II – Espectros
Tabelas

Processamento Digital de Sinais

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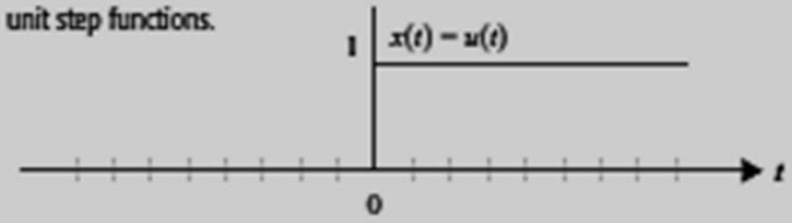
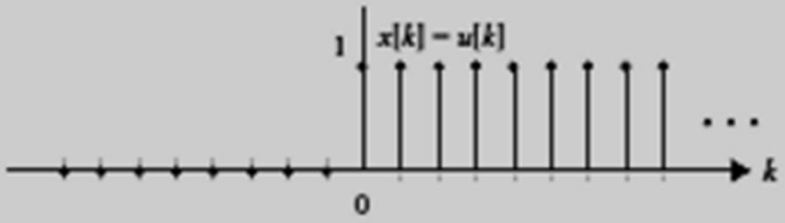
Alguns sinais mais comuns em PDS

Name	*Continuous*	*Discrete*
Unit Step function	$u(t)=\begin{cases}1, & t \geq 0\\ 0, & t<0\end{cases}$	$u[n]=\begin{cases}1, & n \geq 0\\ 0, & n<0\end{cases}$
Ramp signal	$r(t)=\begin{cases}t, & t \geq 0\\ 0, & t<0\end{cases}$	$r[n]=nu(n)=\begin{cases}n, & n \geq 0\\ 0, & n<0\end{cases}$
Impulse function	$\delta(t)=0, t \neq 0$	$\delta[n]=\begin{cases}1, & n=0\\ 0, & otherwise\end{cases}$
Rectangular pulse function	$rect\left(\frac{t}{\tau}\right)=\begin{cases}1, & \lvert t\rvert \leq \tau/2\\ 0, & \lvert t\rvert > \tau/2\end{cases}$	$rect[\frac{n}{2N}]=\begin{cases}1, & \lvert n\rvert \leq N\\ 0, & \lvert n\rvert > N\end{cases}$
Triangular pulse	$tri\left(\frac{t}{\tau}\right)=\begin{cases}1-\lvert\frac{t}{\tau}\rvert, & t \leq \lvert\tau\rvert\\ 0, & t > \lvert\tau\rvert\end{cases}$	$tri[\frac{n}{N}]=\begin{cases}1-\frac{\lvert n\rvert}{N}, & \lvert n\rvert \leq N\\ 0, & elsewhere\end{cases}$
Signum signal	$Sgn(t)=\begin{cases}1, & t>0\\ -1, & t<0\end{cases}$	$Sgn[n]=\begin{cases}1, & n>0\\ -1, & n<0\end{cases}$
Sinusoidal signal	$x(t)=\sin(2\pi f_0 t+\theta)$	$X[n]=\sin(2\pi f_0 n+\theta)$
Sinc function	$\text{sinc}\ (\omega_0 t)=\frac{\sin(\pi\omega_0 t)}{\pi\omega_0 t}$	$\text{sinc}\ [\omega_0 n]=\frac{\sin(\pi\omega_0 n)}{\pi\omega_0 n}$

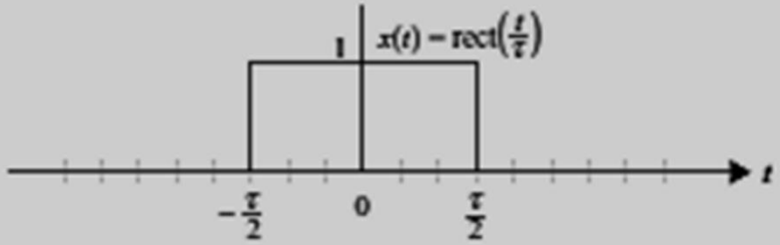
unit step functions.

