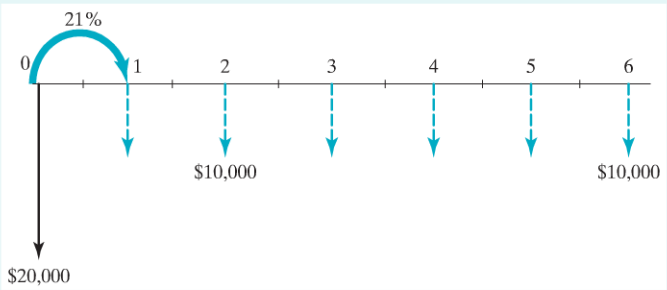

Figure 3.51 A cash flow diagram with a skipping cash flow pattern.

The present-worth calculation is

$$P = \$20{,}000 + \$10{,}000(P/A, 21\%, 6)$$
$$= \$52{,}446$$

SUMMARY

- Money has a time value because it can earn more money over time. A number of terms involving the time value of money were introduced in this chapter:

  Interest is the cost of money. More specifically, it is a cost to the borrower and an earning to the lender, above and beyond the initial sum borrowed or loaned.

  Interest rate is a percentage periodically applied to a sum of money to determine the amount of interest to be added to that sum.

  Simple interest is the practice of charging an interest rate only to an initial sum.

  Compound interest is the practice of charging an interest rate to an initial sum *and* to any previously accumulated interest that has not been withdrawn from the initial sum. Compound interest is by far the most commonly used system in the real world.

  Economic equivalence exists between individual cash flows and/or patterns of cash flows that have the same value. Even though the amounts and timing of the cash flows may differ, the appropriate interest rate makes them equal.

- The compound-interest formula, perhaps the single most important equation in this text, is

$$F = P(1 + i)^N$$

  where P is a present sum, i is the interest rate, N is the number of periods for which interest is compounded, and F is the resulting future sum. All other important interest formulas are derived from this one.

- **Cash flow diagrams** are visual representations of cash inflows and outflows along a timeline. They are particularly useful for helping us detect which of the following five patterns of cash flow is represented by a particular problem.

  1. **Single payment.** A single present or future cash flow.
  2. **Uniform series.** A series of flows of equal amounts at regular intervals.
  3. **Linear-gradient series.** A series of flows increasing or decreasing by a fixed amount at regular intervals.

4. **Geometric-gradient series.** A series of flows increasing or decreasing by a fixed percentage at regular intervals.
5. **Irregular series.** A series of flows exhibiting no overall pattern. However, patterns might be detected for portions of the series.

- **Cash flow patterns** are significant because they allow us to develop **interest formulas**, which streamline the solution of equivalence problems. Table 3.6 summarizes the important interest formulas that form the foundation for all other analyses you will conduct in engineering economic analysis.

PROBLEMS

Types of Interest

3.1 You deposit $10,000 in a savings account that earns 7.5% simple interest per year. What is the minimum number of years you must wait to double your balance? Suppose instead that you deposit the $10,000 in another savings account that earns 7% interest compounded yearly. How many years will it take now to double your balance?

3.2 Compare the interest earned by $5,000 for five years at 6% simple interest with that earned by the same amount for five years at 6% compounded annually.

3.3 You are considering investing $3,000 at an interest rate of 8% compounded annually for five years or investing the $3,000 at 9% per year simple interest for five years. Which option is better?

3.4 You are about to borrow $10,000 from a bank at an interest rate of 10% compounded annually. You are required to make five equal annual repayments in the amount of $2,638 per year, with the first repayment occurring at the end of year 1. Show the interest payment and principal payment in each year.

Equivalence Concept

3.5 Suppose you have the alternative of receiving either $18,000 at the end of five years or P dollars today. Currently, you have no need for money, so you could deposit the P dollars in a bank that pays 5% interest. What value of P would make you indifferent in your choice between P dollars today and the promise of $18,000 at the end of five years?

***3.6** Suppose that you are obtaining a personal loan from your uncle in the amount of $25,000 (now) to be repaid in three years to cover some of your college expenses. If your uncle usually earns 8% interest (annually) on his money, which is invested in various sources, what minimum lump-sum payment three years from now would make your uncle happy?

3.7 If you deposited $100 now $(n = 0)$ and $200 two years from now $(n = 2)$ in a savings amount that pays 10% annual interest, how much would you have at the end of year 10?

