Introduccion EDO con Python

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Abstract En este NoteBook encontraras codigo para manejar EDO con el lenguaje de programacion Python.

```python
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import solve_ivp

# ==========================================
# 1. User Definitions & Configuration
# ==========================================

# Configuration Parameters
CONFIG = {
    'x_range': (-3, 3),       # Domain for x
    'y_range': (-3, 3),       # Domain for y
    'grid_density': 25,       # Density of arrows in vector field
    'num_integral_curves': 25, # Number of curves for Figure 2
    'specific_ics': [-3,-2,-1,-0.5,0,0.5,1,2,3], # Specific initial conditions
y(x_start) for Figure 3
    'resolution': 500         # Resolution for numerical integration steps
}

# ==========================================
# 2. Visualization Functions
# ==========================================

def plot_vector_field(f, x_lim, y_lim, density=20):
    """
    Figure 1: Generates the Vector Field (Slope Field).

    Mathematical Context:
    At every point (x, y), the ODE defines a slope m = f(x,y).
    The vector at that point is <1, f(x,y)>.
    We normalize these vectors to show direction without magnitude distortion.
    """
```

```
    fig = plt.figure(figsize=(10, 8))

    # Create a grid of points
    x = np.linspace(x_lim[0], x_lim[1], density)
    y = np.linspace(y_lim[0], y_lim[1], density)
    X, Y = np.meshgrid(x, y)

    # Calculate vector components
    # dx is constant (1), dy is the function output
    U = np.ones_like(X)
    V = f(X, Y)

    # Normalize the arrows (make them all unit length for clarity)
    # Magnitude N = sqrt(U^2 + V^2)
    N = np.sqrt(U**2 + V**2)
    U = U / N
    V = V / N

    # Plot Quiver
    plt.quiver(X, Y, U, V, angles='xy', scale_units='xy', scale=3,
color='#555555', width=0.003)

    # Styling
    plt.title("Figure 1: Vector Field (Direction Field)", fontsize=16)
    plt.xlabel("x", fontsize=14)
    plt.ylabel("y", fontsize=14)
    plt.xlim(x_lim)
    plt.ylim(y_lim)
    plt.grid(True, linestyle='--', alpha=0.6)
    plt.axhline(0, color='black', linewidth=1)
    plt.axvline(0, color='black', linewidth=1)

    # Add equation text
    plt.text(x_lim[0] + 0.5, y_lim[1] - 0.5, r"$\frac{dy}{dx} = f(x,y)$",
             fontsize=14, bbox=dict(facecolor='white', alpha=0.8))

    plt.tight_layout()
    plt.show()

def plot_integral_curves(f, x_lim, y_lim, num_curves=20):
    """
    Figure 2: Plots a dense set of integral curves to show flow.

    Method:
    We define a range of initial conditions along the left boundary (x_min)
    and integrate forward to x_max using Runge-Kutta 4(5).
    """
    fig = plt.figure(figsize=(10, 8))
```

```
    # Generate initial conditions along the left edge
    y0_values = np.linspace(y_lim[0], y_lim[1], num_curves)
    x_span = x_lim

    # Evaluation points for smooth curves
    t_eval = np.linspace(x_lim[0], x_lim[1], CONFIG['resolution'])

    # Plot background vector field faintly for context
    x_grid = np.linspace(x_lim[0], x_lim[1], 20)
    y_grid = np.linspace(y_lim[0], y_lim[1], 20)
    X, Y = np.meshgrid(x_grid, y_grid)
    U = np.ones_like(X)
    V = f(X, Y)
    N = np.sqrt(U**2 + V**2)
    plt.quiver(X, Y, U/N, V/N, alpha=0.2, color='gray')

    # Solve and plot each curve
    print(f"Generating {num_curves} integral curves...")
    for y0 in y0_values:
        # solve_ivp requires function signature fun(t, y)
        sol = solve_ivp(f, x_span, [y0], t_eval=t_eval, method='RK45')

        if sol.success:
            plt.plot(sol.t, sol.y[0], '-', color='teal', alpha=0.6,
linewidth=1.5)

    # Styling
    plt.title("Figure 2: Integral Curves (General Flow)", fontsize=16)
    plt.xlabel("x", fontsize=14)
    plt.ylabel("y", fontsize=14)
    plt.xlim(x_lim)
    plt.ylim(y_lim)
    plt.grid(True, linestyle='--', alpha=0.6)
    plt.axhline(0, color='black', linewidth=1)
    plt.axvline(0, color='black', linewidth=1)

    plt.tight_layout()
    plt.show()

def plot_specific_solutions(f, x_lim, y_lim, initial_conditions):
    """
    Figure 3: Plots specific, labeled numerical solutions.

    Use Case:
    Highlighting specific behaviors based on exact starting points.
    """
    fig = plt.figure(figsize=(10, 8))
```

```
    x_span = x_lim
    t_eval = np.linspace(x_lim[0], x_lim[1], CONFIG['resolution'])

    colors = plt.cm.viridis(np.linspace(0, 0.9, len(initial_conditions)))

    print("Generating specific solutions...")
    for i, y0 in enumerate(initial_conditions):
        sol = solve_ivp(f, x_span, [y0], t_eval=t_eval, method='RK45')

        if sol.success:
            label_text = f"$y({x_lim[0]}) = {y0}$"
            plt.plot(sol.t, sol.y[0], linewidth=2.5, color=colors[i],
label=label_text)

            # Mark the initial condition point
            plt.scatter([x_lim[0]], [y0], color=colors[i], s=50, zorder=5)

    # Styling
    plt.title("Figure 3: Specific Numerical Solutions", fontsize=16)
    plt.xlabel("x", fontsize=14)
    plt.ylabel("y", fontsize=14)
    plt.xlim(x_lim)
    plt.ylim(y_lim)
    plt.grid(True, linestyle='--', alpha=0.6)
    plt.legend(title="Initial Conditions", loc='best', frameon=True,
shadow=True)
    plt.axhline(0, color='black', linewidth=1)
    plt.axvline(0, color='black', linewidth=1)

    plt.tight_layout()
    plt.show()

# ==========================================
# 3. Main Execution Block
# ==========================================

def plots_ODE(ode_f):
    print("--- Differential Equation Visualizer ---")
    print(f"ODE: dy/dx = x - y")
    print(f"Domain: x in {CONFIG['x_range']}, y in {CONFIG['y_range']}")

    # 1. Plot Vector Field
    plot_vector_field(
        ode_f,
        CONFIG['x_range'],
        CONFIG['y_range'],
        density=CONFIG['grid_density']
```

```python
    )

    # 2. Plot Integral Curves (Flow)
    plot_integral_curves(
        ode_f,
        CONFIG['x_range'],
        CONFIG['y_range'],
        num_curves=CONFIG['num_integral_curves']
    )

    # 3. Plot Specific Solutions
    plot_specific_solutions(
        ode_f,
        CONFIG['x_range'],
        CONFIG['y_range'],
        initial_conditions=CONFIG['specific_ics']
    )
```

```python
def ode_f1(x, y):
    """
    Defines the First-Order ODE: dy/dx = f(x, y).

    Example: dy/dx = x - y

    Parameters:
        x (float): Independent variable (often time).
        y (float): Dependent variable.

    Returns:
        float: The derivative dy/dx at (x, y).
    """
    return -0.5*y
```

```python
def ode_f2(x, y):
    """
    Defines the First-Order ODE: dy/dx = f(x, y).

    Example: dy/dx = y^2

    Parameters:
        x (float): Independent variable (often time).
        y (float): Dependent variable.

    Returns:
        float: The derivative dy/dx at (x, y).
```

```python
    """
    return y**2
```

```python
def ode_f3(x, y):
    """
    Defines the First-Order ODE: dy/dx = f(x, y).

    Example: dy/dx =  y - sin(x)

    Parameters:
        x (float): Independent variable (often time).
        y (float): Dependent variable.

    Returns:
        float: The derivative dy/dx at (x, y).
    """
    return y-np.sin(x)
```

```python
def ode_f4(x, y):
    """
    Defines the First-Order ODE: dy/dx = f(x, y).

    Example: dy/dx =  -x**2+np.sin(y)

    Parameters:
        x (float): Independent variable (often time).
        y (float): Dependent variable.

    Returns:
        float: The derivative dy/dx at (x, y).
    """
    return -x**2+np.sin(y)
```

```python
def ode_f5(x, y):
    """
    Defines the First-Order ODE: dy/dx = f(x, y).

    Example: dy/dx = x**2-y

    Parameters:
        x (float): Independent variable (often time).
        y (float): Dependent variable.

    Returns:
        float: The derivative dy/dx at (x, y).
```

```
    """
    return x**2-y
```

```
def ode_f6(x, y):
    """
    Defines the First-Order ODE: dy/dx = f(x, y).

    Example: dy/dx =  np.sin(x-y)

    Parameters:
        x (float): Independent variable (often time).
        y (float): Dependent variable.

    Returns:
        float: The derivative dy/dx at (x, y).
    """
    return np.sin(x-y)
```

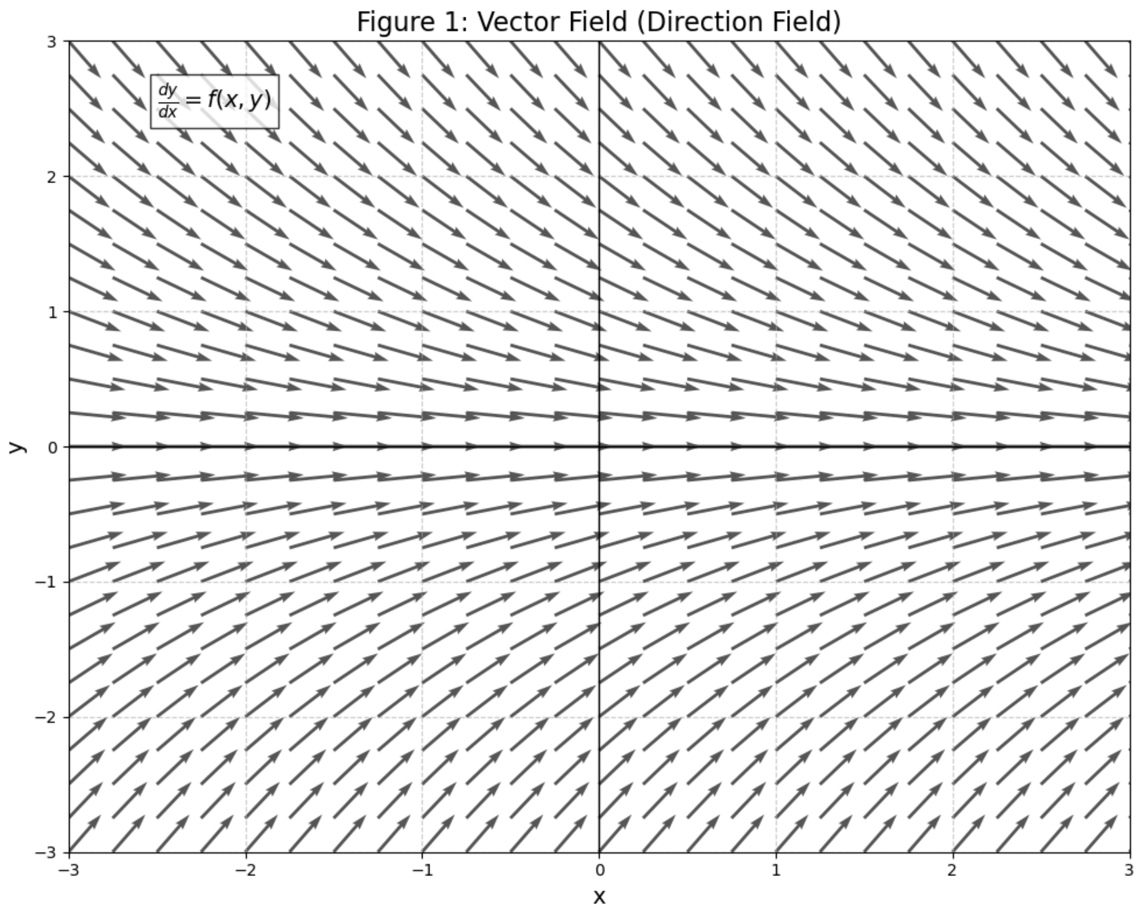
```
def ode_f7(x, y):
    """
    Defines the First-Order ODE: dy/dx = f(x, y).

    Example: dy/dx =  -6*x*y

    Parameters:
        x (float): Independent variable (often time).
        y (float): Dependent variable.

    Returns:
        float: The derivative dy/dx at (x, y).
    """
    return -6*x*y
```

```
def ode_f8(x, y):
    """
    Defines the First-Order ODE: dy/dx = f(x, y).

    Example: dy/dx = (4-2*x)/(3*y**2-5)

    Parameters:
        x (float): Independent variable (often time).
        y (float): Dependent variable.

    Returns:
        float: The derivative dy/dx at (x, y).
```

