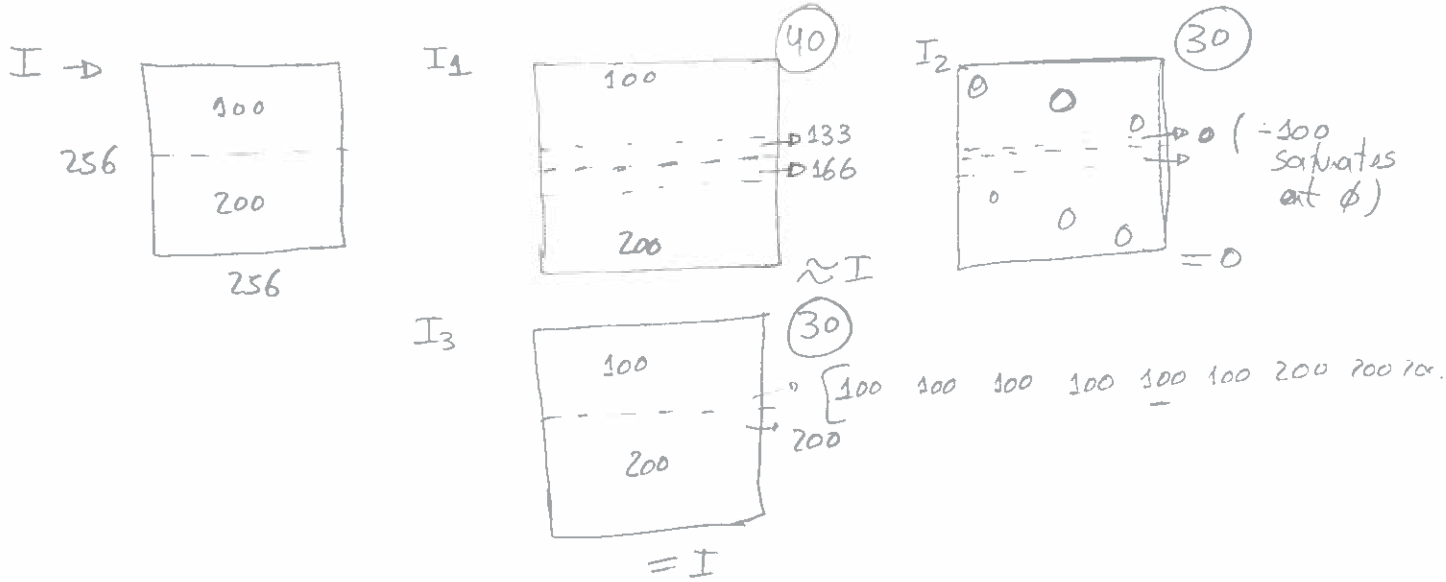
$y = \text{int}[2{,}55x - 255]$

$x = 140 \rightarrow y = \text{int}[2{,}55 \times 140 - 255] = 102$

$x = 150 \rightarrow y = \text{int}[2{,}55 \times 150 - 255] = 127$ or 128

(4) $w_1 = \frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$ $w_2 = \begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$ $w_3 = $ mediana $\{3\times3\}$

a) $I \rightarrow$



b) # multiplications $= 256 \times 256 \times 9 = 589\ 824$ 50

sums $= 256 \times 256 \times 8 = 524\ 288$ 50

c) the difference between I and I_1 is the two central rows

For I, we have

$\rightarrow$ 100 100 100

$\rightarrow$ 200 200 --- 200

256

on these rows $\Rightarrow E_I = 256 \times 100^2 + 256 \times 200^2 = 12\ 800\ 000$

For I_1, we have

$\rightarrow$ 133 133

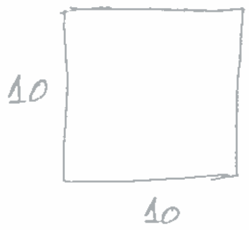
$\rightarrow$ 166 - - 166

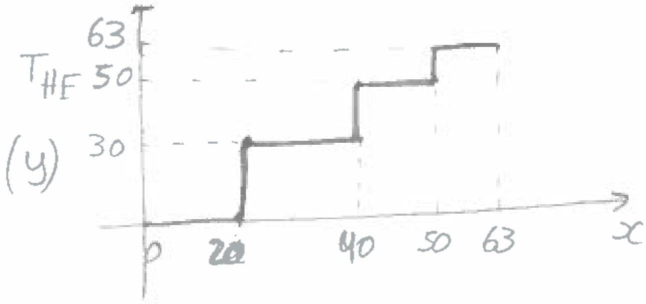
$\Rightarrow E_{I_2} = 256 \times 133^2 + 256 \times 166^2 = 11\ 582\ 720$

thus $E_{I_1} < E_I \iff E_I > E_{I_1}$ 100

(ii)

