Constructed Facilities Laboratory
Department of Civil, Construction, and Environmental Engineering

Technical Report
No. IS-07-01

CUMBIA

SET OF CODES FOR THE ANALYSIS OF REINFORCED CONCRETE MEMBERS

Prepared by:

Mr. Luis A. Montejo, ***Research Assistant***
Dr. Mervyn J. Kowalsky, ***Principle Investigator***

Prepared for:

Alaska Department of Transportation
and Public Facilities
Juneau, Alaska

February 2007

Constructed Facilities Laboratory
2414 Campus Shore Drive
North Carolina State University
Raleigh, NC 27695-7533
Tel: (919) 513-1733
Fax: (919) 513-1765
Email: cfl@ncsu.edu
Web Site: www.cfl.ncsu.edu

CUMBIA

(from the Colombian folklore)

Set of Codes for the Analysis of Reinforced Concrete Members

Theory and User Guide

By

Luis A. Montejo
Research Assistant

Supervised by

Mervyn J. Kowalsky
Associate Professor

This report is part of the PhD dissertation of the author. The project was supported by the Department of Transportation of Alaska and the Department of Civil, Construction and Environmental Engineering at North Carolina State University.

Department of Civil, Construction and Environmental Engineering
North Carolina State University
Raleigh, NC.
2007

DISCLAIMER

There are **no warranties**, explicit or implicit, that the codes contained in this collection are free of error, or are consistent with any particular standard of accuracy, or that they will meet your requirements for any particular application. They should not be relied on for any purpose where incorrect results could result in loss of property or personal injury. If you do use the codes for any such purpose it is at your own risk. The authors disclaim all liability of any kind, direct or consequential, resulting from your use of these codes.

If you find that something does not seems to work correctly with the codes, please e-mail a notification of your problem to ~~lumontv@yahoo.com.ar.~~. If you use CUMBIA to produce scientific articles, we would appreciate being informed of the article's title, authors and place of appearance. CUMBIA is available at **no charge** from ~~www.geocities.com\lumontv\eng~~.

luis.montejo@upr.edu
https://sites.google.com/a/upr.edu/montejo/

1. INTRODUCTION

CUMBIA is a set of *Matlab* codes to perform monotonic moment-curvature analysis and force-displacement response of reinforced concrete members of rectangular or circular section. An axial load – moment interaction analysis can also be performed. The section analysis is performed by tabulating moment and curvature of the member section for increasing levels of concrete strain. The member response is obtained from the section moment-curvature results along with an equivalent plastic hinge length, as presented by Priestley, Seible and Calvi (1996). Shear deformations are computed following the procedure described in Priestley, Calvi and Kowalsky (2006). The shear strength envelope for the member is calculated using the revised UCSD shear model (Kowalsky and Priestley, 2000). The on set of buckling is determined according to two different models, one propose by Moyer and Kowalsky (2003) and the other proposed by Berry and Eberhard (2005).

The constitutive models for the concrete and steel can be easily specified by the user. Nonetheless, the code has some default models. The default models for the unconfined and confined models are those proposed by Mander, Priestley and Park (1988). There are two models for the steel, one is the same used by the King program (1986) and the other is the proposed by Raynor et al. (2002). The code allows the analysis of members subjected to axial load (tension or compression) and single or double bending.

2. MATERIAL MODELS

The constitutive models for the concrete (confined and unconfined) and reinforcing steel can be specified by the user or the default models can be used.

2.1 Default Models

2.1.1 Model for the Confined Concrete

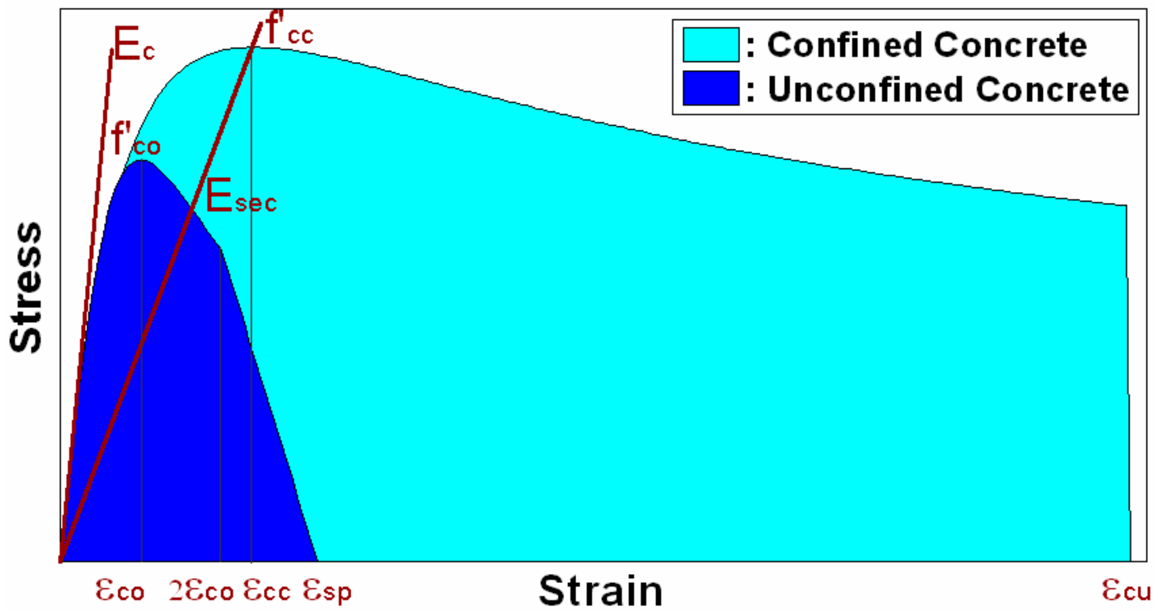

Figure 1. Mander model for confined and unconfined concrete

Mander et al. (1988) have proposed a unified stress-strain approach for confined concrete (figure 1) applicable to circular and rectangular sections. Since experimental results have shown that the ultimate concrete strain calculated based on the Mander model to be consistently conservative by 50% (Kowalsky, 2000), the original Mander expression for ε_{cu} is modified as shown in equation 6. The longitudinal compressive stress f_c is given by:

$$f_c = \frac{f'_{cc}\,xr}{r-1+x^r} \tag{1}$$

where:

$$x = \frac{\varepsilon_c}{\varepsilon_{cc}} \tag{2}$$

ε_c : longitudinal compressive concrete strain

$$\varepsilon_{cc} = \varepsilon_{co}\left[1 + 5\left(\frac{f'_{cc}}{f'_{co}} - 1\right)\right] \tag{3}$$

ε_{co} : unconfined concrete strain

f'_{co} : unconfined concrete stress

$$r = \frac{E_c}{E_c - E_{\sec}} \tag{4}$$

$$E_{\sec} = \frac{f'_{cc}}{\varepsilon_{cc}} \tag{5}$$

$$\varepsilon_{cu} = 1.4\left(0.004 + \frac{1.4\rho_s f_{yh}\varepsilon_{su}}{f'_{cc}}\right) \tag{6}$$

For circular sections:

$$f'_{cc} = f'_{co}\left(-1.254 + 2.254\sqrt{1 + \frac{7.94 f'_l}{f'_{co}}} - 2\frac{f'_l}{f'_{co}}\right) \tag{7}$$

$$f'_l = \frac{1}{2}k_e\rho_s f_{yh} \tag{8}$$

f_{yh} : yielding stress of transverse steel

$$\rho_s = \frac{4A_{sp}}{d_s s} \tag{9}$$

A_{sp} : cross section area of spiral or hoop

d_s : diameter of the core (center to center of spirals)

$$k_e = \frac{\left(1 - \frac{s'}{2d_s}\right)^2}{1 - \rho_{cc}} \quad \text{for circular hoops} \tag{10}$$

$$k_e = \frac{1 - \frac{s'}{2d_s}}{1 - \rho_{cc}} \qquad \text{for circular spirals} \tag{11}$$

s' : clear distance between spirals or hoops

ρ_{cc} : ratio of area of longitudinal reinforcement to area of core section

For rectangular sections:

$$f'_{lx} = k_e \rho_x f_{yh} \ , \quad f'_{ly} = k_e \rho_y f_{yh} \tag{12}$$

$$\rho_x = \frac{A_{sx}}{sH_c} \ , \ \rho_y = \frac{A_{sy}}{sB_c} \tag{13}$$

A_{sx} , A_{sy} : total area of transverse steel running in the x and y directions, respectively.

B_c , H_c : core dimensions to centerline of perimeter hoop in the x and y directions.

s : distance between hoops, center to center

$$k_e = \frac{\left(1 - \sum_{i=1}^{n} \frac{(w'_i)^2}{6B_c H_c}\right)\left(1 - \frac{s'}{2B_c}\right)\left(1 - \frac{s'}{2H_c}\right)}{(1 - \rho_{cc})} \tag{14}$$

w'_i : i^{th} clear distance between longitudinal adjacent bars properly restrained

To determine the confined concrete compressive strength in rectangular sections, Mander et al. (1988) proposed to use a constitutive model involving a specified ultimate strength surface for multiaxial compressive stress. Instead of use an ultimate strength surface, *CUMBIA* uses equation 7 with an average effective lateral confined stress of: $f'_l = 0.5(f'_{lx} + f'_{ly})$ as proposed by King (1986).

2.1.1.1 Lightweight concrete

A constitutive model for members made of lightweight concrete is also available. The model is the proposed by Kowalsky et al. (2000) and is based on the traditional Mander model for normal weight concrete. It is well known that the confined concrete compression strength for lightweight concrete is significantly less than for normal weight concrete. The confined concrete compression strength developed by Mander et al. over predicts the strength of confined lightweight concrete. In the modified model instead of use equation 7, it is suggested to calculate the confined strength as: $f'_{cc} = f'_{co}\left(1+2\,f'l/f'_{co}\right)$. It is also recommended to use a strain at maximum unconfined stress of 0.004 and to calculate the concrete elastic modulus based on the unit weight of the concrete.

2.1.2 Model for the Unconfined Concrete

The unconfined concrete follow the same curve that the confined concrete (equation 1) with a lateral confined stress $f'_l = 0$. The part of the falling branch for strains larger than $2\varepsilon_o$ is assumed to be a straight line which reaches zero at ε_{sp}.

