SPEA-V-202

Contemporary Economic Issues in Public Affairs

Mathematics for Economic Analysis

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Outline: Basic Math Concepts for Economics

Functions

- Review definition
- Table and graphical representation
- Linear functions: slope and intercept
- Functions as relations of variables.

Solving Systems of Equations

- Conditions for solution
- Methods for solving systems of equations
- Examples

Functions: Introduction

- Functions f are rules of correspondence. For any value x in the domain, it assigns a unique value y in the range/codomain.
- Think it as a matching operator. Each number on X gets paired with a number on Y. Each pair will be a coordinate (x, y).

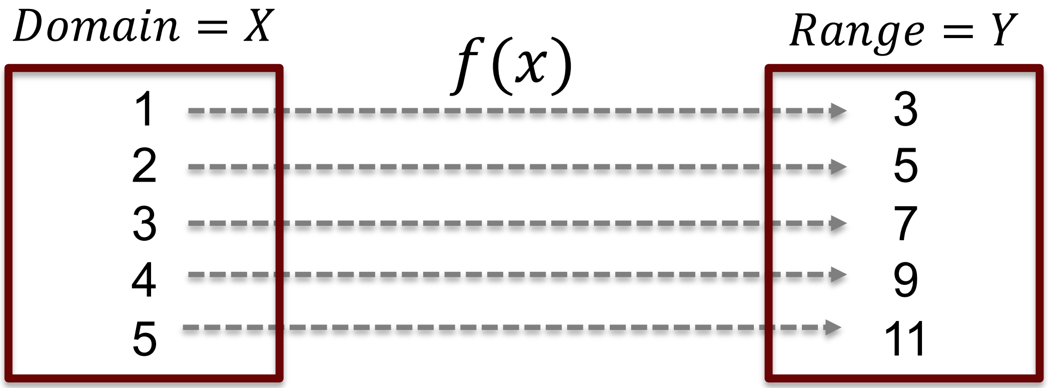
$Domain = X$ $\qquad f(x) \qquad$ $Range = Y$

$$1 \rightarrow 3$$
$$2 \rightarrow 5$$
$$3 \rightarrow 7$$
$$4 \rightarrow 9$$
$$5 \rightarrow 11$$

Functions: Intuition

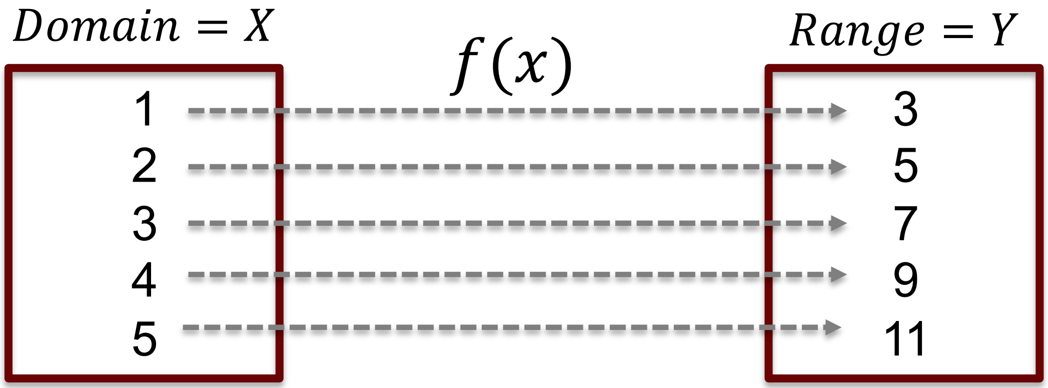
- You can also think functions as input-output relations. The function transforms every number on the domain to a number on the range. That's the rationale behind function notation.

- $y = f(x)$ says that y is the result of applying process f to x.



Functions: Example

- In our example, what is the f doing to x to transform it into y ?
- If we look closely, each value y results from multiplying x by 2 and adding 1. In math notation: $f(x) = 2x + 1$
- Since $\boldsymbol{y}$ **depends** on the value of $\boldsymbol{x}$, it is called the **dependent variable** while $\boldsymbol{x}$ is often referred as the **independent variable**.



