

Ejercicios Problemas Laplace

a) $f(t) = \begin{cases} 4, & 0 \leq t < 1 \\ 4-3t, & 1 \leq t < 2 \\ 0, & t \geq 2 \end{cases}$

$f(t) = 4 - t \cdot U(t-1) + 3(t-1) \cdot U(t-1)$

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$\mathcal{L}\{f(t)\} = \mathcal{L}\{4 - t \cdot U(t-1) + 3(t-1) \cdot U(t-1)\}$

$\frac{4}{s} - \mathcal{L}\{t \cdot U(t-1)\} + 3 \mathcal{L}\{(t-1) \cdot U(t-1)\}$

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b) $f(t) = e^{5t}$

$\mathcal{L}\{f(t)\} = \mathcal{L}\{e^{5t}\} = \frac{1}{s-5}$

c) $f(t) = t e^{5t}$

$\mathcal{L}\{f(t)\} = \mathcal{L}\{t e^{5t}\} = \frac{1}{(s-5)^2}$

d) $f(t) = \sin t \cdot U(t - \frac{\pi}{2})$

$\mathcal{L}\{f(t)\} = \mathcal{L}\{\sin t \cdot U(t - \frac{\pi}{2})\}$

$F(s) = e^{-\frac{\pi}{2}s} \mathcal{L}\{\sin(t + \frac{\pi}{2})\}$

$F(s) = e^{-\frac{\pi}{2}s} \mathcal{L}\{\cos(t)\}$

$F(s) = e^{-\frac{\pi}{2}s} \left(\frac{s}{s^2 + 1} \right)$

e) $e^{-t} \cos 2t$

$\mathcal{L}\{f(t)\} = \mathcal{L}\{e^{-t} \cos 2t\}$

$\frac{s+1}{(s+1)^2 + 4}$

Ejercicios Segunda Sección

a) $\frac{1}{s^2-1}$

$\mathcal{L}^{-1}\left\{\frac{1}{s^2-1}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{4} \left(\frac{1}{s-1} - \frac{1}{s+1} \right) \right\}$

$\frac{1}{4} e^{\frac{t}{2}}$

b) $\frac{4}{s^2-9}$

$\mathcal{L}^{-1}\left\{\frac{4}{s^2-9}\right\} = \mathcal{L}^{-1}\left\{2 \cdot \frac{2}{s^2-9}\right\}$

$2 \sinh(2t)$

c) $\frac{s}{s^2-2s+1}$

$\mathcal{L}^{-1}\left\{\frac{s}{s^2-2s+1}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{(s-1)^2}\right\}$

$\frac{A}{s-1} + \frac{B}{(s-1)^2} = \frac{s}{(s-1)^2}$

$A(s-1) + B = s$

$A = 1$

$-A + B = 0$

$B = 1$

$e^t + t e^t = e^t(1+t)$

Ejercicios Tercera Sección

a) $y''' - 3y'' + 2y' = 0$

$y(0) = 1, y'(0) = 0, y''(0) = 0$

$s^3 Y(s) - s^2 Y(0) - s Y'(0) - Y''(0) = 0$

$s^3 Y(s) - s^2(1) - s(0) - 0 = 0$

$s^3 Y(s) - s^2 = 0$

$Y(s) = \frac{s^2}{s^3-3s+2}$

$Y(s) = \frac{s^2-3s+2}{(s-1)(s-2)(s+1)}$

$s+2 \left| \frac{s^2-3s+2}{(s-1)(s-2)(s+1)} \right|$

$A(s-1) + B(s-2) + C(s+1) = s^2-3s+2$

$A(s-1) + B(s-2) + C(s+1) = s^2-3s+2$

$A+B+C=1$

$-A+B+C=0$

$A-2B+C=3$

$B=C-2, A=-7$

$B=3, C=5$

$\mathcal{L}^{-1}\left\{\frac{7}{s+2} + \frac{3}{s-1} + \frac{5}{s-2}\right\}$

$-7e^{-2t} + 3e^t + 5e^{2t} = Y(t)$

b) $y''' - y'' + y' - y = 0$

$y(0) = 1, y'(0) = 0, y''(0) = 0$

$s^3 Y(s) - s^2 Y(0) - s Y'(0) - Y''(0) = 0$

$s^3 Y(s) - s^2(1) - s(0) - 0 = 0$

$s^3 Y(s) - s^2 = 0$

$Y(s) = \frac{s^2}{s^3-s^2+s-1}$

$Y(s) = \frac{s^2-1}{s^3-s^2+s-1}$

$s+1 \left| \frac{s^2-1}{s^3-s^2+s-1} \right|$

$\frac{A}{s+1} + \frac{B(s-1)}{s^2-1} = \frac{s^2-1}{(s+1)(s-1)^2}$

$A(s+1) + B(s-1) = s^2-1$

$A+B=1$

$-B+C=1$

$A-C=1$