Circuit Theory and Electronics Fundamentals

T2

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Contents

1 Introduction

The aim of this assignment is to properly analyze and study a RC circuit composed of a voltage source, a dependent voltage source, a dependent current source, a capacitor and seven resistors. The main focus is to create a comparison between the theoretical analysis of the circuit and a simulation using the software NGspice.

The circuit is represented with resort to *LibreOffice Draw* and can be viewed in figure 1.

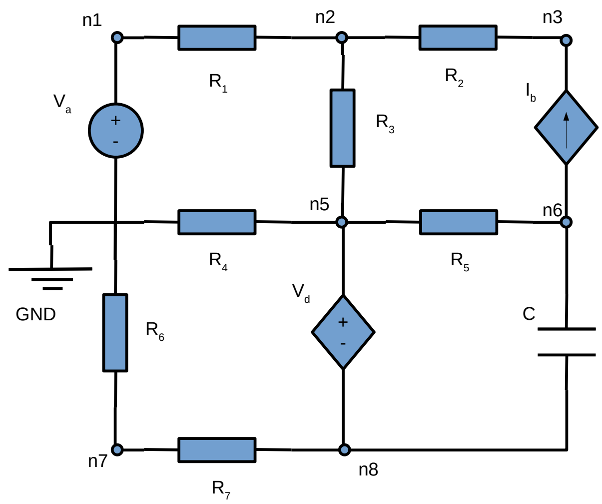

Figure 1: Studied Circuit

In Section 2, a theoretical analysis of the circuit is presented. In Section 3, the circuit is analysed by simulation, and the results are compared to the theoretical results obtained in Section 2. The conclusions of this study are outlined in Section 4.

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analyzed theoretically. In the table below the known values are presented. The units are $kOhm$, V, mS, μF.

R_1	1.01080769792
R_2	2.07664633274
R_3	3.12595649013
R_4	4.18722214507
R_5	3.08416992010
R_6	2.00179338129
R_7	1.04556537884
V_s	5.00120775651
C	1.03136220214
K_b	7.12593545434
K_d	8.24048597287

Table 1: Known Data

2.1 For $t < 0$

According to the equation given in the description of the assignment, $v_s = V_s$ when $t < 0$. In this situation the capacitor behaves like an open circuit, which means we can consider $I_c = 0$.

Using the known values and the deduction above, it is possible to solve the circuit, using the node method. The following system is the one used to solve this circuit.

$$\begin{cases} V_1 = V_s \\ G_1(V_1 - V_2) + G_2(V_3 - V_2) + G_3(V_5 - V_2) = 0 \\ G_2(V_2 - V_3) + K_b(V_2 - V_5) = 0 \\ V_4 = 0 \\ G_1(V_2 - V_1) + G_4(V_5 - V_4) + \frac{V_8 - V_5}{K_d} = 0 \\ G_5(V_5 - V_6) + K_b(V_5 - V_2) = 0 \\ G_6(V_4 - V_7) + \frac{V_8 - V_5}{K_d} = 0 \\ G_7(V_7 - V_8) + \frac{V_8 - V_5}{K_d} = 0 \end{cases}$$

With the aid of Octave to solve the system, the following values are obtained (table 2 and 3).

V_1	5.001208e+00
V_2	4.776340e+00
V_3	4.292647e+00
V_4	0.000000e+00
V_5	4.809026e+00
V_6	5.527391e+00
V_7	-1.853734e+00
V_8	-2.821966e+00

Table 2: Voltages in all nodes in V

I_1	-2.224637e-01
I_2	-2.329201e-01
I_3	1.045640e-02
I_4	1.148500e+00
I_5	2.329201e-01
I_6	-9.260366e-01
I_7	-9.260366e-01
I_s	-2.224637e-01
I_b	-2.329201e-01
I_c	0.000000e+00
I_d	9.260366e-01

Table 3: Currents in all branches in mA

2.2 For $t = 0$

In this situation, the capacitor is replaced by a voltage source Vx, that imposes a voltage of the same value of that that was determined in the previous situation for the capacitor. It is then possible to reach a value for R_eq as seen from the capacitor because it is now possible to solve a new system of equations (2.2) in which $vs = 0$ that allows the determination of I_x, this being the current passing through the capacitor.

Translating the system below into a matrix, it is possible to determine R_{eq}.