2.1.3 King model for the Reinforcing Steel

The stress-strain relation for the reinforcing steel (figure 2a) is the same used by the King program (1986)

$$
\begin{aligned}
f_s &= E_s\varepsilon_s && \varepsilon_s \le \varepsilon_y \\
f_s &= f_y && \varepsilon_y < \varepsilon_s < \varepsilon_{sh} \\
f_s &= f_y\left[\frac{m(\varepsilon_s-\varepsilon_{sh})+2}{60(\varepsilon_s-\varepsilon_{sh})+2}+\frac{(\varepsilon_s-\varepsilon_{sh})(60-m)}{2(30r+1)^2}\right] && \varepsilon_{sh} < \varepsilon_s \le \varepsilon_{sm}
\end{aligned}
\tag{15}
$$

where:

$$m = \frac{(f_{su}/f_y)(30r+1)^2-60r-1}{15r^2} \tag{16}$$

$$r = \varepsilon_{su} - \varepsilon_{sh} \tag{17}$$

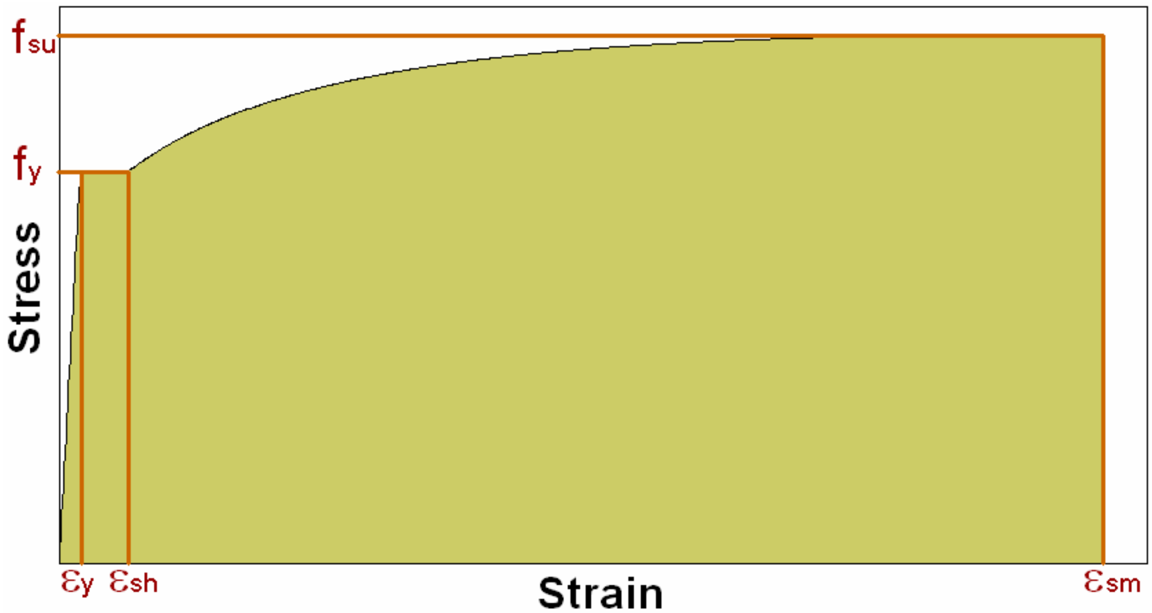

Figure 2a. King constitutive model for the reinforcing steel

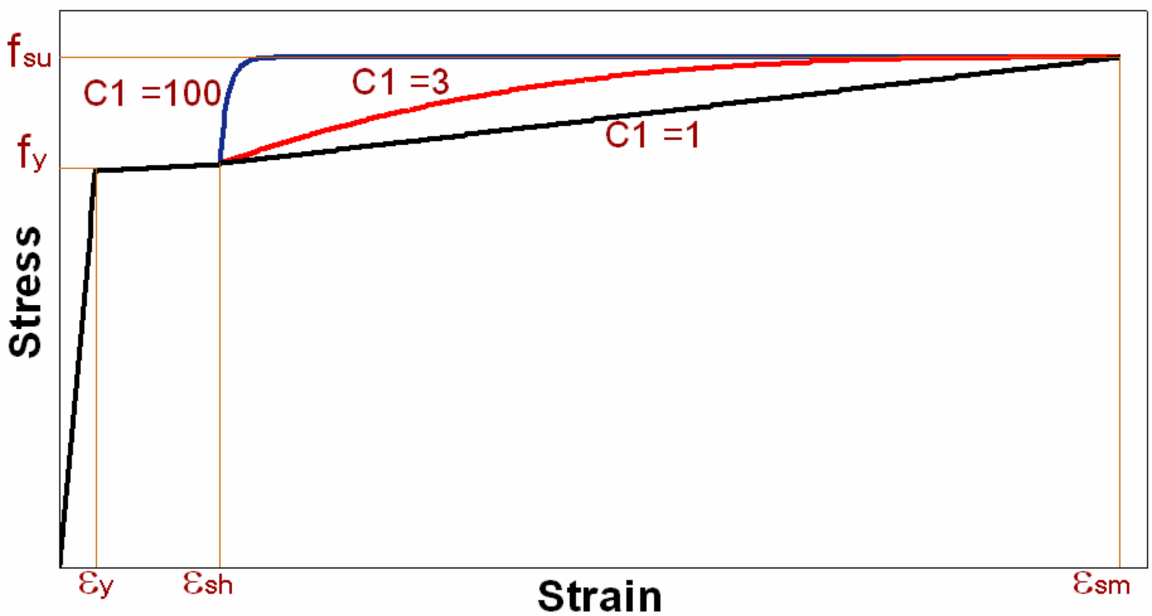

Figure 2b. Raynor constitutive model for the reinforcing steel

2.1.4 Raynor Model for the Reinforcing Steel

As proposed by Raynor et al. (2002):

$$
\begin{aligned}
f_s &= E_s \varepsilon_s && \varepsilon_s \le \varepsilon_y \\
f_s &= f_y + (\varepsilon_s - \varepsilon_y) E_y && \varepsilon_y < \varepsilon_s < \varepsilon_{sh} \qquad (15b) \\
f_s &= f_u - (f_u - f_{sh}) \left(\frac{\varepsilon_{sm} - \varepsilon_s}{\varepsilon_{sm} - \varepsilon_{sh}} \right)^{C1} && \varepsilon_{sh} < \varepsilon_s \le \varepsilon_{sm}
\end{aligned}
$$

where:

$$\varepsilon_y = \frac{f_y}{E} \tag{16b}$$

$$f_{sh} = f_y + \left(\varepsilon_{sh} - \varepsilon_y\right)E_y \tag{17b}$$

E_y is the slope of the yield plateau and C1 is the parameter that defines the curvature of the strain hardening curve (Figure 2b).

2.2 User Specified Material Models

Additional to the default material models the user can also specify its own material model. The models should be predefined and saved in a text (.txt) file. The text file should consist of two columns, one with the strains and the other with the corresponding stresses. Perhaps, the file containing a bilinear elastoplastic model for the steel should looks like:

0	0
ε_y	f_y
ε_{sm}	f_y

The files with the user defined material models should be saved in the folder "models". This folder already contains three examples with bilinear models for the unconfined and confined concrete and the reinforcing steel. When using materials models specified by the user, the values of the unconfined concrete compressive stress, yielding steel stress and ultimate steel stress need to be specified.

3. SECTION ANALYSIS

The section analysis is performed by tabulating moment and curvature of the member section for increasing levels of concrete strain, an iterative procedure is used in order to find the depth of the neutral axis to satisfy equilibrium at each level of concrete strain. The program stops when the concrete strain in the core exceeds the maximum concrete compressive strain, the tension strain in the steel bars exceeds the maximum steel strain or there is a suddenly lost of strength. If desired the user can also specify the code to perform an axial load – moment interaction analysis (if this option is specified the program could take few minutes to run), the strain limits to generate the integration diagram are specified by the user.

The program may also stops if the maximum number of iteration is reached or if the specified number of concrete layers is too high. The default maximum number of iterations is 1000 and the default number of concrete layers is 40. The user can change these values directly from the code by changing the values in the variables "*itermax*" and "*ncl*" (figure 3). If this does not solve the problem, the allowable "*tolerance*" can also be changed.

```

% ===========================================================================================
% ===========================================================================================
% ====================              END OF INPUT DATA             ===========================
% ===========================================================================================
% ===========================================================================================



% control parameters:

itermax     = 1000;                     % max number of iterations
ncl         = 40;                       % # of concrete layers
tolerance   = 0.001;                    % x fpc x Ag
dels        = 0.0001;                   % delta strain for default material models

```

Figure 3. Control parameters

4. MEMBER RESPONSE

The member response is obtained using the plastic hinge method proposed by Priestley, Seible and Calvi (1996). The plastic hinge method replaces the real curvature distribution with an equivalent curvature distribution in order to facilitate the application of the moment area method to find the displacements in the member (figure 4). The length of the equivalent plastic hinge (L_p) is defined as the length over which the maximum curvature can be assumed to be constant. Equations 18 and 19 are used to calculate the equivalent plastic hinge length and the strain penetration length (L_{sp}), respectively.

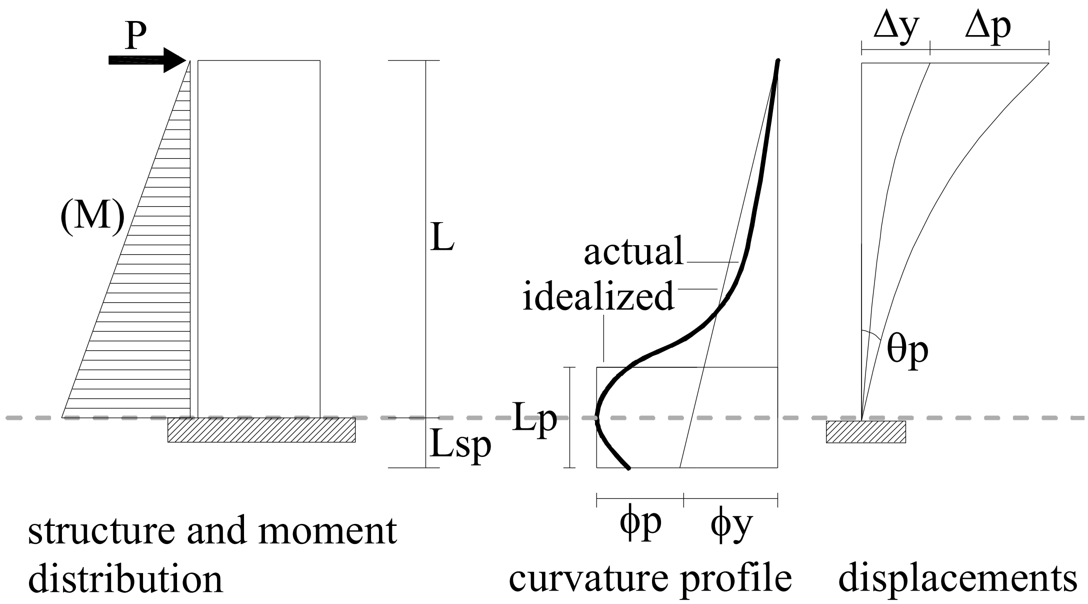

Figure 4. Plastic hinge method

$$L_p = kL_c + L_{sp} \geq 2L_{sp} \tag{18}$$

$$L_{sp} = 0.022 f_s d_{bl} \quad f_s \leq fy \tag{19}$$

where:

L_c: length from the critical section to the point of contraflexure

f_s : tension stress in the longitudinal bars

f_y : longitudinal bars yielding stress

d_{bl} : longitudinal bars diameter

$$k = 0.2\left(\frac{f_{su}}{f_y}\right) - 1 \le 0.08$$

An effective length of the member is then defined as:

$$L_{eff} = L + L_{sp} \qquad \text{single bending} \tag{20}$$

$$L_{eff} = L + 2L_{sp} \qquad \text{double bending} \tag{21}$$

Flexural displacement before cracking is calculated as $\Delta_f = L^2/3$ or $\Delta_f = L^2/6$ for single and double bending, respectively. The flexural displacement of the member before yielding and after cracking is calculated as:

$$\Delta_f = \frac{\phi L_{eff}^{\ 2}}{3} \qquad \text{single bending} \tag{22}$$

$$\Delta_f = \frac{\phi L_{eff}^{\ 2}}{6} \qquad \text{double bending} \tag{23}$$

The flexural displacement beyond yield is given by:

$$\Delta_f = \left[\phi - \phi_y^{'}\frac{M}{M_y^{'}}\right] L_p\left(L + L_{sp} - 0.5L_p\right) + \Delta_y^{'}\frac{M}{M_y^{'}} \qquad \text{single bending}$$

$$\Delta_f = \left[\phi - \phi_y^{'}\frac{M}{M_y^{'}}\right] L_p\left(L + 2\left(L_{sp} - 0.5L_p\right)\right) + \Delta_y^{'}\frac{M}{M_y^{'}} \qquad \text{double bending} \tag{24}$$

where $\phi_y^{'}$, $M_y^{'}$ and $\Delta_y^{'}$ are the curvature, moment and displacement for the first yielding, respectively. Note that the yield displacement is scaled to the current moment, *M*, to account for additional column elastic flexibility.