5 a) I





x	y THE
20	30
40	50
50	63

50

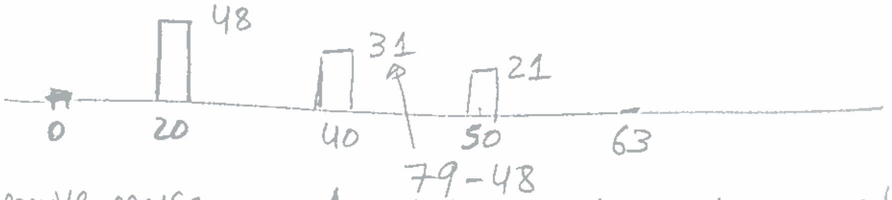
$20 \;\; 30 \longrightarrow \;\; 30 = \frac{h_a(20)}{100} \times 63 \rightarrow h_a(20) = \frac{100 \times 30}{63} = 47{,}6 \approx 48$

$40 \;\; 50 \longrightarrow \;\; 30 = \frac{h_a(40)}{100} \times 63 \rightarrow h_a(40) = \frac{100 \times 50}{63} = 79{,}3 \approx 79$

$50 \;\; 63 \longrightarrow \;\; 63 = \frac{h_a(50)}{100} \times 63 \iff h_a(50) = 100$

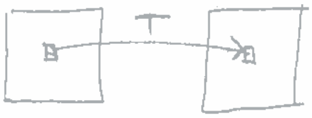
$h_a \rightarrow$ histogram accumulated

50



b) In order to remove noise or to detect edges, we need to analyze the neighborhood of a pixel. That is carried out with spatial filtering techniques. 50

The intensity transformation techniques process pixel by pixel and do not analyze the neighborhood of each pixel. 50

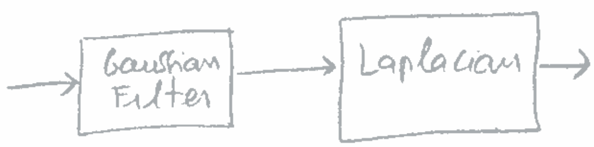


c) Laplacian — Laplacian of Gaussian

(i)

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

For instance



35

Low-Pass filter to remove the noise

(ii) more complexity 30

(iii) Edge detection
Edge enhancement
for noiseless images

Edge detection
Edge enhancement
for noisy images

35

⑥ $I_x = \begin{bmatrix} 5 & 4 & -2 \\ 6 & 3 & 2 \\ 3 & 0 & 0 \end{bmatrix}$ $I_y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ -1 & 3 & 0 \end{bmatrix}$

a) $\nabla I = \sqrt{I_x^2 + I_y^2} = \begin{bmatrix} \sqrt{25+1} & \sqrt{16} & \sqrt{4} \\ \sqrt{36} & \sqrt{9} & \sqrt{5} \\ \sqrt{10} & \sqrt{9} & 0 \end{bmatrix} = \begin{bmatrix} \sqrt{26} & 4 & 2 \\ 6 & 3 & \sqrt{5} \\ \sqrt{10} & 3 & 0 \end{bmatrix}$ 50

$= \begin{bmatrix} 5{,}099 & 4 & 2 \\ 6 & 3 & 2{,}23 \\ 3{,}16 & 3 & 0 \end{bmatrix}$ $> 3{,}5$

$\theta_I = \operatorname{atan}\left(\frac{I_y}{I_x}\right) = \begin{bmatrix} \operatorname{atan}\left(\frac{1}{5}\right) & \operatorname{atan}\left(\frac{0}{4}\right) & \operatorname{atan}\left(\frac{0}{-2}\right) \\ \operatorname{atan}\left(\frac{0}{6}\right) & \operatorname{atan}\left(\frac{0}{3}\right) & \operatorname{atan}\left(\frac{1}{2}\right) \\ \operatorname{atan}\left(-\frac{1}{3}\right) & \operatorname{atan}\left(\frac{3}{0}\right) & \operatorname{atan}\left(\frac{0}{0}\right) \end{bmatrix}$

Não definido (Undefined)

$= \begin{bmatrix} 0{,}197 & 0 & \pi \\ 0 & 0 & 0{,}463 \\ -0{,}32 & 1{,}57 & \text{ind} \end{bmatrix}$ 50

b) $\nabla I > 3{,}5 \rightarrow$ Edge map $= \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ 100