R_{eq} is necessary because it enables the determination of the natural solution of the system through the RC constant. This application will be demonstrated in the next exercise.

$$\begin{cases} V_1 = 0 \\ G_1(V_1 - V_2) + G_2(V_3 - V_2) + G_3(V_5 - V_2) = 0 \\ G_2(V_2 - V_3) + K_b(V_2 - V_5) = 0 \\ V_4 = 0 \\ G_1(V_2 - V_1) + G_4(V_5 - V_4) + \frac{V_8 - V_5}{K_d} = 0 \\ G_5(V_5 - V_6) + K_b(V_5 - V_2) + \frac{R_{eq}}{V_x} = 0 \\ G_6(V_4 - V_7) + \frac{V_8 - V_5}{K_d} = 0 \\ G_7(V_7 - V_8) + \frac{V_8 - V_5}{K_d} = 0 \\ V_6 - V_8 = V_x \end{cases}$$

Using the values computed by Octave once again, tables 4 and 5 are built.

V_1	0.000000e+00
V_2	0.000000e+00
V_3	-0.000000e+00
V_4	0.000000e+00
V_5	0.000000e+00
V_6	8.349357e+00
V_7	1.983039e-17
V_8	2.293301e-17
R_eq	3.084170e+00

Table 4: Voltages in all nodes in V

I_1	0.000000e+00
I_2	-0.000000e+00
I_3	0.000000e+00
I_4	0.000000e+00
I_5	2.707165e+00
I_6	9.906310e-18
I_7	2.967416e-18
I_s	0.000000e+00
I_b	0.000000e+00
I_x	2.707165e+00
I_d	-2.782969e-18

Table 5: Currents in all branches in mA

The value for V_x was estabilished as $V_6 - V_8$ being V_6 and V_8 determined in the subsection above.

2.3 Natural Solution

The objective for this part of the assignment was to determine the natural solution of V_6. Using the information calculated in the previous subsection, V_x specifically, it is possible with the use of 1 to achieve the goal, given this is a RC circuit, as shown in figure 2.

$$V_{6n} = Ae^{\frac{-t}{\tau}} \qquad \tau = R_{eq}C \tag{1}$$

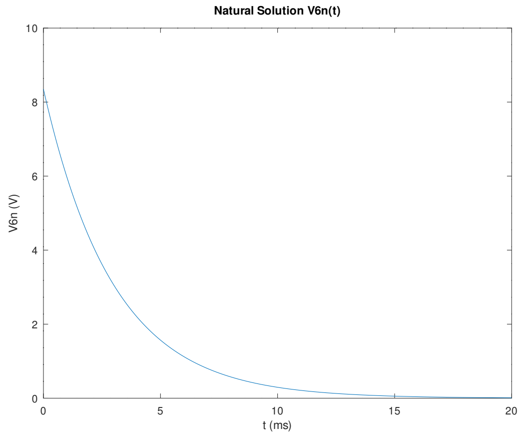

Figure 2: Natural Solution

2.4 Forced Solution

In this sector, it is now necessary to determine the forced solution for $V_6f(t)$, with a frequency of 1000Hz.Utilizing the suggestion, it is introduced a phasor with a constant V_s of 1V. It was than ran a similar node analysis to the previous, using V_s as the voltage source, while replacing the capacitance C of the capacitor with its impendance Z. The following equations were then deducted:

$$Z_c = \frac{1}{\omega C j} \qquad \omega = 2\pi f \tag{2}$$

$$\begin{cases} V_1 = j \\ G_1(V_1 - V_2) + G_2(V_3 - V_2) + G_3(V_5 - V_2) = 0 \\ G_2(V_2 - V_3) + K_b(V_2 - V_5) = 0 \\ V_4 = 0 \\ G_1(V_2 - V_1) + G_4(V_5 - V_4) + \frac{V_8 - V_5}{K_d} = 0 \\ G_5(V_5 - V_6) + K_b(V_5 - V_2) + \frac{V_8 - V_5}{Z_c} = 0 \\ G_6(V_4 - V_7) + \frac{V_8 - V_5}{K_d} = 0 \\ G_7(V_7 - V_8) + \frac{V_8 - V_5}{K_d} = 0 \end{cases}$$