A simplified approach for calculating the shear stiffness in the elastic range of response based on the cracked section stiffness suggested by Priestley et al. (2006) is used to calculate the shear deformations. The premise of this approach is that the elastic shear stiffness is reduced approximately in proportion to the flexural stiffness. After shear cracking and before the section reaches its nominal moment, shear deflections are computed by considering the shear flexibility of an equivalent strut-and-tie model. The procedure is described in equations 25 to 35 where A_g and I_g are the gross area and moment of inertia of the section.

$E_c = 5000\sqrt{f'c} \quad (Mpa)$	concrete modulus of elasticity	(25)
$G = 0.43E_c$	concrete shear modulus	(26)
$A_s = 0.9A_g$	shear effective area (circular section)	(27)
$A_s = \frac{5}{6}A_g$	shear effective area (rectangular section)	(28)
$I_{eff} = \dfrac{M_y^{'}}{E_c\phi_y^{'}}$	effective moment of inertia	(29)
$k_{sg} = \dfrac{GA_s}{L}$	shear stiffness	(30)
$k_{seff} = k_{sg}\dfrac{I_{eff}}{I_g}$	effective shear stiffness	(31)

The shear deformations of the columns before shear cracking are then calculated using equation 32 where V is the shear force acting in the member. Shear cracking is assumed to occur when the applied shear is larger than the shear strength of the concrete V_c, which is obtained from Equation 39 with $\gamma = 0.29$ (i.e. the initial concrete strength).

$$\Delta_s = \frac{V}{k_{seff}}, \quad V < V_c \qquad (32)$$

After shear cracking occurs and before the nominal moment is reached, the shear stiffness is calculated using equation 33 or 34, where B and d are the effective width and depth of the section.

$$k_{scr} = \frac{0.25\rho_y}{0.25 + 10\rho_y} E_s Bd \qquad \text{rectangular sections} \tag{33}$$

$$k_{scr} = \frac{0.25(\pi\rho_s/8)}{0.25 + 10(\pi\rho_s/8)} E_s 0.64D^2 \qquad \text{circular sections} \tag{34}$$

The shear displacement beyond yield is assumed to increase proportional to the flexural displacement. The total displacement in the member is given then by equation 35.

$$\Delta = \Delta_f + \Delta_s \tag{35}$$

5. SHEAR CAPACITY

The shear strength envelope for the member is calculated using the revised UCSD shear model (Kowalsky and Priestley, 2000). The original UCSD model was the first model for assessment of shear strength that included: (1) the effect of the axial load separate from the concrete strength and (2) the degrading of concrete strength with ductility. The revised model intends to take also into account: (1) the effect of concrete compression zone on the mobilization of transverse steel and (2) the influence of the aspect ratio and the longitudinal steel ratio in the shear strength of the concrete. The model expresses the shear strength capacity of the member as the sum of three separate components as shown in equation 36. V_s represents the shear capacity attributed to the steel truss mechanisms, V_p represents the strength attributed to the axial load and V_c represents the strength of the concrete shear resisting mechanism.

$$V = V_s + V_p + V_c \tag{36}$$

$$V_s = \frac{\pi}{2} A_{sp} f_{yh} \frac{D - clb + \frac{d_h}{2} - c}{s} \cot(\theta) \qquad \text{circular section} \tag{37}$$

$$V_s = A_{sx} f_{yh} \frac{H - clb + \frac{d_h}{2} - c}{s} \cot(\theta) \qquad \text{rectangular section} \tag{38}$$

where clb is the cover to the longitudinal bar, d_h is the diameter of the transverse steel and c is the depth of the neutral axis at M_n.

$$V_s = \alpha\beta\gamma\sqrt{f'_c}\left(0.8A_g\right) \tag{39}$$

$$1 \le \alpha = 3 - \frac{M}{VD} \le 1.5 \tag{40}$$

$$\beta = 0.5 + 20\rho_l \le 1 \tag{41}$$

$$0.05 \le \gamma = 0.37 - 0.04\mu_\Delta \le 0.29 \qquad \text{uniaxial bending} \tag{42}$$

$$0.05 \le \gamma = 0.33 - 0.04\mu_\Delta \le 0.29 \qquad \text{biaxial bending} \tag{43}$$

In equations 42 and 43 the yield displacement used to calculate the displacement ductility is calculated taking into account only the flexural deformation. However, the total displacement is calculated taking into account flexural and shear deformations. The variable *M/VD*, where *M* is the moment and *V* the shear at the critical section, is equivalent to the aspect ratio L_c/D, where L_c is the distance from the critical section to the point of contraflexure. For the single bending case $L_c=L$ and for the double bending case it is usually assumed $L_c=L/2$. The variables ρ_l and μ_Δ represent the longitudinal steel ratio and the displacement ductility, respectively.

$$V_p = P\frac{D-c}{2L} \quad P>0 \qquad \text{single bending} \tag{44}$$

$$V_p = P\frac{D-c}{L} \quad P>0 \qquad \text{double bending} \tag{45}$$

$$V_p = 0 \quad P<0 \tag{46}$$

Equations 36 to 46 are used for the assessment of the shear strength of existing structures. For the design of new structures a more conservative approach is used: the axial load is reduced by 15%, the angle of the flexure-shear crack is incremented to 35˚ and a shear strength reduction factor of 0.85 is applied.

6. BUCKLING MODEL

The assessment of reinforcement buckling limit state is implemented in the code following two different methodologies, one is the proposed by Moyer and Kowalsky (2003) and the other is the proposed by Berry and Eberhard (2005). In the Moyer-Kowalsky model, the characteristic compression strain capacity is defined by equation 43. From experimental results the growth strain ε_{sgr} was determined to be 50% of the peak strain after a curvature ductility of 4, linear interpolation was proposed for the evaluation of growth strain between curvature ductilities of 1 (where growth strain is zero) and 4. The allowable steel compression strain is then defined as the characteristic compression strain minus the growth strain (equation 44). The on set of buckling is the defined as the point where the tension strain in the column reach the allowable tension strain (figure 5).

$$\varepsilon_{scc} = -3\left(\frac{s}{d_{bl}}\right)^{-2.5} \tag{47}$$

$$\varepsilon_{sfl} = \varepsilon_{scc} - \varepsilon_{sgr} \tag{48}$$

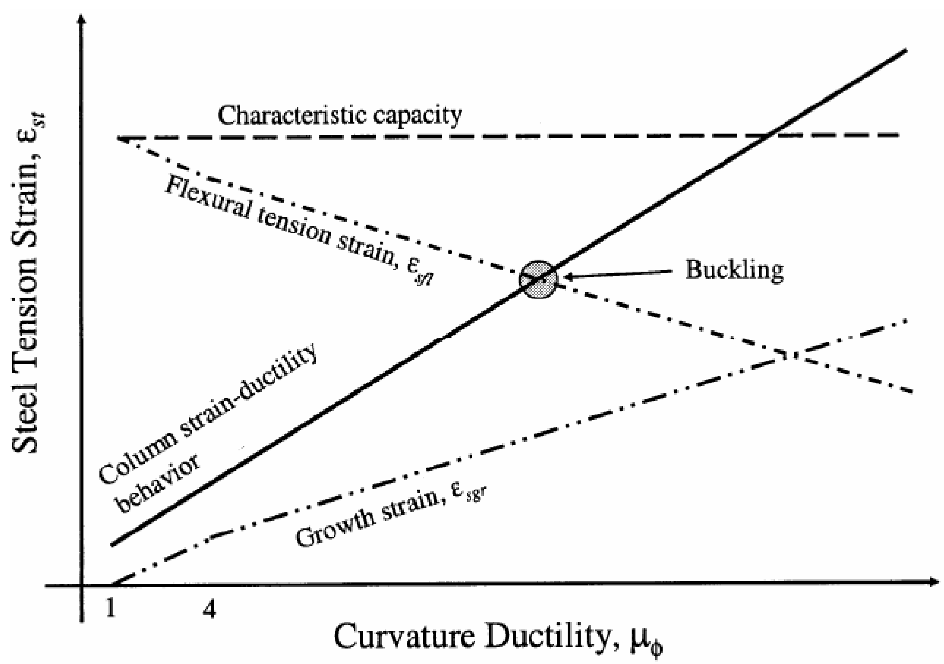

Figure 5. Assessment of reinforcement buckling limit state (from Moyer and Kowalsky, 2003)

In the Berry-Eberhard model, the plastic rotation at the onset of bar buckling is defined as:

$$\theta_{pbb} = C_0 \left(1 + C_1 \rho_{eff}\right) \left(1 + C_2 \frac{P}{A_g f_c^{'}}\right)^{-1} \left(1 + C_3 \frac{L}{D} + C_4 \frac{f_y d_{bl}}{D}\right) \tag{49}$$

$C_0 = 0.019 \quad C_1 = 1.650 \quad C_2 = 1.797 \quad C_3 = 0.012 \quad C_4 = 0.072$ rectangular sections

$C_0 = 0.006 \quad C_1 = 7.190 \quad C_2 = 3.129 \quad C_3 = 0.651 \quad C_4 = 0.227$ circular sections

where $\rho_{eff} = \dfrac{f_{yh}}{f_c^{/}} \rho_s$ and buckling occurs when the plastic rotation in the member reach the value of θ_{pbb}.

7. LIMIT STATES

Deformation limit states are calculated based on the material strain. The default limit values used are presented in the next table. Nonetheless, the user can specify its own limit values.

Limit State	**Concrete strain limit**	**Steel strain limit**
Serviceability	0.004 (compression)	0.015 (tension)
Damage Control	$\frac{2}{3}\varepsilon_{cu}$	0.060 (tension)

8. THE OUTPUT FILE

The output of the program is an .xls file which can be accessed with any *EXCEL* type program. The output file can be divided in seven different parts: (1) section and member properties (figure 6), (2) complete section and member analysis and bilinear approximation (figure 7), (3) key values from the section and member analysis (figure 8), (4) shear failure information (figure 9), (5) buckling information (figure 10), (6) deformation limit states (figure 11), (7) information for non-linear time history analysis (figure 12) and (8) axial load – moment interaction analysis (figure 13).