3.8 At what interest rate are these two transactions in Figure P3.8 equivalent?

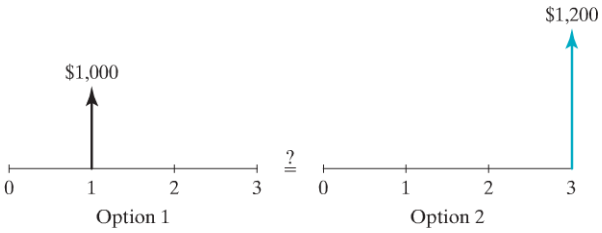

Figure P3.8

Single Payments (Use of *F/P* or *P/F* Factors)

***3.9** The average price of a new home is $180,000. If new home prices are increasing at a rate of 6% per year, how much will a new home cost in 10 years?

3.10 What will be the amount accumulated by each of these present investments?

(a) $7,000 in 5 years at 6% compounded annually

(b) $3,250 in 15 years at 5% compounded annually

(c) $18,000 in 33 years at 8% compounded annually
(d) $20,000 in 8 years at 9% compounded annually

*3.11 You are interested in buying a piece of real property that could be worth $300,000 in 10 years. Assuming that your money is worth 8%, how much would you be willing to pay for the property?

3.12 What is the present worth of these future payments?

(a) $15,500 eight years from now at 14% compounded annually
(b) $18,000 twelve years from now at 4% compounded annually
(c) $20,000 nine years from now at 8% compounded annually
(d) $55,000 four years from now at 11% compounded annually

3.13 For an interest rate of 13% compounded annually, determine the following.

(a) How much can be lent now if $12,000 will be repaid at the end of four years?
(b) How much will be required in five years to repay a $30,000 loan received now?

*3.14 How many years will it take an investment to triple if the interest rate is 6% compounded annually?

3.15 You bought 300 shares of General Electric (GE) stock at $3,200 on April 30, 2009. Your intention is to keep the stock until it doubles in value. If you expect 8% annual growth for GE stock, how many years do you anticipate holding onto the stock? Compare your answer with the solution obtained by the Rule of 72 (discussed in Example 3.10).

3.16 From the interest tables in the text, determine the values of the following factors by interpolation and compare your answers with those obtained by evaluating the *F/P* factor or the *P/F* factor.

(a) The single-payment compound-amount factor for 38 periods at 9.5% interest.
(b) The single-payment present-worth factor for 47 periods at 8% interest.

*3.17 Warren Buffett's Berkshire Hathaway Company went public in 1965. The public offering price was $18 per share. The stock was trading at $92,400 on April 10, 2009, and the market value of the firm was $97.8 billion.

(a) What is the firm's average annual compound growth rate over last 44 years?
(b) If Mr. Buffett's company continues to grow at the historical growth rate, what will be his company's total market value when he reaches the age of 100? (Buffett celebrated his 78th birthday in 2009.)

Uneven Payment Series

3.18 Consider the following sequence of deposits and withdrawals over a period of four years. If you earn 10% interest on your deposits, how much will you able to withdraw at the end of four years?

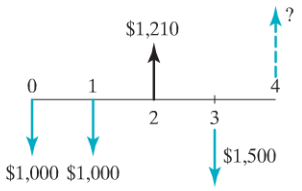

Figure P3.18

3.19 If you desire to withdraw the amounts given in Table P3.19 over the next five years from a savings account that earns 7% interest compounded annually, how much do you need to deposit now?

TABLE P3.19

N	Amount
2	$25,000
3	33,000
4	46,000
5	38,000

3.20 If $2,000 is invested now, $2,500 two years from now, and $3,000 four years from now at an interest rate of 6% compounded annually, what will be the total amount in 10 years?

3.21 A local newspaper headline blared, "Bo Smith Signed for $30 Million." A reading of the article revealed that on April 1, 2010, Bo Smith, the former record-breaking running back from Football University, signed a $30 million package with the Dallas Rangers. The terms of the contract were

$3 million immediately, $2.4 million per year for the first five years (with the first payment after one year) and $3 million per year for the next five years (with the first payment at year 6). If Bo's interest rate is 8% per year, what would his contract be worth at the time he signs it?

3.22 How much invested now at 6% would be just sufficient to provide three payments, with the first payment in the amount of $8,000 occurring two years hence, then $6,000 five years hence, and finally $4,000 seven years hence?