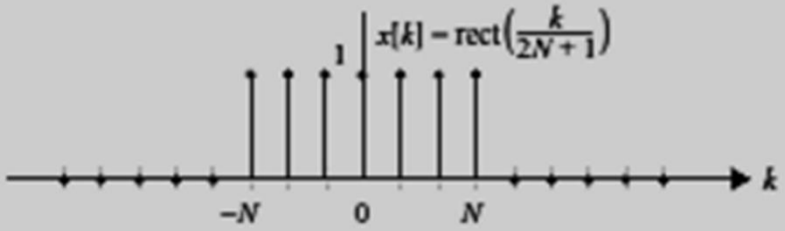




rectangular pulses.

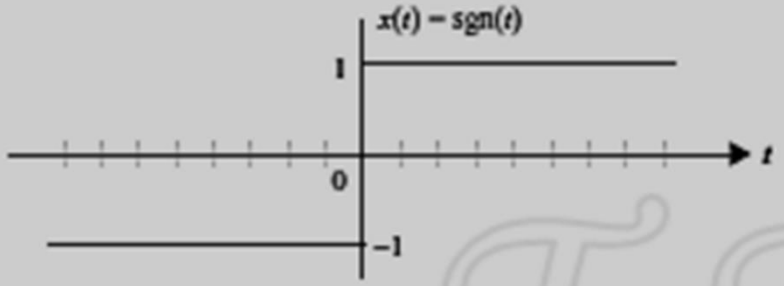
(b)

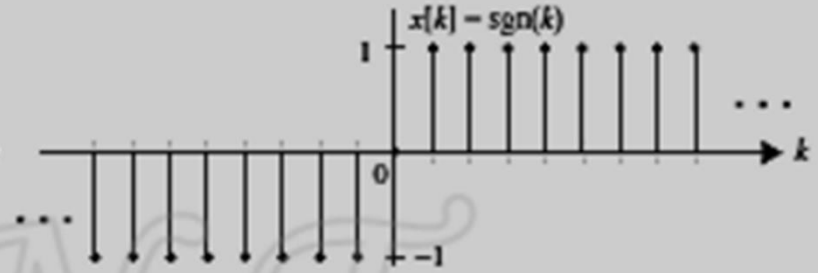




signum functions.

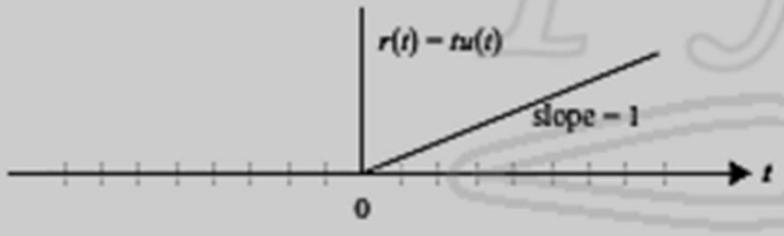
(d)