7.1 Section and Member Properties

In this section the geometrical properties and reinforcement details of the member being analyzed is display. The longitudinal steel ratio is defined as the area of longitudinal steel over the gross area of the section. The transverse steel ratio is defined by equation 9 for circular sections. In the case of rectangular sections an average transverse steel ration of the ratios defined in equation 13 is displayed. The axial load ratio is defined as:

$$ALR = \frac{P}{f_c^{'} A_g} \tag{50}$$

3	Circular Section
4	
5	Diameter: 1000.0 mm
6	cover to longitudinal bars: 50.0 mm
7	number of longitudinal bars: 22
8	diameter of longitudinal bars: 25.0 mm
9	diameter of transverse steel: 9.0 mm
10	spacing of transverse steel: 120.0 mm
11	type of tranverse reinforcement: spirals
12	axial load: 400.00 kN
13	concrete compressive strength: 35.00 MPa
14	long steel yielding stress: 460.00 MPa
15	long steel max. stress: 620.00 MPa
16	transverse steel yielding stress: 400.00 MPa
17	Member Length: 3000.0 mm
18	Single Bending
19	Biaxial Bending
20	Longitudinal Steel Ratio: 0.014
21	Transverse Steel Ratio: 0.002
22	Axial Load Ratio: 0.015

Figure 6. Output file – section and member properties

7.2 Complete Section and Member Analysis and Bilinear Approximation

The results obtained following the procedures described in sections 3 to 5 are tabulated in this section. The table presents the following information:

Column 1: Cover concrete compression strain (Notice that values greater than the maximum unconfined concrete compression strain are displayed. However, once the maximum value is reached, the program considers that the cover concrete is not longer there).

Column 2: Core concrete compression strain.

Column 3: Neutral axis depth in mm.

Column 4: Tensile steel strain in mm.

Column 5: Moment of the critical section in kN-m.

Column 6: Curvature of the critical section in 1/m.

Column 7: Lateral force applied in kN.

Column 8: Shear displacement in m.

Column 9: Flexural displacement in m.

Column 10: Total displacement at the top of the member in m.

Column 11: Shear capacity in kN (for assessment of existing structures).'

Column 12: Shear capacity in kN (for design of new structures).

24	Cover	Core	N.A	Steel	Moment	Curvature	Force	Sh displ.	Fl displ.	Total displ.	Shear(asse	Shear(design)
25	Strain	Strain	[mm]	Strain	[kN-m]	[1/m]	[kN]	[m]	[m]	[m]	[kN]	[kN]
26	0	0	0	0	0	0	0	0	0	0	1322.53	954.65
27	0.0001	0.00009	451.25	-0.00011	175.42	0.00022	58.47	0.00007	0.00078	0.00085	1322.53	954.65
28	0.0002	0.00017	349.17	-0.00034	324.73	0.00057	108.24	0.00012	0.00202	0.00214	1322.53	954.65
29	0.0003	0.00026	315.12	-0.00059	462.48	0.00095	154.16	0.00017	0.00336	0.00353	1322.53	954.65
30	0.0004	0.00034	307.33	-0.00082	641.98	0.0013	213.99	0.00024	0.00459	0.00483	1322.53	954.65
31	0.0005	0.00042	298.24	-0.00107	791.38	0.00168	263.79	0.0003	0.00591	0.00621	1322.53	954.65
32	0.0006	0.00051	293.06	-0.00132	942.06	0.00205	314.02	0.00035	0.00722	0.00757	1322.53	954.65
33	0.0007	0.00059	289.42	-0.00157	1088.58	0.00242	362.86	0.00041	0.00853	0.00894	1322.53	954.65
34	0.0008	0.00067	286.53	-0.00182	1229.78	0.00279	409.93	0.00046	0.00985	0.01031	1322.53	954.65
35	0.0009	0.00076	285.1	-0.00206	1376.56	0.00316	458.85	0.00051	0.01114	0.01165	1322.53	954.65
36	0.001	0.00084	284.39	-0.0023	1522.82	0.00352	507.61	0.00057	0.0124	0.01297	1322.53	954.65
37	0.0011	0.00092	280.85	-0.00257	1629.05	0.00392	543.02	0.00061	0.01388	0.01448	1322.53	954.65
38	0.0012	0.001	275.97	-0.00288	1710.57	0.00435	570.19	0.00064	0.0152	0.01583	1322.53	954.65
39	0.0013	0.00108	270.5	-0.00321	1770.31	0.00481	590.1	0.00066	0.01638	0.01704	1322.53	954.65
40	0.0014	0.00116	265.14	-0.00355	1814.19	0.00528	604.73	0.00068	0.01745	0.01813	1320.82	953.4
41	0.0015	0.00124	260.53	-0.0039	1864.28	0.00576	621.43	0.0007	0.01859	0.01928	1313.39	947.96
42	0.0016	0.00131	254.73	-0.00429	1891.13	0.00628	630.38	0.00071	0.0196	0.02031	1306.79	943.12
43	0.0018	0.00147	246.57	-0.00504	1951.02	0.0073	650.34	0.00073	0.02164	0.02236	1293.52	933.4
44	0.002	0.00162	238.68	-0.00586	1989.17	0.00838	663.06	0.00074	0.02359	0.02433	1280.86	924.12
45	0.0025	0.00199	223.93	-0.00797	2056.8	0.01116	685.6	0.00077	0.02836	0.02913	1249.87	901.42
46	0.003	0.00236	212.73	-0.01022	2068.68	0.0141	689.56	0.00077	0.03292	0.03369	1220.45	879.86
47	0.0035	0.00274	208.51	-0.01224	2155.58	0.01679	718.53	0.0008	0.0377	0.03851	1189.41	857.11
48	0.004	0.00312	206.43	-0.01417	2192.2	0.01938	730.73	0.00082	0.04193	0.04275	1162.03	837.05
49	0.0045	0.00351	205.92	-0.01599	2223.5	0.02185	741.17	0.0009	0.04595	0.04684	1135.63	817.71
50	0.005	0.00389	205.4	-0.01782	2237.35	0.02434	745.78	0.00097	0.04984	0.05081	1110.04	798.96
51	0.006	0.00468	207.21	-0.02115	2256.39	0.02896	752.13	0.00111	0.057	0.05811	1062.96	764.47
52	0.007	0.00551	213.25	-0.02377	2259.76	0.03282	753.25	0.00123	0.0629	0.06413	1024.17	736.05
53	0.008	0.00632	216.75	-0.0266	2254.08	0.03691	751.36	0.00135	0.06905	0.0704	983.71	706.4
54	0.009	0.00714	220.3	-0.0293	2271.55	0.04085	757.18	0.00147	0.07518	0.07665	943.4	676.86
55	0.01	0.00796	222.79	-0.03208	2282.41	0.04489	760.8	0.00159	0.08139	0.08298	902.57	646.95
56	0.012	0.0096	227.57	-0.03743	2293	0.05273	764.33	0.00182	0.09339	0.09521	823.7	589.16
57	0.014	0.01127	233.05	-0.04232	2315.45	0.06007	771.82	0.00204	0.10472	0.10676	749.2	534.57
58												
59	Bilinear Approximation:											
60												
61	Curvature	Moment	Displ.	Force								
62	[1/m]	[kN-m]	[m]	[kN]								
63	0	0	0	0								
64	0.00506	2192.2	0.01868	730.73								
65	0.06007	2315.45	0.10676	771.82								

Figure 7. Output file – complete section and member response and bilinear approximation

In the bilinear approximation the value for the equivalent yielding curvature is extrapolated from the value of the first yielding curvature:

$$\phi_y = \phi_y^{'} \frac{M_n}{M_y^{'}} \tag{51}$$

where M_n is the nominal moment of the section and is defined as the moment at which the cover concrete compression strain is 0.04 or the steel tension strain is 0.015, whichever occurs first.

7.3 Key Values

Some important values from the section and member response are presented in this part. Take into account that up to this moment none shear or buckling assessment have been done, so this values are "potential values" that will be acceptable only if a shear or buckling failure does not occur.

67	*** concrete strain exceeds maximum ***				
68					
69	Moment for First Yielding: 1524.15 kN-m				
70	Curvature for First Yielding: 0.00352 1/m				
71	Potential Section Nominal Moment: 2192.20 kN-m				
72	Equivalent Curvature: 0.00506 1/m				
73	Potential Section Curvature Ductility: 11.86				
74	Potential Displacement Ductility: 5.71				

Figure 8. Output file – key values

7.4 Shear Failure Information

If a shear failure occurs, the type of shear failure (ductile or brittle) will be indicated in this section. The force, moment, displacement, curvature and ductility at the point of failure are also displayed.

76	*** shear failure at some ductility ***				
77					
78	Displacement for Shear Failure: 0.10357 m				
79	Displacement Ductility at Shear Failure: 5.54				
80	Force for Shear Failure: 769.75 kN				
81	Curvature for Shear Failure: 0.05805 1/m				
82	Curvature Ductility at Shear Failure: 11.46				
83	Moment for Shear Failure: 2309.25 kN-m				

Figure 9. Output file – shear failure information

7.5 Buckling Failure Information

If buckling is predicted to occur by any of the implemented models; the force, moment, displacement, curvature and ductility at the point of failure is presented in this section.

85	Moyer - Kowalsky buckling model:
86	
87	Curvature Ductility for Buckling: 11.06
88	Curvature at Buckling: 0.05602 m
89	Displacement Ductility at Buckling: 5.37
90	Displacement at Buckling: 0.10038 m
91	Force for Buckling: 767.68 kN
92	Moment for Buckling: 2303.05 kN

Figure 10. Output file – buckling failure information

7.6 Potential limit states

All the information regarding the potential deformation limit states of the member being analyzed is displayed in this section. They are called "potential" because the structure will only reach those limit states if no buckling or shear happen before. The ultimate concrete strain predicted by the original Mander model is also displayed at the end of this section.

96	== Potential Deformation Limit States (serviceability/damage control/ultimate) ==							
97								
98	Cover	Steel	Moment	Force	Curvature	Curvature	Displaceme	Displacement
99	Strain	Strain	[kN-m]	[kN]	[1/m]	Ductility	[m]	Ductility
100	0.004	-0.01417	2192.2	730.73	0.01938	3.83	0.04275	2.29
101	0.00807	-0.02678	2255.24	751.75	0.03717	7.34	0.07082	3.79
102	0.014	-0.04232	2315.45	771.82	0.06007	11.86	0.10676	5.71
103								
104	Deformation Limit States Citeria :							
105	serviceability concrete strain: 0.0040							
106	serviceability steel strain: -0.0150							
107	damage control concrete strain: 0.0081							
108	damage control steel strain: -0.0600							
109								
110	Original Mander Model Ultimate Concrete Strain: 0.0081							

Figure 11. Output file – potential limit states

7.7 Information for non-linear THA

This section presents the information that is usually required if a non-linear time history analysis is wanted to be performed.

112	for non-linear THA:
113	
114	E: 29580398915.50 Pa
115	G: 12719571533.66 Pa
116	A: 0.7854 m2
117	I: 0.014632 m4
118	Bi-Factor: 0.005
119	Hinge Length: 0.506 m
120	Tension Yield: 4967643.38 N
121	Compression Yield: 31666143.41 N
122	Moment Yield: 2192199.87 N-m

Figure 12. Output file – information for non-linear THA

7.8 Axial load – moment interaction analysis

If this kind of analysis is specified this section will present the information of the interaction surface. First column corresponds to the moment in kN-m and the second column to the axial load in kN. Important values that are usually required when performing a non-linear THA are also listed at the end. The interaction surface will correspond to the concrete and steel strain limits specified by the user.

