TAREA. III
Factor Integrante, Bernoulli, Riccati

I. Factor integrante

a) $\frac{dy}{dx} = \frac{\operatorname{sen}y}{x\cos y - \operatorname{sen}^2 y} \rightarrow \frac{dx}{dy} = \frac{x\cos y - \operatorname{sen}^2 y}{\operatorname{sen}y}$

$\frac{dx}{dy} = x\cot y - \operatorname{sen}y \rightarrow x' + P(y)x = f(y)$

$x' + (-x\cot y) = -\operatorname{sen}y$

$\text{Factor} = e^{\int -\cot y\,dy} = e^{-\int \cot y\,dy} = e^{-\ln \operatorname{sen}y}$

$= \frac{1}{\operatorname{sen}y} = \csc y$

$\frac{dx}{dy}\csc y - x\cot y\csc y = -\operatorname{sen}y\csc y$

$\frac{dx}{dy}\csc y - x\cot y\csc y = -1$

$\frac{d}{dy}\left[x\csc y\right] = -1$

$x\csc y = -y + C$

$\underline{x = \operatorname{sen}y(C - y)}$

b) $\frac{dI}{dt} = \frac{t - tI}{t^2+1}$

$I' = \frac{t}{t^2+1} - \frac{t}{t^2+1}I$

$I' + I\frac{t}{t^2+1} = \frac{t}{t^2+1} \rightarrow \text{Factor: } e^{\int \frac{t}{t^2+1}dt} = e^{\frac{1}{2}\ln U} = \sqrt{t^2+1}$

$u = t^2+1$

$du = 2t\,dt$

$I'\sqrt{t^2+1} + I\frac{\sqrt{t^2+1}\,t}{t^2+1} = \frac{\sqrt{t^2+1}\,t}{t^2+1}$

$\frac{d\left[\sqrt{t^2+1}\,I\right]}{dx} = \frac{t}{\sqrt{t^2+1}}$ $\quad u = t^2+1$, $du = 2t\,dt$

$I\sqrt{t^2+1} = \frac{1}{2}\frac{u^{1/2}}{\frac{1}{2}}$

$I\sqrt{t^2+1} = \sqrt{t^2+1} + C$

$\underline{I = 1 + \frac{C}{\sqrt{t^2+1}}}$

c) $(x + x^3 \operatorname{sen}(2y))\,dy - 2y\,dx = 0 \rightarrow$ No es lineal

$(x + x^3 \operatorname{sen}(2y))\,dy = 2y\,dx$

$$\frac{dy}{dx} = \frac{2y}{(x + x^3 \operatorname{sen}(2y))}$$

$$\frac{dy}{dx} - \frac{2y}{(x + x^3 \operatorname{sen}(2y))} = 0$$

No se resuelve por factor integrante

$$y' + P(x)y = q(x)y^n$$

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$$z = y^{1-n}$$

Bernoulli, resolver por método de elección.

a) $x^2 \dfrac{dy}{dx} + y^2 = xy$

$$x^2 \frac{dy}{dx} - xy = -y^2$$

$$y' - \frac{y}{x} = \frac{-y^2}{x^2} \longrightarrow z = y^{1-2} = \frac{1}{y} \rightarrow y = \frac{1}{z}$$

$$y' = \frac{-z'}{z^2}$$

$$\frac{-z'}{z^2} - \frac{1}{zx} = -\frac{\left(\frac{1}{z}\right)^2}{x^2}$$

$$\frac{-z'}{z^2} - \frac{1}{zx} = \frac{-\frac{1}{z^2}}{x^2} = \frac{-1}{x^2z^2} \leftarrow \text{Todo por } -z^2$$

$$z' + \frac{z}{x} = \frac{1}{x^2} \rightarrow \text{No separable}$$

$$\frac{dz}{dx} + \frac{z}{x} = \frac{1}{x^2} \rightarrow dz + dx\left(\frac{z}{x} - \frac{1}{x^2}\right) = 0$$

No son homogeneas
Ni exactas

Factor: $e^{\int \frac{1}{x} dx} = e^{\ln x} = x$

$$\frac{dz}{dx}x + z = \frac{1}{x}$$

$$\frac{d}{dx}\left[\frac{z}{x}\right] = \frac{1}{x}$$

$$\frac{z}{x} = \ln x + C$$

$$z = \frac{\ln x + C}{x}$$

$$\underline{y = \frac{x}{\ln x + C}}$$

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b) $\frac{x\,dy}{dx} - (1+x)y = xy^2$

$z = y^{1-n} = \frac{1}{y} \Rightarrow y = \frac{1}{z}$

$y' = \frac{-z'}{z^2}$

$y' - (1+x)\frac{y}{x} = y^2$

$\frac{-z'}{z^2} - \frac{(1+x)}{xz} = \frac{1}{z^2}$

$z' + \frac{z(1+x)}{x} = -1$

Factor: $e^{\int \frac{(1+x)}{x}dx} = e^{\int \frac{1}{x}dx}\, e^{\int dx} = xe^x$

$z'xe^x + \frac{z'xe^x(1+x)}{x} = -xe^x$

$\frac{d}{dx}[zxe^x] = -\int xe^x dx$

$zxe^x = -(e^x - xe^x + c) =$

$zxe^x = -e^x + xe^x + c$

$z = \frac{-e^x(1-x)+c}{xe^x}$

$y = \frac{-xe^x}{e^x(1-x)+c}$

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3. Resuelva ecuación de Riccati.

a) $\dfrac{dy}{dx} = -\dfrac{1}{x^2} - \dfrac{y}{x} + y^2, \quad y_1 = \dfrac{2}{x}$

$y_1' = \dfrac{-2}{x^2}$

1. Comprobar

$$\frac{-2}{x^2} = \frac{-1}{x^2} - \frac{2}{x^2} + \frac{4}{x^2}$$

$$\frac{-2}{x^2} \neq \frac{1}{x^2} \leftarrow \text{Solución Parcial}$$