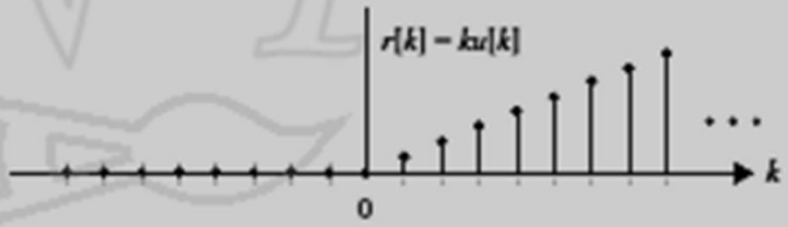


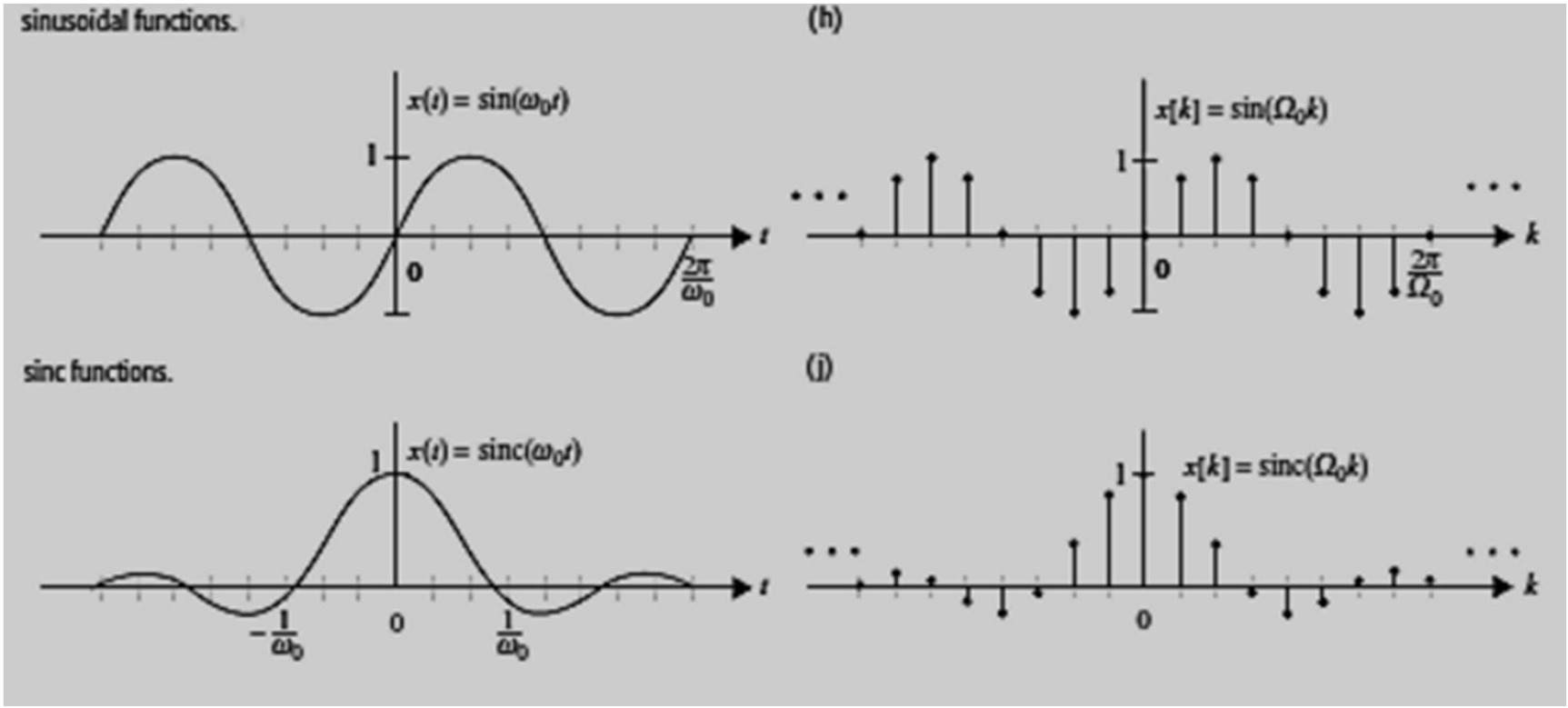


ramp functions.

(f)






sinusoidal functions.
(h)
$x(t) = \sin(\omega_0 t)$
$x[k] = \sin(\Omega_0 k)$
1
0
$\frac{2\pi}{\omega_0}$
$\frac{2\pi}{\Omega_0}$
t
k
sinc functions.
(j)
$x(t) = \mathrm{sinc}(\omega_0 t)$
$x[k] = \mathrm{sinc}(\Omega_0 k)$
$-\frac{1}{\omega_0}$
$\frac{1}{\omega_0}$

Algumas

Tabelas úteis

Definições

$$\mathbf{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$

$$\omega_0 = \frac{2\pi}{T_0} = 2\pi f_0$$

$$X_k = \frac{1}{T_0} \int_{T_0} x(t) e^{-jk\omega_0 t} dt$$

$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{j2\pi k f_0 t} = \sum_{k=-\infty}^{\infty} X_k e^{jk\omega_0 t}$$

