SUMMARY

- The **rate of return (ROR)** is the interest rate earned on unrecovered project balances such that an investment's cash receipts make the terminal project balance equal to zero. The rate of return is an intuitively familiar and understandable measure of project profitability that many managers prefer to NPW or other equivalence measures.
- Mathematically, we can determine the rate of return for a given project cash-flow series by locating an interest rate that equates the net present worth of the project's cash flows to zero. This break-even interest rate is denoted by the symbol i^*.
- The **internal rate of return (IRR)** is another term for ROR, which stresses the fact that we are concerned with the interest earned on the portion of the project that is internally invested, not those portions released by (borrowed from) the project.
- To apply rate-of-return analysis correctly, we need to classify an investment as either simple or nonsimple. A **simple investment** is defined as an investment in which the initial cash flows are negative and only one sign change in the net cash flow occurs, whereas a **nonsimple investment** is an investment for which more than one sign change in the cash flow series occurs. Multiple i^*'s occur only in nonsimple investments. However, not all nonsimple investments will have multiple i^*'s. In this regard,
  1. The possible presence of multiple i^*'s (rates of return) can be predicted by
     - The net cash-flow sign test
     - The accumulated cash-flow sign test

     When multiple rates of return cannot be ruled out by the two methods, it is useful to generate an NPW profile to approximate the value of i^*.
  2. All i^* values should be exposed to the **net-investment test**. Passing this test indicates that i^* is an internal rate of return and is therefore a suitable measure of project profitability. Failing to pass the test indicates project borrowing, a situation that requires further analysis with the use of an **external interest rate**.
  3. **Return-on-invested-capital** analysis uses one rate (the firm's MARR) on externally invested balances and solves for another rate (i^*) on internally invested balances.
- For a pure investment, i^* is the rate of return that is internal to the project. For a mixed investment, the RIC calculated with the use of the external interest rate (or MARR) is the true IRR; so the decision rule is as follows.

  If IRR $>$ MARR, accept the project.
  If IRR $=$ MARR, remain indifferent.
  If IRR $<$ MARR, reject the project.

  IRR analysis yields results consistent with NPW and other equivalence methods.
- In properly selecting among alternative projects by IRR analysis, **incremental investment** must be used. In creating an incremental investment, we always subtract the lower-cost investment from the higher-cost one. Basically, you want to know that the extra investment required can be justified on the basis of the extra benefits generated in the future.

PROBLEMS

Note: *The symbol i* represents the interest rate that makes the net present value of the project in question equal to zero. The symbol IRR represents the internal rate of return of the investment. For a simple investment, IRR = i*. For a nonsimple investment, i* is generally not equal to IRR.*

Concept of Rate of Return

7.1 Suppose that you invest $1,000 in stock, which is called your financial asset. One year later, your investment yields $1,245. What is the rate of return of your investment?

7.2 You are going to buy a new car worth $24,500. The dealer computes your monthly payment to be $514.55 for 60 months of financing. What is the dealer's effective rate of return on this loan transaction?

7.3 You wish to sell a bond that has a face value of $1,000. The bond bears an interest rate of 7%, which is payable semiannually. Four years ago, the bond was purchased at $950. At least an 8% annual return on the investment is desired. What must be the minimum selling price of the bond now in order to make the desired return on the investment?

***7.4** In 1970, Wal-Mart offered 300,000 shares of its common stock to the public at a price of $16.50 per share. Since that time, Wal-Mart has had 11 two-for-one stock splits. On a purchase of 100 shares at $16.50 per share on the company's first offering, the number of shares has grown to 204,800 shares worth $10,887,600 in January 2010. What is the return on investment for investors who purchased the stock in 1970 (over a 40-year ownership period)? Assume that no dividends were received during that period.

7.5 Johnson Controls spent more than $2.5 million retrofitting a government complex and installing a computerized energy-management system for the State of Massachusetts. As a result, the state's energy bill dropped from an average of $6 million a year to $3.5 million. Moreover, both parties will benefit from the 10-year life of the contract. Johnson recovers half the money it saved in reduced utility costs (about $1.2 million a year over 10 years); Massachusetts has more money to spend on other things. What is the rate of return realized by Johnson Controls in this energy-control system?

***7.6** Suppose that you invest $50,000 in a restaurant business. One year later, you sell half of this business to a partner for $100,000. Then a year later, the business is in the red, and you have to pay $40,000 to close the business. What is the rate of return on your investment from this restaurant business?

