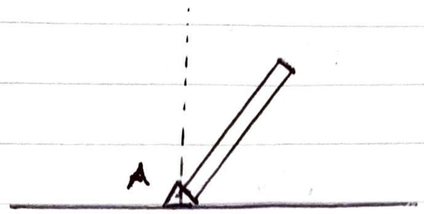
Ej 17.

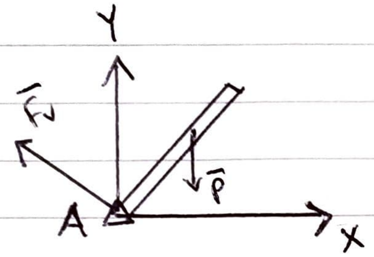
$L = 50\,cm = 0{,}5\,m$

$M = 10\,kg$



$I^{cm} = \frac{1}{12} ML^2$

Poniendo el SRI fijo en A tal que las fuerzas que pasan por A no hacen torque y gira en torno al eje situado en A.

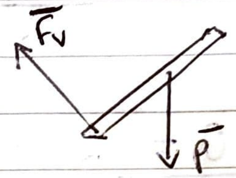


Interacciones

Barra – Tierra : P

Barra – Pivote : F_v

DCL



2da Ley de Newton

$$\Sigma F = ma \quad , \quad \Sigma \delta^A = I^A \gamma$$

Dado que hace con respecto de A, hallo I^A

$$I^A = I^{cm} + Md^2 = \frac{1}{12} ML^2 + M\left(\frac{L}{2}\right)^2 = \left(\frac{1}{12} + \frac{1}{4}\right) ML^2$$

$$\boxed{I^A = \frac{1}{3} ML^2}$$

$$\bar{\delta}_{Fv} = \bar{r}_A \times F_v = 0 \times F_v = 0$$

$$\bar{r}_{cm} = \frac{L}{2}(\text{sen}\,\beta\,\hat{\imath} + \cos\beta\,\hat{\jmath})$$

$$\bar{\delta}_P = \bar{r}_{cm/A} \times \bar{P} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ L/2\,\text{sen}\,\beta & L/2\,\cos\beta & 0 \\ 0 & -mg & 0 \end{vmatrix} = \frac{-L}{2}\,\text{sen}\,\beta \cdot mg$$

$$\Sigma \bar{\delta}^A = I^A \bar{\gamma}$$

$$\gamma = \frac{-\frac{\cancel{L}}{2}\,\text{sen}\,\beta\,\cancel{m}g}{\cancel{M}L^{\cancel{2}}/3} = \frac{-3}{2}\,\frac{\text{sen}\,\beta\,g}{L}$$

$$\boxed{\gamma = \frac{-3}{2}\frac{\operatorname{sen}\beta\, g}{L}}$$

El vector aceleración angular tiene sentido negativo.

$$\gamma = \frac{-3}{2}\frac{\operatorname{sen}(60)\cdot 10\,m/s^2}{0{,}5m} = -25{,}981\ 1/s^2 \approx -26\ 1/s^2$$

b) La velocidad del centro de masa

Suponiendo que en la posición vertical $\Omega(0) = 0$

Dado que $W_{Fv} = 0$ porque su desplazamiento es nulo

$W_{FNC} = \Delta E_m = E_{mf} - E_{mi} = 0$, poniendo $E_{pg} = 0$ fijo a la altura , sobre el piso.

$$\cancel{E_{ci}}^{0} + E_{pi} = E_{cf} + E_{pf}$$

$$mg\frac{L}{2} = \frac{1}{2}mV_{cm}^2 + \frac{I^{cm}}{2}w^2 + mg\frac{L}{2}\cos\beta$$