Using the following equation:

$$V_{complex_i} = V_i e^{-jphase(i)} \tag{3}$$

Now, with resort to angle and abs commands in Octave, the tables below were made:

V_1	1.000000e+00
V_2	9.550373e-01
V_3	8.583221e-01
V_3	0.000000e+00
V_5	9.615729e-01
V_6	5.662518e-01
V_7	3.706573e-01
V_8	5.642569e-01

Table 6: Complex Amplitudes

Ph_1	1.570796e+00
Ph_2	1.570796e+00
Ph_3	1.570796e+00
Ph_4	0.000000e+00
Ph_5	1.570796e+00
Ph_6	-1.423112e+00
Ph_7	-1.570796e+00
Ph_8	-1.570796e+00

Table 7: Phases

It is possible, therefore, to conclude that:

$$V_{6f} = 5.662518e - 01e^{-1.423112j} \tag{4}$$

2.5 Natural and Forced Superimposed

In this section, it is necessary to convert the phasor into real time functions, in order to find a function to evaluate V_6 for $t > 0$, which was achieved by adding the natural and forced solutions, for a frequency of 1000Hz. The equation for $V_s(t)$ for $t > 0$ was already on the initial circuit diagram. The two equations are:

$$V_{ifinal} = V_{in} + V_{if} \tag{5}$$

So, calculating V_{6final}:

$$V_6(t) = e^{\frac{-t}{R_{eq}C}} + Ae^{-jphase(i)} \tag{6}$$

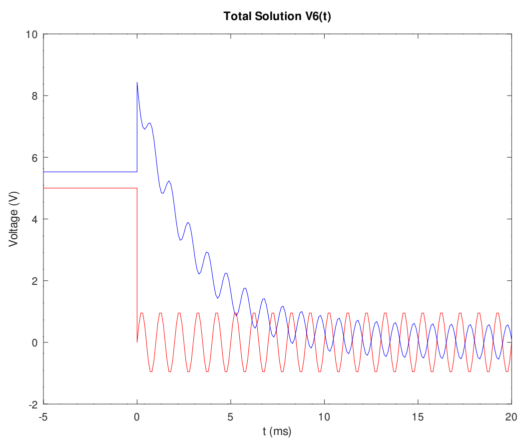

Figure 3: Natural and Forced Superimposed

2.6 Frequency Response

The main focus of this procedure is to determine the frequency response $v_6(f)$, $v_c(f)$ and $v_s(f)$ for a range from 0.1Hz to 1Mhz (in a logarithmic scale)

The graphs for phase and magnitude were then plotted, with the magnitude (in dB) being calculated with the aid of the Octave abs function. The phase is calculated with the angle function of Octave, but not without being converted from radians to degrees.

$$Z = \frac{1}{2\pi fCj} \tag{7}$$

with f being a logharitmic scale vector from -1 to 6 with 100 entries.

The system of equations used was:

$$\begin{cases} V_1 = j \\ G_1(V_1 - V_2) + G_2(V_3 - V_2) + G_3(V_5 - V_2) = 0 \\ G_2(V_2 - V_3) + K_b(V_2 - V_5) = 0 \\ V_4 = 0 \\ G_1(V_2 - V_1) + G_4(V_5 - V_4) + \frac{V_8 - V_5}{K_d} = 0 \\ G_5(V_5 - V_6) + K_b(V_5 - V_2) + \frac{V_8 - V_5}{Z_c} = 0 \\ G_6(V_4 - V_7) + \frac{V_8 - V_5}{K_d} = 0 \\ G_7(V_7 - V_8) + \frac{V_8 - V_5}{K_d} = 0 \end{cases}$$

The V_1, $V_6 - V_8$ and V_6 calculated were then assigned to $Vsfre(k)$, $Vxfre(k)$ and $V6fre(k)$, respectively

Lastly, the two following graphics were plotted using a base 10 logarithmic scale for frequencies, the logarithmic value of the abs of the variables stated above and the angle of these complex variables, converted to degrees.