Functions: Example

- **Class example:** (i) fill out the table according to the domain X and the function f; (ii) express in words what function f does to x.
- **Domain:** positive numbers from 1 to 5. $X = \{x \mid x = 1,2,3,4,5\}$
- **Function:** $y = f(x) = 10x + 5$

Domain X	Range Y
1	
2	
3	
4	
5	

Functions: Example

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Domain X	Range Y
1	15
2	25
3	35
4	45
5	55

Functions: Graphical Representation

- Both domain and range will be usually the positive part of the real line.
- Hence, we can represent functions in the XY-coordinate plane.

$$y = f(x) = 10x + 5$$

Domain X	Range Y
1	15
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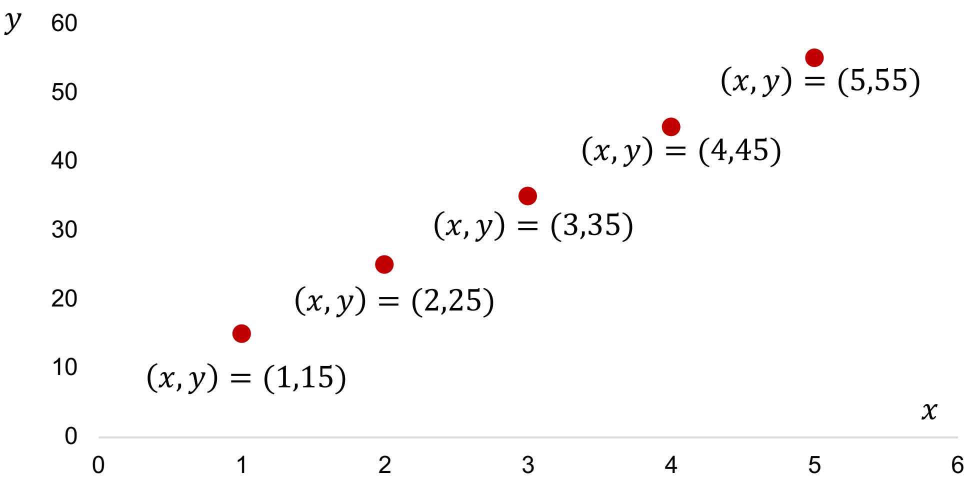

Functions: Graphical Representation

- Connecting the dots, we obtain the line representation of the function.
- In this course, we will only work with linear functions (like this one).

$$y = f(x) = 10x + 5$$

Domain X	Range Y
1	15
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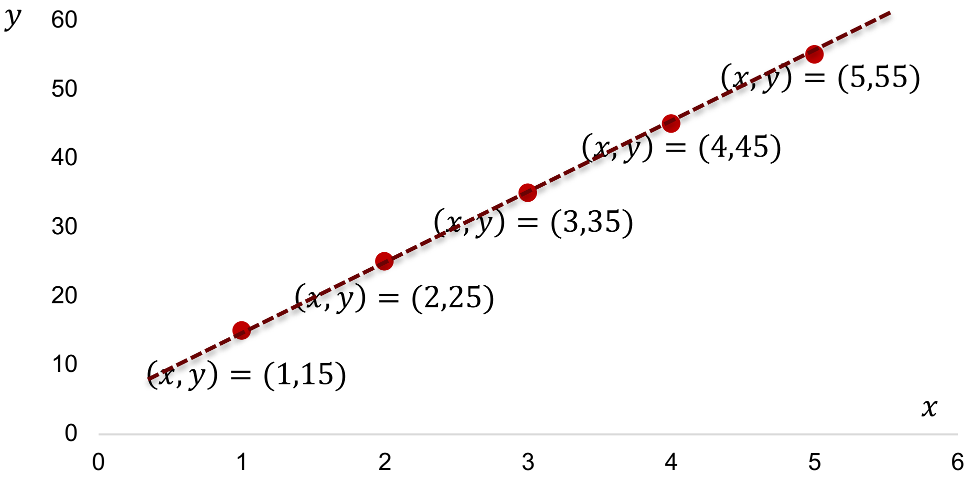


Linear Functions

- The general formula to draw a line in the XY plane is:

$$y = f(x) = mx + b$$

- **Intercept (b):** where the line crosses the $y - axis$
- **Slope (m):** steepness of the line. Also expresses the rate of change.
- m and b could be either positive or negative.
- Given the values of these two parameters, we can draw any line!