Dado que rota en torno a A, $V_{cm} = w\cdot\frac{L}{2} \Rightarrow w = \frac{2V_{cm}}{L}$

$$2\left(\cancel{m}g\frac{L}{2}\right) = \left(\frac{1}{2}\cancel{m}V_{cm}^2 + \frac{1}{2}\frac{\cancel{ML^2}}{12}\frac{4V_{cm}^2}{\cancel{L^2}} + \cancel{m}g\frac{L}{2}\cos\beta\right)\cdot 2$$

$$gL = V_{cm}^2 + \frac{1}{3}V_{cm}^2 + gL\cos\beta$$

$$\frac{4}{3}V_{cm}^2 = gL - gL\cos\beta$$

$$\boxed{V_{cm} = \sqrt{\frac{3}{4}gL(1-\cos\beta)}} = \sqrt{\frac{3}{4}10\cdot 0{,}5[1-\cos(60)]}$$

$V_{cm} = 1{,}370\,m/s$

c) Impulso Angular L

$$\bar{L} = \bar{r} \times \bar{p}$$

$$\bar{P}_{sist} = M\bar{V}_{cm}$$

Describiendo la trayectoria del CM como una circunferencia con respecto al pivote

$$\bar{r}_{cm/A} = L/2\,(\operatorname{sen}\beta\,\hat{\imath} + \cos\beta\,\hat{\jmath})$$

$$\bar{V}_{cm/A} = \frac{L}{2}\Omega(\cos\beta\,\hat{\imath} - \operatorname{sen}(\beta)\,\hat{\jmath})$$

Entonces

$$\bar{L} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ L/2\operatorname{sen}\beta & L/2\cos\beta & \\ mL/2\,\Omega\cos\beta & mL/2\,\Omega\operatorname{sen}\beta & \end{vmatrix} = -m\frac{L^2}{4}\Omega\operatorname{sen}^2\beta - \frac{L^2}{4}\Omega\cos^2\beta\, m = -\frac{L^2}{4}\Omega m\underbrace{(\cos^2\beta + \operatorname{sen}^2\beta)}_{1}$$

$$\boxed{\bar{L} = -\frac{L^2}{4}\Omega\,\hat{k}}$$

$$V_{cm} = \Omega \cdot L/2$$

$$\Omega = \frac{2V_{cm}}{L}$$

$$\bar{L} = -\frac{(0{,}5m)^{\cancel{2}}}{4} \cdot \frac{2\,V_{cm}}{\cancel{(0{,}5m)}} \cdot 10\,Kg$$

$$\bar{L} = -\frac{10}{4}V_{cm}\hat{k} = -\frac{10Kg}{4}1{,}370\,\frac{m^2}{s^2}\cdot\hat{k} = -3{,}425\,Kg\,\frac{m^2}{s^2}\,\hat{k}$$

d) La fuerza que el pivote A le ejercia a la barre

$$\Sigma F = m a_{cm}$$

$$\bar{F}_V + \bar{P} = m\,\bar{a}_{cm}$$

$$\bar{F}_V = +m\,\bar{a}_{cm} - \bar{P}$$

Por la condición de rigidez

$$V_{cm} = \omega L/2$$

$$a_{cm} = \delta L/2$$

$$\begin{matrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 0 & -1 \\ 0 & L/2 & 0 \end{matrix}$$

$$\bar{a}_{cm/A} = -\frac{L}{2}\Omega^2(\operatorname{sen}\beta\,\hat{\imath} + \cos\beta\,\hat{\jmath})$$

$$F_{vx} = ma_x = m\,-\frac{L}{2}\,\Omega^2 \operatorname{sen}\beta = -10\,\text{Kg}\cdot\frac{0{,}5\,\text{m}}{2}\left(\frac{2\cdot 1{,}370\,\text{m/s}}{0{,}5\,\text{m}}\right)^2\cdot \operatorname{sen}(60)$$

$$F_{vx} = -65{,}02\,\text{N}$$

$$F_{vy} = ma_y + mg = m-\frac{L}{2}\,\Omega^2\cos\beta + mg = -10\,\text{Kg}\cdot\frac{0{,}5}{2}\left(\frac{2\cdot 1{,}370\,\text{m/s}}{0{,}5\,\text{m}}\right)^2\cos(60) + 10\cdot$$

$$F_{vy} = 137{,}538\,\text{N}$$

$$\bar{F}_v = -65{,}02\,\text{N}\,\hat{\imath} + 137{,}538\,\text{N}\,\hat{\jmath}$$