Equal Payment Series

***3.23** What is the future worth of a series of equal year-end deposits of $2,000 for 15 years in a savings account that earns 8% annual interest if the following were true?

(a) All deposits are made at the *end* of each year?

(b) All deposits are made at the *beginning* of each year?

3.24 You are paying into a mutual fund that earns 6% compound interest. If you are making an annual contribution of $10,000, how much will be in the funds in 20 years?

3.25 What is the future worth of the following series of payments?

(a) $6,000 at the end of each year for 5 years at 8% compounded annually

(b) $4,000 at the end of each year for 12 years at 6.25% compounded annually

(c) $9,000 at the end of each year for 20 years at 9.45% compounded annually

(d) $3,000 at the end of each year for 12 years at 11.75% compounded annually

***3.26** What equal annual series of payments must be paid into a sinking fund to accumulate the following amounts?

(a) $32,000 in 15 years at 8% compounded annually

(b) $55,000 in 10 years at 6% compounded annually

(c) $35,000 in 20 years at 7% compounded annually

(d) $8,000 in 4 years at 11% compounded annually

3.27 You want to save money from your business operation to replace a truck that has been used in delivery. The truck will be replaced after 10 years from now and the replacement cost would be about $50,000. If you earn 6% interest on your savings, how much must you deposit at the end of each year to meet the needs?

3.28 Part of the income that a machine generates is put into a sinking fund to replace the machine when it wears out. If $2,000 is deposited annually at 6% interest, how many years must the machine be kept before a new machine costing $35,000 can be purchased?

3.29 A no-load (commission-free) mutual fund has grown at a rate of 11% compounded annually since its beginning. If it is anticipated that it will continue to grow at that rate, how much must be invested every year so that $15,000 will be accumulated at the end of five years?

3.30 To help you reach a $5,000 goal five years from now, your father offers to give you $500 now. You plan to get a part-time job and make five additional deposits, one at the end of each year. (The first deposit is made at the end of the first year.) If all your money is deposited in a bank that pays 7% interest, how large must your annual deposit be?

***3.31** What equal annual payment series is required to repay the following present amounts?

(a) $12,000 in 6 years at 4% interest compounded annually

(b) $3,500 in 7 years at 6.7% interest compounded annually

(c) $6,500 in 5 years at 3.5% interest compounded annually

(d) $32,000 in 15 years at 8.5% interest compounded annually

3.32 From the interest tables in Appendix A, determine the values of the following factors by interpolation and compare your results with those obtained from evaluating the *A/P* and *P/A* interest formulas.

(a) The capital-recovery factor for 38 periods at 6.25% interest.

(b) The equal-payment series present-worth factor for 85 periods at 9.25% interest.

3.33 You have borrowed $50,000 at an interest rate of 12%. Equal payments will be made over a three-year period. (The first payment will be made at the end of the first year.) What will the annual payment be, and what will the interest payment be for the second year?

3.34 You borrowed $10,000 from a bank with the agreement that you repay the balance in 10 equal

annual installments at an interest rate of 9%. What should be the required annual payment size?

*3.35 What is the present worth of the following series of payments?

(a) $1,000 at the end of each year for 8 years at 6.8% compounded annually

(b) $3,500 at the end of each year for 12 years at 9.5% compounded annually

(c) $1,900 at the end of each year for 9 years at 8.25% compounded annually

(d) $9,300 at the end of each year for 5 years at 7.75% compounded annually

3.36 You are considering buying a piece of industrial equipment to automate a part of your production process. This automation will save labor costs by as much as $35,000 per year over 10 years. The equipment costs $200,000. Should you purchase the equipment if your interest rate is 12%?

3.37 Recently, an NFL quarterback agreed to an eight-year, $50 million contract that at the time made him one of the highest-paid players in professional football history. The contract included a signing bonus of $11 million. The agreement called for annual salaries of $2.5 million in 2005, $1.75 million in 2006, $4.15 million in 2007, $4.90 million in 2008, $5.25 million in 2009, $6.2 million in 2010, $6.75 million in 2011, and $7.5 million in 2012. The $11 million signing bonus was pro-rated over the course of the contract, so that an additional $1.375 million was paid each year over the eight-year contract period.

(a) How much was the quarterback's contract actually worth at the time of signing?

(b) For the signing bonus portion, suppose that the quarterback was allowed to take either the pro-rated payment option as just described, or a lump-sum payment option in the amount of $8 million at the time he signed the contract. Should he have taken the lump-sum option instead of the pro-rated one? Assume that his interest rate is 6%.

