

Feedback — Week 3: Context Free Language

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You submitted this homework on **Sun 11 Oct 2015 6:53 PM CEST**. You got a score of **6.00** out of **6.00**.

Question 1

Let G be the grammar:

$$S \rightarrow SS \mid (S) \mid \epsilon$$

$L(G)$ is the language BP of all strings of balanced parentheses, that is, those strings that could appear in a well-formed arithmetic expression. We want to prove that $L(G) = BP$, which requires two inductive proofs:

1. If w is in $L(G)$, then w is in BP.
2. If w is in BP, then w is in $L(G)$.

We shall here prove only the first. You will see below a sequence of steps in the proof, each with a reason left out. These reasons belong to one of three classes:

A)

Use of the inductive hypothesis.

B)

Reasoning about properties of grammars, e.g., that every derivation has at least one step.

C)

Reasoning about properties of strings, e.g., that every string is longer than any of its proper substrings.

The proof is an induction on the number of steps in the derivation of w . You should decide on the reason for each step in the proof below, and then identify from the available choices a correct pair consisting of a step and a kind of reason (A, B, or C).

Basis: One step.

(1)

The only 1-step derivation of a terminal string is $S \Rightarrow \epsilon$ because _____

(2)

ϵ is in BP because _____

Induction: An n -step derivation for some $n > 1$.

(3)

The derivation $S \Rightarrow^n w$ is either of the form

(a) $S \Rightarrow SS \Rightarrow^{n-1} w$ or of the form

(b) $S \Rightarrow (S) \Rightarrow^{n-1} w$

because _____

Case (a):

(4)

$w = xy$, for some strings x and y such that $S \Rightarrow^p x$ and $S \Rightarrow^q y$, where $p < n$ and $q < n$ because _____

(5)

x is in BP because _____

(6)

y is in BP because _____

(7)

w is in BP because _____

Case (b):

(8)

$w = (z)$ for some string z such that $S \Rightarrow^{n-1} z$ because _____

(9)

z is in BP because _____

(10)

w is in BP because _____

Your Answer	Score	Explanation
<input type="radio"/> (5) for reason C		
<input type="radio"/> (2) for reason A		
<input type="radio"/> (5) for reason B		
<input checked="" type="radio"/> (5) for reason A	✓ 1.00	
Total	1.00 / 1.00	

Question 2

Which of the following grammars derives a subset of the language:
 $\{x \mid x \text{ contains } a \text{ and } c \text{ in proportion } 4:3 \text{ and there are no two consecutive } c\text{'s}\}$?

Your Answer	Score	Explanation
<input type="radio"/> $S \rightarrow \epsilon \quad S \rightarrow SaScSaScSaSaSaS$		
<input checked="" type="radio"/> $S \rightarrow \epsilon \quad S \rightarrow aScSaScSaScSaS$	✓ 1.00	
<input type="radio"/> $S \rightarrow \epsilon \quad S \rightarrow aacacac \quad S \rightarrow SaScSaScSaScSa$		
<input type="radio"/> $S \rightarrow \epsilon \quad S \rightarrow aScScaSaScSaS$		

Total

1.00 / 1.00

Question 3

Here are eight simple grammars, each of which generates an infinite language of strings. These strings tend to look like alternating *a*'s and *b*'s, although there are some exceptions, and not all grammars generate all such strings.

- G1: $S \rightarrow abS \mid ab$
- G2: $S \rightarrow SS \mid ab$
- G3: $S \rightarrow aB; B \rightarrow bS \mid a$
- G4: $S \rightarrow aB; B \rightarrow bS \mid b$
- G5: $S \rightarrow aB; B \rightarrow bS \mid ab$
- G6: $S \rightarrow aB \mid b; B \rightarrow bS$
- G7: $S \rightarrow aB \mid a; B \rightarrow bS$
- G8: $S \rightarrow aB \mid ab; B \rightarrow bS$

The initial symbol is *S* in all cases. Determine the language of each of these grammars. Then, find, in the list below, the pair of grammars that define the same language.

Your Answer	Score	Explanation
<input type="radio"/> G2 and G5		
<input type="radio"/> G4 and G6		
<input type="radio"/> G4 and G7		
<input checked="" type="radio"/> G1 and G2	✓ 1.00	
Total	1.00 / 1.00	

Question 4

For the grammar:

- $S \rightarrow AB \mid CD$ $A \rightarrow BC \mid a$ $B \rightarrow AC \mid C$

C → AB | CD

D → AC | d

1. Find the generating symbols. Recall, a grammar symbol is *generating* if there is a derivation of at least one terminal string, starting with that symbol.
2. Eliminate all *useless productions* --- those that contain at least one symbol that is not a generating symbol.
3. In the resulting grammar, eliminate all symbols that are not *reachable* --- they appear in no string derived from S.

In the list below, you will find several statements about which symbols are generating, which are reachable, and which productions are useless. Select the one that is FALSE.

Your Answer	Score	Explanation
<input checked="" type="radio"/> A → a is useless.	✓ 1.00	
<input type="radio"/> d is generating.		
<input type="radio"/> S is not generating.		
<input type="radio"/> C → CD is useless.		
Total	1.00 / 1.00	

Question 5

Here is a context-free grammar:

S → AB | CD

A → BG | ∅

B → AD | ε

C → CD | 1

D → BB | E

E → AF | B1

F → EG | ∅C

G → AG | BD

Find all the nullable symbols (those that derive ε in one or more steps). Then, identify the true statement from the list below.

Your Answer	Score	Explanation
<input type="radio"/> C is nullable.		

<input type="radio"/> G is not nullable.		
<input checked="" type="radio"/> E is not nullable.	✓	1.00
<input type="radio"/> B is not nullable.		
Total		1.00 / 1.00

Question 6

Here is a context-free grammar:

```
S → AB | CD
A → BG | ∅
B → AD | ε
C → CD | 1
D → BB | E
E → AF | B1
F → EG | ∅C
G → AG | BD
```

Find all the nullable symbols, and then use the construction given in the slides to modify the grammar's productions so there are no ε-productions. The language of the grammar should change only in that ε will no longer be in the language. Which of the following productions is in the modified grammar?

Your Answer	Score	Explanation
<input type="radio"/> $G \rightarrow AG \mid BD \mid A \mid G \mid B \mid D \mid \epsilon$		
<input type="radio"/> $C \rightarrow CD \mid C$		
<input checked="" type="radio"/> $G \rightarrow AG \mid BD \mid A \mid G \mid B \mid D$	✓ 1.00	
<input type="radio"/> $A \rightarrow BG \mid \emptyset \mid G \mid \epsilon$		
Total		1.00 / 1.00