129	Interaction Surface			
130				
131	Moment	Axial Load		
132	[kN-m]	[kN]		
133	0	-4967.64		
134	337.15	-4470.88		
135	962.81	-2980.59		
136	1550.55	-1490.29		
137	2074.59	0		
138	2443.76	1374.45		
139	2786.79	2748.89		
140	3095.45	4123.34		
141	3363.08	5497.79		
142	3577.78	6872.23		
143	3752.19	8246.68		
144	3881.29	9621.13		
145	3961.77	10995.57		
146	3980.74	12370.02		
147	3946.46	13744.47		
148	3859.79	15118.91		
149	3750.15	16493.36		
150	3614.96	17867.81		
151	2962.5	22166.3		
152	2280.53	25332.91		
153	1394.24	28499.53		
154	0	31666.14		
155				
156	NLTHA Approximation:			
157				
158	PTY: -4967.6 kN			
159	PCY: 31666.1 kN			
160	PB: 12370.0 kN		MB: 3980.7 kN-m	
161	(1/3)PB: 4123.3 kN		(1/3)MB: 3095.4 kN-m	
162	(2/3)PB: 8246.7 kN		(2/3)MB: 3752.2 kN-m	

Figure 13. Output file – interaction surface information

8. EXAMPLES

8.1 Example 1: *CUMBIACIR*

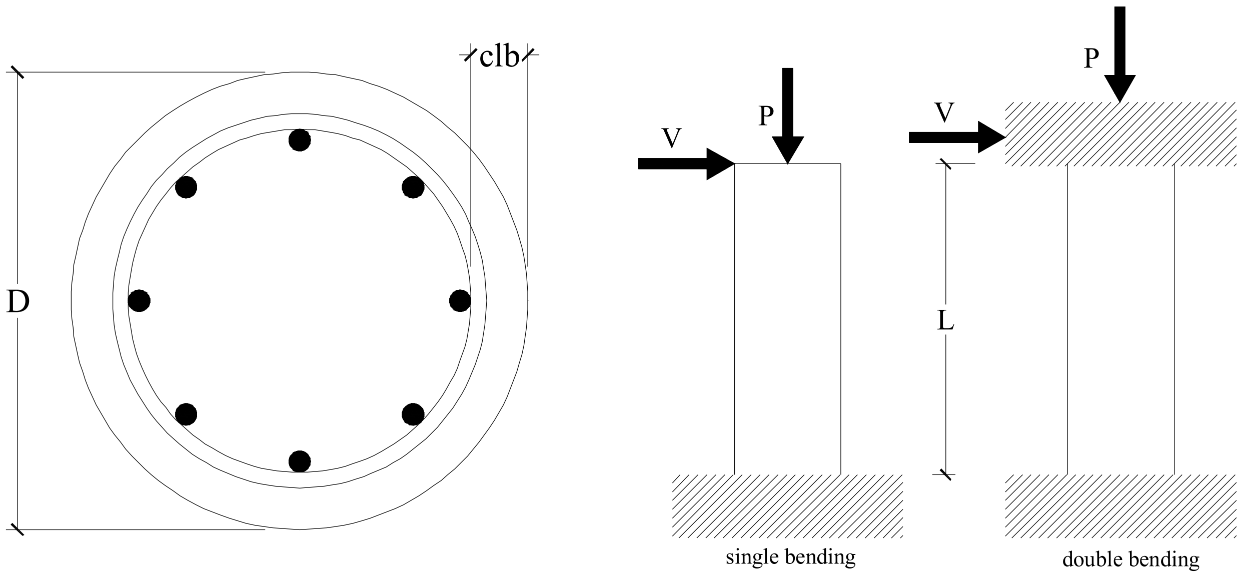

Figure 14. Section and member geometry for the circular column.

The first example is a circular column in single-biaxial bending and with an axial load in compression:

Section diameter: 1000 mm

Cover to longitudinal bars: 50 mm

Member clear length: 3000 mm

Single bending

Biaxial bending

# of longitudinal bars: 22

Diameter of longitudinal bars: 25 mm

Diameter of transverse reinforcement: 9 mm

Type of transverse reinforcement: Spirals

Spacing of transverse steel: 120 mm

Axial load: 400 kN Compression
Concrete compressive stress: 35 MPa
Longitudinal steel yielding stress: 460 MPa
Transverse steel yielding stress: 400 MPa
Steel modulus of elasticity: 200000 MPa
Material models: use default.

Since the default material models will be used, some other material properties are required:

Maximum steel stress (after stress hardening): 620 MPa
Steel strain for strain hardening: 0.008
Longitudinal steel ultimate strain: 0.12
Unconfined concrete strain: 0.002
Transverse steel ultimate strain: 0.12
Unconfined concrete maximum strain: 0.0064

Limit States: use default

The input data is entered in the program as displayed in figure 15. The program generates eight different plots (figures 16 to 23) with the materials constitutive models used, the moment-curvature and force-displacement response, the onset of buckling assessment and the axial load – moment interaction analysis. The output file generated will have the name CUMBIACIR EX 1.xls and can be found in the *CUMBIA* folder, this file is the same shown in figures 6 to 13. In this case the onset of buckling and shear failure are predicted to occur almost at the same displacement of 102 mm.

```
name = 'CUMBIACIR EX 1' %identifies actual work, the output file will be name.xls

interaction = 'y';  % if you want to also perform an axial load - moment interaction
                    % analysis type 'y', otherwise type 'n

% section properties:

D       = 1000;                  % section diameter (mm)
clb     = 50;                    % cover to longitudinal bars (mm)

% member properties

L             = 3000;                % member clear length (mm)
bending       = 'single';            % single or double
ductilitymode = 'biaxial';           % biaxial or uniaxial

% reinforcement details:

nbl     = 22;              % number of longitudinal bars
Dbl     = 25;              % long. bar diameter (mm)
Dh      = 9;               % diameter of transverse reinf. (mm)
type    = 'spirals';       % 'spirals' or 'hoops'*
s       = 120;             % spacing of transverse steel (mm)*

% aplieed loads:

P      =  400;               % axial load kN (-) tension (+)compression

% material models (input the 'name' of the file with the stress-strain relationship
% to use the default models: Mander model for confiend and unconfined concrete and
% King model for the steel, type 'mc', 'mu' or 'ks'):

confined   = 'mc';
unconfined = 'mu';
rebar      = 'ks';

% material properties

fpc     = 35;             % concrete compressive strength (MPa)
fy      = 460;            % long steel yielding stress (MPa)
fyh     = 400;            % transverse steel yielding stress (MPa)
Es      = 200000;         % steel modulus of elasticity

fsu     = 620;             % steel max stress (MPa)*
esh     = 0.008;           % steel strain for strain hardening*
esu     = 0.12;            % long. steel maximum strain*
eco     = 0.002;           % unconfined strain*
esm     = 0.12;            % max transv. steel strain*
espall  = 0.0064;          % max uncon. conc. strain*


% *this information is used only if the default material models are selected

% Deformation Limit States:

ecser = 0.004;     esser = 0.015;   % serviceability
ecdam = 'mander'; esdam = 0.060;   % damage control (to use the ultimate concrete
```

Figure 15. Input data for CumbiaCir example 1.

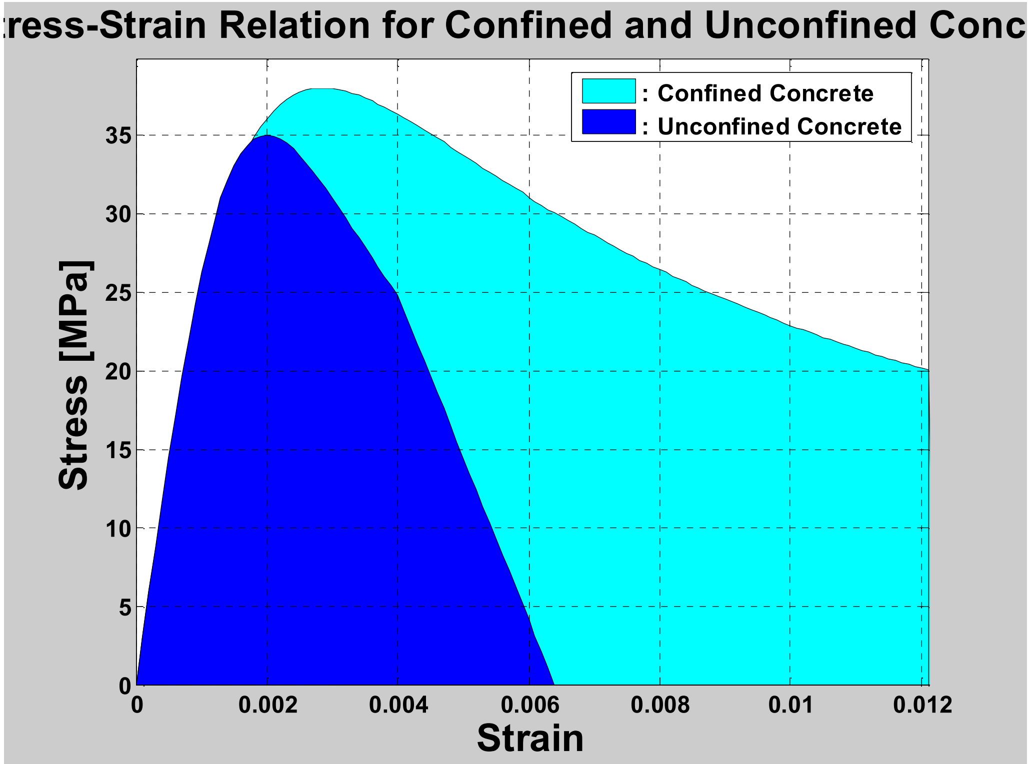

Figure 16. Model for the concrete - CumbiaCir example 1.

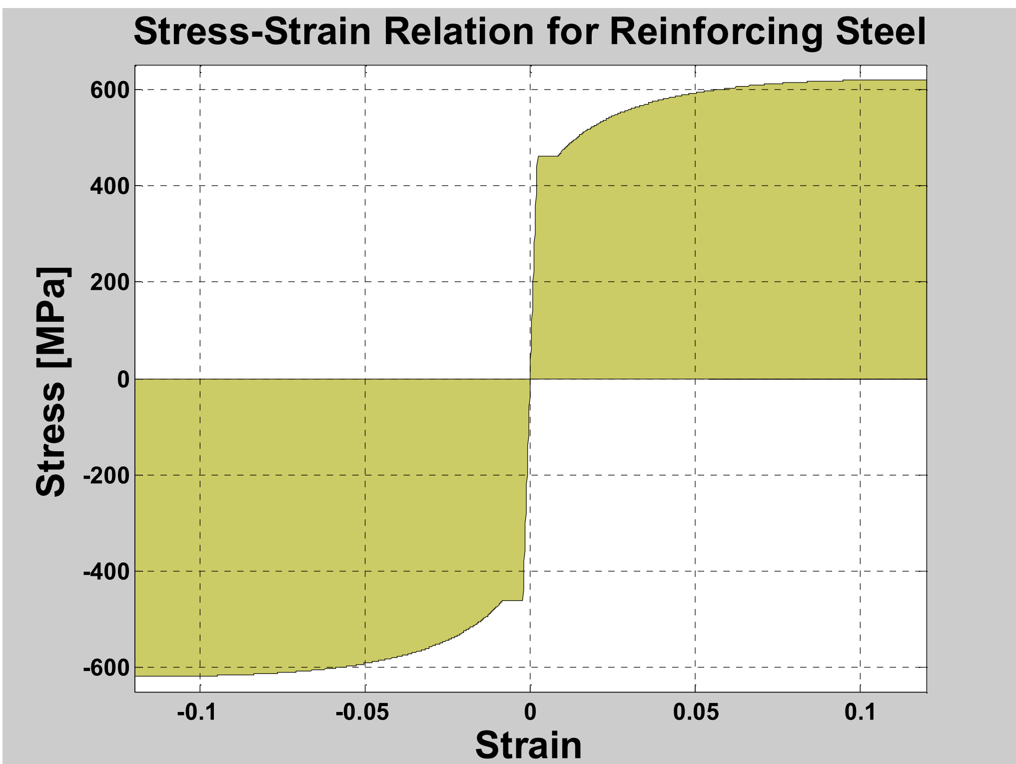

Figure 17. Model for the reinforcing steel - CumbiaCir example 1.