Linear-Gradient Series

3.38 An individual deposits an annual bonus into a savings account that pays 8% interest compounded annually. The size of the bonus increases by $3,000 each year, and the initial bonus amount was $10,000. Determine how much will be in the account immediately after the fifth deposit.

3.39 Five annual deposits in the amounts of $7,500, $6,000, $4,500, $3,000, and $1,500, in that order, are made into a fund that pays interest at a rate of 8% compounded annually. Determine the amount in the fund immediately after the fifth deposit.

3.40 Compute the value of P in the accompanying cash flow diagram, assuming that $i = 9$.

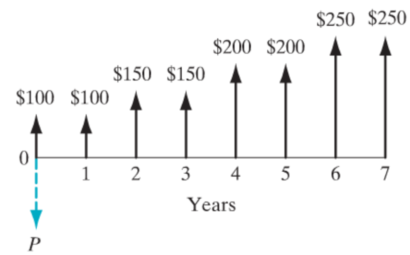

Figure P3.40

3.41 What is the equal payment series for 12 years that is equivalent to a payment series of $15,000 at the end of the first year, decreasing by $1,000 each year over 12 years? Interest is 8% compounded annually.

*3.42 How much do you have to deposit now in your savings account that earns a 6% annual interest if you want to withdraw the annual payment series in the figure below?

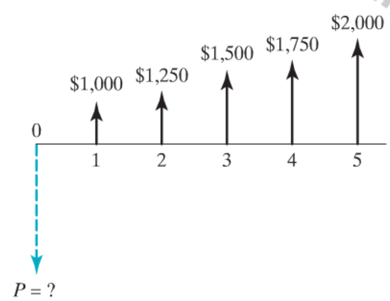

Figure P3.42

Geometric-Gradient Series

3.43 Kersey Manufacturing Co., a small fabricator of plastics, needs to purchase an extrusion molding machine for $120,000. Kersey will borrow money from a bank at an interest rate of 9% over five years.

Kersey expects its product sales to be slow during the first year, but to increase subsequently at an annual rate of 10%. Kersey therefore arranges with the bank to pay off the loan on a "balloon scale," which results in the lowest payment at the end of the first year and each subsequent payment being just 10% over the previous one. Determine the five annual payments.

***3.44** Joe's starting salary as a mechanical engineer is around $60,000. Joe is planning to place a total of 10% of his salary each year in the mutual fund. Joe expects a 5% salary increase each year for the next 30 years of employment. If the mutual fund will average 7% annual return over the course of his career, what can Joe expect at retirement?

3.45 Suppose that an oil well is expected to produce 100,000 barrels of oil during its first year in production. However, its subsequent production (yield) is expected to decrease by 10% over the previous year's production. The oil well has a proven reserve of 1,000,000 barrels.

(a) Suppose that the price of oil is expected to be $60 per barrel for the next several years. What would be the present worth of the anticipated revenue stream at an interest rate of 12% compounded annually over the next seven years?

(b) Suppose that the price of oil is expected to start at $60 per barrel during the first year, but to increase at the rate of 5% over the previous year's price. What would be the present worth of the anticipated revenue stream at an interest rate of 12% compounded annually over the next seven years?

(c) Consider part (b) again. After three years' production, you decide to sell the oil well. What would be a fair price?

3.46 A city engineer has estimated the annual toll revenues from a newly proposed highway construction over 20 years as follows:

$$A_n = (\$2{,}000{,}000)(n)(1.06)^{n-1}$$
$$n = 1, 2, \ldots, 20$$

To validate the bond, the engineer was asked to present the estimated total present value of toll revenue at an interest rate of 6%. Assuming annual compounding, find the present value of the estimated toll revenue.

3.47 What is the amount of 10 equal annual deposits that can provide five annual withdrawals? A first withdrawal of $12,000 is made at the end of year 11 and subsequent withdrawals increase at the rate of 8% per year over the previous year's withdrawal. Determine the amounts from the following rates.

(a) The interest rate is 12% compounded annually?

(b) The interest rate is 9% compounded annually?

Various Interest Factor Relationships

3.48 By using only those factors given in interest tables, find the values of the factors that follow, which are not given in your tables. Show the relationship between the factors by using factor notation, and calculate the value of the factor. Then compare the solution you obtained by using the factor formulas with a direct calculation of the factor values.

