“TAREA #5”
ITESO

ITESO, Universidad Jesuita de Guadalajara

MÉTODOS DE OPTIMIZACIÓN
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Problem 1: Dual formulation

Consider the linear programming problem

$\max\ c^T x$

$s.t.\ Ax \leq b$

$x \geq 0$

where $A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 5 & 4 & 3 & 2 & 1 \end{bmatrix}$, $\quad b = \begin{bmatrix} 3 \\ 14 \end{bmatrix}$, $\quad$ and $c^T = [6 \quad 5 \quad 4 \quad 3 \quad 5]$.

Solve this problem using the following strategy:

1. Find the dual of the above primal linear program. The dual has only two variables. Solve the dual by hand after drawing a graph of its feasible set.

Primal:

$\max\ z = 6x_1 + 5x_2 + 4x_3 + 3x_4 + 5x_5$

$$s.t. \begin{cases} x_1 + x_2 + x_3 + x_4 + x_5 \leq 3 \\ 5x_1 + 4x_2 + 3x_3 + 2x_4 + x_5 \leq 14 \\ x_1,\ x_2,\ x_3,\ x_4,\ x_5,\ x_6 \geq 0 \end{cases}$$

$y_1[x_1 + x_2 + x_3 + x_4 + x_5] + y_2[5x_1 + 4x_2 + 3x_3 + 2x_4 + x_5] \leq 3y_1 + 14y_2$

$x_1[y_1 + 5y_2] + x_2[y_1 + 4y_2] + x_3[y_1 + 3y_2] + x_4[y_1 + 2y_2] + x_5[y_1 + y_2] \leq 3y_1 + 14y_2$

Dual:

$\min\ z = 3y_1 + 14y_2$

$$s.t. \begin{cases} y_1 + 5y_2 \geq 6 \\ y_1 + 4y_2 \geq 5 \\ y_1 + 3y_2 \geq 4 \\ y_1 + 2y_2 \geq 3 \\ y_1 + y_2 \geq 5 \\ y_1,\ y_2 \geq 0 \end{cases}$$

Una vez dibujada la gráfica vemos que hay 2 vértices posibles.

El cruce de $x + y \geq 5$ con el eje y y el cruce de $x + y \geq 5$ con $x + 5y \geq 6$.

① Vértice 1: $x + y = 5$

Si $x = 0 \rightarrow y = 5$
Vértice $(0, 5)$

② $x + y = 5$
$x + 5y = 6$

$-4y = -1$
$y = 0.25$

$x + 0.25 = 5$
$x = 4.75$

③ Evaluamos:
$(0, 5) \rightarrow 3(0) + 14(5) = 70$
$(4.75, 0.25) \rightarrow 3(4.75) + 14(0.25) = 17.75$

Valores que minimizan:
$y_1 = 4.75$
$y_2 = 0.25$

2. Using the optimal solution to the dual problem and the optimality conditions, determine what primal constraints are binding and what primal variables must be zero at an optimal solution. Using this information, determine the optimal solution to the primal linear program.

Para obtener soluciones del primal revisamos las condiciones del dual.
$(y_1^* = 4.75,\ y_2^* = 0.25)$

$(A^T y^* - c)^T x^* = 0$

Sustituimos valores de y_1^* y y_2^* para saber cuales x_i^* deben ser 0.

$x_1^*[4.75 + 1.25 - 6] = 0$
$x_2^*[4.75 + 1 - 5] = 0$
$x_3^*[4.75 + 0.75 - 4] = 0$
$x_4^*[4.75 + 0.5 - 3] = 0$
$x_5^*[4.75 + 0.25 - 5] = 0$

Entonces solo x_1^* y x_5^* son distintos de 0, para obtener su valor resolvemos el sistema.

$x_1 + x_5 = 3$
$5x_1 + x_5 = 14$

$-4x_1 = -11$

$x_1 = 2.75$

$2.75 + x_5 = 3$
$x_5 = 0.25$

Entonces:

$x_1^* = 2.75$
$x_5^* = 0.25$
$x_2^*,\ x_3^*,\ x_4^* = 0$

Problem 2: An Investment Problem - Retirement Planning Services
Sam Givens is a financial analyst for Retirement Planning Services, Inc. who specializes in designing retirement income portfolios for retirees using corporate bonds. She has just completed a consultation with a client who expects to have $\$750{,}000$ in liquid assets to invest when she retires next month. Sam and her client agreed to consider upcoming bond issues from the following six companies:

Company	Return	Years to Maturity	Rating
Acme Chemical	8.65%	11	1-Excellent
DynaStar	9.50%	10	3-Good
Eagle Vision	10.00%	6	4-Fair
MicroModeling	8.75%	10	1-Excellent
OptiPro	9.25%	7	3-Good
Sabre Systems	9.00%	13	2-Very Good

The column labeled "Return" in this table represents the expected annual yield on each bond, the column labeled "Years to Maturity" indicates the length of time over which the bonds will be payable, and the column labeled "Rating" indicates an independent underwriter's assessment of the quality or risk associated with each issue.
Sam believes that all of the companies are relatively safe investments. However, to protect her client's income, Sam and her client agreed that no more than 25% of her money should be invested in any one investment and at least half of her money should be invested in long-term bonds that mature in ten or more years. Also, even though DynaStar, Eagle Vision, and OptiPro offer the highest returns, it was agreed that no more than 35% of the money should be invested in these bonds because they also represent the highest risks (i.e., they were rated lower than "very good").
Sam needs to determine how to allocate her client's investments to maximize her income while meeting their agreed upon investment restrictions.
For this problem, then, present:

1. The linear programming model that represents the given situation. That is the primal problem by defining all decision variables, the objective function, and the constraints.