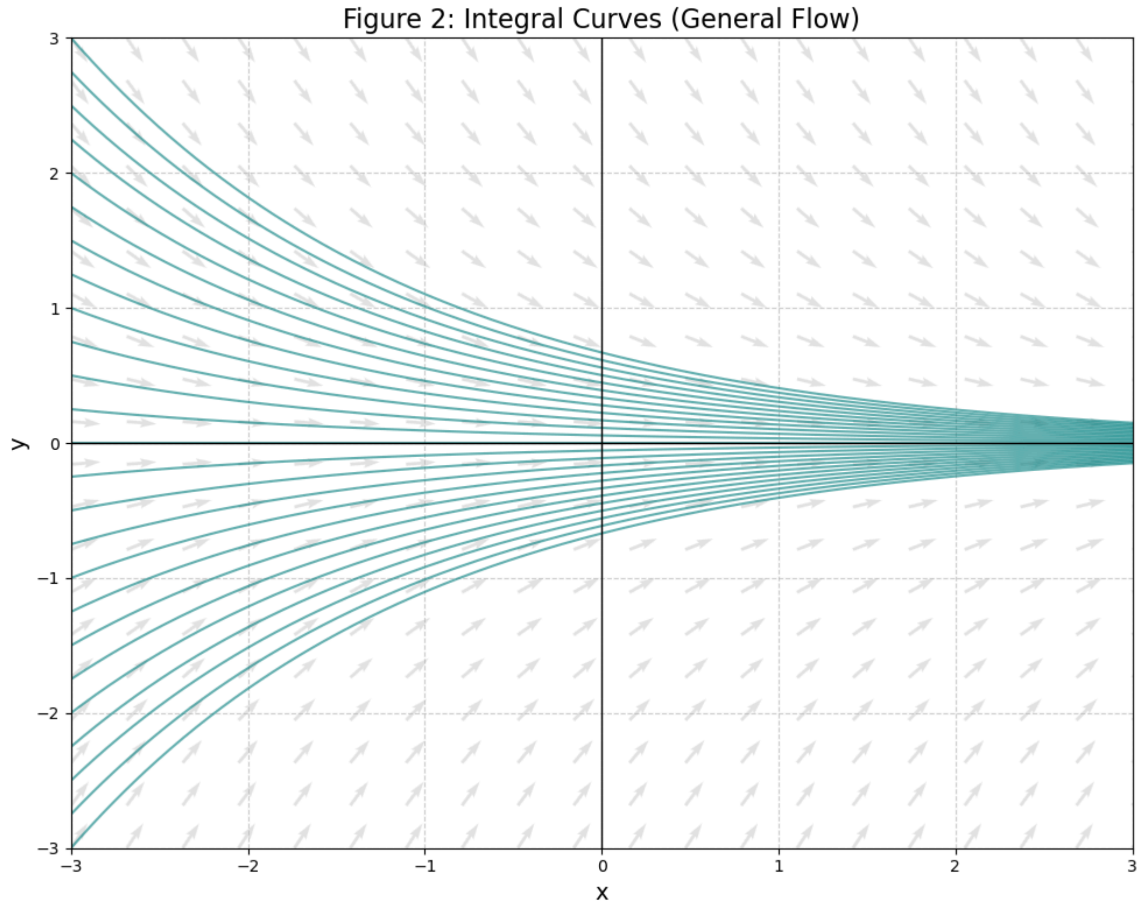
```
    """
    return (4-2*x)/(3*y**2-5)
```

```
plots_ODE(ode_f1)
```

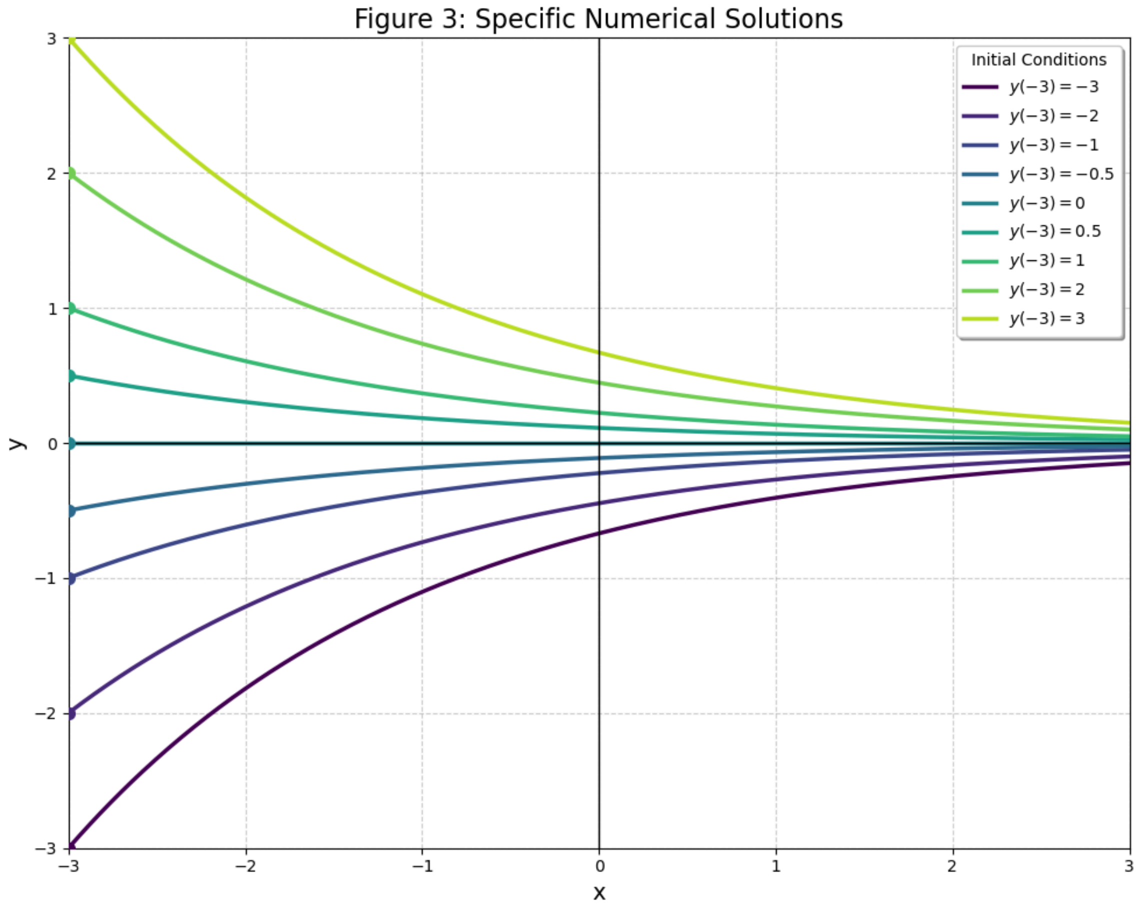
```
--- Differential Equation Visualizer ---
ODE: dy/dx = x - y
Domain: x in (-3, 3), y in (-3, 3)
```




```
Generating 25 integral curves...
```

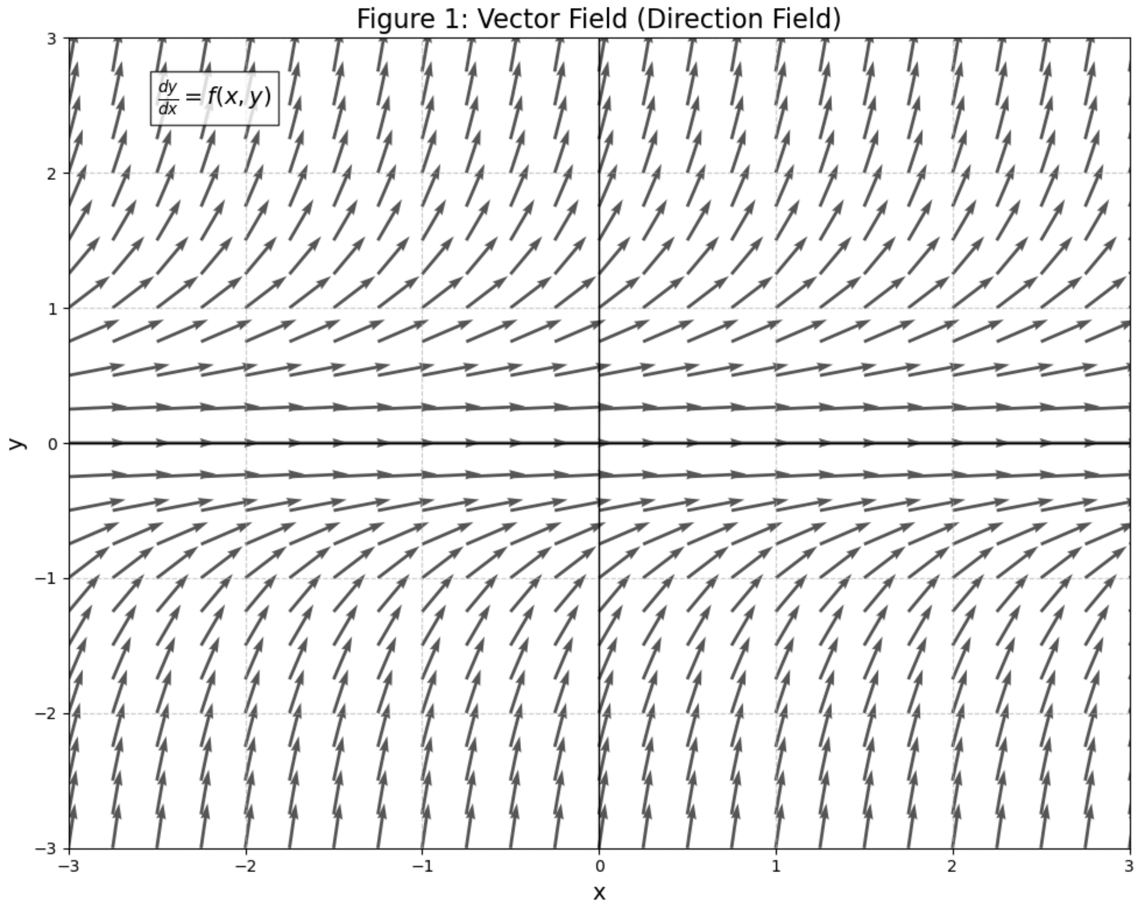



```
Generating specific solutions...
```