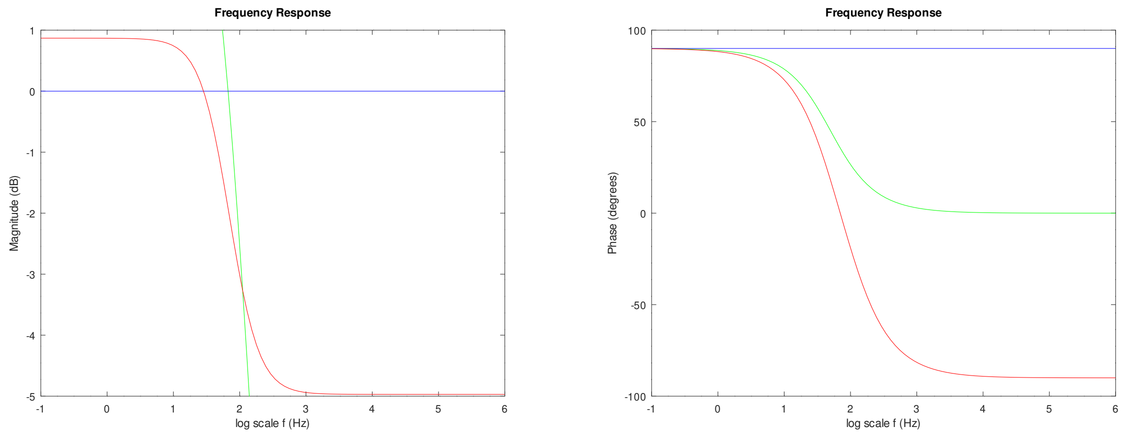

Figure 4: Magnitude (dB) - Frequency (Hz) / Phase (Degrees) - Frequency (Hz)

3 Simulation Analysis

3.1 For $t < 0$

Through the usage of NGSpice, the first simulation carried out was with a circuit similar to the one analyzed in the first section of the theoretical analysis. For simulation purposes, note that an extra voltage source was introduced into the circuit, in order to control the current passing through the dependent voltage source, acting as an anmeter. The results obtained are shown in the table 8 below.

Name	Value [A or V]
@c1[i]	0.000000e+00
@g1[i]	-2.32920e-04
@r1[i]	-2.22464e-04
@r2[i]	-2.32920e-04
@r3[i]	-1.04564e-05
@r4[i]	1.148500e-03
@r5[i]	2.329201e-04
@r6[i]	-9.26037e-04
@r7[i]	-9.26037e-04
n1	5.001208e+00
n2	4.776340e+00
n3	4.292647e+00
n4	0.000000e+00
n5	4.809026e+00
n6	5.527391e+00
n7	-1.85373e+00
n8	-2.82197e+00

Table 8: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

3.2 For $t = 0$

Simulation number 2 sole purpose is to determine a value for I_x, which was achieved by setting v_s to 0 and substituting the previously existing capacitor with a voltage source V_x. The remaining circuit remains equal to before. Ultimately, this component will impose the same potencial difference between nodes 6 and nodes 8 and will be able to operate at $t = 0$. Conducting this simulation is essential to subsequently determine the natural response of the system, through the calculations of values for V_x and I_x. The results obtained are shown in the table below.

Name	Value [A or V]
@g1[i]	0.000000e+00
@r1[i]	0.000000e+00
@r2[i]	0.000000e+00
@r3[i]	0.000000e+00
@r4[i]	0.000000e+00
@r5[i]	2.707165e-03
@r6[i]	0.000000e+00
@r7[i]	0.000000e+00
n1	0.000000e+00
n2	0.000000e+00
n3	0.000000e+00
n4	0.000000e+00
n5	0.000000e+00
n6	8.349357e+00
n7	0.000000e+00
n8	0.000000e+00

Table 9: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

3.3 Natural Solution

The third step of these simulations consisted in making a transient analysis to determine voltage variation in node 6 as the capacitor discharges for the time interval [0,20]ms. The simulated natural response of the circuit is shown in the figure below.

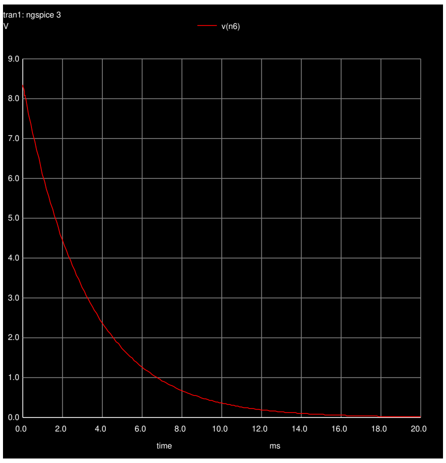

Figure 5: Natural Solution

3.4 Total Solution

Utilizing the same script as in simulation number 3, the fourth simulation conducted intends to evaluate the total response (natural and forced) on node 6 and voltage source Vs for a frequency of 1kHz. The time interval studied is the same as before, [0,20]ms. The simulation results are represented in the following figure 6.