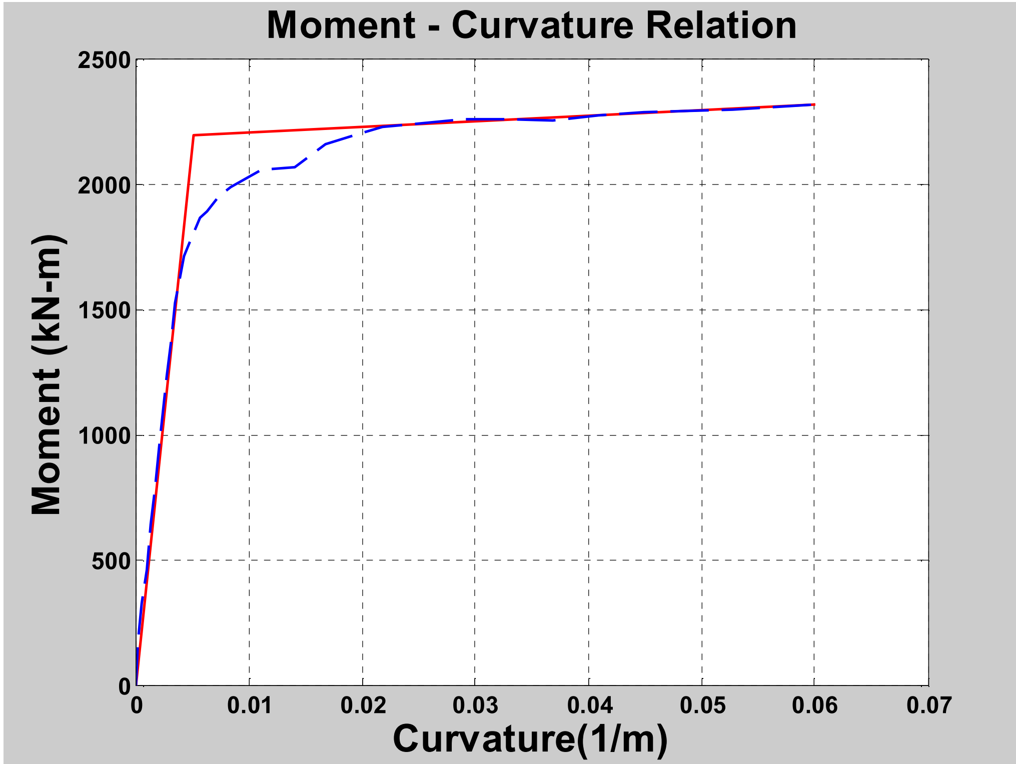

Figure 18. Moment-curvature relation - CumbiaCir example 1.

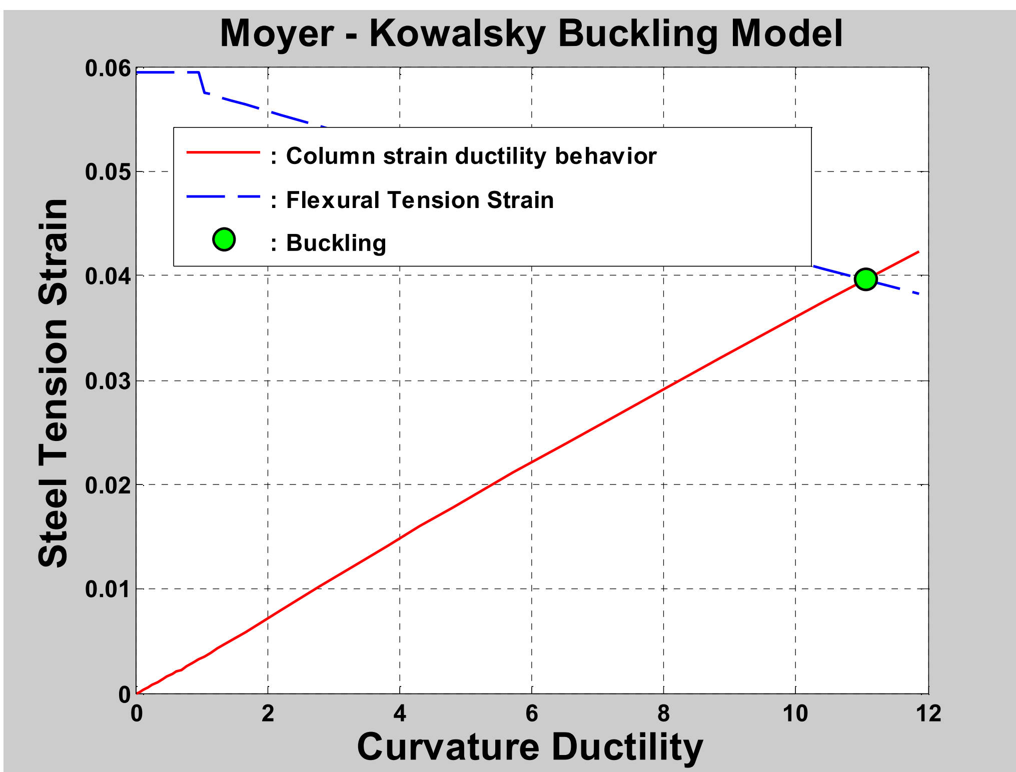

Figure 19. Assessment of buckling (Moyer-Kowalsky) - CumbiaCir example 1.

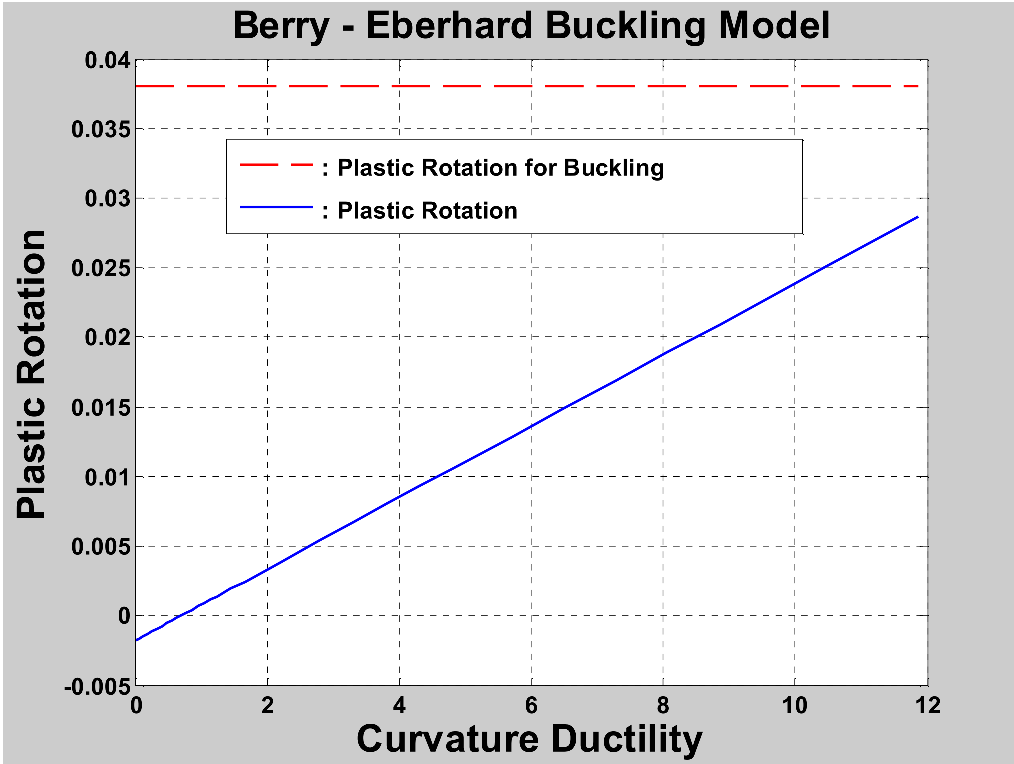

Figure 20. Assessment of buckling (Berry - Eberhard) - CumbiaCir example 1.

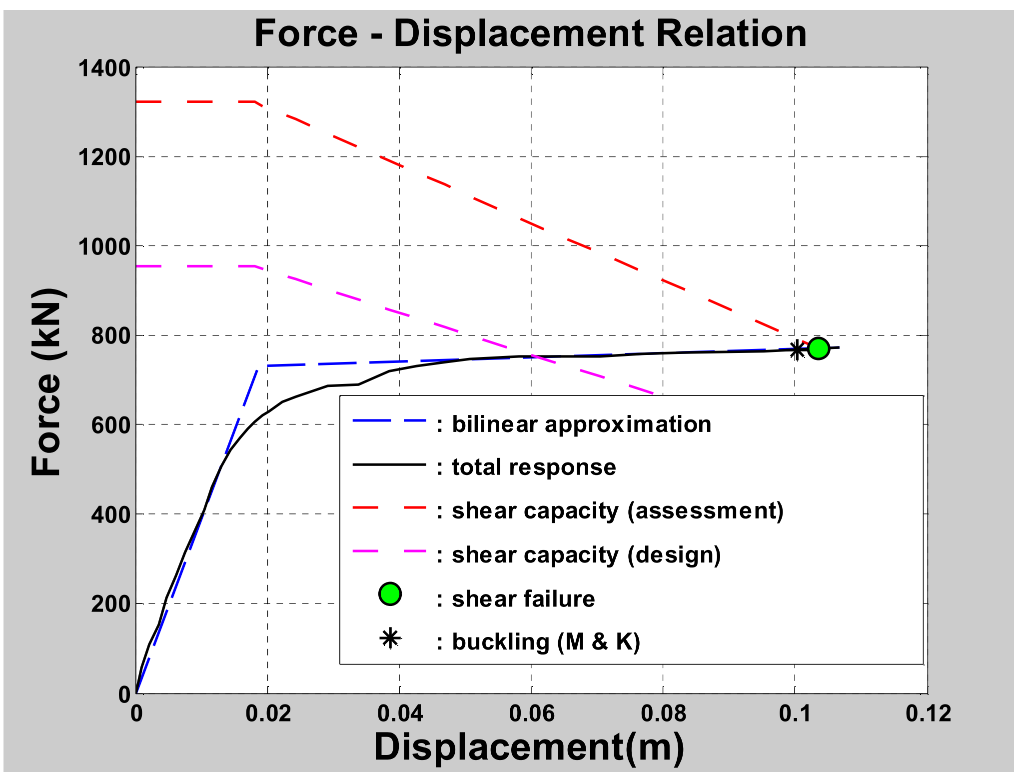

Figure 21. Member response - CumbiaCir example 1.