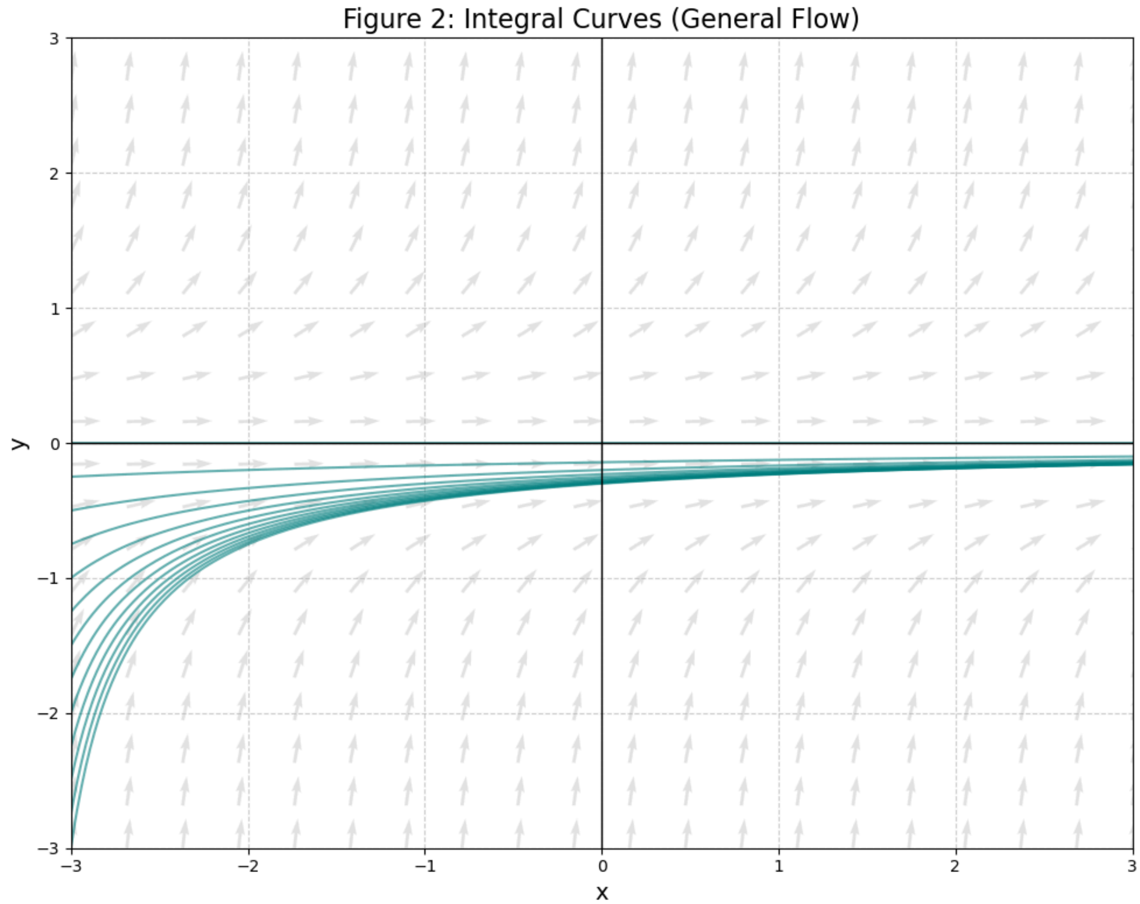



```
plots_ODE(ode_f2)
```

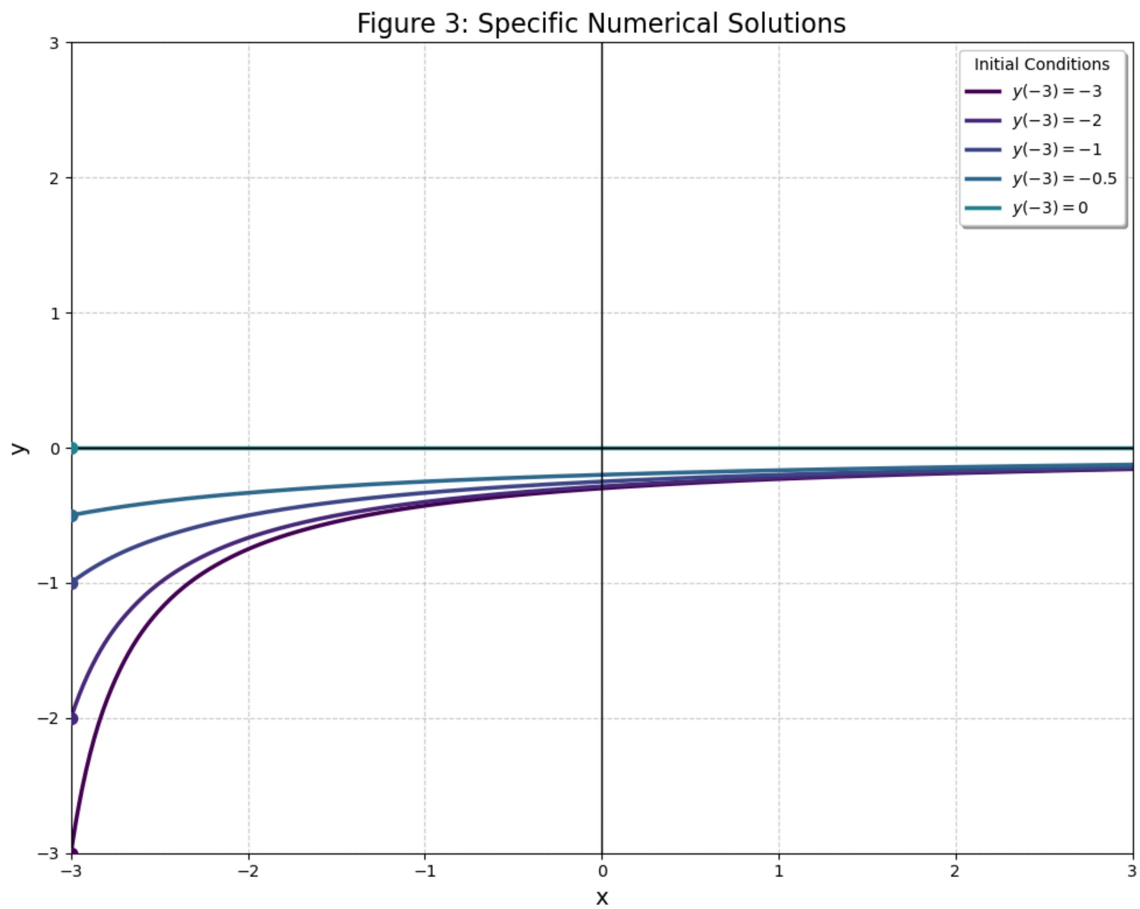
```
--- Differential Equation Visualizer ---
ODE: dy/dx = x - y
Domain: x in (-3, 3), y in (-3, 3)
```




```
Generating 25 integral curves...
```

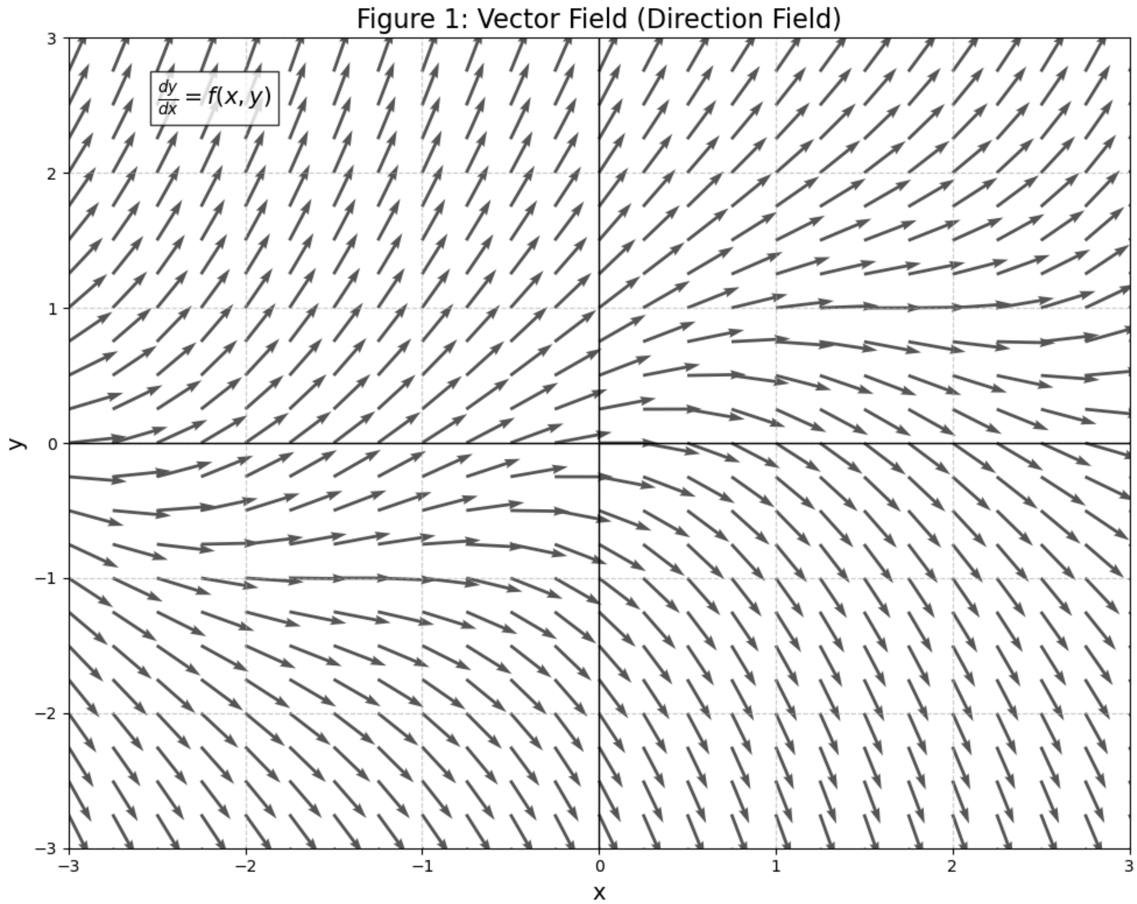



```
Generating specific solutions...
```