Primal:

$$\max\ z = 0.0865x_1 + 0.095x_2 + 0.1x_3 + 0.0875x_4 + 0.0925x_5 + 0.09x_6$$

$$s.t.\begin{cases} x_1 + x_2 + x_4 + x_6 \geq 375{,}000 \\ x_2 + x_3 + x_5 \leq 262{,}500 \\ x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \leq 750{,}000 \\ 0 \leq x_1,\ \ x_2,\ \ x_3,\ \ x_4,\ \ x_5,\ \ x_6 \leq 187{,}500 \end{cases}$$

x_1: Dinero invertido en Acme Chemical
x_2: Dinero invertido en DynaStar
x_3: Dinero invertido en Eagle Vision
x_4: Dinero invertido en MicroModeling
x_5: Dinero invertido en OptiPro
x_6: Dinero invertido en Sabre Systems

2. Derive the dual problem from the primal problem and elaborate on the entire process.

Primal:

$$\max\ z = 0.0865x_1 + 0.095x_2 + 0.1x_3 + 0.0875x_4 + 0.0925x_5 + 0.09x_6$$

$$s.t.\begin{cases} -x_1 - x_2 - x_4 - x_6 \leq -375{,}000 \\ x_2 + x_3 + x_5 \leq 262{,}500 \\ x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \leq 750{,}000 \\ 0 \leq x_1,\ \ x_2,\ \ x_3,\ \ x_4,\ \ x_5,\ \ x_6 \leq 187{,}500 \end{cases}$$

$$y_1[-x_1 - x_2 - x_4 - x_6] + y_2[x_2 + x_3 + x_5] + y_3[x_1 + x_2 + x_3 + x_4 + x_5 + x_6] \leq -375{,}000y_1 + 262{,}500y_2 + 750{,}000y_3$$
$$x_1[-y_1 + y_3] + x_2[-y_1 + y_2 + y_3] + x_3[y_2 + y_3] + x_4[-y_1 + y_3] + x_5[y_2 + y_3] + x_6[-y_1 + y_3] \leq -375{,}000y_1 + 262{,}500y_2 + 750{,}000y_3$$

Dual:

$$\min\ z = -375{,}000y_1 + 262{,}500y_2 + 750{,}000y_3$$

$$s.t.\begin{cases} -y_1 + y_3 \geq 0.0865 \\ -y_1 + y_2 + y_3 \geq 0.095 \\ y_2 + y_3 \geq 0.1 \\ -y_1 + y_3 \geq 0.0875 \\ y_2 + y_3 \geq 0.0925 \\ -y_1 + y_3 \geq 0.09 \\ y_1,\ \ y_2,\ \ y_3 \geq 0 \end{cases}$$

3. Write the primal problem and the dual problem in a matrix form. Explain the relationship between the two problems.

Primal:

$\max\ c^T x$
$s.t.\ Ax \leq b$
$x \geq 0$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} \qquad c = \begin{bmatrix} 0.0865 \\ 0.095 \\ 0.1 \\ 0.0875 \\ 0.0925 \\ 0.09 \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & -1 & 0 & -1 & 0 & -1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 375{,}000 \\ 262{,}500 \\ 750{,}000 \end{bmatrix}$$

Dual:

$\min\ b^T y$

$s.t.\ A^T y \geq c$

$y \geq 0$

$$y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

El problema dual se deriva del primal, donde en lugar de maximizar, se minimiza. El funcional de costo del primal se vuelve las restricciones del lado derecho del problema dual y las restricciones del lado derecho del primal se vuelven los coeficientes del funcional de costo del dual. Como se minimiza, las restricciones en el dual se vuelven mayor o igual. Para esto se transpone la matriz A, donde están los coeficientes de las restricciones del lado izquierdo del problema primal.

4. Solve both the primal and dual problems using the Excel solver. Take advantage of the matrix expression previously obtained in order to improve the way problems are solved using Excel. For example, a consequence of a good analysis is that the shadow prices provided by the Excel sensitivity analysis would all have the same sign.

Primal:

x1	x2	x3	x4	x5	x6
112500	75000	187500	187500	0	187500

Dual:

y1	y2	y3
0	0.0085	0.0865

Problem 3: A Multi-Period Cash Flow Problem - Taco-Viva

Taco-Viva is a small but growing restaurant chain specializing in Mexican fast food. The management of the company has decided to build a new location in Wilmington, North Carolina, and wants to establish a construction fund (or sinking fund) to pay for the new facility. Construction of the restaurant is expected to take six months and cost $\$800{,}000$. Taco-Viva's contract with the construction company requires it to make payments of $\$250{,}000$ at the end of the second and fourth months, and a final payment of $\$300{,}000$ at the end of the sixth month when the restaurant is completed. The company can use four investment opportunities to establish the construction fund; these investments are summarized in the following table:

Investment	Available in Month	Months to Maturity	Yield at Maturity
A	1, 2, 3, 4, 5, 6	1	1.8%
B	1, 3, 5	2	3.5%
C	1, 4	3	5.8%
D	1	6	11.0%

The table indicates that investment A will be available at the beginning of each of the next six months, and funds invested in this manner mature in one month with a yield of 1.8%. Funds can be placed in investment C only at the beginning of months 1 and/or 4, and mature at the end of three months with a yield of 5.8%. The management of Taco-Viva needs to determine the investment plan that allows them to meet the required schedule of payments while placing the least amount of money in the construction fund.
For this problems, then, present:

1. The linear programming model that represents the given situation. That is the primal problem by defining all decision variables, the objective function, and the constraints.

Primal:
$\min\ z = A_1 + B_1 + C_1 + D_1$

$$s.t. \begin{cases} 1.018A_1 - A_2 = 0 \\ 1.018A_2 + 1.035B_1 - A_3 - B_2 = 250 \\ 1.018A_3 + 1.058C_1 - A_4 - C_2 = 0 \\ 1.018A_4 + 1.035B_2 - A_5 - B_3 = 250 \\ 1.018A_5 - A_6 = 0 \\ 1.018A_6 + 1.035B_3 + 1.058C_2 + 1.11D_1 = 300 \\ A_i,\ B_i,\ C_i,\ D_i \geq 0 \end{cases}$$

A_i: Dinero invertido en Inversión A, $i = \{1,2,3,4,5,6\}$
B_i: Dinero invertido en Inversión B, $i = \{1,2,3\}$
C_i: Dinero invertido en Inversión C, $i = \{1,2\}$
D_i: Dinero invertido en Inversión D, $i = \{1\}$

2. Write the primal problem in a matrix form.

Primal:
$\max\ c^T x$
$s.t.\ Ax = b$

$x \geq 0$

$$x = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \\ B_1 \\ B_2 \\ B_3 \\ C_1 \\ C_2 \\ D_1 \end{bmatrix} \qquad c = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \qquad b = \begin{bmatrix} 0 \\ 250 \\ 0 \\ 250 \\ 0 \\ 300 \end{bmatrix}$$

$$A = \begin{bmatrix} 1.018 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.018 & -1 & 0 & 0 & 0 & 1.035 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.018 & -1 & 0 & 0 & 0 & 0 & 0 & 1.058 & -1 & 0 \\ 0 & 0 & 0 & 1.018 & -1 & 0 & 0 & 1.035 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.018 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.018 & 0 & 0 & 1.035 & 0 & 1.058 & 1.11 \end{bmatrix}$$

3. Solve the primal problem using the Excel solver. Take advantage of the matrix expression previously obtained in order to improve the way problems are solved using Excel.