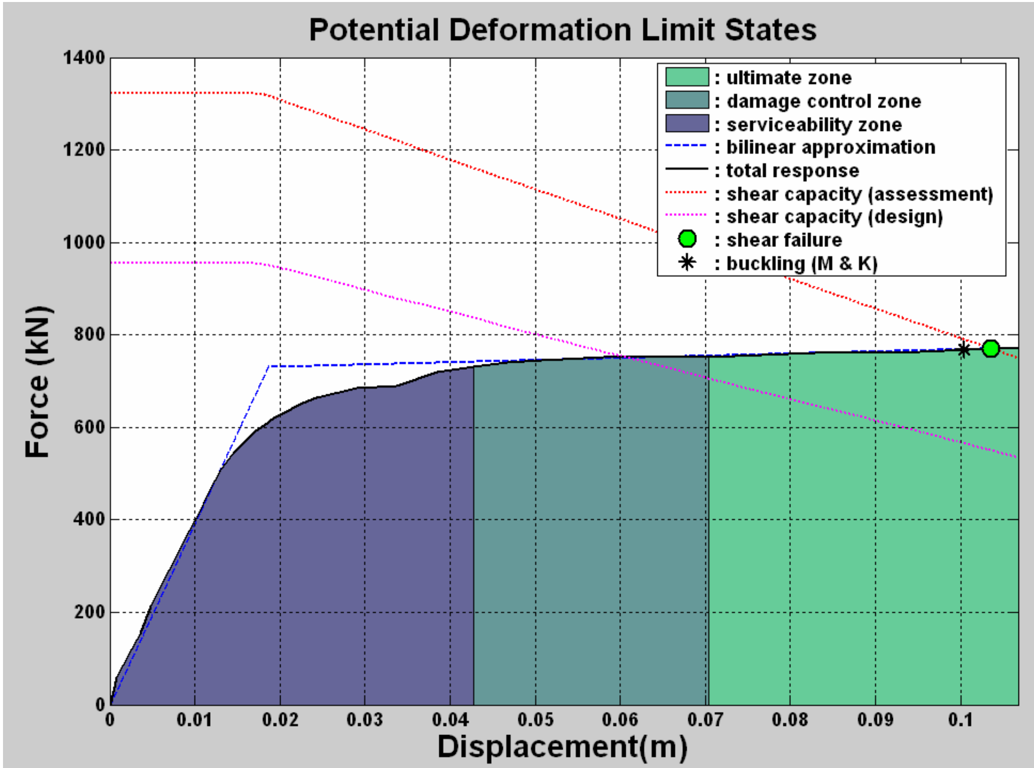

Figure 22. Potential deformation limit states - CumbiaCir example 1.

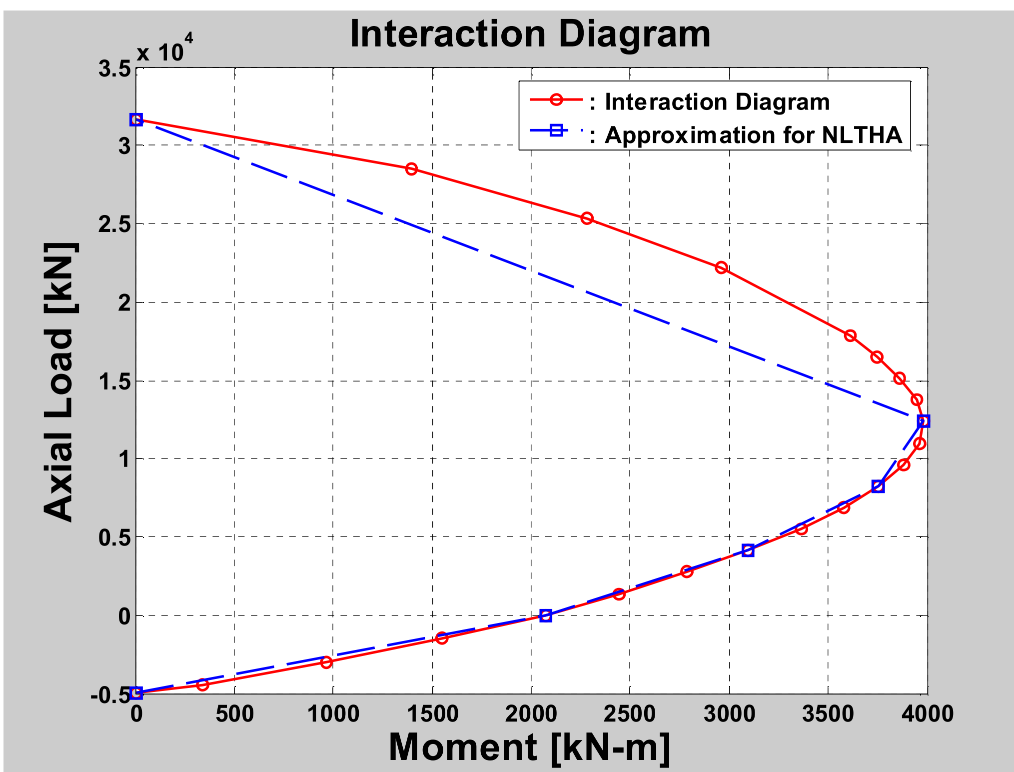

Figure 23. Interaction diagram - CumbiaCir example 1.

8.2 Example 2: *CUMBIARECT*

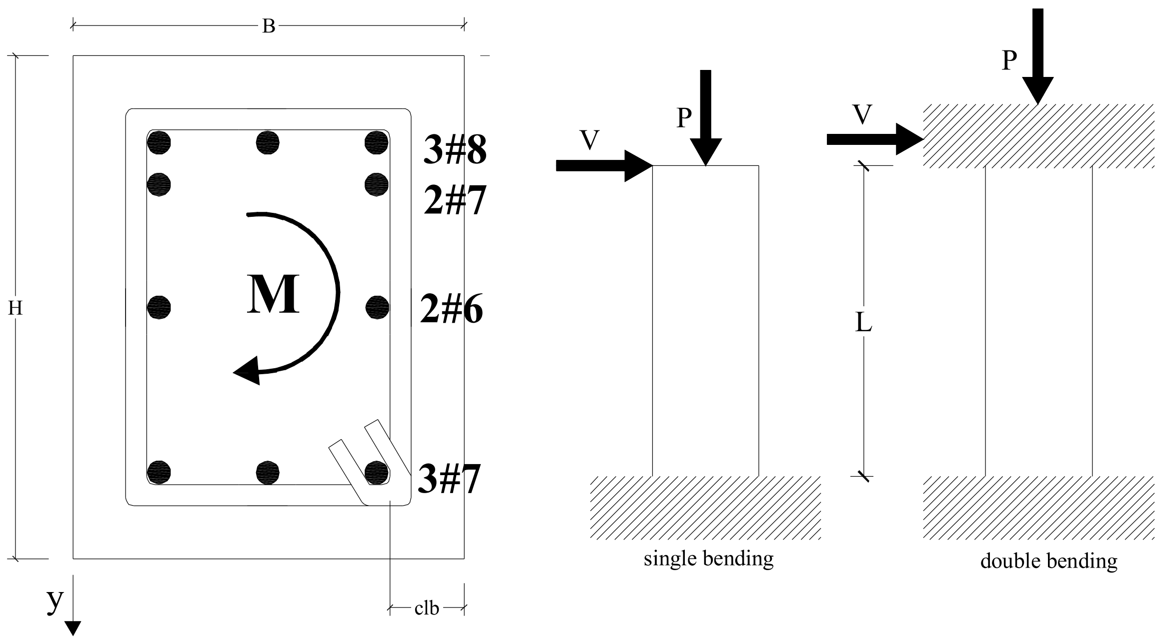

Figure 24. Section and member geometry for the rectangular column.

This example is a rectangular column in single-uniaxial bending and with an axial load in tension; the section geometry is displayed in Figure 24, noticed that tension bars are in the top of the section:

Section Dimension: 400 mm (H) x 300 mm (B)

Cover to longitudinal bars: 40 mm

Number of legs of the transverse steel in the x direction: 2

Number of legs of the transverse steel in the y direction: 2

Member clear length: 1200 mm

Single bending

Longitudinal steel (As shown)

Diameter of transverse reinforcement: 9.5 mm

Type of transverse reinforcement: Closed Hoops

Spacing of transverse steel: 120 mm

Axial load: 200 kN Tension

Concrete compressive stress: 28 MPa

Longitudinal steel yielding stress: 450 MPa

Transverse steel yielding stress: 400 MPa

Steel modulus of elasticity: 200000 MPa

Material models: use default.

Since the default material models will be used, some other material properties are required:

Maximum steel stress (after stress hardening): 600 MPa

Steel strain for strain hardening: 0.008

Longitudinal steel ultimate strain: 0.12

Unconfined concrete strain: 0.002

Transverse steel ultimate strain: 0.12

Unconfined concrete maximum strain: 0.0064

The input data is entered in the program as displayed in figure 25. Figures 26 and 27 show the results obtained. In this case the member fails by shear at a displacement of 63.5mm.

```
H     = 400;                                     % section height (mm)- perp to x
B     = 300;                                     % section width  (mm)- perp to y
ncx   = 2;                                       % # legs transv. steel x_dir (confinement)
ncy   = 2;                                       % # legs transv. steel y_dir (shear)
clb   = 40;                                      % cover to longitudinal bars (mm)

% member properties

L        = 1200;                             % member clear length (mm)
bending = 'single';                          % single or double
ductilitymode = 'uniaxial';                   % biaxial or uniaxial

% longitudinal reinforcement details, MLR is a matrix composed by
% [distance from the top to bar center (mm) - # of bars - bar diameter (mm)] each row
% corresponds to a layer of reinforcement:

MLR=[52.7 3 25.4
     102  2 22.2
     200  2 19
     349  3 22.2];

% transverse reinforcement details

Dh       = 9.5;                                        % diameter of transverse reinf. (mm)
s        = 120;                                         % spacing of transverse steel (mm)*

% applieed loads:

P       = -200;                 % axial load kN (-) tension (+)compression

confined    = 'mc';
unconfined = 'mu';
rebar       = 'ks';


wi    = [0];                                     % vector with clear distances between
                                                 % periferical longitudinal bars properly
                                                 % restrained or enter zero for automatical
                                                 % calculation(used only if the mander model is

% material properties

fpc      = 28;                  % concrete compressive strength (MPa)
Ec       = 0;                   % concrete modulus of elasticity (MPa) or
                                % input 0 for automatic calculation using
                                % 5000(fpc)^0.5
eco      = 0.002;               % unconfined strain (usually 0.002 for normal weight or 0.0
esm      = 0.12;                % max transv. steel strain (usually ~0.10-0.15)*
espall   = 0.0064;              % max uncon. conc. strain (usually 0.0064)

fy       = 450;                 % long steel yielding stress (MPa)
fyh      = 400;                 % transverse steel yielding stress (MPa)
Es       = 200000;              % steel modulus of elasticity
fsu      = 600;                 % long steel max stress (MPa)*
esh      = 0.008;               % long steel strain for strain hardening (usually 0.008)*
esu      = 0.12;                 % long. steel maximum strain (usually ~0.10-0.15)*
```

Figure 25. Input data for CumbiaRect example 2.