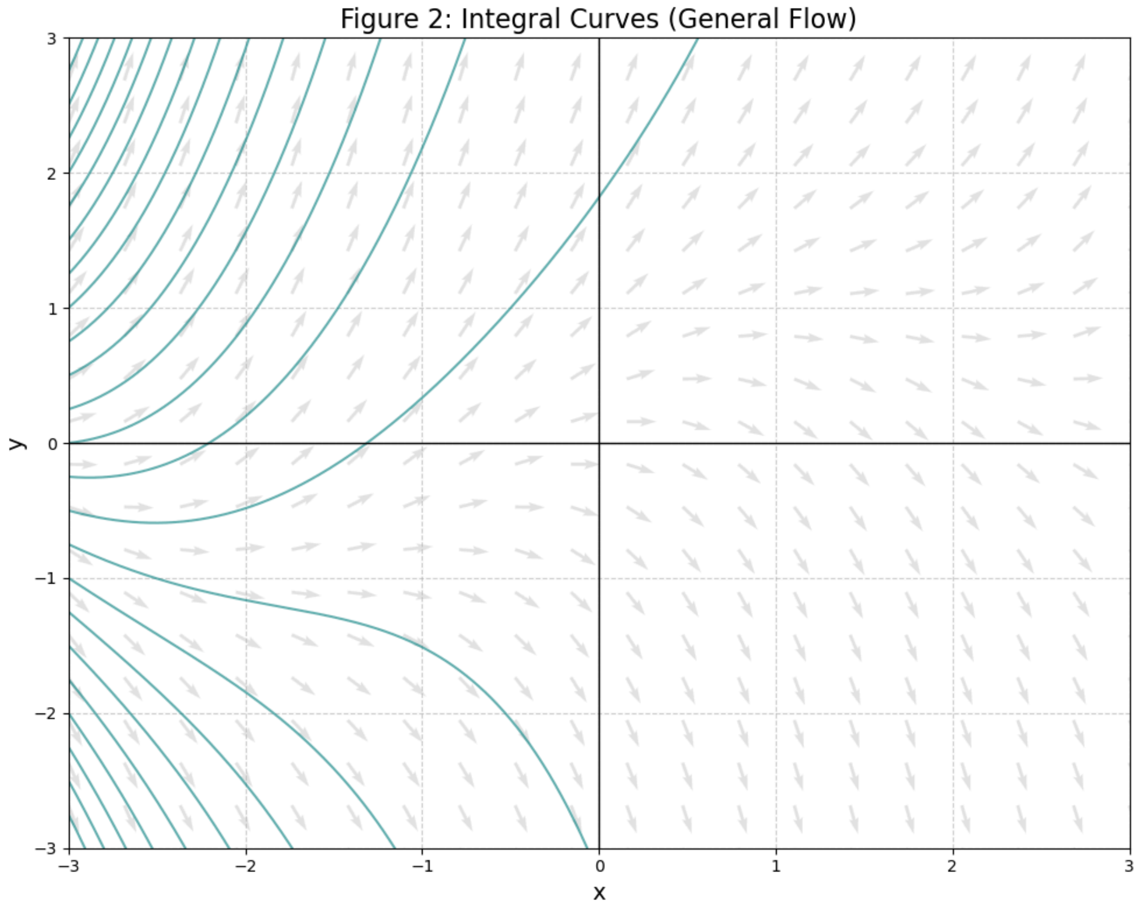



```
plots_ODE(ode_f3)
```

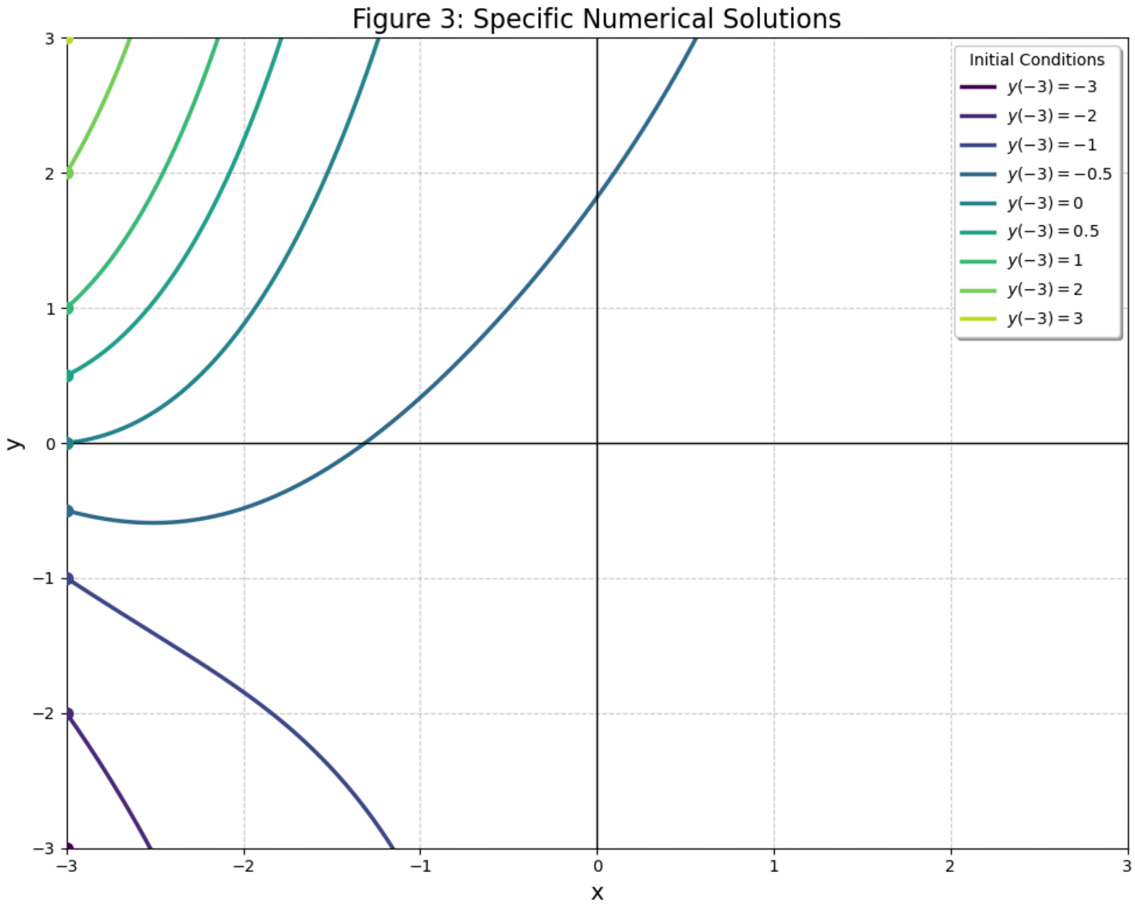
```
--- Differential Equation Visualizer ---
ODE: dy/dx = x - y
Domain: x in (-3, 3), y in (-3, 3)
```




```
Generating 25 integral curves...
```

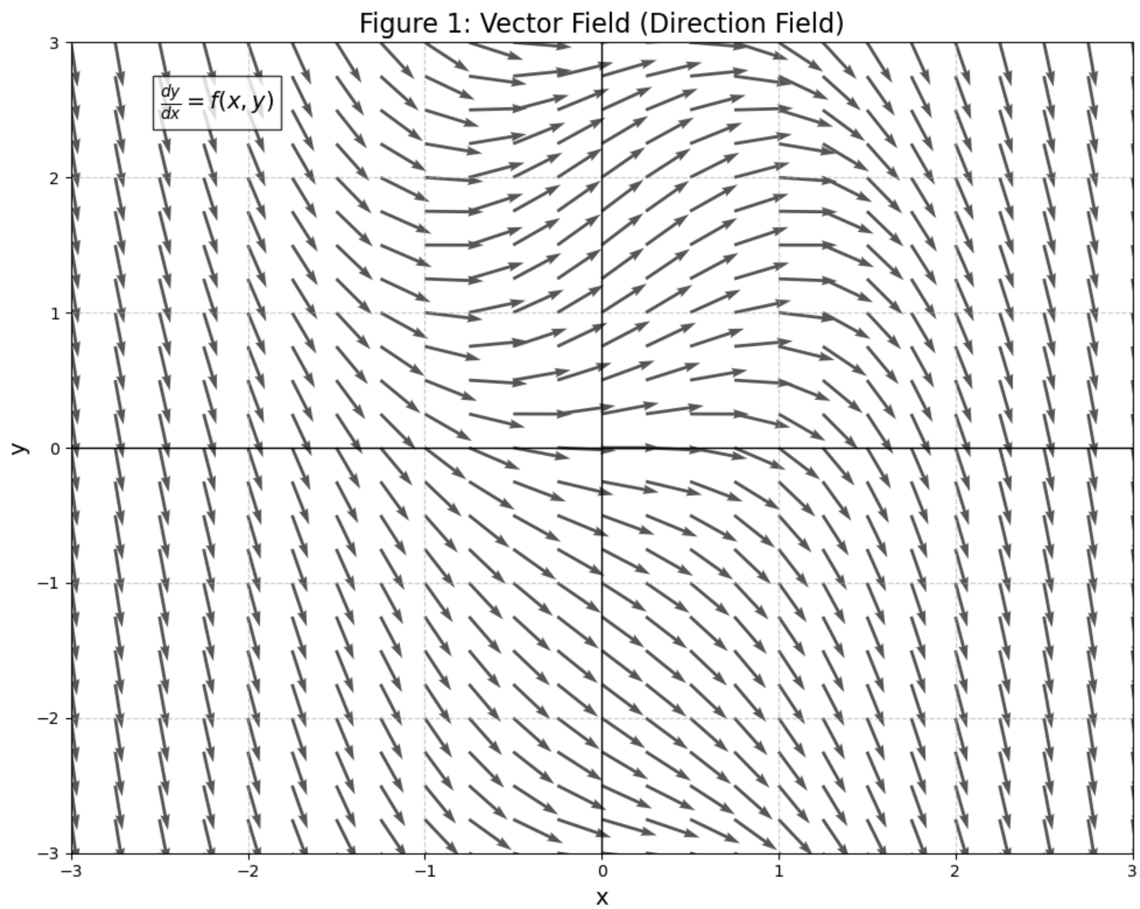



```
Generating specific solutions...
```