Linear Functions: Steps to Draw the Line

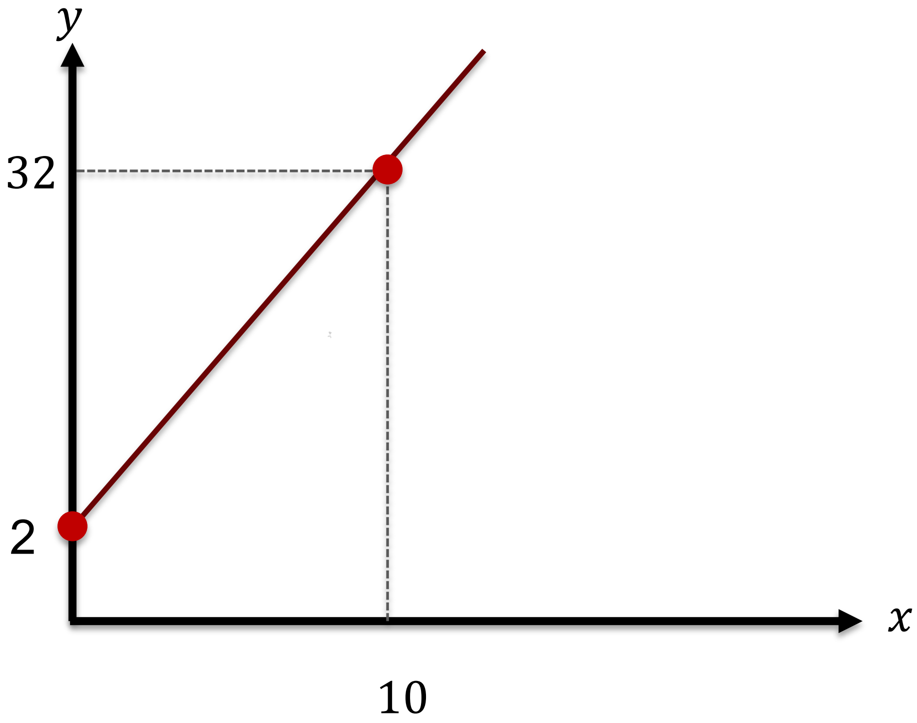
$$f(x) = 2 + 3x$$

1. **Step 1:** Identify the intercept b in the $y - axis$. This is the first point of the line.

2. **Step 2:** Compute any other value of the function. Say $x = 10$

   When $x = 10, f(x) = 2 + (3 * 10) = 32$

3. **Step 3:** Connect both points.



Linear Functions: An Example

$$f(x) = 20 - 2x$$

1. **Step 1:** Identify the intercept b in the $y - axis$. This is the first point of the line.

2. **Step 2:** Compute any other value of the function.

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Linear Functions: An Example