Sinal Periódico	Coeficientes da série de Fourier
$x(t)=\sum_{k=-\infty}^{\infty} X_k e^{jk\omega_0 t}$	$X_k=\frac{1}{T_0}\int_{T_0} x(t)e^{-jk\omega_0 t}dt$
$x(t), y(t)$ periódicos período T_0	X_k, Y_k
$ax(t)+by(t)$	aX_k+bY_k
$x(t-t_0)$	$X_k e^{-j\omega_0 k t_0}$
$e^{j\omega_0 k_0 t}x(t)$	X_{k-k_0}
$x(-t)$	X_{-k}
$x(t)*y(t)$	$X_k Y_k$
$x(t)y(t)$	$X_k * Y_k$
$\frac{dx}{dt}$	$jk\omega_0 X_k$

$x(t)$ **sinal real**	$\begin{cases} X_k = X_{-k}^* \\ \mathrm{Re}\{X_k\} = \mathrm{Re}\{X_{-k}\} \\ \mathrm{Im}\{X_k\} = -\mathrm{Im}\{X_{-k}\} \\ \lvert X_k \rvert = \lvert X_{-k} \rvert \\ \arg\{X_k\} = -\arg\{X_{-k}\} \end{cases}$
Potência de um sinal periódico	$P_x = \frac{1}{T_0}\int_0^{T_0} \lvert x(t) \rvert^2 \, dt$
Teorema de Parseval	$\frac{1}{T_0}\int_0^{T_0} \lvert x(t) \rvert^2 \, dt = \sum_{k=-\infty}^{\infty} \lvert X_k \rvert^2$

Sinal	Coeficientes da série de Fourier
$x(t) = \sum_{k=-\infty}^{+\infty} X_k e^{jk\omega_0 t}$	$X_k = \frac{1}{T_0} \int_{T_0} x(t) e^{-jk\omega_0 t} dt$
$x(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$	$X_k = \frac{1}{T}$
$x(t) = 1$	$\begin{cases} X_0 = 1 \\ X_k = 0, \ k \neq 0 \end{cases}, \quad \forall T_0 > 0$
$x(t) = e^{j\omega_0 t}$	$\begin{cases} X_1 = 1 \\ X_k = 0, \ k \neq 1 \end{cases}$

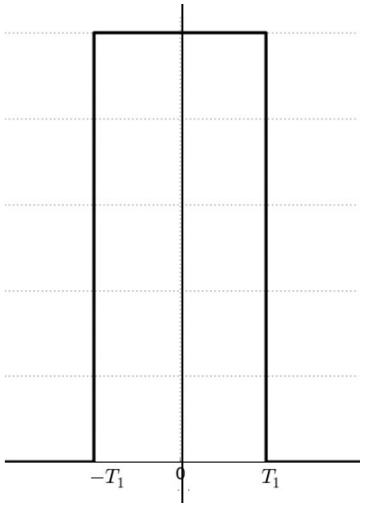
$$x(t)=\cos(\omega_0 t)$$

$$\begin{cases} X_1 = X_{-1} = 0.5 \\ X_k = 0, \quad k \neq 1,-1 \end{cases}$$

$$x(t)=\sin(\omega_0 t)$$

$$\begin{cases} X_1 = -X_{-1} = \dfrac{1}{2j} \\ X_k = 0, \quad k \neq 1,-1 \end{cases}$$

$$x(t)=\begin{cases} 1 \quad , \quad |t| < T_1 \\ 0 \quad , \quad T_1 < |t| < \dfrac{T_0}{2} \end{cases}$$