A1	A2	A3	A4	A5	A6	B1	B2	B3	C1	C2	D1
241.237296	245.579568	0	245.579568	0	0	0	0	0	500.126128	283.553875	0

Problem 4: Modifying the Taco-Viva Problem to Account for Risk

In investment problems like this, it is not uncommon for decision makers to place limits on the amount of risk they are willing to assume. For instance, suppose that the chief financial officer (CFO) for Taco-Viva assigned the following risk ratings to each of the possible investments on a scale from 1 to 10 (where 1 represents the least risk and 10 the greatest risk). We also will assume that the CFO wants to determine an investment plan where the weighted average risk level does not exceed 5.

Investment	Risk Rating
A	1
B	3
C	8
D	6

For this problems, then, present:

1. The linear programming model that represents the given situation. That is the primal problem by defining all decision variables, the objective function, and the constraints.

Primal:

$\min\ z = A_1 + B_1 + C_1 + D_1$

$$
s.t. \begin{cases}
1.018A_1 - A_2 \leq 0 \\
-1.018A_1 + A_2 \leq 0 \\
1.018A_2 + 1.035B_1 - A_3 - B_2 \leq 250 \\
-1.018A_2 - 1.035B_1 + A_3 + B_2 \leq -250 \\
1.018A_3 + 1.058C_1 - A_4 - C_2 \leq 0 \\
-1.018A_3 - 1.058C_1 + A_4 + C_2 \leq 0 \\
1.018A_4 + 1.035B_2 - A_5 - B_3 \leq 250 \\
-1.018A_4 - 1.035B_2 + A_5 + B_3 \leq -250 \\
1.018A_5 - A_6 \leq 0 \\
-1.018A_5 + A_6 \leq 0 \\
1.018A_6 + 1.035B_3 + 1.058C_2 + 1.11D_1 \leq 300 \\
-1.018A_6 - 1.035B_3 - 1.058C_2 - 1.11D_1 \leq -300 \\
A_1 + A_2 + A_3 + A_4 + A_5 + A_6 + 3B_1 + 3B_2 + 3B_3 + 8C_1 + 8C_2 + 6D_1 \leq 5(A_i + B_i + C_i + D_i) \\
A_i,\ \ B_i,\ \ C_i,\ \ D_i \geq 0
\end{cases}
$$

A_i: Dinero invertido en Inversión A, $i = \{1, 2, 3, 4, 5, 6\}$
B_i: Dinero invertido en Inversión B, $i = \{1, 2, 3\}$
C_i: Dinero invertido en Inversión C, $i = \{1, 2\}$
D_i: Dinero invertido en Inversión D, $i = \{1\}$

2. Write the primal problem in a matrix form.

Primal:

$\max\ c^T x$
$s.t.\ Ax \leq b$
$x \geq 0$

$$
x = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \\ B_1 \\ B_2 \\ B_3 \\ C_1 \\ C_2 \\ D_1 \end{bmatrix} \qquad c = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \qquad b = \begin{bmatrix} 0 \\ 0 \\ 250 \\ -250 \\ 0 \\ 0 \\ 250 \\ -250 \\ 0 \\ 0 \\ 300 \\ -300 \\ 5(A_i + B_i + C_i + D_i) \end{bmatrix}
$$

$$A = \begin{bmatrix} 1.018 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1.018 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.018 & -1 & 0 & 0 & 0 & 1.035 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1.018 & 1 & 0 & 0 & 0 & -1.035 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.018 & -1 & 0 & 0 & 0 & 0 & 0 & 1.058 & -1 & 0 \\ 0 & 0 & -1.018 & 1 & 0 & 0 & 0 & 0 & 0 & -1.058 & 1 & 0 \\ 0 & 0 & 0 & 1.018 & -1 & 0 & 0 & 1.035 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1.058 & 1 & 0 & 0 & -1.035 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.018 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1.018 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.018 & 0 & 0 & 1.035 & 0 & 1.058 & 1.11 \\ 0 & 0 & 0 & 0 & 0 & -1.018 & 0 & 0 & -1.035 & 0 & -1.058 & -1.11 \\ 1 & 1 & 1 & 1 & 1 & 1 & 3 & 3 & 3 & 8 & 8 & 6 \end{bmatrix}$$

3. Solve the primal problem using the Excel solver. Take advantage of the matrix expression previously obtained in order to improve the way problems are solved using Excel.

A1	A2	A3	A4	A5	A6	B1	B2	B3	C1	C2	D1
241.237296	245.579568	0	245.579568	1.0397E-15	0	0	0	0	500.126128	283.553875	0

Problem 5: Project Selection

Star Oil Company is considering five different investment opportunities. The cash outflows and net present values (in millions of dollars) are given in the following table:

	Investment ($)				
	1	2	3	4	5
Time 0 cash outflow	11	53	5	5	29
Time 1 cash outflow	3	6	5	1	34
NPV	13	16	16	14	39

Cash Flows and Net Present Value for Investments in Capital Budgeting

Star Oil has $\$40$ million available for investment now (time 0); it estimates that one year from now (time 1) $\$20$ million will be available for investment. Star Oil may purchase any fraction of each investment. In this case, the cash outflows and NPV are adjusted accordingly. For example, if Star Oil purchases one-fifth of investment 3, then a cash outflow of $\frac{1}{5}(5) = \$1$ million would be required at time 0, and a cash outflow of $\frac{1}{5}(5) = \$1$ million would be required at time 1. The one-fifth share of investment 3 would yield an NPV of $\frac{1}{5}(16) = \$3.2$ million. Star Oil wants to maximize the NPV that can be obtained by investing in investments $1 - 5$. Formulate an LP that will help achieve this goal. Assume that any funds left over at time 0 cannot be used at time 1.