For example, $(F/P, 8\%, 38) = (F/P, 8\%, 30)(F/P, 8\%, 8) = 18.6253$.

(a) (*P/F*, 8%, 67)

(b) (*A/P*, 8%, 42)

(c) (*P/A*, 8%, 135)

3.49 Prove the following relationships among the following interest factors.

(a) $(F/P, i, N) = i(F/A, i, N) + 1$

(b) $(P/F, i, N) = 1 - (P/A, i, N)i$

(c) $(A/F, i, N) = (A/P, i, N) - i$

(d) $(A/P, i, N) = i/[1 - (P/F, i, N)]$

(e) $(P/A, i, N \to \infty) = 1/i$

(f) $(A/P, i, N \to \infty) = i$

(g) $(P/G, i, N \to \infty) = 1/i^2$

Equivalence Calculations

3.50 Find the present worth of the cash receipts where $i = 12\%$ compounded annually with only four interest factors.

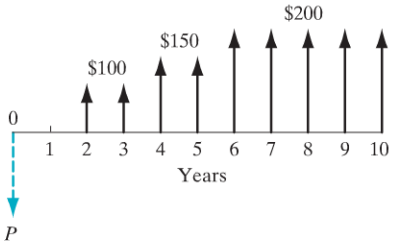

Figure P3.50

*3.51 Find the equivalent present worth of the cash receipts where $i = 8\%$. In other words, how much do you have to deposit now (with the second deposit in the amount of $200 at the end of the first year) so that you will be able to withdraw $200 at the end of second year, $120 at the end of third year, and so forth if the bank pays you an 8% annual interest on your balance?

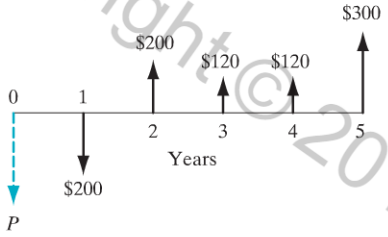

Figure P3.51

3.52 What value of A makes the two annual cash flows equivalent at 13% interest compounded annually?

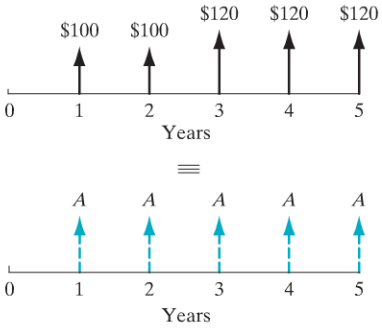

Figure P3.52

3.53 The two cash flow transactions shown in the accompanying cash flow diagram are said to be equivalent at 6% interest compounded annually. Find the unknown value of X that satisfies the equivalence.

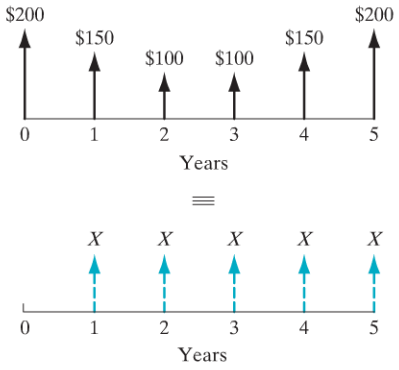

Figure P3.53

3.54 Solve for the present worth of this cash flow using at most three interest factors at 10% interest compounded annually.

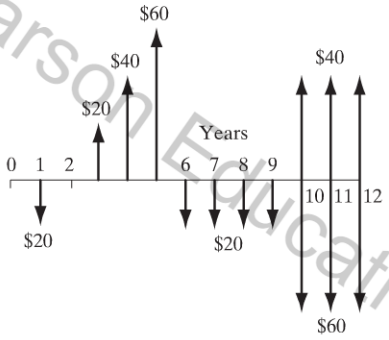

Figure P3.54

3.55 From the accompanying cash flow diagram, find the value of C that will establish the economic equivalence between the deposit series and the withdrawal series at an interest rate of 8% compounded annually.

(a) $1,335
(b) $862
(c) $1,283
(d) $828

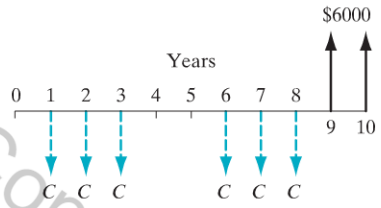

Figure P3.55

3.56 The following equation describes the conversion of a cash flow into an equivalent equal payment series with $N = 10$:

$$A = [800 + 20(A/G, 6\%, 7)] \times (P/A, 6\%, 7)(A/P, 6\%, 10) + [300(F/A, 6\%, 3) - 500](A/F, 6\%, 10)$$

Reconstruct the original cash flow diagram.