7.7 Pablo Picasso's 1905 portrait *Boy with a Pipe* sold for $104.2 million in an auction at Sotheby's Holdings, Inc., on June 24, 2004, shattering the existing record for art and ushering in a new era in pricing for 20th-century paintings. The Picasso, sold by the philanthropic Greentree Foundation, cost Mr. Whitney about $30,000 in 1950. Determine the annual rate of appreciation of the artwork over 54 years.

Investment Classification and Calculation of i^*

7.8 Consider four investments with the sequences of cash flows given in Table P7.8.

TABLE P7.8

	Net Cash Flow			
n	Project A	Project B	Project C	Project D
0	−$22,000	−$28,000	$35,000	−$56,500
1	30,000	32,000	−18,000	2,500
2	20,000	32,000	−18,000	6,459
3	10,000	−22,000	−18,000	78,345

(a) Identify all the simple investments.
(b) Identify all the nonsimple investments.
(c) Compute i^* for each investment.
(d) Which project has no rate of return?

7.9 Consider the infinite cash-flow series with repeated cash-flow patterns given in Table P7.9.

TABLE P7.9

n	A_n
0	−$1,000
1	500
2	900
3	800
4	700
5	500
6	900
7	800
8	700
⋮	⋮

Determine i^* for this infinite cash-flow series.

7.10 Consider the investment projects given in Table P7.10.

(a) Classify each project as either simple or non-simple.

(b) Use the quadratic equation to compute i^* for project A.

(c) Obtain the rate(s) of return for each project by plotting the NPW as a function of the interest rate.

7.11 Consider the projects given in Table P7.11.

(a) Classify each project as either simple or non-simple.

(b) Identify all positive i^*'s for each project.

(c) For each project, plot the present worth as a function of the interest rate (i).

7.12 Consider the financial data for a project given in Table P7.12.

TABLE P7.10

	Project Cash Flow				
n	A	B	C	D	E
0	−$150	−$200	−$300	−$70	−$60
1	60	80	$200	120	−120
2	150	80	10	40	−50
3	—	60	50	40	0
4	—	60	−180	−200	160
5	—	—	60	40	150
6	—	—	50	30	140
7	—	—	500	—	130

TABLE P7.11

	Net Cash Flow			
n	A	B	C	D
0	−$2,000	−$1,800	−$10,000	−$2,500
1	100	800	5,600	−1,360
2	100	600	4,900	4,675
3	100	500	−3,500	2,288
4	2,100	700	7,000	—
5	—	—	−1,400	—
6	—	—	2,100	—
7	—	—	900	—

TABLE P7.12

Initial investment	$90,000
Project life	6 years
Salvage value	$10,000
Annual revenue	$27,000
Annual expenses	$8,000

(a) What is i^* for this project?
(b) If the annual expense increases at a 7% rate over the previous year's expenses, but the annual income is unchanged, what is the new i^*?
(c) In part (b), at what annual rate will the annual income have to increase to maintain the same i^* obtained in part (a)?

***7.13** Consider two investments, A and B, with the sequences of cash flows given in Table P7.13.

TABLE P7.13

	Net Cash Flow	
n	Project A	Project B
0	−$30,000	−$30,000
1	2,000	11,000
2	6,000	10,000
3	10,000	10,000
4	24,000	8,000
5	26,000	5,000

(a) Compute i^* for each investment.
(b) Plot the present-worth curve for each project on the same chart, and find the interest rate that makes the two projects equivalent.

7.14 Consider the investment projects given in Table P7.14.

(a) For each project, apply the sign rule to predict the number of possible i^*'s.
(b) For each project, plot the NPW profile as a function of i between 0 and 200%.
(c) For each project, compute the value(s) of i^*.

7.15 Consider an investment project with the cash flows given in Table P7.15.

(a) Find the IRR for this investment.
(b) Plot the present worth of the cash flow as a function of i.
(c) On the basis of the IRR criterion, should the project be accepted at MARR = 15%?

Mixed Investments

7.16 Consider the investment projects given in Table P7.16.

Assume that MARR = 12% in the following questions.

(a) Compute i^* for each investment. If the problem has more than one i^*, identify all of them.
(b) Compute IRR(true) for each project.
(c) Determine the acceptability of each investment.

7.17 Consider the investment projects given in Table P7.17.

(a) Use the quadratic formula to compute i^* for A.
(b) Classify each project as either simple or non-simple.