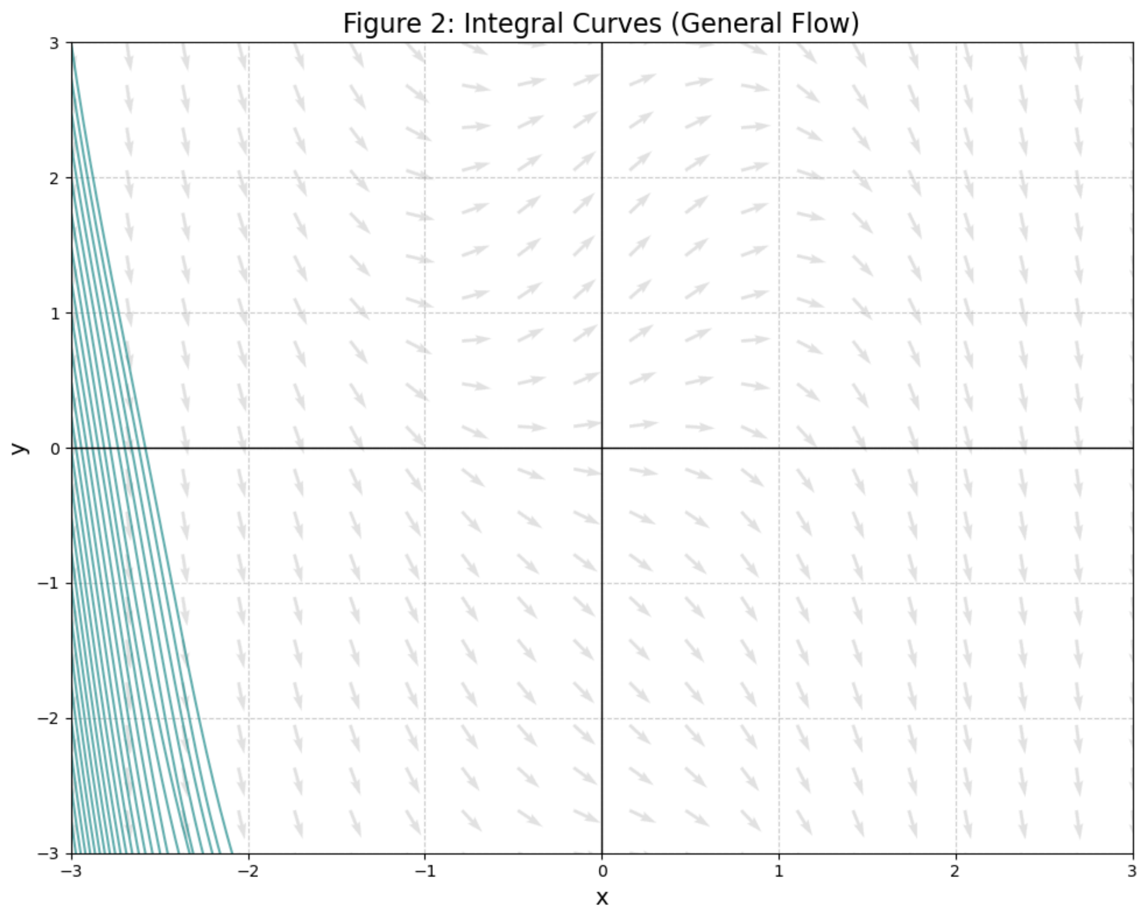



```
plots_ODE(ode_f4)
```

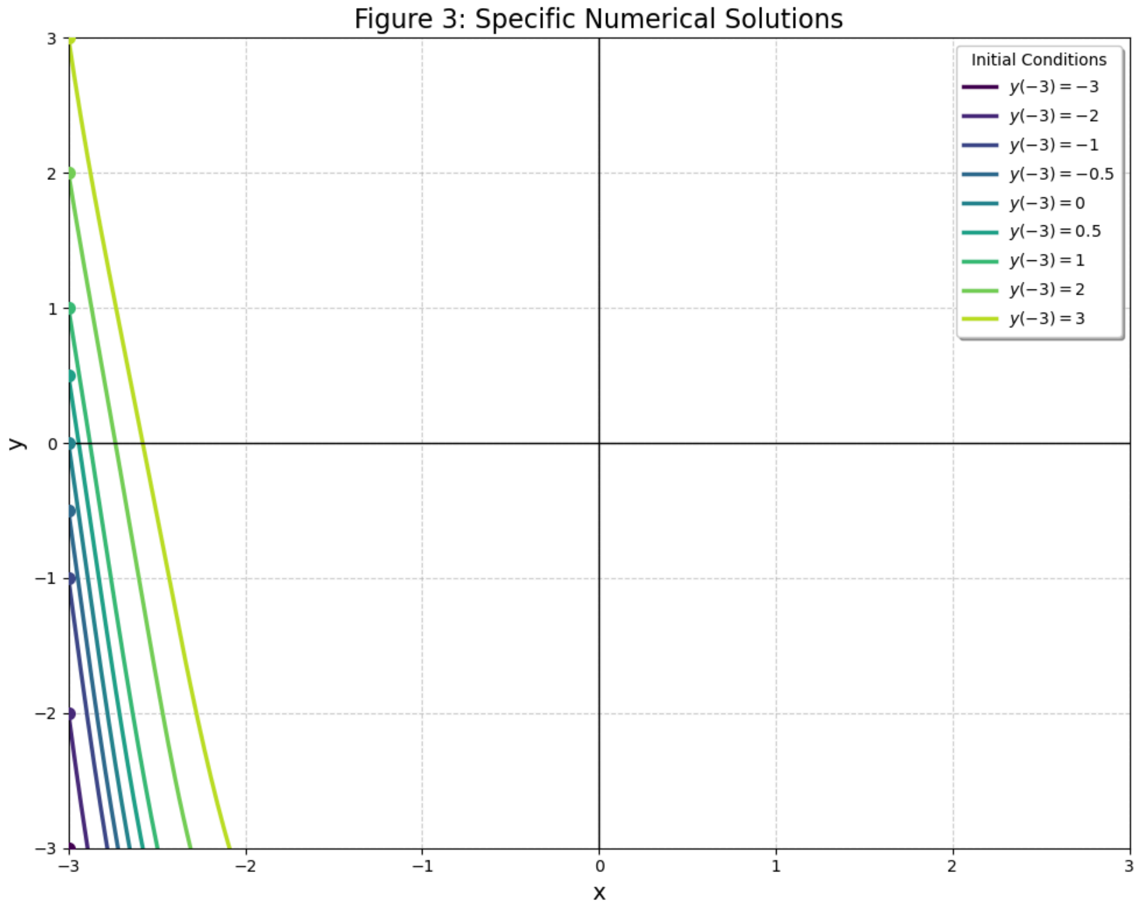
```
--- Differential Equation Visualizer ---
ODE: dy/dx = x - y
Domain: x in (-3, 3), y in (-3, 3)
```




```
Generating 25 integral curves...
```

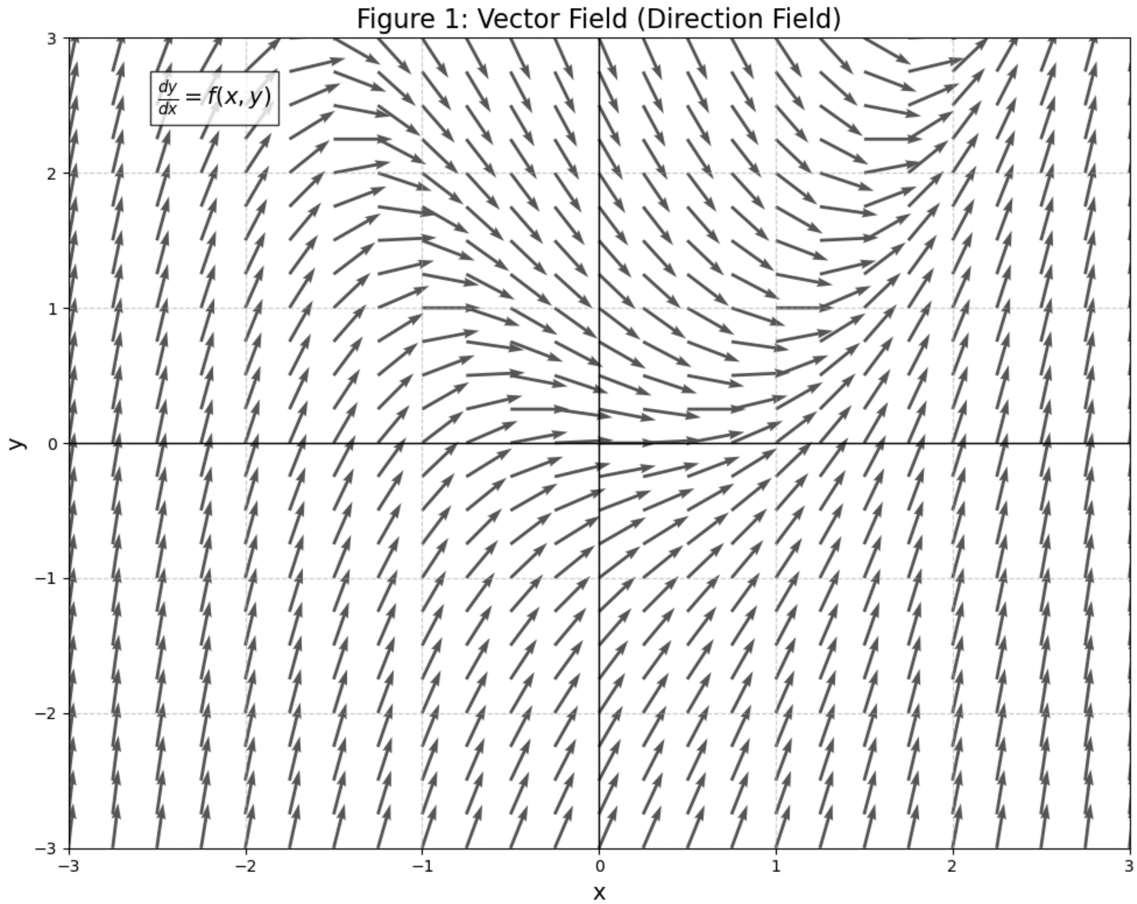



```
Generating specific solutions...
```