3.57 Consider the cash flow shown in the accompanying diagram. What value of C makes the inflow series equivalent to the outflow series at an interest rate of 10%?

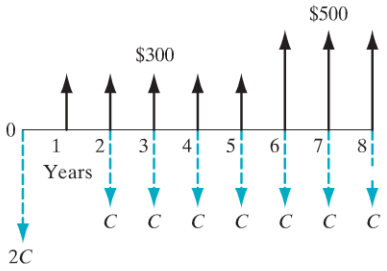

Figure P3.57

3.58 Find the value of X so that the two cash flows shown in the diagram are equivalent for an interest rate of 8%.

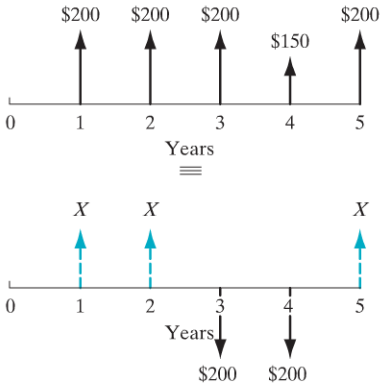

Figure P3.58

3.59 What single amount at the end of the fifth year is equivalent to a uniform annual series of $3,000 per year for 10 years if the interest rate is 9% compounded annually?

3.60 From the following list, identify all the correct equations used in computing either the equivalent present worth (P) or future worth (F) for the cash flow shown at $i = 10\%$.

(a) $P = R(P/A, 10\%, 6)$
(b) $P = R + R(P/A, 10\%, 5)$
(c) $P = R(P/F, 10\%, 5) + R(P/A, 10\%, 5)$
(d) $F = R(F/A, 10\%, 5) + R(F/P, 10\%, 5)$
(e) $F = R + R(F/A, 10\%, 5)$
(f) $F = R(F/A, 10\%, 6)$
(g) $F = R(F/A, 10\%, 6) - R$

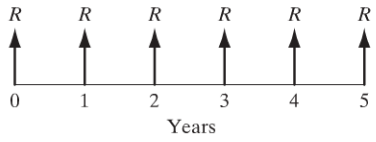

Figure P3.60

3.61 On the day his baby is born, a father decides to establish a savings account for the child's college education. Any money that is put into the account will earn an interest rate of 8% compounded annually.

The father will make a series of annual deposits in equal amounts on each of his child's birthdays from the 1st through the 18th, so that the child can make four annual withdrawals from the account in the amount of $30,000 on each birthday. Assuming that the first withdrawal will be made on the child's 18th birthday, which of the following equations are correctly used to calculate the required annual deposit?

(a) $A = (\$30{,}000 \times 4)/18$

(b) $A = \$30{,}000(F/A, 8\%, 4) \times (P/F, 8\%, 21)(A/P, 8\%, 18)$

(c) $A = \$30{,}000(P/A, 8\%, 18) \times (F/P, 8\%, 21)(A/F, 8\%, 4)$

(d) $A = [\$30{,}000(P/A, 8\%, 3) + \$30{,}000](A/F, 8\%, 18)$

(e) $A = \$30{,}000\big[(P/F, 8\%, 18) + (P/F, 8\%, 19) + (P/F, 8\%, 20) + (P/F, 8\%, 21)\big](A/P, 8\%, 18)$

3.62 Find the equivalent equal payment series (A) using an *A/G* factor such that the two cash flows are equivalent at 10% compounded annually.

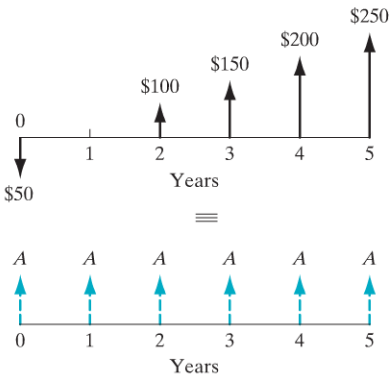

Figure P3.62

3.63 Consider the following cash flow.

TABLE P3.63

Year End	Payment
0	$500
1–5	$1,000

In computing F at the end of year 5 at an interest rate of 12%, which of the following equations is *incorrect*?