TABLE P7.14

	Project Cash Flows					
n	A	B	C	D	E	F
0	−$150	−$100	−$200	−$180	−$100	−$100
1	200	470	−100	0	300	300
2	0	720	200	0	250	−100
3	400	−360	250	500	−40	400

TABLE P7.15

n	Net Cash Flow
0	−$220,000
1	94,000
2	144,000
3	72,000

TABLE P7.16

	Net Cash Flow		
n	Project 1	Project 2	Project 3
0	−$1,000	−$1,000	−$1,000
1	2,500	1,960	1,400
2	−840	950	−200
IRR	?	?	?

(c) Apply the cash-flow sign rules to each project, and determine the number of possible positive i^*'s. Identify all projects having a unique i^*.

(d) Compute the IRRs for projects B through E. Assume MARR = 12%.

(e) Apply the net-investment test to each project.

7.18 You established an investment fund with an initial deposit of $5,000. You contributed another $5,000 at the end of the first year. The fund value, including interest, is $11,500 at the end of two years. Find the internal rate of return from your investment fund.

7.19 Consider the investment projects given in Table P7.19.

Assume that MARR = 12% in the following questions:

(a) Identify the i^*('s) for each investment. If the project has more than one i^*, identify all of them.

(b) Which project(s) is (are) a mixed investment?

TABLE P7.17

	Project Cash Flow				
n	A	B	C	D	E
0	−$150	−$150	−$150	−$100	$200
1	150	40	30	60	100
2	36	40	90	0	−500
3	—	70	−120	40	−500
4	—	70	—	30	200
5	—	—	—	20	600

TABLE P7.19

	Net Cash Flow		
n	Project 1	Project 2	Project 3
0	−$8,500	−$5,000	−$1,500
1	10,000	10,000	6,000
2	10,000	30,000	−6,000
3	—	−40,000	—

(c) Compute the IRR for each project.
(d) Determine the acceptability of each project.

7.20 Consider the investment projects given in Table P7.20.

Assume that MARR = 12% for the following questions.
(a) Identify the pure investment(s).
(b) Identify the mixed investment(s).
(c) Determine the IRR for each investment.
(d) Which project would be acceptable?

***7.21** By outbidding its competitors, Trane Image Processing (TIP), a defense contractor, received a contract worth $7,300,000 to build navy flight simulators for U.S. Navy pilot training over two years. With some defense contracts, the U.S. government makes an advance payment when the contract is signed, but in this case, the government will make two progressive payments: $4,300,000 at the end of the first year and the $3,000,000 balance at the end of the second year. The expected cash outflows required to produce the simulators are estimated to be $1,000,000 now, $2,000,000 during the first year, and $4,320,000 during the second year. The expected net cash flows from this project are summarized in Table P7.21.

In normal situations, TIP would not even consider a marginal project such as this one. However, hoping that the company can establish itself as a technology leader in the field, management felt that it was worth outbidding its competitors. Financially, what is the economic worth of outbidding the competitors for this project?

(a) Determine whether this project is or is not a mixed investment.
(b) Compute the IRR for this investment. Assume MARR = 12%.
(c) Should Boeing accept the project?

7.22 Consider the investment projects given in Table P7.22.

(a) Compute i^* for project A. If there is more than one i^*, identify all of them.
(b) Identify the mixed investment(s).

TABLE P7.20

	Net Cash Flow		
n	Project A	Project B	Project C
0	−$1,000	−$1,250	−$300
1	4,000	500	1,230
2	0	500	−1,674
3	−2,000	500	756
4	—	500	—
IRR	(−21.07%, 286.62%)	(21.86%)	(20%, 40%, 50%)

TABLE P7.21

Year	Cash Inflow	Cash Outflow	Net Cash Flow
0	—	$1,000,000	−$1,000,000
1	$4,300,000	2,000,000	2,300,000
2	3,000,000	4,320,000	−1,320,000

TABLE P7.22

	Net Cash Flow		
n	Project A	Project B	Project C
0	−$1,000	—	−$1,500
1	2,060	−250	1,000
2	−1,060	200	−800
3	—	100	2,000
4	—	50	—
*i**	?	24.86%	17.66%

(c) Assuming that MARR = 10%, determine the acceptability of each project on the basis of the IRR criterion.

7.23 Consider the investment projects given in Table P7.23.

Assume that MARR = 12% in the following questions.

(a) Compute *i** for projects A and B. If the project has more than one *i**, identify all of them.

(b) Classify each project as either a pure or a mixed investment.

(c) Compute the IRR for each investment.

(d) Determine the acceptability of each project.

7.24 Consider an investment project whose cash flows are as in Table P7.24.

(a) Plot the present-worth curve by varying *i* from 0% to 250%.

(b) Is this a mixed investment?