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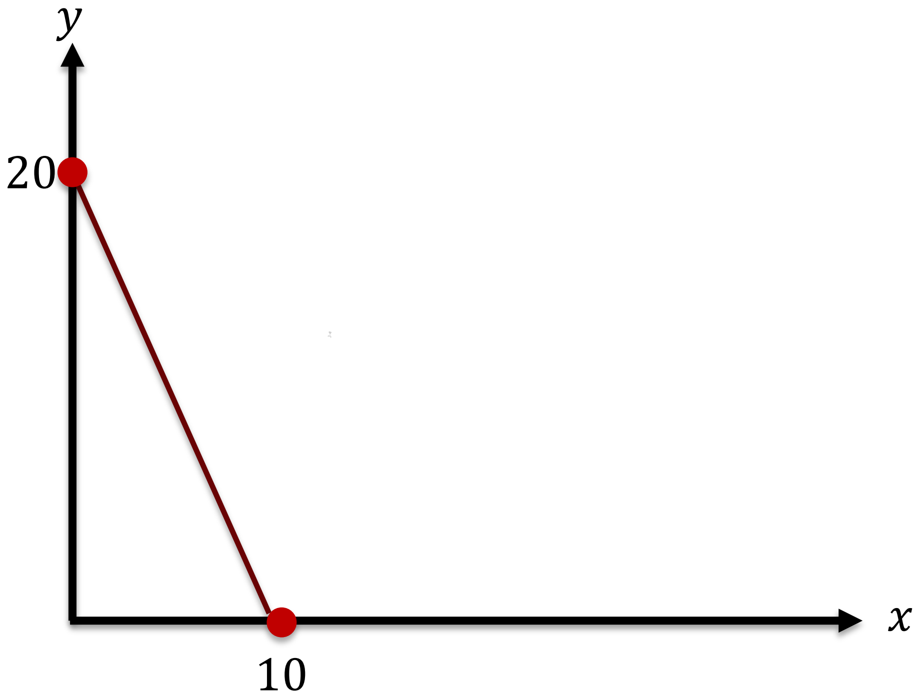
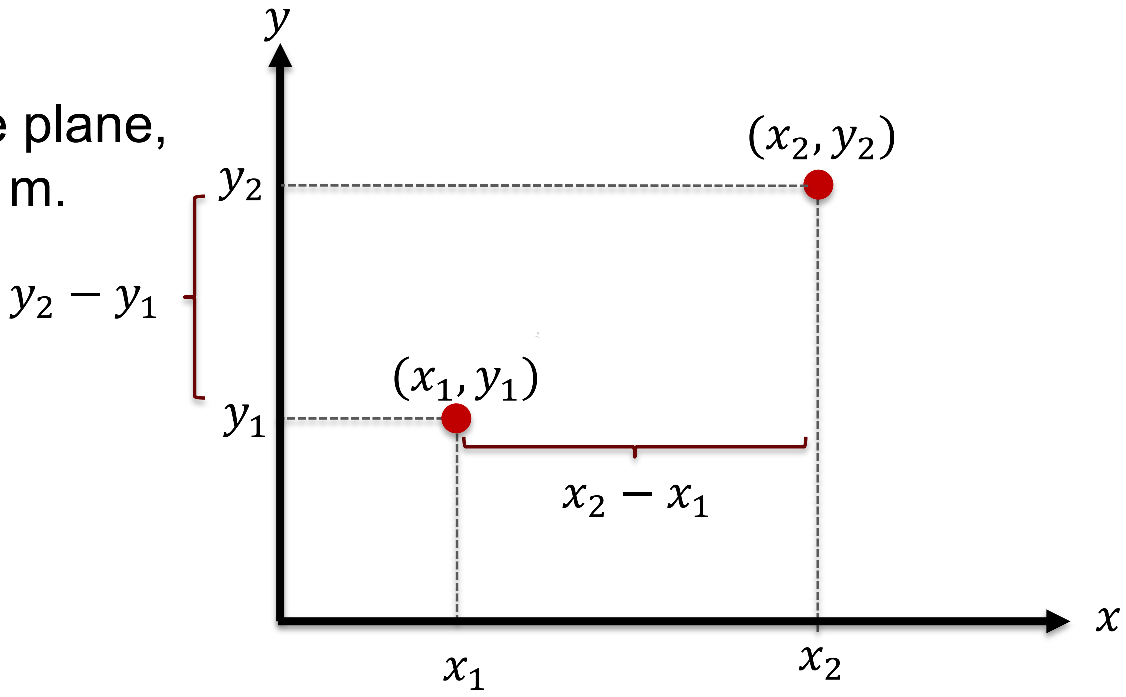


Linear Functions: Slope

$$y = f(x) = mx + b$$

- Given any two points in the plane, we can calculate the slope m.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



Linear Functions: Slope

- The slope also reflects the rate of change.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{Change\ in\ Y}{Change\ in\ X}$$

$$f(x) = 20 - 2x$$

x	f(x)
1	20-(2*1) = 18
5	20-(2*5) = 10
8	20-(2*8) = 4

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - 18}{5 - 1} = \frac{-8}{4} = -2$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 18}{8 - 1} = \frac{-14}{7} = -2$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 10}{8 - 5} = \frac{-6}{3} = -2$$