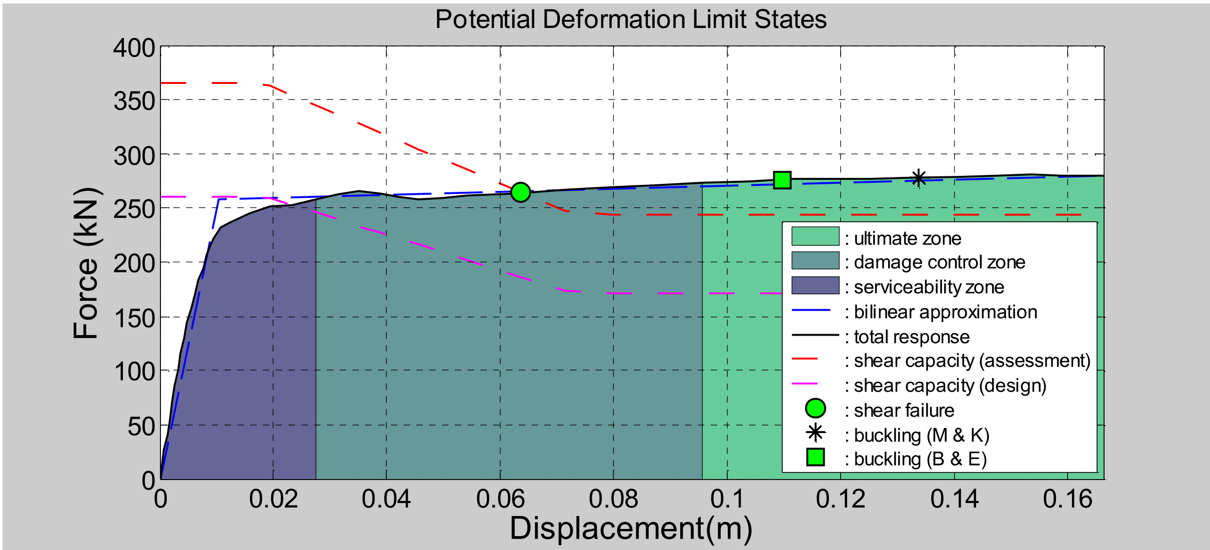

Figure 26. Member response – CumbiaRect example 2.

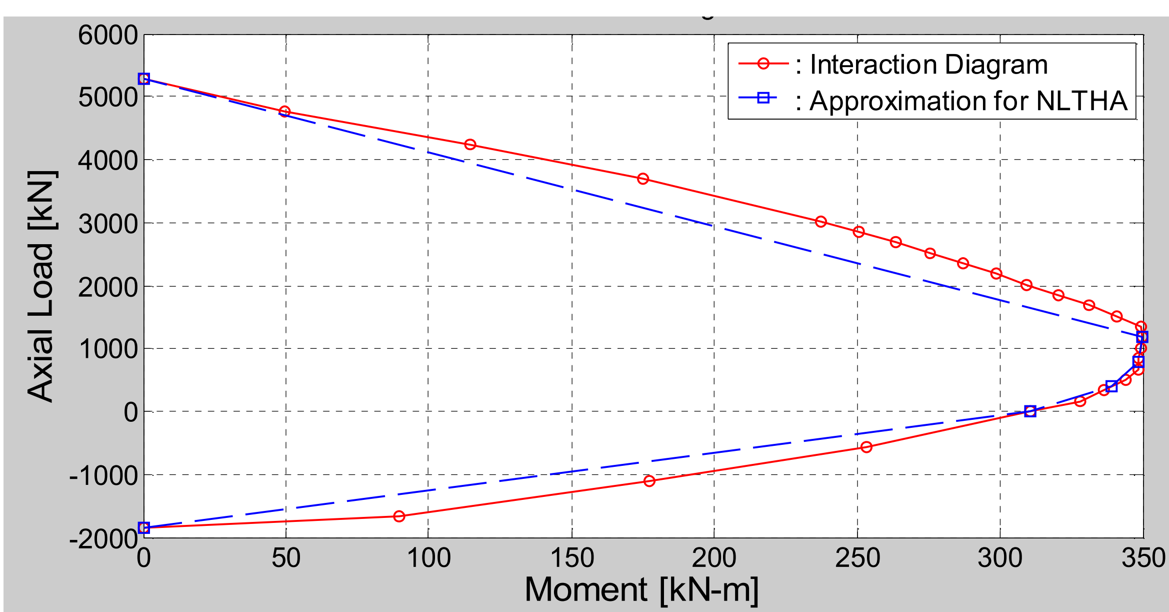

Figure 27. Interaction diagram - CumbiaRect example 2.

8.3 Example 3: ***CUMBIARECT***

This example is identical to example 2 but this time (as shown in figure 28) a tie in the y direction was added. The only that need to be changed from the previous input is the value of *ncy* (from 2 to 3). The results obtained are displayed in figure 29. It can be noticed that the mechanism of failure have changed from shear to buckling.

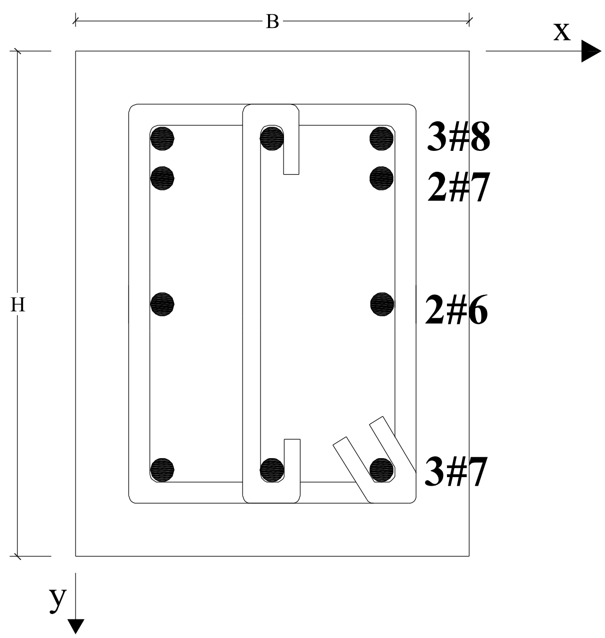

Figure 28. Section geometry for example 3.

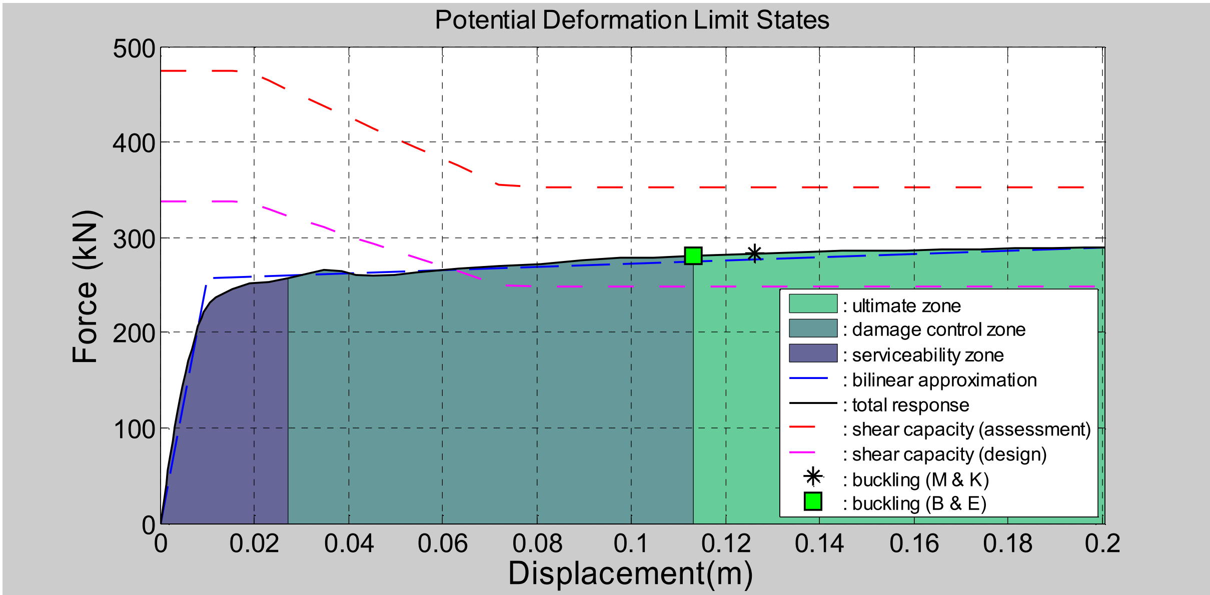

Figure 30. Member response for example 3.

9. REFERENCES

Berry P.B. and Eberhard M.O. Practical Performance Model for Bar Buckling. *ASCE Journal of Structural Engineering.* Vol. 131, No. 7, July 2005.

King D.J., Priestley M.J.N., and Park R. *Computer Programs for Concrete Column Design.* Research Report 86/12, Department of Civil Engineering, University of Canterbury, New Zealand, May 1986.

Kowalsky M.J. and Priestley M.J.N. Improved Analytical Model for Shear Strength of Circular Reinforced Concrete Columns in Seismic Regions. *ACI Structural Journal*, Vol. 97, No. 3, 2000.

Kowalsky M.J. Deformation Limit States for Circular Concrete Reinforced Bridge Columns. *ASCE Journal of Structural Engineering*, Vol. 126, No. 8, August 2000.

Kowalsky, M. J., Priestley, M. J. N., and Seible, F. Dynamic Behavior of Lightweight Concrete Bridges. ACI Structural Journal, 97-S63, 2000.

Mander J.B., Priestley M.J.N., and Park R. Theoretical Stress-Strain Model for Confined Concrete. *ASCE Journal of Structural Engineering,* Vol. 114, No. 8, August 1988.

Moyer M.J. and Kowalsky M.J. Influence of Tension Strain on Buckling of Reinforcement in Concrete Columns. *ACI Structural Journal*, Vol. 100, No. 1, January 2003.

Priestley M.J.N., Calvi G.M., Kowalsky M.J. *Direct Displacement Based Seismic Design of Structures.* IUSS Press, Pavia, Italy, (In preparation).

Priestley M.J.N., Seible F. and Benzoni G. *Seismic Performance of Circular Columns with Low Longitudinal Steel Ratios.* Department of Applied Mechanics and Engineering Sciences, University of California, San Diego, La Jolla, CA 92093, June 1994, Structural Systems Research Project SSRP - 94/08.

Priestley M.J.N., Seible F. and Calvi G.M. *Seismic Design and Retrofit of Bridge Structures.* John Wiley and Sons, New York, 1996.

Raynor D.J., Lehman D.L. and Stanton J.F. Bond Slip Response o Reinforced Bars Grouted in Ducts. *ACI Structural Journal*, Vol. 99, No. 5, 2002.