For this problems, then, present:

1. The linear programming model that represents the given situation. That is the primal problem by defining all decision variables, the objective function, and the constraints.

Primal:

$$\max\ z = 13x_1 + 16x_2 + 16x_3 + 14x_4 + 39x_5$$

$$s.t.\begin{cases} 11x_1 + 53x_2 + 5x_3 + 5x_4 + 29x_5 \leq 40 \\ 3x_1 + 6x_2 + 5x_3 + x_4 + 34x_5 \leq 20 \\ 0 \leq x_1,\ \ x_2,\ \ x_3,\ \ x_4,\ \ x_5 \leq 1 \end{cases}$$

x_1: Dinero invertido en la oportunidad 1
x_2: Dinero invertido en la oportunidad 2
x_3: Dinero invertido en la oportunidad 3
x_4: Dinero invertido en la oportunidad 4
x_5: Dinero invertido en la oportunidad 5

2. Derive the dual problem from the primal problem and elaborate on the entire process.

Primal:

$$\max\ z = 13x_1 + 16x_2 + 16x_3 + 14x_4 + 39x_5$$

$$s.t.\begin{cases} 11x_1 + 53x_2 + 5x_3 + 5x_4 + 29x_5 \leq 40 \\ 3x_1 + 6x_2 + 5x_3 + x_4 + 34x_5 \leq 20 \\ 0 \leq x_1,\ \ x_2,\ \ x_3,\ \ x_4,\ \ x_5 \leq 1 \end{cases}$$

$$y_1[11x_1 + 53x_2 + 5x_3 + 5x_4 + 29x_5] + y_2[3x_1 + 6x_2 + 5x_3 + x_4 + 34x_5] \leq 40y_1 + 20y_2$$
$$x_1[11y_1 + 3y_2] + x_2[53y_1 + 6y_2] + x_3[5y_1 + 5y_2] + x_4[5y_1 + y_2] + x_5[29y_1 + 34y_2] \leq 40y_1 + 20y_2$$

Dual:

$$\min\ z = 40y_1 + 20y_2$$

$$s.t.\begin{cases} 11y_1 + 3y_2 \geq 13 \\ 53y_1 + 6y_2 \geq 16 \\ 5y_1 + 5y_2 \geq 16 \\ 5y_1 + y_2 \geq 14 \\ 29y_1 + 34y_2 \geq 39 \\ y_1,\ \ y_2,\ \ y_3,\ \ y_4,\ \ y_5 \geq 0 \end{cases}$$

3. Write the primal problem and the dual problem in a matrix form. Explain the relationship between the two problems.

Primal:

$\max\ c^T x$
$s.t.\ Ax \leq b$
$x \geq 0$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \qquad c = \begin{bmatrix} 13 \\ 16 \\ 16 \\ 14 \\ 39 \end{bmatrix}$$

$$A = \begin{bmatrix} 11 & 53 & 5 & 5 & 29 \\ 3 & 6 & 5 & 1 & 34 \end{bmatrix} \; b = \begin{bmatrix} 40 \\ 20 \end{bmatrix}$$

Dual:

$\min \; b^T y$

$s.t. \; A^T y \geq c$

$y \geq 0$

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

El problema dual se deriva del primal, donde en lugar de maximizar, se minimiza. El funcional de costo del primal se vuelve las restricciones del lado derecho del problema dual y las restricciones del lado derecho del primal se vuelven los coeficientes del funcional de costo del dual. Como se minimiza, las restricciones en el dual se vuelven mayor o igual. Para esto se transpone la matriz A, donde están los coeficientes de las restricciones del lado izquierdo del problema primal.

4. Solve both the primal and dual problems using the Excel solver. Take advantage of the matrix expression previously obtained in order to improve the way problems are solved using Excel. For example, a consequence of a good analysis is that the shadow prices provided by the Excel sensitivity analysis would all have the same sign.

Primal:

x1	x2	x3	x4	x5	Max
1	0.20085995	1	1	0.28808354	57.4490172

Dual:

y1	y2	y3	y4	y5	y6	y7	Min
0.19041769	0.98464373	7.9514742	0	10.1246929	12.0632678	0	57.4490172

Problem 6: Short-Term Financial Planning

Semicond is a small electronics company that manufactures tape recorders and radios. The per-unit labor costs, raw material costs, and selling price of each product are given in the following table:

	Tape Recorder	Radio
Selling price	$100	$90
Labor cost	$50	$35

Raw material cost	\$30	\$40

Cost Information for Semicond

On December $1, 2002$, Semicond has available raw material that is sufficient to manufacture 100 tape recorders and 100 radios. On the same date, the company's balance sheet is as shown in the following table:

	Assets	Liabilities
Cash	\$10,000	-
Accounts receivable	\$3,000	-
Inventory outstanding	\$7,000	-
Bank loan	-	\$10,000

Balance Sheet for Semicond

Semicond's asset-liability ratio (called the current ratio) is $20{,}000/10{,}000 = 2$. Semicond must determine how many tape recorders and radios should be produced during December. Demand is large enough to ensure that all goods produced will be sold. All sales are on credit, however, and payment for goods produced in December will not be received until February $1, 2003$. During December, Semicond will collect $\$2{,}000$ in accounts receivable, and Semicond must pay off $\$1{,}000$ of the outstanding loan and a monthly rent of $\$1{,}000$. On January $1, 2003$, Semicond will receive a shipment of raw material worth $\$2{,}000$, which will be paid for on February $1, 2003$. Semicond's management has decided that the cash balance on January $1, 2003$, must be at least $\$4{,}000$. Also, Semicond's bank requires that the current ratio at the beginning of January be at least 2. To maximize the contribution to profit from December production, $(revenues\ to\ be\ received) - (variable\ production\ costs)$, what should Semicond produce during December?

For this problems, then, present:

1. The linear programming model that represents the given situation. That is the primal problem by defining all decision variables, the objective function, and the constraints.