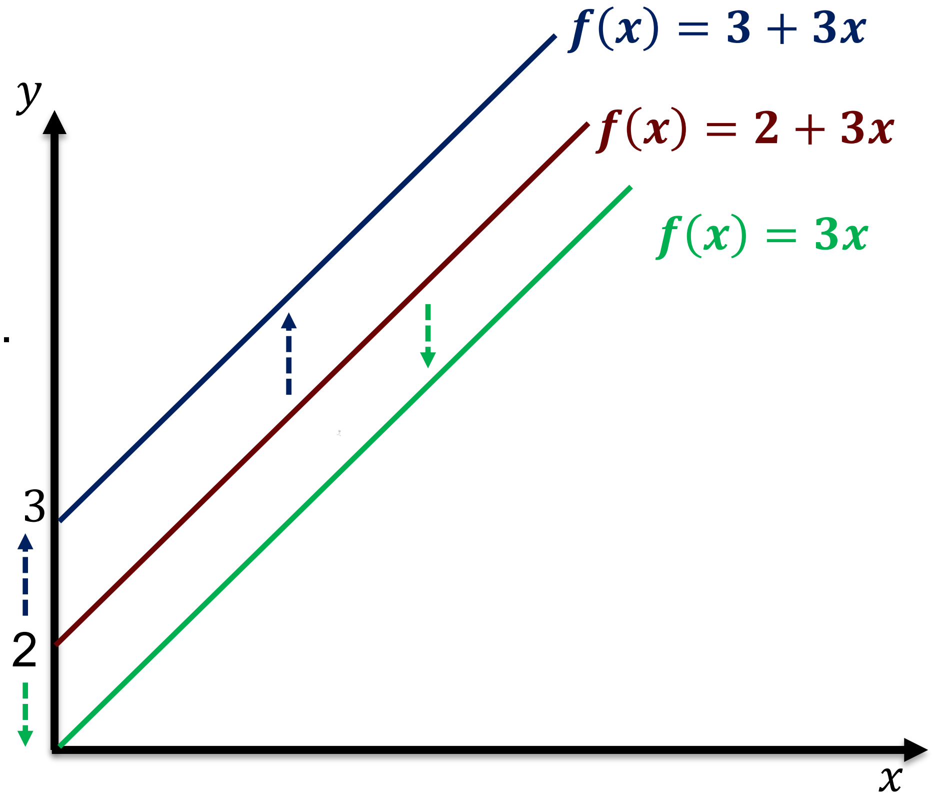
Linear Functions: Shifting a Graph

$$y = f(x) = mx + b$$

$$f(x) = 2 + 3x$$

- **Changes in the intercept just move the graph either up or down.**

$$f(x) = \boldsymbol{b} + 3x$$



Linear Functions: Shifting a Graph

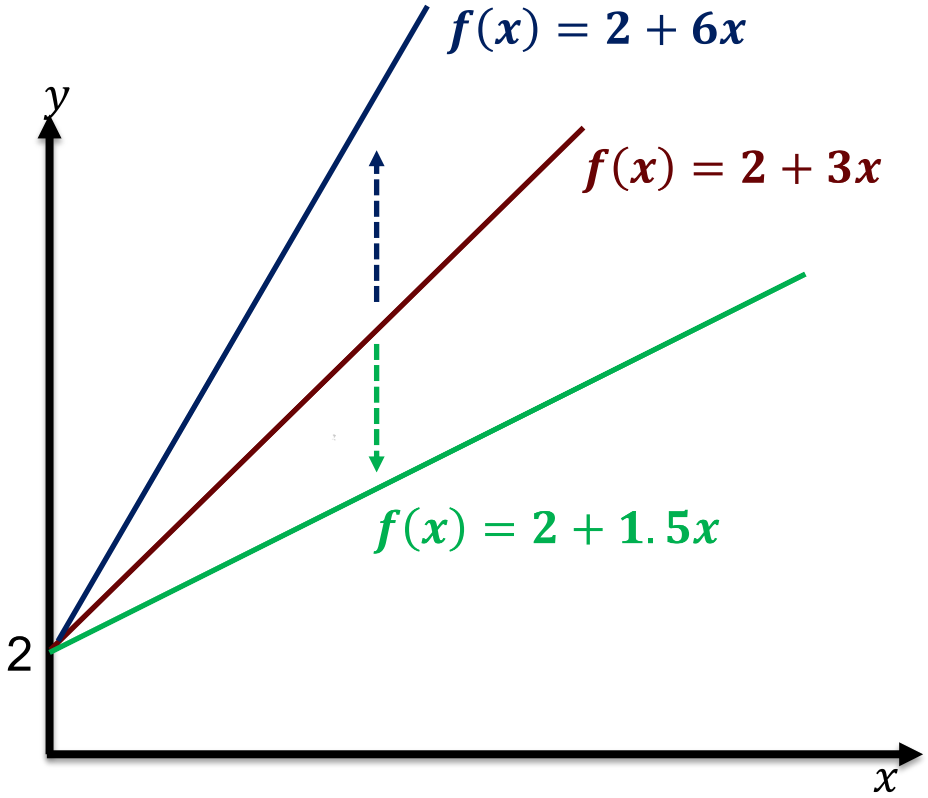
$$y = f(x) = mx + b$$

$$f(x) = 2 + 3x$$

- Changes in the intercept just move the graph either up or down.