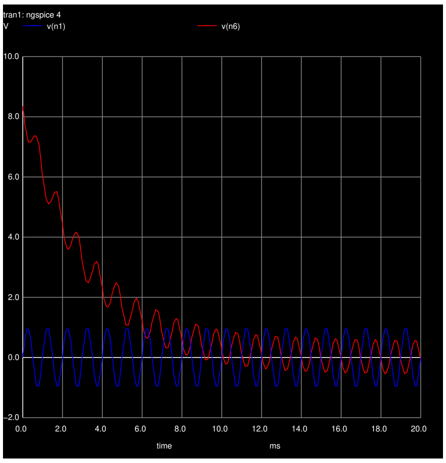

Figure 6: Total Solution

3.5 Frequency Response

Lastly, this frequency analysis aims to compare the frequency response between v_s and the 6th node. The results obtained are represented in the figures below.

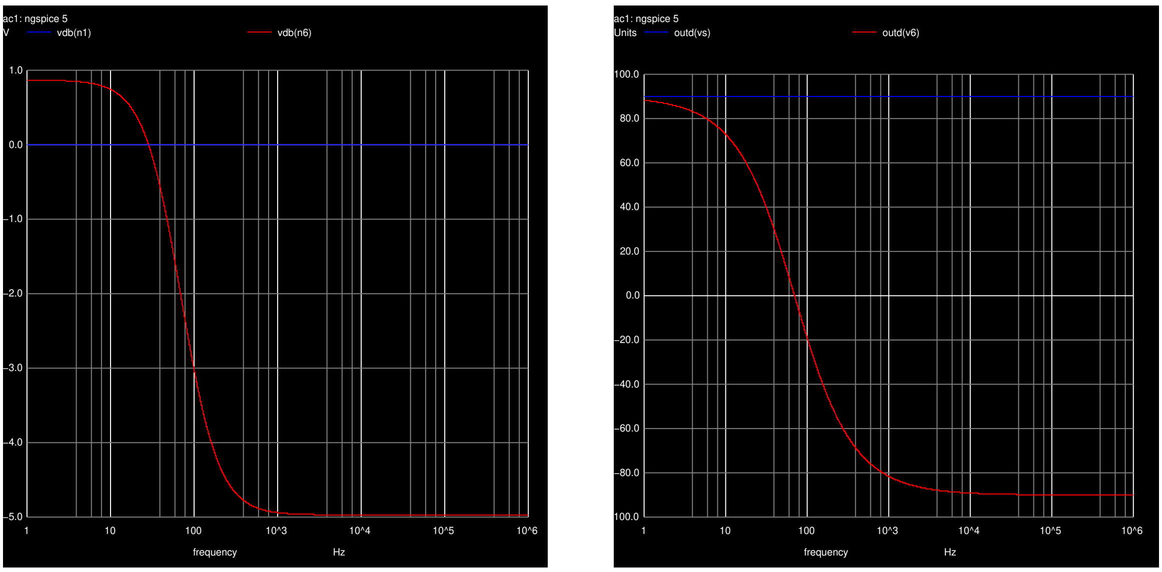

Figure 7: Magnitude (dB) - Frequency (Hz) / Phase (degrees) - Frequency (Hz)

We noticed a significant difference bettween them . The frequency response in V_s is null in opposite to the one seen in n6. This is due to the fact that V_s changes ccording to the frequency, thus remaining constant. V_6 on the other hand changes its value according to V_s showing a frequency analysis that changes through time.

4 Conclusion

It is possible to conclude, similar to the first assignment, that the values of the current in the branches, the voltage in the nodes and the equivalent resistant achieved using theory with the aid of Octave resulted in the same values as the obtained through the NGspice simulation. The difference between them is practically null. Just like in the first lab, this is due to the fact that the circuit is linear, given that he only new element, the capacitor, is also linear.

The assignment was successfully completed.