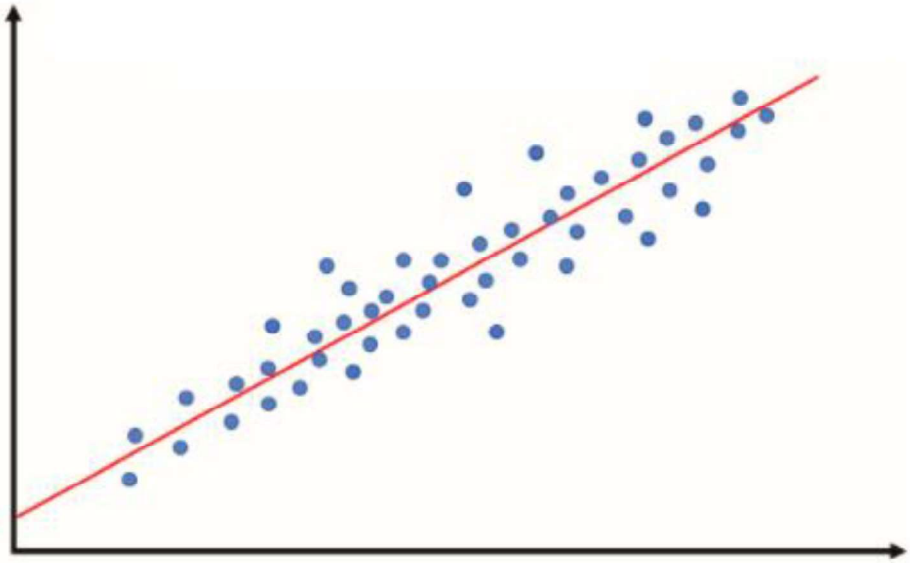
Introduction to Optimization

Before introducing the optimization concepts, we will think about how to solve a couple of problems.

Problem 1: Imagine you have many points in 2D (x, y), and you want to find the line that fits those points (Linear Regression). How can you calculate the line parameters (slope and intercept)?

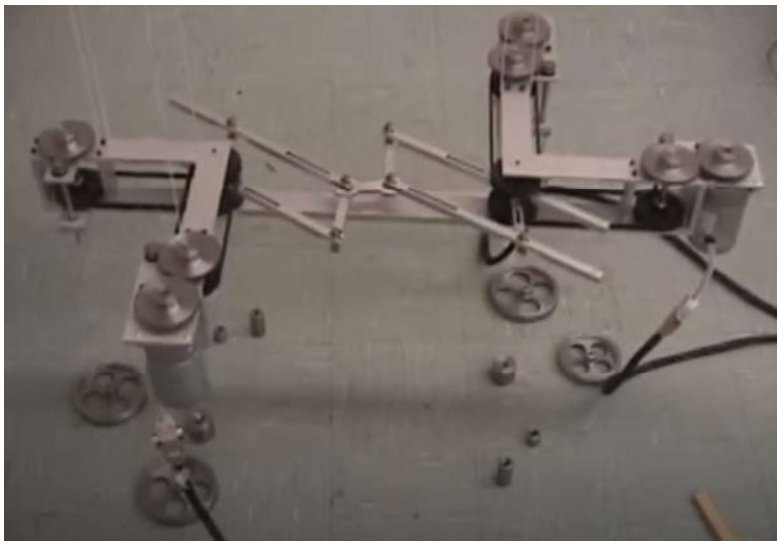


Problem 2: Imagine you have several digital images of a landscape. You want to combinate them to generate a panoramic photo. For each image, you need to calculate the new position (x, y), the rotation angle (θ), or the new size of the image (α).

How can you calculate those values to generate your panoramic image?

Problem 3: Imagine you have an unbalanced mechanism, and you want to add some counterweights to balance it and reduce the shaking force and shaking moment. Shaking force and shaking moment can be seen as functions that, given the positions (x, y) and thickness (t) of the counterweights, return a value of "stress".
Ver video: https://youtu.be/Bk1QmccUaZY

How can you select the positions and thickness of counterweights for balancing the mechanism?



What is optimization?

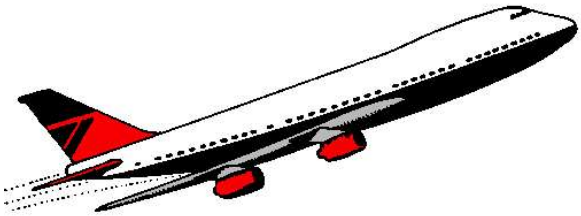
An **optimization** problem consists of **maximizing or minimizing a function by choosing feature values**. Optimization is a discipline of mathematics and mainly uses algorithms for solving problems. It lies in applied mathematics.

The most important in optimization is defining the **objective function**. This performance measurement can distinguish good solutions from bad ones. It represents the quantity we want to maximize or minimize and needs to be a number. For example, it could be money, time, energy, error, force, space, etcetera.