- **Changes in the slope, pivots the line upward or downwards.**

$$f(x) = 2 + \boldsymbol{m}x$$



Linear Functions: Slope

- Slope m is key for economic intuition!

- It tells you the effect of x on y.

- In a linear function, a one unit increase on x, leads to an increase/decrease of $m - units$ on y.

- The function will be increasing if m is positive (positively sloped line).

Algebraic Tip

- Each time you are doing algebraic manipulation of an equation **always remember** that each step you take must be done at both sides of the equation.

$$y = f(x) = 50 - 5x$$

$$y = 50 - 5x$$

- Subtract 50 from both sides

$$y - 50 = 50 - 5x \; - 50$$

$$y - 50 = -5x$$

- Divide by -5 both sides

$$\frac{y - 50}{-5} = -\frac{5x}{-5}$$

$$\frac{-y + 50}{5} = x$$

$$x = g(y) = 10 \; - \frac{1}{5}y$$

Inverse Functions

- Most of the times, functions are expressed $y = f(x)$. However, sometimes we want to know how is x expressed in terms of y.

- If we express $x = g(y)$ then g is the inverse function of f. Take the standard linear function.

$$y(x) = 3x + 5$$

- Now, let's do some algebraic manipulation to get the inverse function:

$$y = 3x + 5$$

$$y - 5 = 3x + 5 - 5$$

$$y - 5 = 3x$$

$$\frac{y - 5}{3} = \frac{3x}{3}$$

$$x(y) = \frac{y - 5}{3}$$

Solving Linear Equations

- For the most part of the course, we will be looking for price and quantity that places the market in equilibrium.

- We will have equations for both the supply and the demand.

$$q_d = 10 - 2p$$

$$q_s = p - 2$$

- In equilibrium, we have that $q_d = q_s = q$

$$q = 10 - 2p$$

$$q = p - 2$$

Solving Linear Equations

- **Conditions for solution:** for a system to have a solution we require to have at least one equation for each unknown variable.
- In order words, the number of equations and variables should be the same.
- For the most part we will work with systems of:
  - 2 equations (supply and demand) and
  - 2 unknowns (price and quantity).

Solving Linear Equations: Substitution

- There are several ways to solve the following system of equations.

$$q = 10 - 2p$$

$$q = -2 + p$$

- **Substitution Method**: rearrange the terms of one equation such that you can plug it into the second equation.

Solving Linear Equations: Substitution

- In this case we already have everything with q in the left-hand side. So we can plug the first equation into the second one, and solve for q.

$$10 - 2p = -2 + p$$

- Place every term with p into the right-hand side. Add $(2 - p)$ to both sides.

$$10 - 2p + (2 - p) = -2 + p + (2 - p)$$

$$12 \ - 3p = 0$$

$$p^* = 4\,; q^* = 2$$

- **Notation Remark:** we will denote the solutions to the system of equations with a star * as a superscript.

Solving Linear Equations: Elimination

- There are several ways to solve the following system of equations.

$$q = 10 - 2p$$

$$q = -2 + p$$

- **Elimination Method**: multiply one equation by one number such that when they are added, you get rid of the of the variables.

Solving Linear Equations: Elimination

$$q = 10 - 2p$$
$$q = -2 + p$$

- For example, multiply the first equation by -1

$$-q = -10 + 2p$$

$$q = -2 + p$$

- Add both equations, q disappears and you can solve for p. Then with any of the equations you can retrieve the equilibrium value of q.

$$0 = -12 + 3p$$
$$3p = 12$$
$$p^* = 4$$
$$q^* = 10 - (2p^*) \rightarrow q^* = 2$$

Solving Linear Equations: Graphic

- There are several ways to solve the following system of equations.