Variables de decisión:

x_1: Cantidad de grabadoras producidas en diciembre
x_2: Cantidad de radios producidas en diciembre

Construcción de restricciones:

Restricción Tape recorders:
$x_1 \leq 100$

Restricción de radios:
$x_2 \leq 100$

Restricción Cash on hand at least $4000 on January 1 2003:
$10000 + 2000 - 1000 - 1000 - 50x_1 - 35x_2$
$10000 - 50x_1 - 35x_2 \geq 4000$
$50x_1 - 35x_2 \leq 6000$

Restricción Current ratio on January 1 2003 must hold 2:
Cash position:
$10000 - 50x_1 - 35x_2$

Account Receivable:
$1000 - 100x_1 - 90x_2$

Inventory outstanding:
$9000 - 30x_1 - 40x_2$

Asset Position:
$Cash\ position\ +\ Account\ Receivable\ +\ Inventory\ outstanding = 20000 + 20x_1 + 15x_2$

Liabilities:
$11000

Current ratio:
$$\frac{20000 + 20x_1 + 15x_2}{11000} \geq 2$$
$20000 + 20x_1 + 15x_2 \geq 22000$
$20x_1 + 15x_2 \geq 2000$

Primal:
$$\max\ z = 20x_1 + 15x_2$$
$$s.t. \begin{cases} 50x_1 - 35x_2 \leq 6000 \\ 20x_1 + 15x_2 \geq 2000 \\ x_1 \leq 100 \\ x_2 \leq 100 \\ x_1,\ x_2 \geq 0 \end{cases}$$

Primal modificado:
$$\max\ z = 20x_1 + 15x_2$$
$$s.t. \begin{cases} 50x_1 - 35x_2 \leq 6000 \\ -20x_1 - 15x_2 \leq -2000 \\ x_1 \leq 100 \\ x_2 \leq 100 \\ x_1,\ x_2 \geq 0 \end{cases}$$

2. Derive the dual problem from the primal problem and elaborate on the entire process.

Primal modificado:

$$\max\ z = 20x_1 + 15x_2$$

$$s.t.\begin{cases} 50x_1 - 35x_2 \leq 6000 \\ -20x_1 - 15x_2 \leq -2000 \\ x_1 \leq 100 \\ x_2 \leq 100 \\ x_1,\ x_2 \geq 0 \end{cases}$$

$$y_1[50x_1 - 35x_2] + y_2[-20x_1 - 15x_2] \leq 6000y_1 - 2000y_2$$
$$x_1[50y_1 - 20y_2] + x_2[-35y_1 - 15y_2] \leq 6000y_1 - 2000y_2$$

Dual:

$$\min\ z = 6000y_1 - 2000y_2$$

$$s.t.\begin{cases} 50y_1 - 20y_2 \geq 20 \\ -35y_1 - 15y_2 \geq 15 \\ y_1,\ y_2 \geq 0 \end{cases}$$

3. Write the primal problem and the dual problem in a matrix form. Explain the relationship between the two problems.

Primal:

$$\max\ c^T x$$
$$s.t.\ Ax \leq b$$
$$x \geq 0$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad c = \begin{bmatrix} 20 \\ 15 \end{bmatrix}$$
$$A = \begin{bmatrix} 50 & -35 \\ -20 & -15 \end{bmatrix} \quad b = \begin{bmatrix} 6000 \\ -2000 \end{bmatrix}$$

Dual:

$$\min\ b^T y$$
$$s.t.\ A^T y \geq c$$
$$y \geq 0$$

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

El problema dual se deriva del primal, donde en lugar de maximizar, se minimiza. El funcional de costo del primal se vuelve las restricciones del lado derecho del problema dual y las restricciones del lado derecho del primal se vuelven los coeficientes del funcional de costo del dual. Como se minimiza, las restricciones en el dual se vuelven mayor o igual. Para esto se transpone la matriz A, donde están los coeficientes de las restricciones del lado izquierdo del problema primal.

4. Solve both the primal and dual problems using the Excel solver. Take advantage of the matrix expression previously obtained in order to improve the way problems are solved using Excel. For example, a consequence of a good analysis is that the shadow prices provided by the Excel sensitivity analysis would all have the same sign.

Primal:

x1	x2	Max
50	100	2500

Dual:

y1	y2	y3	y4	Min
0.4	0	0	1	2500

Problem 7: Multi-Period Cash Flow Management - Finco Investment Corporation

Finco Investment Corporation must determine investment strategy for the firm during the next three years. Currently (time 0), \$100,000 is available for investment. Investments A, B, C, D, and E are available. The cash flow associated with investing \$1 in each investment is given in the following table:

	Cash Flow (\$) at Time *			
	0	1	2	3
A	-1	+0.50	+1	0
B	0	-1	+0.50	+1
C	-1	+1.2	0	0
D	-1	0	0	+1.9
E	0	0	-1	+1.5

⁎ Note: Time 0 = present; time 1 = 1 year from now; time 2 = 2 years from now; time 3 = 3 years from now.

For example, \$1 invested in investment B requires a \$1 cash outflow at time 1 and returns 50 cents at time 2 and \$1 at time 3. To ensure that the company's portfolio is diversified, Finco requires that at most \$75,000 be placed in any single investment. In addition to investments A-E, Finco can earn interest at 8% per year by keeping uninvested cash in money market funds. Returns from investments may be immediately reinvested. For example, the positive cash flow received from investment C at time 1 may immediately be reinvested in investment B. Finco cannot borrow funds, so the cash available for investment at any time is limited to cash on hand.

Formulate an LP that will maximize cash on hand at time 3.

For this problems, then, present:

1. The linear programming model that represents the given situation. That is the primal problem by defining all decision variables, the objective function, and the constraints.