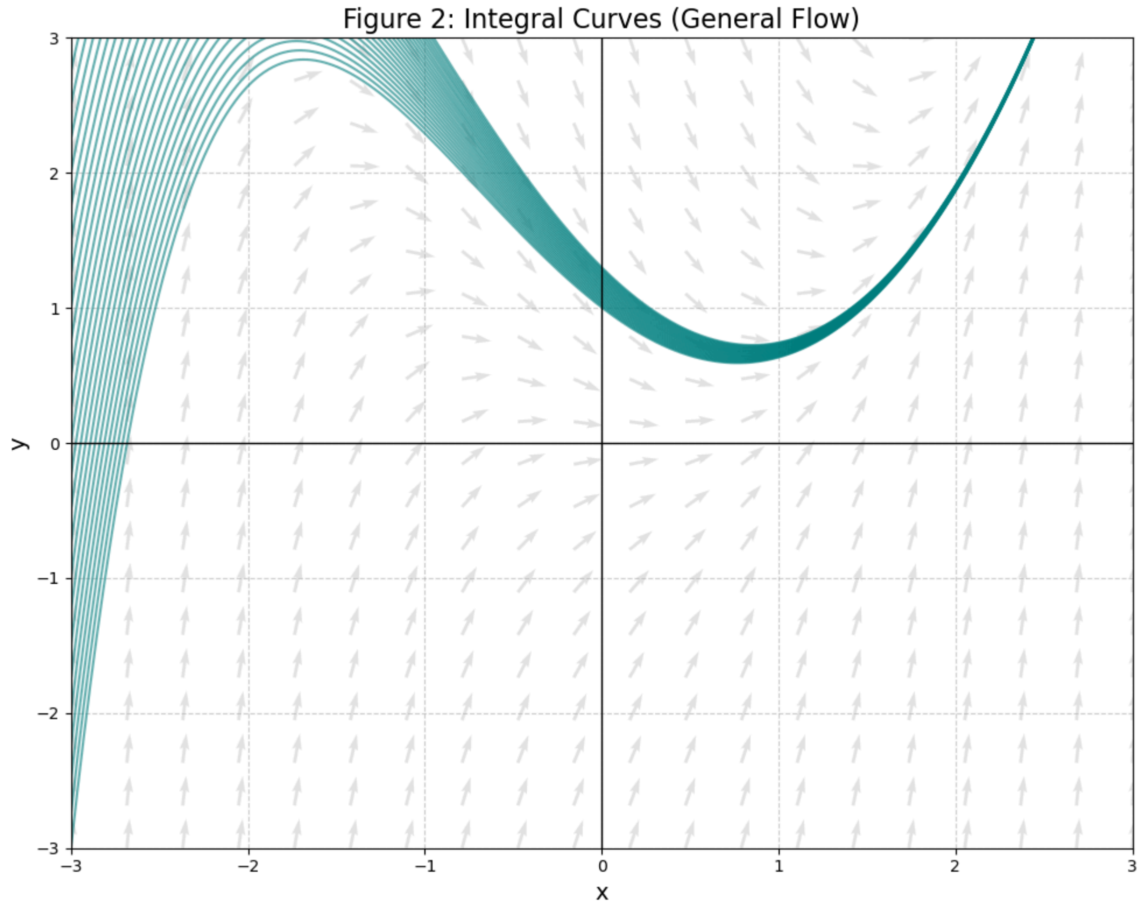



```
plots_ODE(ode_f5)
```

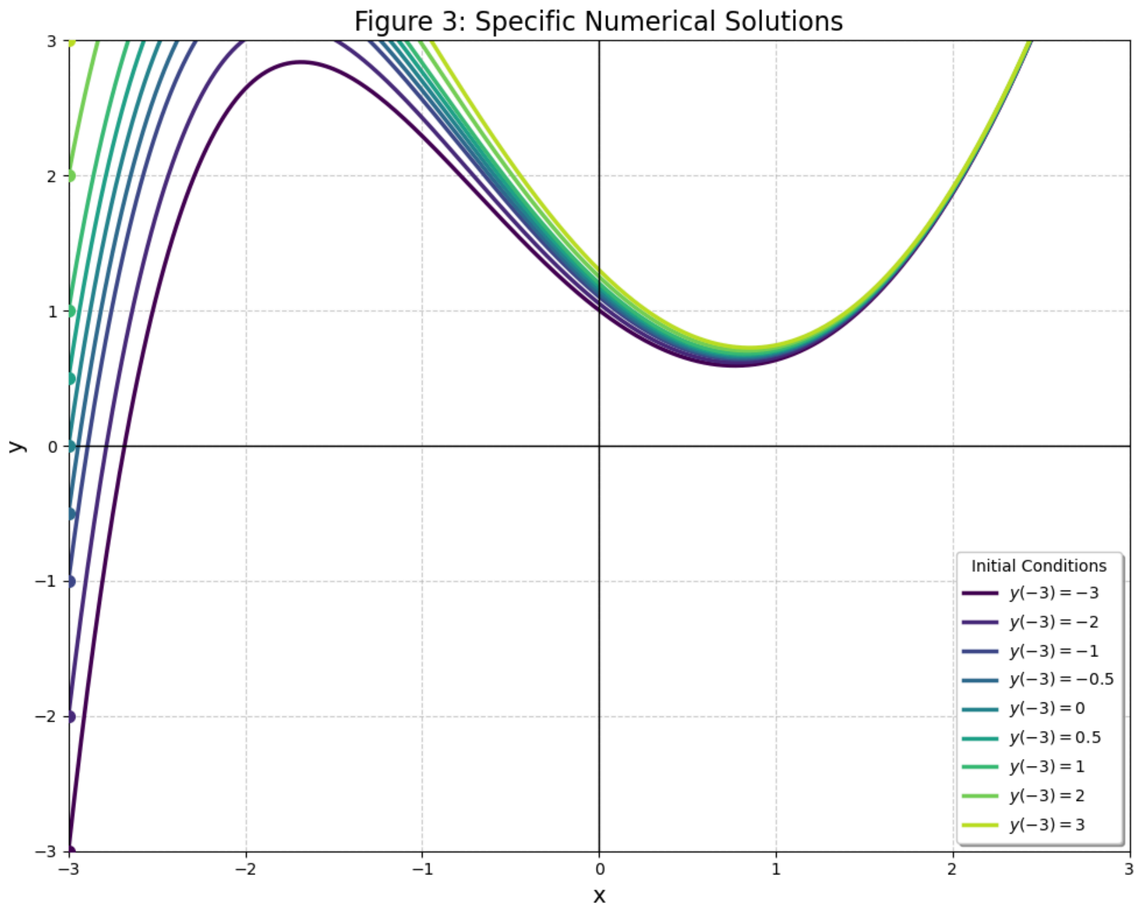
```
--- Differential Equation Visualizer ---
ODE: dy/dx = x - y
Domain: x in (-3, 3), y in (-3, 3)
```




```
Generating 25 integral curves...
```

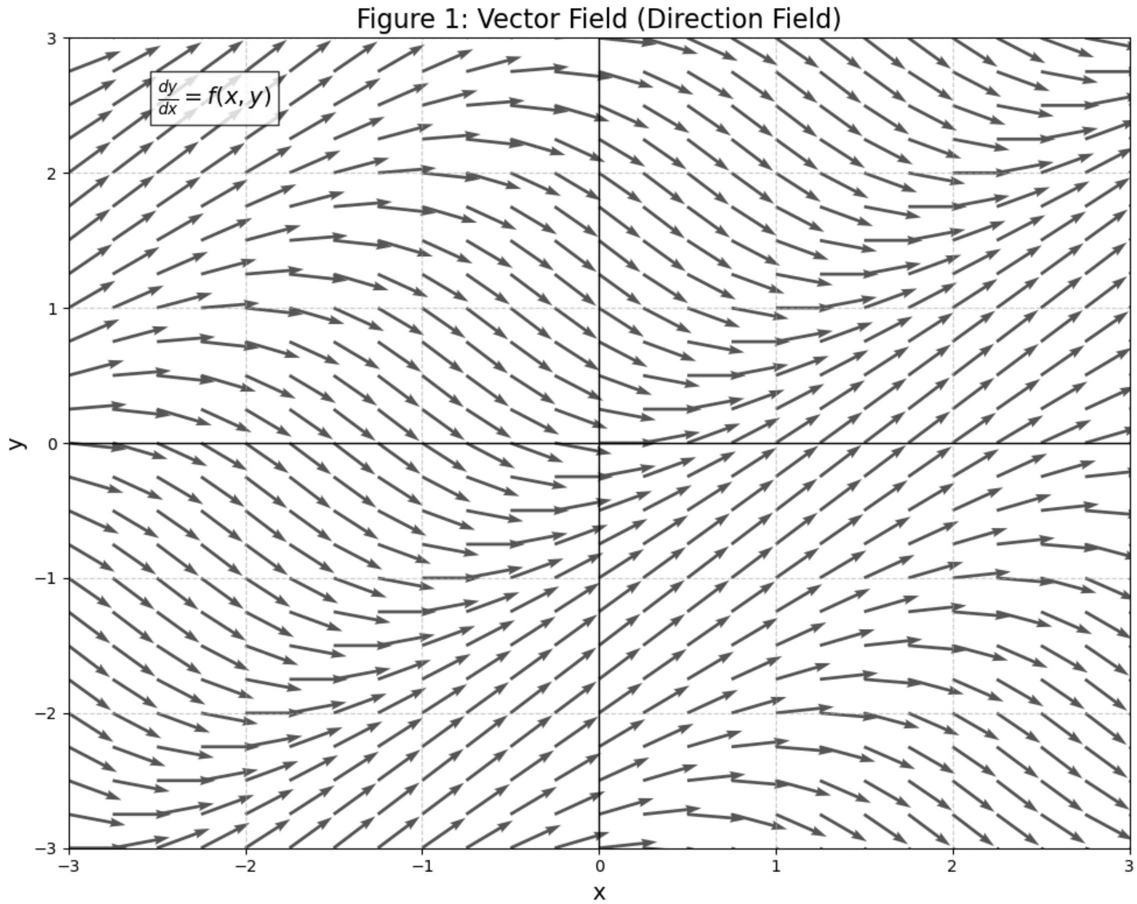



```
Generating specific solutions...
```



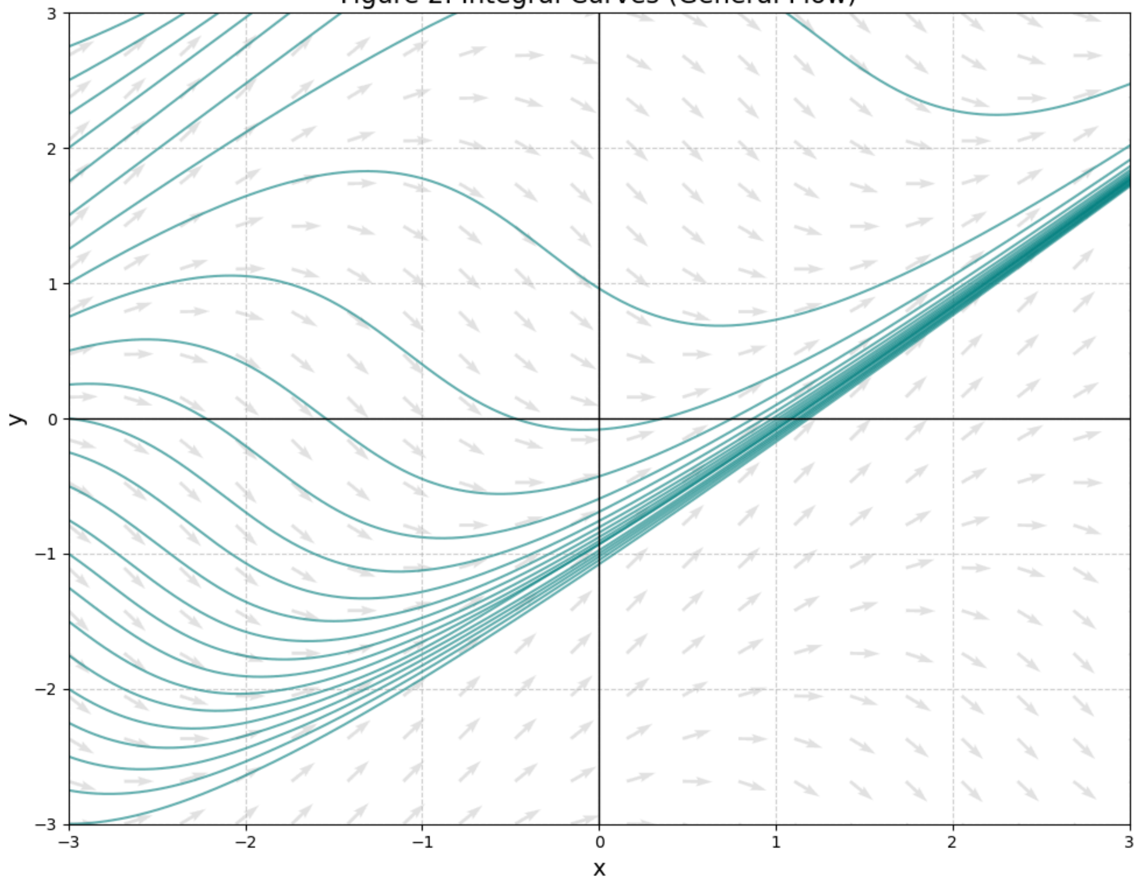

```
plots_ODE(ode_f6)
```

```
--- Differential Equation Visualizer ---
ODE: dy/dx = x - y
Domain: x in (-3, 3), y in (-3, 3)
```