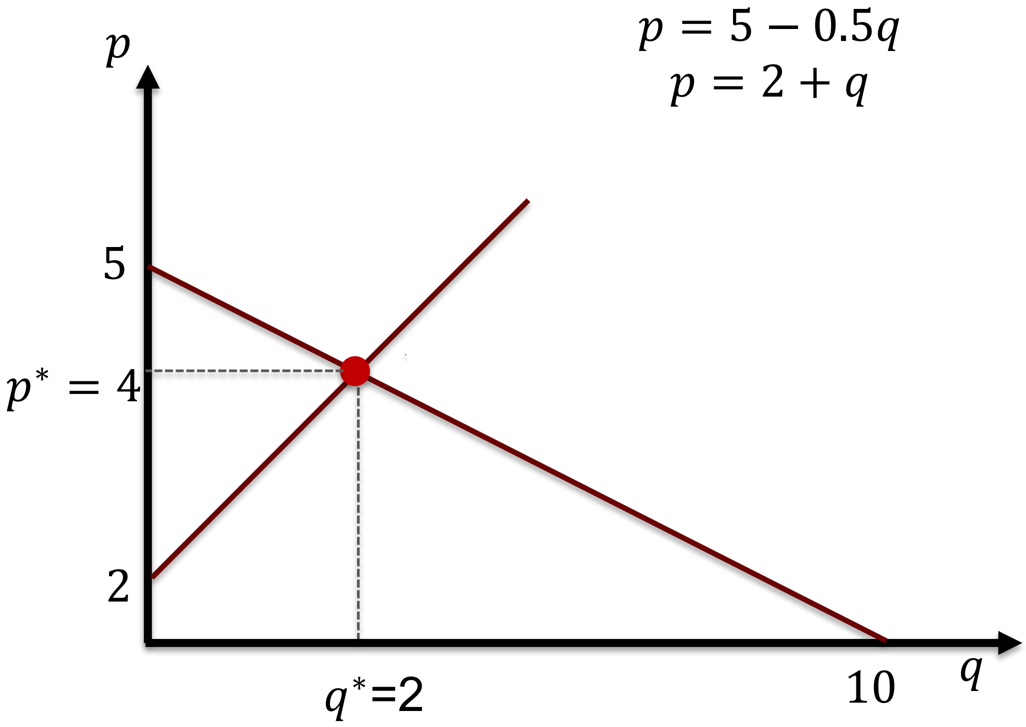
$$q = 10 - 2p$$

$$q = -2 + p$$

- **Graphic Method**: draw the lines and find at which points they intersect. For that, you need to first express the functions as $p = f(q)$ (or $y = f(x)$)

$$p = 5 - 0.5q$$
$$p = 2 + q$$

Solving Linear Equations: Graphic



- If the functions are pretty and you draw them carefully, the graphic method is perhaps the easiest.
- For some systems, the exact solution requires doing the math.

Solving Linear Equations: Example

- **Homework:** solve the following system of equations with the method you prefer.

$$p = 20 - 2q$$
$$p = 5 + 3q$$

- Verify the solution is given by q=3 and p = 14.

For next class:

- **Homework:** Solve practice exercises from Assignment 0.
- Complete this week's discussion forum.
- **Readings:** Mankiw Chapters 4 and 5.
- **Next Episode:** Demand Curve

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Multivariate Functions

- Functions can depend on more than one argument.
- In notation, we can just say variable y depends on $x, w, and\ z$.

$$y = f(x, w, z)$$

- You can think of them as more complex processes, or even recipes.
- *To make one strawberry milkshake you need: 2 glasses of milk, 3 scoops of ice cream, and 1 spoon of whipped cream.*

$$milkshake = 2\ milk + 3\ icecream + 1\ whipped\ cream$$

Multivariate Functions

- The intuition for the slope remains the same: <u>the number that multiplies each argument tells you the nature of the relation between the independent and dependent variables.</u>

$$y = f(x, w, z) = Ax + Bw + Cz$$

$$y = f(x, w, z) = 2x + 3w - z$$

- From this we know that y:

  ✓ Increases along with x and w: because A = 2 and B =3. Both positive.

  ✓ Decreases with z: because C = -1. Negative.

Multivariate Functions

- **Example:** Demand for burgers depends on the own-price of burgers, average income of people in town, and the price of salads.

$$Q_b = f(P_b, I, P_s) = -2P_b + 5I + 3P_s$$

- From this we know that demand for burgers Q_b:
  - ✓ Decreases with the price of burgers.
  - ✓ Increases along with income and the price of salads.

Multivariate Functions

- When we deal with this numerically, we will assign specific fixed values for the arguments we are not directly interested in. We will call this, keeping everything else constant or *caeteris paribus.*

$$Q_b = f(P_b, I, P_s) = -2P_b + 5I + 3P_s$$

$$I = 10; P_s = 5$$

$$Q_b = -2P_b + (5 * 10) + (3 * 5) = 65 - 2P_b$$

$$Q_b = f(P_b, I, P_s) = f(P_b) = 65 - 2P_b$$