Primal:

$$\max\ z = B + 1.9D + 1.5E + 1.08S_2$$

$$s.t. \begin{cases} A + C + D + S_0 = 100000 \\ 0.5A + 1.2C + 1.08S_0 = B + S_1 \\ A + 0.5B + 1.08S_1 = E + S_2 \\ 0 \le A, B, C, D, E \le 75000 \\ S_1,\ \ S_2 \ge 0 \end{cases}$$

A: Dinero invertido en A
B: Dinero invertido en B
C: Dinero invertido en C
D: Dinero invertido en D
E: Dinero invertido en E
S_i: Dinero invertido en money market funds, $i = \{0, 1, 2\}$

Primal modificado:

$$\max\ z = B + 1.9D + 1.5E + 1.08S_2$$

$$s.t. \begin{cases} A + C + D + S_0 = 100000 \\ 0.5A + 1.2C + 1.08S_0 - B - S_1 = 0 \\ A + 0.5B + 1.08S_1 - E - S_2 = 0 \\ 0 \le A, B, C, D, E \le 75000 \\ S_1,\ \ S_2 \ge 0 \end{cases}$$

2. Write the primal problem in a matrix form.

Primal:

$\max\ c^T x$
$s.t.\ Ax \le b$
$x \ge 0$

$$x = \begin{bmatrix} A \\ B \\ C \\ D \\ E \\ S_0 \\ S_1 \\ S_2 \end{bmatrix} \qquad c = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1.9 \\ 1.5 \\ 0 \\ 0 \\ 1.08 \end{bmatrix} \qquad b = \begin{bmatrix} 100000 \\ 0 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0.5 & -1 & 1.2 & 0 & 0 & 1.08 & -1 & 0 \\ 1 & 0.5 & 0 & 0 & -1 & 0 & 1.08 & -1 \end{bmatrix}$$

3. Solve the primal problem using the Excel solver. Take advantage of the matrix expression previously obtained in order to improve the way problems are solved using Excel.

Primal:

A	B	C	D	E	s0	s1	s2	Max
60000	30000	0	40000	75000	0	0	0	218500

Problem 8: Bond Dedication *I*

Suppose a pension fund needs to cover some liabilities in the next six years. Cash requirements (in million $) are:

Year	1	2	3	4	5	6
Required	100	200	800	100	800	1200

Suppose the pension fund can invest in ten government bonds with the cash flows and current prices in the following table:

	1	2	3	4	5	6	Price
Bond 1	10	10	10	10	10	110	109
Bond 2	7	7	7	7	7	107	94.8
Bond 3	8	8	8	8	8	108	99.5
Bond 4	6	6	6	6	106	-	93.1
Bond 5	7	7	7	7	107	-	97.2
Bond 6	5	5	5	105	-	-	92.9
Bond 7	10	10	110	-	-	-	110
Bond 8	8	8	108	-	-	-	104
Bond 9	7	107	-	-	-	-	102
Bond 10	100	-	-	-	-	-	95.2

Find the least expensive portfolio of bonds whose cash flows will be sufficient to cover the cash requirements. Assume surplus cash can be carried from one year to the next but earn no interest.

For this problem, then, present:

1. The linear programming model that represents the given situation. That is the primal problem by defining all decision variables, the objective function, and the constraints.

Primero se plantea el problema con holguras:

Primal:

$\min\ z = 109x_1 + 94.8x_2 + 99.5x_3 + 93.1x_4 + 97.2x_5 + 92.9x_6 + 110x_7 + 104x_8 + 102x_9 + 95.2x_{10}$

$$
s.t.\begin{cases}
10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 5x_6 + 10x_7 + 8x_8 + 7x_9 + 100x_{10} = 100\,M + S_1 \\
10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 5x_6 + 10x_7 + 8x_8 + 107x_9 + S_1 = 200\,M + S_2 \\
10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 5x_6 + 110x_7 + 108x_8 + S_2 = 800\,M + S_3 \\
10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 105x_6 + S_3 = 100\,M + S_4 \\
10x_1 + 7x_2 + 8x_3 + 106x_4 + 107x_5 + S_4 = 800\,M + S_5 \\
110x_1 + 107x_2 + 108x_3 + S_5 = 1200\,M + S_6 \\
x_1,\ x_2,\ x_3,\ x_4,\ x_5,\ x_6,\ x_7,\ x_8,\ x_9,\ x_{10} \geq 0 \\
S_1,\ S_2,\ S_3,\ S_4,\ S_5,\ S_6 \geq 0
\end{cases}
$$

x_1: Cantidad de bonos tipo 1
x_2: Cantidad de bonos tipo 2
x_3: Cantidad de bonos tipo 3
x_4: Cantidad de bonos tipo 4
x_5: Cantidad de bonos tipo 5
x_6: Cantidad de bonos tipo 6
x_7: Cantidad de bonos tipo 7
x_8: Cantidad de bonos tipo 8
x_9: Cantidad de bonos tipo 9
x_{10}: Cantidad de bonos tipo 10
S_i: Excedente, $i = \{1,2,3,4,5,6\}$

Después se acomodan las variables de un solo lado de las restricciones para su solución.

Primal:
$\min\ z = 109x_1 + 94.8x_2 + 99.5x_3 + 93.1x_4 + 97.2x_5 + 92.9x_6 + 110x_7 + 104x_8 + 102x_9 + 95.2x_{10}$

$$
s.t.\begin{cases}
10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 5x_6 + 10x_7 + 8x_8 + 7x_9 + 100x_{10} - S_1 = 100\,M \\
10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 5x_6 + 10x_7 + 8x_8 + 107x_9 + S_1 - S_2 = 200\,M \\
10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 5x_6 + 110x_7 + 108x_8 + S_2 - S_3 = 800\,M \\
10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 105x_6 + S_3 - S_4 = 100\,M \\
10x_1 + 7x_2 + 8x_3 + 106x_4 + 107x_5 + S_4 - S_5 = 800\,M \\
110x_1 + 107x_2 + 108x_3 + S_5 - S_6 = 1200\,M \\
x_1,\ x_2,\ x_3,\ x_4,\ x_5,\ x_6,\ x_7,\ x_8,\ x_9,\ x_{10} \geq 0 \\
S_1,\ S_2,\ S_3,\ S_4,\ S_5,\ S_6 \geq 0
\end{cases}
$$