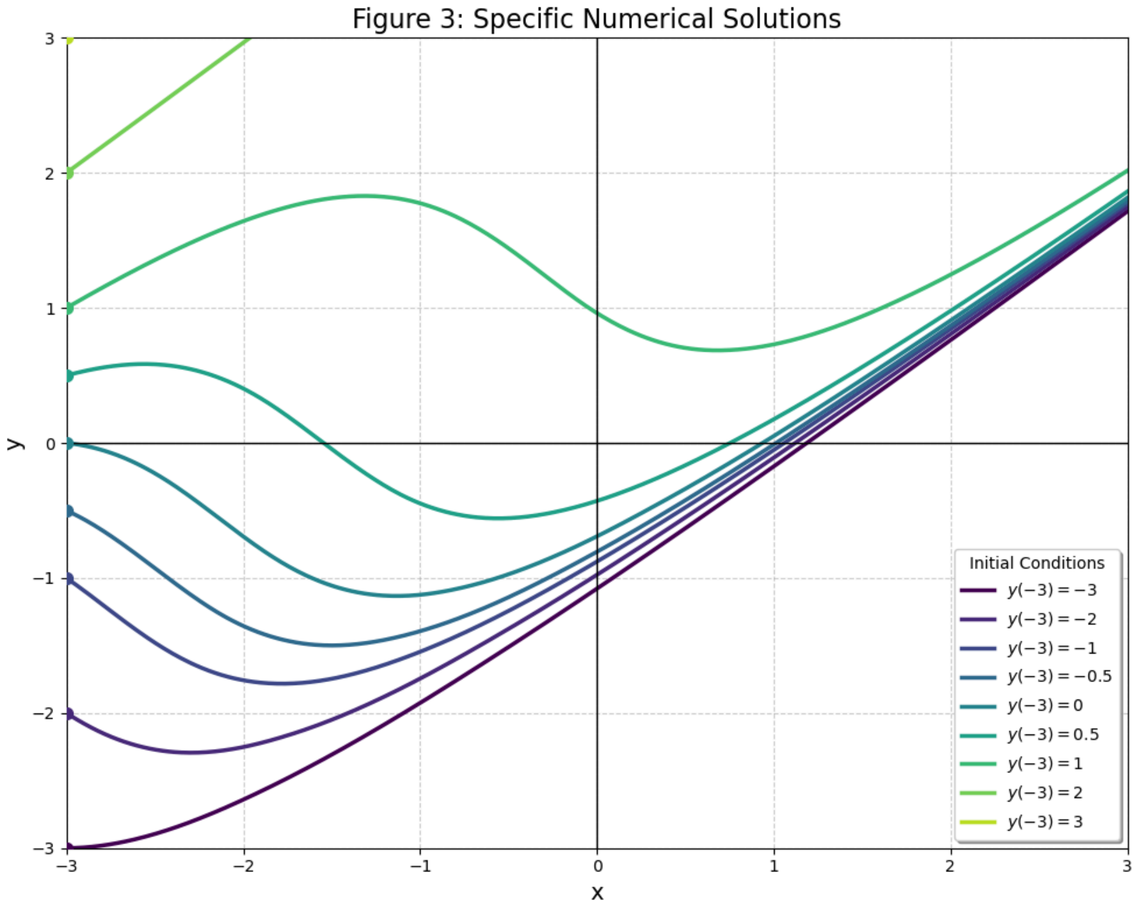



```
Generating 25 integral curves...
```

Figure 2: Integral Curves (General Flow)



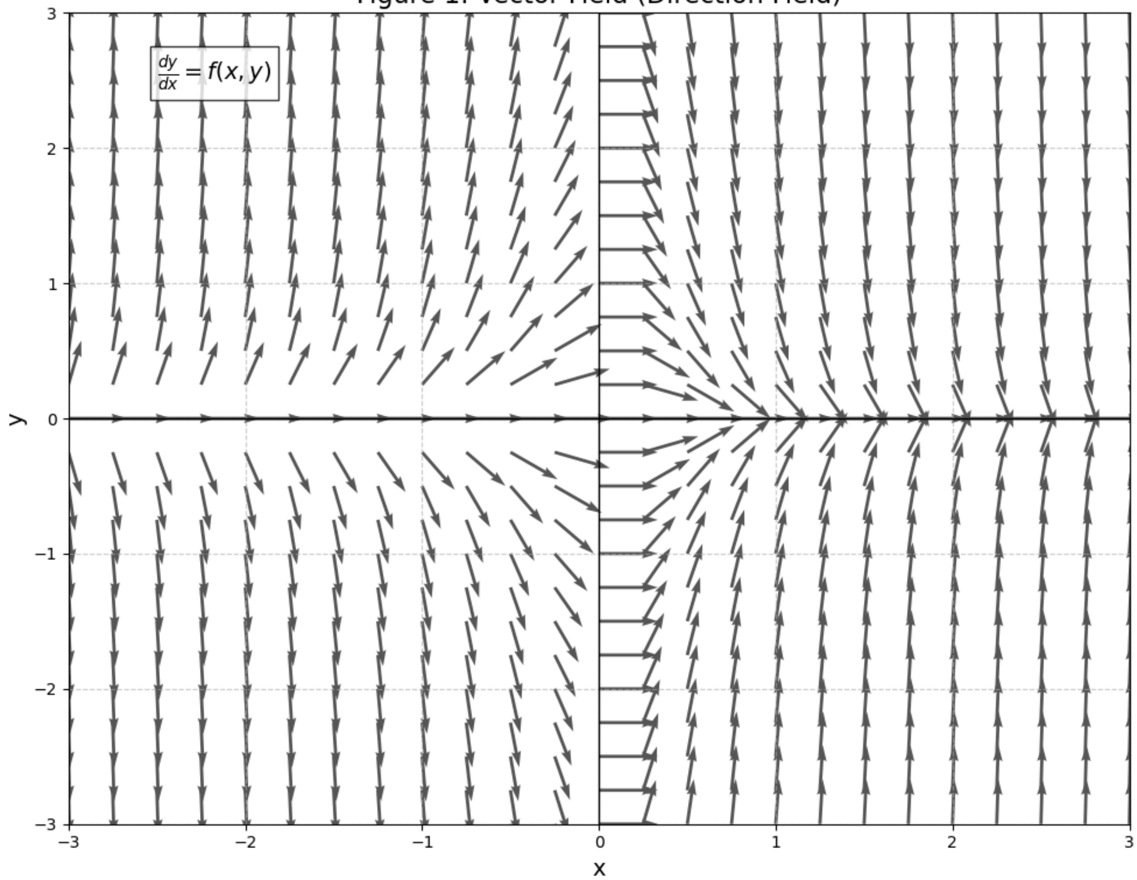

```
Generating specific solutions...
```



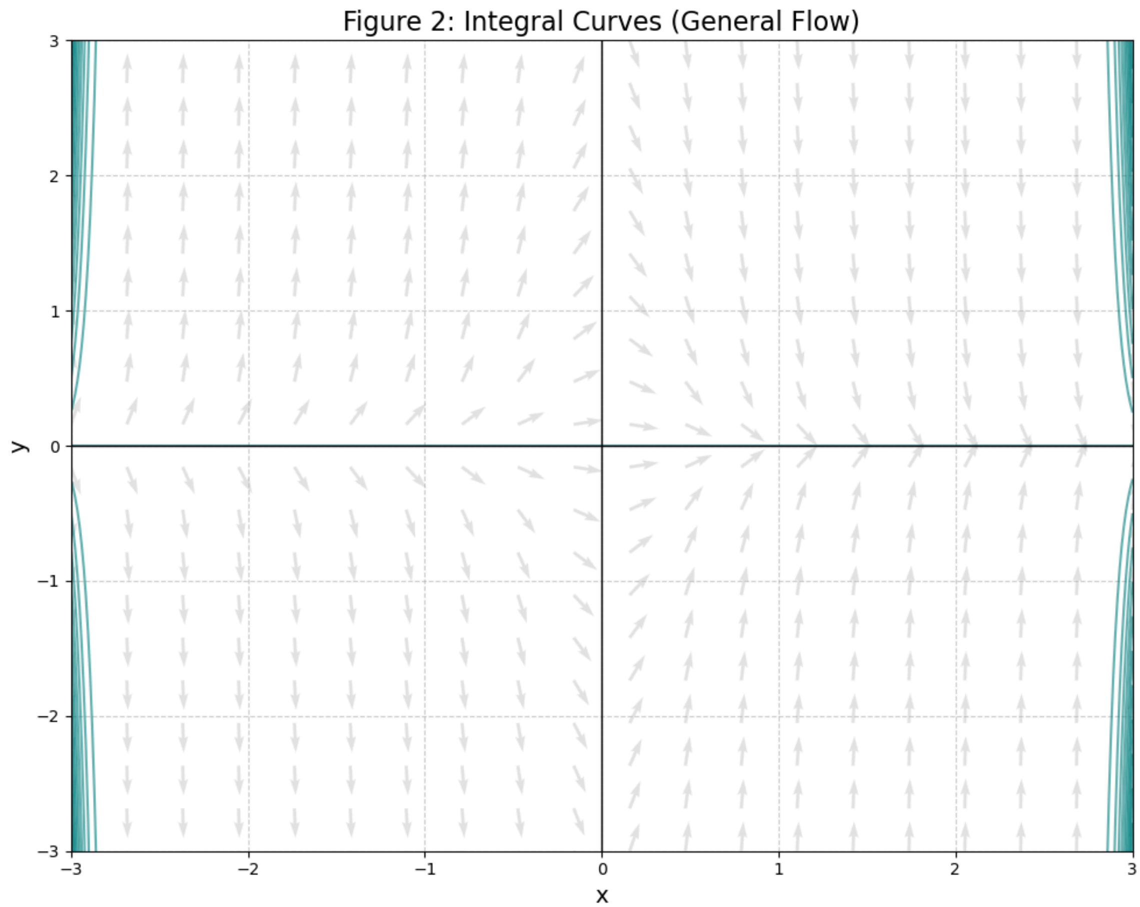

```
plots_ODE(ode_f7)
```

```
--- Differential Equation Visualizer ---
ODE: dy/dx = x - y
Domain: x in (-3, 3), y in (-3, 3)
```