Sometimes the feature values need to accomplish some requirements. For example, the number of people working in a process needs to be more or equal to 1. These requirements are called **constraints**.

Applications

Transportation systems



Definition of distribution centers location

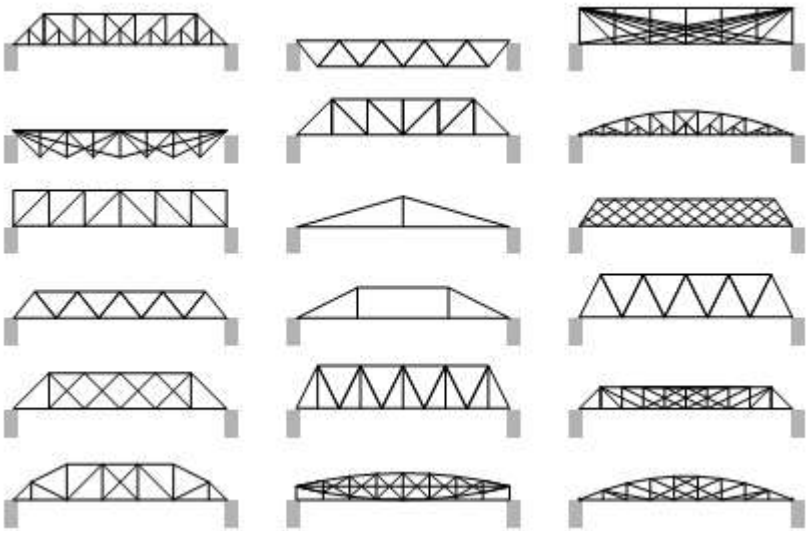
Panoramic images

Images fusion

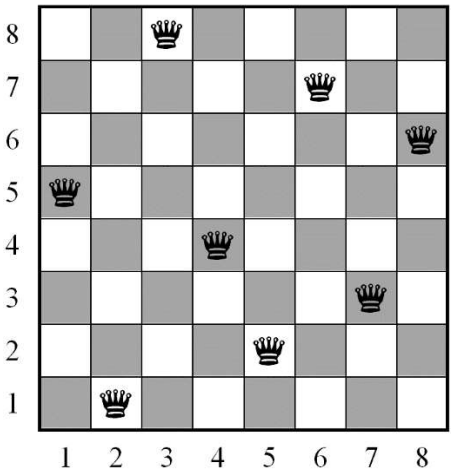
House design plans

Robotic arms design

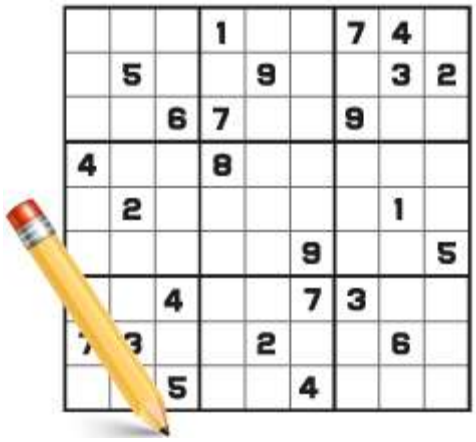
Bridge structures design



8 queens problem



Sudoku



Academic schedule design

Time	Mon	Tue	Wed
7:30 AM	Breakfast	Breakfast	Breakfast
8:00 AM	Business: Lecture Bldg B, Rm 256	Physics: Lab Bldg J, Rm 309	Business: Lecture Bldg B, Rm 256
8:30 AM			
9:00 AM	Applied Math Bldg H, Rm 100		Applied Math Bldg H, Rm 100
9:30 AM			
10:00 AM			
10:30 AM			
11:00 AM			
11:30 AM			

Optimization types

By the variables' types:
- Discrete
  - Knapsack problem
  - Shortest path
  - Traveling salesman problem
- Continuous
  - Panoramic images
  - Robotic arms design

By the characteristics of the problems:

Constrains satisfaction problems: Consists of finding at least a solution that satisfies the constraints.

		Objective function	
		Yes	No
Constraints	Yes	**Constrained optimization problem** Knapsack problem	**Constraint satisfaction problems** 8 queen problem
	No	**Free optimization problem** Linear Regression	**There is no problem**

Techniques for optimization solving problems

Deterministic: These methods or techniques get the same result, in all the executions with the same inputs. Generally, they use mathematical tools and theorems: calculus, discrete mathematics, etc.

Heuristics: A heuristic is an approach for solving a specific problem. We say that a method is a heuristic if it comes from an idea that it is not strong fundamental.

Metaheuristics: algorithms for solving problems that deterministic techniques cannot solve. Generally, these algorithms are inspired by nature, like the evolutive process, insect behaviors, etc. The results are not warranted; however, they generate very good solutions. They are based on a stochastic process; this is, they can return different results even if they were executed with the same inputs.