2. Write the primal problem in a matrix form.

Primal:
$\min\ c^T x$
$s.t.\ Ax = b$
$x \geq 0$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{bmatrix} \quad c = \begin{bmatrix} 109 \\ 94.8 \\ 99.5 \\ 93.1 \\ 97.2 \\ 92.9 \\ 110 \\ 104 \\ 102 \\ 95.2 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad b = \begin{bmatrix} 100 \\ 200 \\ 800 \\ 100 \\ 800 \\ 1200 \end{bmatrix}$$

$$A = \begin{bmatrix} 10 & 7 & 8 & 6 & 7 & 5 & 10 & 8 & 7 & 100 & -1 & 0 & 0 & 0 & 0 & 0 \\ 10 & 7 & 8 & 6 & 7 & 5 & 10 & 8 & 107 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 10 & 7 & 8 & 6 & 7 & 5 & 110 & 108 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 10 & 7 & 8 & 6 & 7 & 105 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 10 & 7 & 8 & 106 & 107 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 110 & 107 & 108 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

3. Solve the primal problem using the Excel solver. Take advantage of the matrix expression previously obtained in order to improve the way problems are solved using Excel.

x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	s1	s2	s3	s4	s5	s6
1	11214953	0	6633842	3	0	10	6008673	0	0	66377238	32754476	14	18307768	22	103

Problem 9: Bond Dedication II

You need to create a portfolio to cover the following stream of liabilities for the next six future dates:

Date	1	2	3	4	5	6
Required	500	200	800	200	800	1200

You may purchase the bonds in the following table:

	Year						
Bond	1	2	3	4	5	6	Price
1	10	10	10	10	10	110	109
2	7	7	7	7	7	107	94.8
3	8	8	8	8	8	108	99.5
4	6	6	6	6	106	-	93.1

5	7	7	7	7	107	-	97.2
6	6	6	6	106	-	-	96.3
7	5	5	5	105	-	-	92.9
8	10	10	110	-	-	-	110
9	8	8	108	-	-	-	104
10	6	6	106	-	-	-	101
11	10	110	-	-	-	-	107
12	7	107	-	-	-	-	102
13	100	-	-	-	-	-	95.2

On the other hand, the term structure of risk-free interest rates is:

Date	1	2	3	4	5	6
Rate	5.04%	5.94%	6.36%	7.18%	7.89%	8.39%

For this problem, then:

1. Formulate a linear programming model to find the lowest-cost long-only dedicated portfolio that covers the stream of liabilities with the bonds above. Assume surplus balances can be carried from one date to the next but earn no interest. What is the cost of your portfolio? What is the composition of your portfolio?

Primero se plantea el problema con holguras:

Primal:

$$\min\ z = 109x_1 + 94.8x_2 + 99.5x_3 + 93.1x_4 + 97.2x_5 + 96.3x_6 + 92.9x_7 + 110x_8 + 104x_9 + 101x_{10} + 107x_{11} + 102x_{12} + 95.2x_{13} + S_1 + S_2 + S_3 + S_4 + S_5 + S_6$$

$$s.t.\begin{cases} 10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 6x_6 + 5x_7 + 10x_8 + 8x_9 + 6x_{10} + 10x_{11} + 7x_{12} + 100x_{13} = 500 + S_1 \\ 10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 6x_6 + 5x_7 + 10x_8 + 8x_9 + 6x_{10} + 110x_{11} + 107x_{12} + S_1 = 200 + S_2 \\ 10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 6x_6 + 5x_7 + 110x_8 + 108x_9 + 106x_{10} + S_2 = 800 + S_3 \\ 10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 106x_6 + 105x_7 + S_3 = 200 + S_4 \\ 10x_1 + 7x_2 + 8x_3 + 106x_4 + 107x_5 + S_4 = 800 + S_5 \\ 110x_1 + 107x_2 + 108x_3 + S_5 = 1200 + S_6 \\ x_1,\ x_2,\ x_3,\ x_4,\ x_5,\ x_6,\ x_7,\ x_8,\ x_9,\ x_{10},\ x_{11},\ x_{12},\ x_{13} \geq 0 \\ S_1,\ S_2,\ S_3,\ S_4,\ S_5,\ S_6 \geq 0 \end{cases}$$

x_1: Cantidad de bonos tipo 1
x_2: Cantidad de bonos tipo 2
x_3: Cantidad de bonos tipo 3
x_4: Cantidad de bonos tipo 4
x_5: Cantidad de bonos tipo 5
x_6: Cantidad de bonos tipo 6
x_7: Cantidad de bonos tipo 7
x_8: Cantidad de bonos tipo 8
x_9: Cantidad de bonos tipo 9
x_{10}: Cantidad de bonos tipo 10
x_{11}: Cantidad de bonos tipo 11

x_{12}: Cantidad de bonos tipo 12
x_{13}: Cantidad de bonos tipo 13
S_i: Excedente, $i = \{1, 2, 3, 4, 5, 6\}$

Después se acomodan las variables de un solo lado de las restricciones para su solución.

Primal:

$$\min z = 109x_1 + 94.8x_2 + 99.5x_3 + 93.1x_4 + 97.2x_5 + 96.3x_6 + 92.9x_7 + 110x_8 + 104x_9 + 101x_{10} + 107x_{11} + 102x_{12} + 95.2x_{13} + S_1 + S_2 + S_3 + S_4 + S_5 + S_6$$

$$s.t. \begin{cases} 10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 6x_6 + 5x_7 + 10x_8 + 8x_9 + 6x_{10} + 10x_{11} + 7x_{12} + 100x_{13} - S_1 = 500 \\ 10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 6x_6 + 5x_7 + 10x_8 + 8x_9 + 6x_{10} + 110x_{11} + 107x_{12} + S_1 - S_2 = 200 \\ 10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 6x_6 + 5x_7 + 110x_8 + 108x_9 + 106x_{10} + S_2 - S_3 = 800 \\ 10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 106x_6 + 105x_7 + S_3 - S_4 = 200 \\ 10x_1 + 7x_2 + 8x_3 + 106x_4 + 107x_5 + S_4 - S_5 = 800 \\ 110x_1 + 107x_2 + 108x_3 + S_5 - S_6 = 1200 \\ x_1,\ x_2,\ x_3,\ x_4,\ x_5,\ x_6,\ x_7,\ x_8,\ x_9,\ x_{10},\ x_{11},\ x_{12},\ x_{13} \geq 0 \\ S_1,\ S_2,\ S_3,\ S_4,\ S_5,\ S_6 \geq 0 \end{cases}$$

El costo del portafolio es de $2830.1

Composición:

x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12	x13	s1	s2	s3	s4	s5	s6
9	0	2	0	6	1	0	0	5	1	0	0	3	0	0	0	54	2	8

Con excedentes de:

s1	s2	s3	s4	s5	s6
88	76	64	0	24	1

2. Suppose that the stream of liabilities changes to:

Date	1	2	3	4	5	6
Required	100	200	800	500	800	1200

Find the new optimal dedicated portfolio and determine the new implied term structure. Is it different from the one you obtained in the previous part? Can you provide an intuitive explanation for the difference or lack thereof?