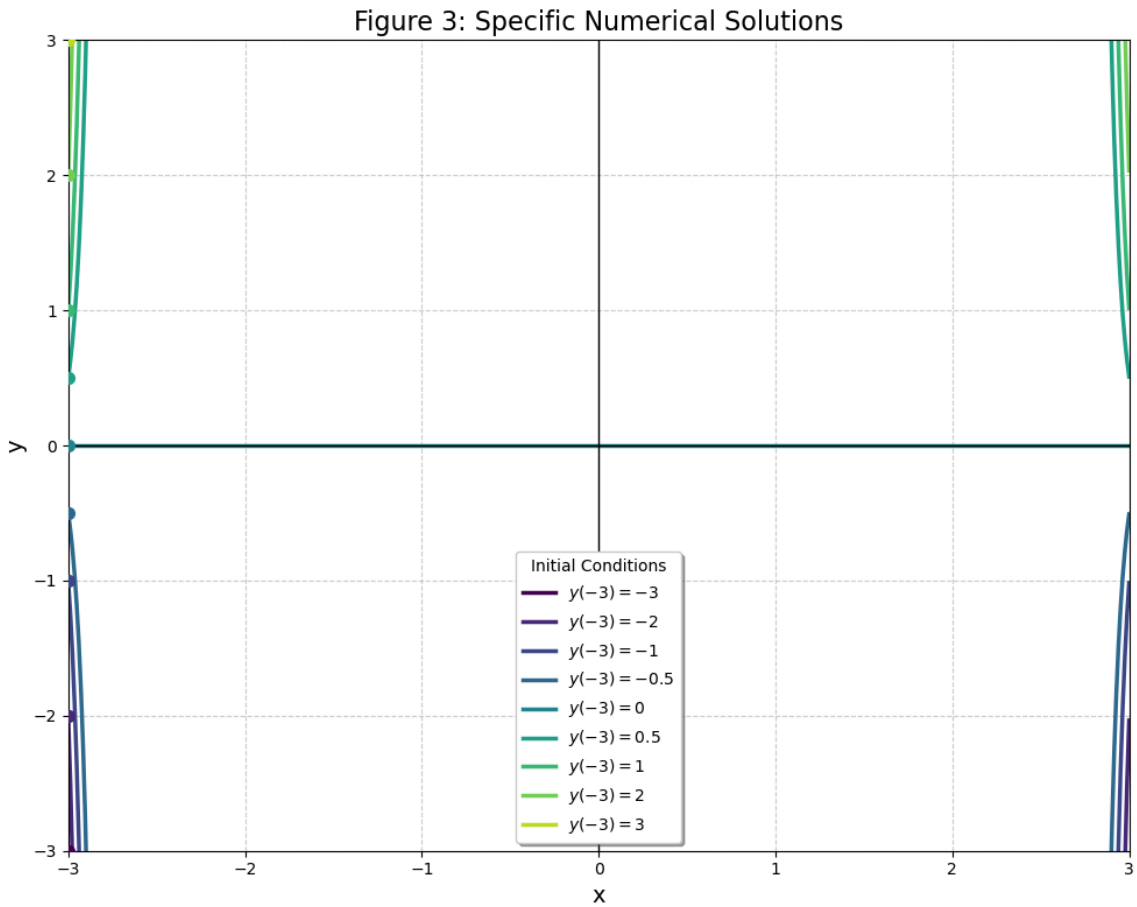
Figure 1: Vector Field (Direction Field)




```
Generating 25 integral curves...
```

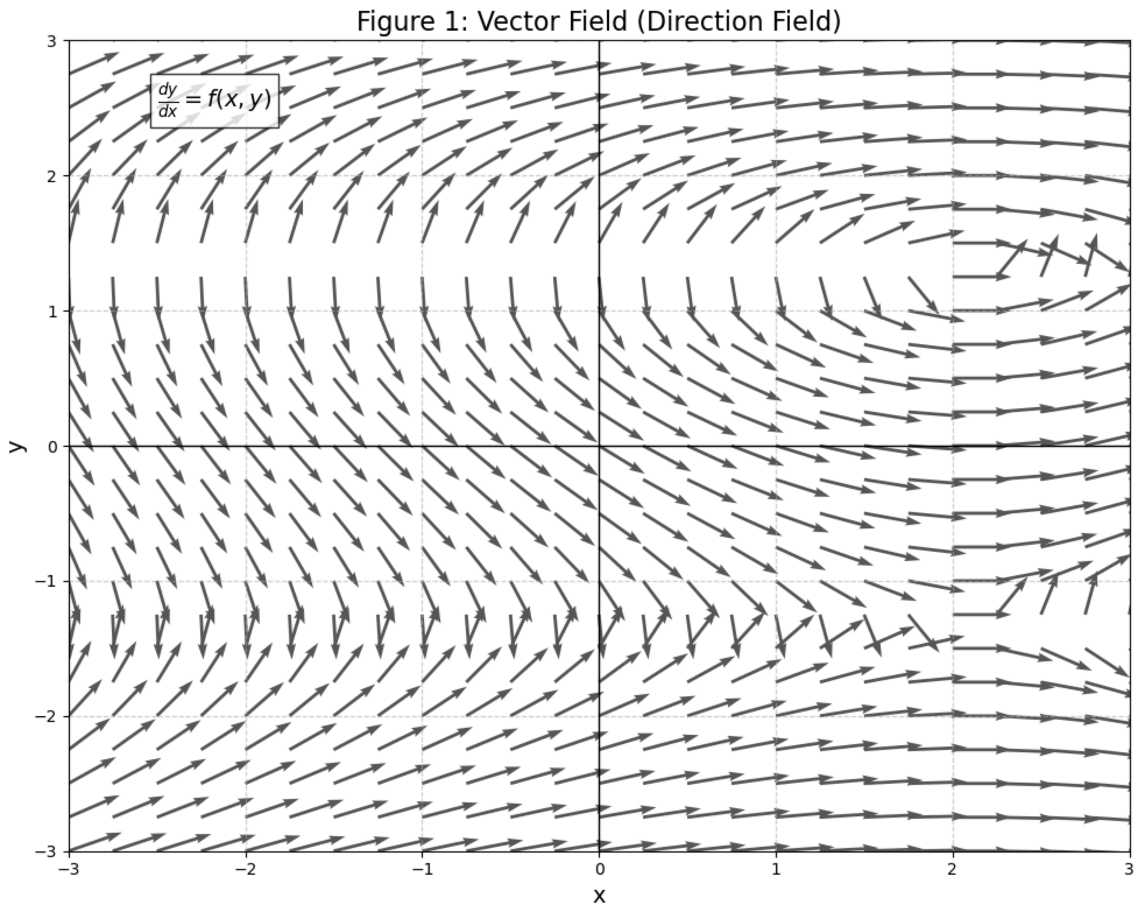



```
Generating specific solutions...
```

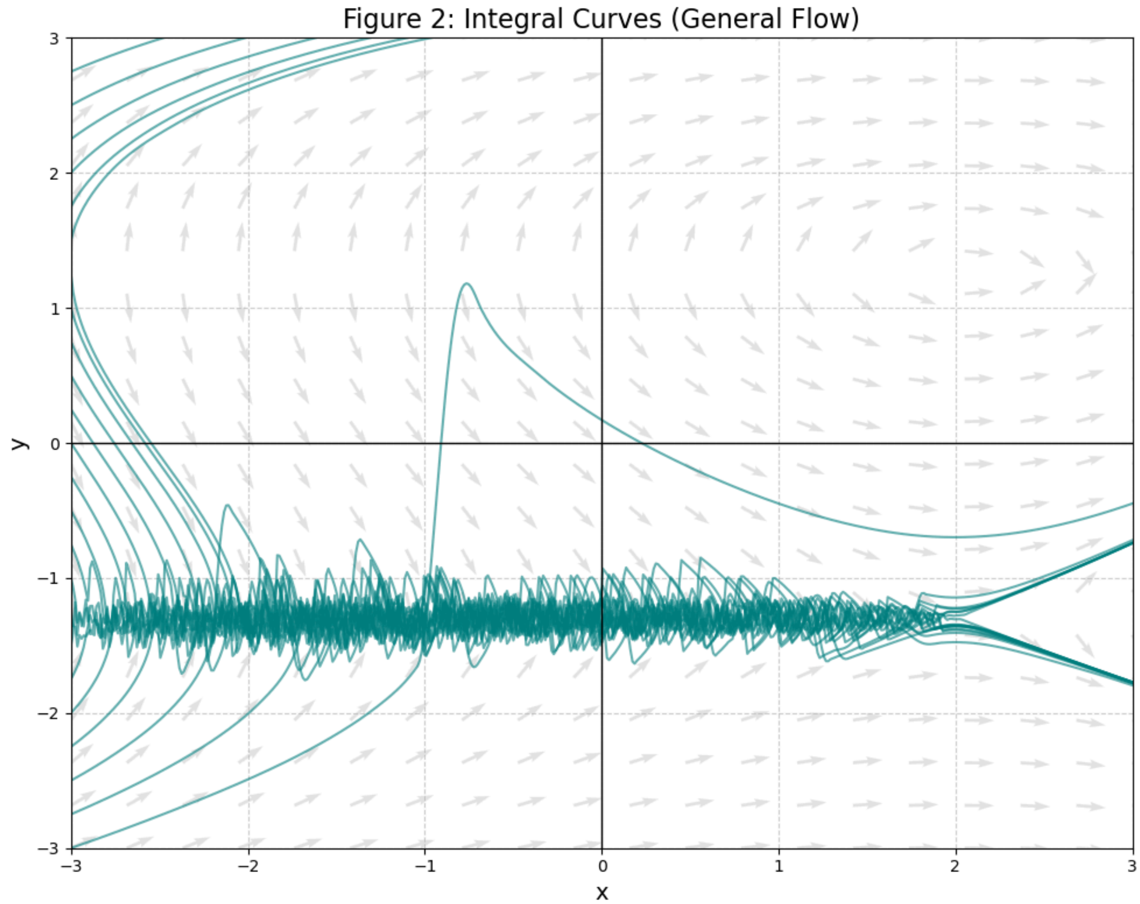



```
plots_ODE(ode_f8)
```

```
--- Differential Equation Visualizer ---
ODE: dy/dx = x - y
Domain: x in (-3, 3), y in (-3, 3)
```




```
Generating 25 integral curves...
```




```
Generating specific solutions...
```

Figure 3: Specific Numerical Solutions