$$X_k = \frac{2T_1}{T_0}\,\mathrm{sinc}\left(k\,\frac{2T_1}{T_0}\right)$$

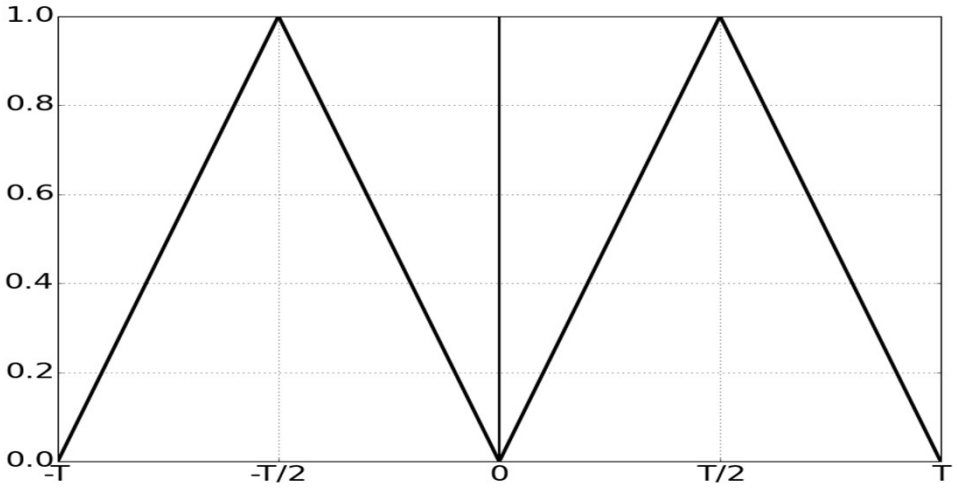
$$x(t) = u\left(t + \frac{T}{2}\right) - u\left(t - \frac{T}{2}\right)$$

$$X_k = T\,\mathrm{sinc}\left(f_0 T\right)$$

$$x(t) = \sin\left(\omega_0 t\right)$$

$$\begin{cases} X_1 = -X_{-1} = \dfrac{1}{2j} \\ X_k = 0, \quad k \neq 1, -1 \end{cases}$$