El costo del portafolio es de $2796.6

Composición:

x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12	x13
0	11	0	2	5	2	0	2	4	0	0	0	0

Con excedentes de:

s1	s2	s3	s4	s5	s6
88	76	64	0	24	1

La estructura del portafolio es distinta a la del anterior, pues se compra distinta cantidad para cada tipo de bono, esto se debe a que los pagos son distintos en algunos periodos, además la cantidad a pagar total es menor, por lo que hace sentido que el costo del portafolio sea menor y su composición distinta al anterior.

Problem 10: Short-Term Financial Planning

A company will face the following cash requirements in the next eight quarters (positive entries represent cash needs while negative entries represent cash surpluses):

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8
100	500	100	-600	-500	200	600	-900

The company has three borrowing possibilities.

1. A 2-year loan available at the beginning of Q1, with a 1% interest per quarter.
2. The other two borrowing opportunities are available at the beginning of every quarter: a 6-month loan with a 1.8% interest per quarter, and a quarterly loan with a 2.5% interest for the quarter.
3. Any surplus can be invested at a 0.5% interest per quarter.

Formulate a linear program that maximizes the wealth of the company at the beginning of Q9.

Primal:

$$\max\ z = w_8 - y_8 - z_8$$

$$s.t.\begin{cases} x_1 + y_1 + z_1 - w_1 = 100 \\ -0.01x_1 - 0.018y_1 + y_2 - 1.025z_1 + z_2 + 1.005w_1 - w_2 = 500 \\ -0.01x_1 - 1.018y_1 - 0.018y_2 + y_3 - 1.025z_2 + z_3 + 1.005w_2 - w_3 = 100 \\ -0.01x_1 - 1.018y_2 - 0.018y_3 + y_4 - 1.025z_3 + z_4 + 1.005w_3 - w_4 = -600 \\ -0.01x_1 - 1.018y_3 - 0.018y_4 + y_5 - 1.025z_4 + z_5 + 1.005w_4 - w_5 = -500 \\ -0.01x_1 - 1.018y_4 - 0.018y_5 + y_6 - 1.025z_5 + z_6 + 1.005w_5 - w_6 = 200 \\ -0.01x_1 - 1.018y_5 - 0.018y_6 + y_7 - 1.025z_6 + z_7 + 1.005w_6 - w_7 = 600 \\ -1.01x_1 - 1.018y_6 - 0.018y_7 + y_8 - 1.025z_7 + z_8 + 1.005w_7 - w_8 = -900 \\ 0 \le x_1 \le 400 \\ 0 \le y_1,\ y_2,\ y_3,\ y_4,\ y_5,\ y_6,\ y_7,\ y_8 \le 400 \\ 0 \le z_1,\ z_2,\ z_3,\ z_4,\ z_5,\ z_6,\ z_7,\ z_8 \le 400 \\ w_1,\ w_2,\ w_3,\ w_4,\ w_5,\ w_6,\ w_7,\ w_8 \ge 0 \end{cases}$$

x_1: Dinero pedido en préstamo a 2 años
y_i: Dinero pedido en préstamo a 6 meses, $i = \{1, 2, 3, 4, 5, 6, 7, 8\}$
z_i: Dinero pedido en préstamo trimestral, $i = \{1, 2, 3, 4, 5, 6, 7, 8\}$
w_i: Excedente para inversión, $i = \{1, 2, 3, 4, 5, 6, 7, 8\}$

Then, generate the sensitivity report and consider:

Celda	Nombre	Final Valor	Sombra Precio	Restricción Lado derecho	Permisible Aumentar	Permisible Reducir
AA10	Q1 total	100	-1.071621954	100	196.5174129	199.5024876
AA11	Q2 total	500	-1.066290501	500	197.5	101
AA12	Q3 total	100	-1.045654263	100	76.9165584	109.2150754
AA13	Q4 total	-600	-1.020150501	-600	78.83947236	1E+30
AA14	Q5 total	-500	-1.015075125	-500	176.1985937	1E+30
AA15	Q6 total	200	-1.010025	200	177.0795867	1E+30
AA16	Q7 total	600	-1.005	600	177.9649846	1E+30
AA17	Q8 total	-900	-1	-900	868.1321965	1E+30

1. Suppose the cash requirement in Q2 is 300 (instead of 500). How would this affect the wealth in Q9?

Para este caso se reduce nuestra deuda $200, esta reducción se multiplica por el precio sombra para obtener el efecto que provoca para Q9. Entonces temenos que: (-200)(-1.066) = $213.2581. Esta cantidad se le suma a nuestro resultado de Q9. Por lo que al inicio del Q9 tendríamos $1081.3903.

En nuestro caso no es posible esto ya que el máximo a reducir permitido es de $101.

2. Suppose the cash requirement in Q2 is 100 (instead of 500). Can the sensitivity report be used to determine the wealth in Q9?

Para este caso se reduce nuestra deuda $400, esta reducción se multiplica por el precio sombra para obtener el efecto que provoca para Q9. Entonces tenemos que: (-400)(-1.066) = $426.516201. Esta cantidad se le suma a nuestro resultado de Q9. Por lo que al inicio del Q9 tendríamos $1294.6484.

En nuestro caso no es posible esto ya que el máximo a reducir permitido es de $101.

3. One of the company's suppliers may allow deferred payments of $\$50$ from Q3 to Q4. What would be the value of this?

Si esto ocurre significa que en Q3 ahora debo $50, entonces tendría un cambio en Q9 de: (-50) (-1.0456) = $52.2827132.
Ahora el pago de $50 se recorre a Q4, por lo que ese mes ganaría $550, teniendo en Q9 un cambio de: (-50) (-1.0201) = -$51.007525.

Esto me da un cambio total para Q9 de $1.275188126, teniendo un total de $869.4073846.