(a) $F = \$1{,}000(F/A, 12\%, 5) - \$500(F/P, 12\%, 5)$

(b) $F = \$500(F/A, 12\%, 6) + \$500(F/A, 12\%, 5)$

(c) $F = \big[\$500 + \$1{,}000(P/A, 12\%, 5)\big] \times (F/P, 12\%, 5)$

(d) $F = \big[\$500(A/P, 12\%, 5) + \$1{,}000\big] \times (F/A, 12\%, 5)$

3.64 Consider the cash flow series given. In computing the equivalent worth at $n = 4$, which of the following equations is *incorrect*?

(a) $V_4 = \big[\$100(P/A, i, 6) - \$100(P/F, i, 4)\big](F/P, i, 4)$

(b) $V_4 = \$100(F/A, i, 3) + \$100(P/A, i, 2)$

(c) $V_4 = \$100(F/A, i, 4) - \$100 + \$100(P/A, i, 2)$

(d) $V_4 = \big[\$100(F/A, i, 6) - \$100(F/P, i, 2)\big](P/F, i, 2)$

Figure P3.64

3.65 Henry Cisco is planning to make two deposits: $25,000 now and $30,000 at the end of year 6. He wants to withdraw C at the end of each year for the first six years and $(C + \$1{,}000)$ each year for the next six years. Determine the value of C if the deposits earn 10% interest compounded annually.

(a) $7,711

(b) $5,794

(c) $6,934

(d) $6,522

Solving for an Unknown Interest Rate or Unknown Interest Periods

3.66 At what rate of interest compounded annually will an investment double in five years?

3.67 Determine the interest rate (i) that makes the pairs of cash flows shown economically equivalent.

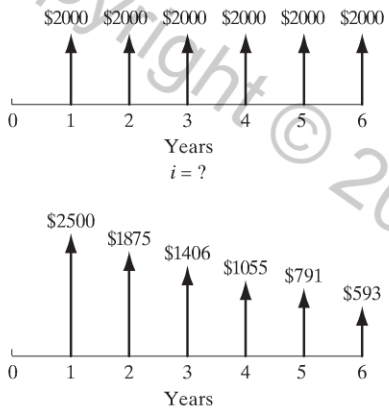

Figure P3.67

***3.68** You have $10,000 available for investment in stock. You are looking for a growth stock whose value can grow to $35,000 over five years. What kind of growth rate are you looking for?

3.69 How long will it take to save $1 million if you invest $2,000 each year at 6%?

3.70 You may have already won $2 million! Just peel the game piece off the Instant Winner Sweepstakes ticket, and mail it to us along with your order for subscriptions to your two favorite magazines. As a grand prize winner, you may choose between a $1 million cash prize paid immediately or $100,000 per year for 20 years—that's $2 million! Suppose that, instead of receiving one lump sum of $1 million, you decide to accept the 20 annual installments of $100,000. If you are like most jackpot winners, you will be tempted to spend your winnings to improve your lifestyle during the first several years. Only after you get this type of spending "out of your system" will you save later sums for investment purposes. Suppose that you are considering the following two options.

Option 1: You save your winnings for the first 7 years and then spend every cent of the winnings in the remaining 13 years.

Option 2: You do the reverse, spending for 7 years and then saving for 13 years.

If you can save winnings at 7% interest, how much would you have at the end of 20 years, and what interest rate on your savings will make these two options equivalent?

3.71 Read the following letter from a magazine publisher.

> Dear Parent:
>
> Currently your *Growing Child/Growing Parent* subscription will expire with your 24-month issue. To renew on an annual basis until your child reaches 72 months would cost you a total of $63.84 ($15.96 per year). We feel it is so important for you to continue receiving this material until the 72nd month that we offer you an opportunity to renew now for $57.12. Not only is this a savings of 10% off the regular rate, but it is an excellent inflation hedge for you against increasing rates in the future. Please act now by sending $57.12.

(a) If your money is worth 6% per year, determine whether this offer can be of any value.

(b) What rate of interest would make you indifferent between the two renewal options?

Short Case Studies

ST3.1 A judge denied a 94-year-old woman's attempt to force the Massachusetts Lottery Commission to pay her entire $5.6 million winnings up front on the grounds that she otherwise won't live long enough to collect it all. The ruling means that the commission can pay Louise Outing, a retired waitress, in installments over 20 years. After an initial gross payment of $283,770, Outing would be