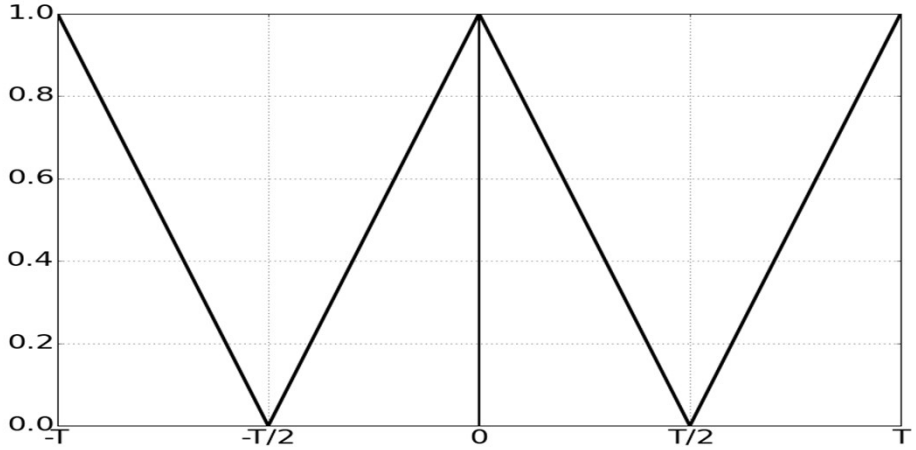
Triangular 1



$$X_0 = 0.5$$

$$X_k = -\frac{1}{k^2\pi^2}\left(1 - (-1)^k\right)$$

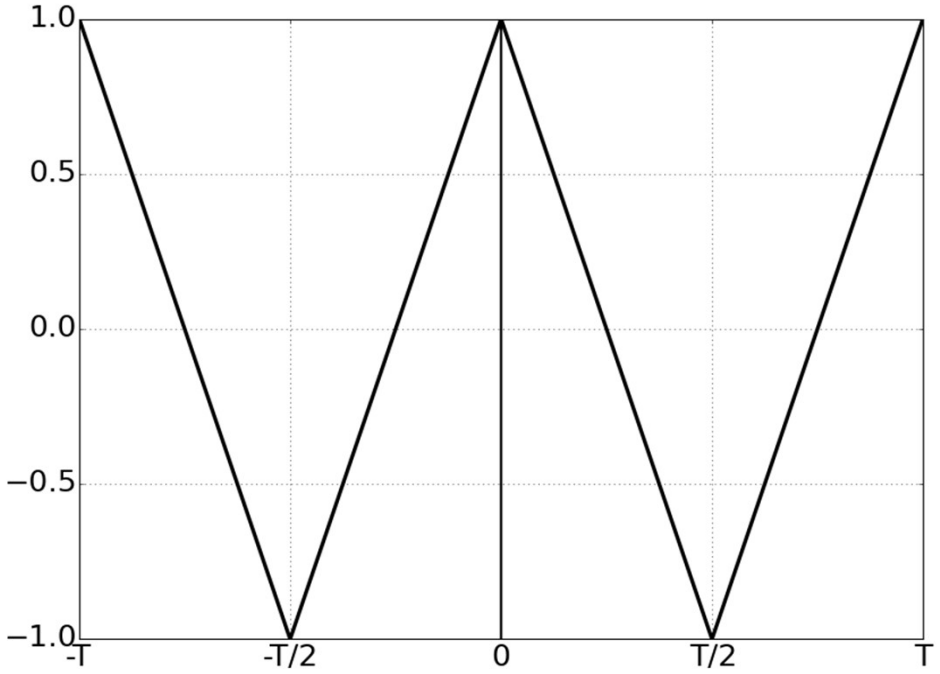
Triangular 2



$$X_0 = 0.5$$

$$X_k = \frac{1}{k^2\pi^2}\left(1-(-1)^k\right)$$

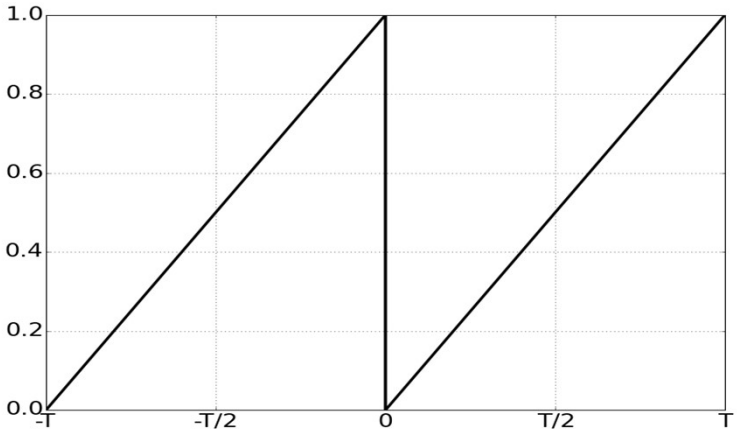
Triangular 3



$$X_0 = 0$$

$$X_k = \frac{2}{k^2\pi^2}\left(1-(-1)^k\right)$$

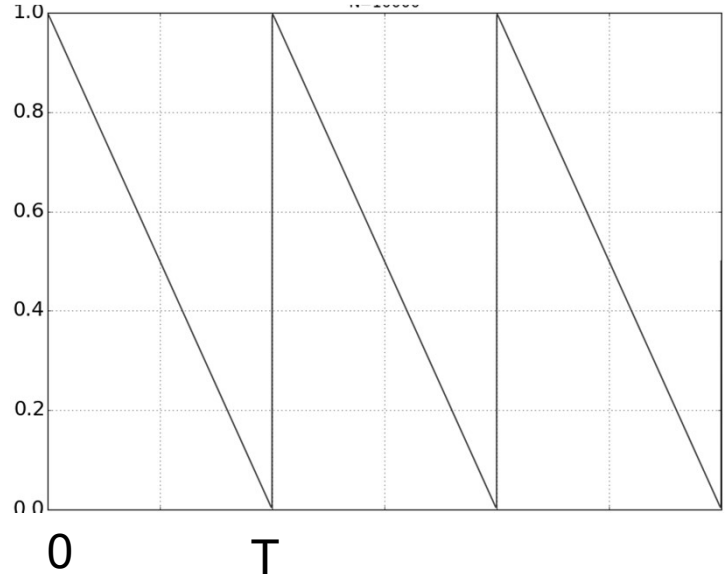
Triangular 4



$$X_0 = 0.5$$

$$X_k = -\frac{1}{jk\pi}$$

Triangular 5



$$X_0 = 0.5$$

$$X_k = \frac{1}{jk\pi}$$