(c) Should the investment be accepted at MARR = 18%?

TABLE P7.24

n	Net Cash Flow
0	−$8,000
1	10,000
2	30,000
3	−40,000

7.25 Consider the cash flows of a certain project given in Table P7.25.

TABLE P7.23

	Net Cash Flow				
n	A	B	C	D	E
0	−$1,000	−$5,000	−$2,000	−$2,000	−$1,000
1	3,100	20,000	1,560	2,800	3,600
2	−2,200	12,000	944	−200	−5,700
3	—	−3,000	—	—	3,600
*i**	?	?	18%	−92.45%, 32.45%	35.39%

TABLE P7.25

n	Net Cash Flow
0	−$150,000
1	465,000
2	−330,000

The project's *i**'s are computed as 10% and 100%, respectively. The firm's MARR is 8%.

(a) Show why this investment project fails the net-investment test.

(b) Compute the IRR, and determine the acceptability of this project.

7.26 Consider the investment projects given in Table P7.26.

TABLE P7.26

	Net Cash Flow		
n	Project 1	Project 2	Project 3
0	−$1,000	−$1,000	−$1,000
1	−1,000	1,600	1,500
2	4,000	−300	−500
3	3,000	−200	2,000

Which of the following statements is correct?

(a) All projects are nonsimple investments.

(b) Project 3 should have three real rates of return.

(c) All projects will have a unique positive real rate of return.

(d) None of the above.

IRR Analysis

7.27 You are considering an investment that costs $2,000. It is expected to have a useful life of three years. You are very confident about the revenues during the first two years but you are unsure about the revenue in year 3. If you hope to make at least a 10% rate of return on your investment ($2,000), what should be the minimum revenue in year 3?

Year	Cash Flow
0	−$2,000
1	$1,000
2	$1,200
3	*X*

***7.28** Agdist Corporation distributes agricultural equipment. The board of directors is considering a proposal to establish a facility to manufacture an electronically controlled "intelligent" crop sprayer invented by a professor at a local university. This crop sprayer project would require an investment of $10 million in assets and would produce an annual after-tax net benefit of $1.8 million over a service life of eight years. All costs and benefits are included in these figures. When the project terminates, the net proceeds from the sale of the assets will be $1 million. Compute the rate of return of this project. Is this a good project at MARR = 10%?

7.29 Consider an investment project with the cash flows given in Table P7.29.

Compute the IRR for this investment. Is the project acceptable at MARR = 10%?

7.30 Consider the cash flow of a certain project given in Table P7.30.

TABLE P7.29

n	Cash Flow
0	−$15,000
1	0
2	14,520
3	3,993

TABLE P7.30

n	Net Cash Flow
0	−$2,500
1	700
2	900
3	*X*

If the project's IRR is 10%,

(a) Find the value of X.

(b) Is this project acceptable at MARR = 8%?

7.31 A company invests $2000 at the beginning of a five-year project. At the end of every year for the first three years, the project generates $500. At the end of the fourth year, the project generates no money. At the end of the fifth year, the project is terminated. How much must the project generate at the end of the fifth year to realize a 13% return on the initial investment?

***7.32** You are considering a luxury apartment building project that requires an investment of $14,500,000. The building has 50 units. You expect the maintenance cost for the apartment building to be $350,000 the first year and $400,000 the second year. The maintenance cost will continue to increase by $50,000 in subsequent years. The cost to hire a manager for the building is estimated to be $85,000 per year. After five years of operation, the apartment building can be sold for $16,000,000. What is the annual rent per apartment unit that will provide a return on investment of 15%? Assume that the building will remain fully occupied during its five years of operation.

7.33 A machine costing $35,000 to buy and $4,000 per year to operate will save mainly labor expenses in packaging over six years. The anticipated salvage value of the machine at the end of the six years is $5,000. To receive a 12% return on investment (rate of return), what is the minimum required annual savings in labor from this machine?

7.34 Champion Chemical Corporation is planning to expand one of its propylene manufacturing facilities. At $n = 0$, a piece of property costing $1.5 million must be purchased to build a plant. The building, which needs to be expanded during the first year, costs $3 million. At the end of the first year, the company needs to spend about $4 million on equipment and other start-up costs. Once the building becomes operational, it will generate revenue in the amount of $3.5 million during the first operating year. This will increase at the annual rate of 5% over the previous year's revenue for the next nine years. After 10 years, the sales revenue will stay constant for another three years before the operation is phased out. (It will have a project life of 13 years after construction.) The expected salvage value of the land at the end of the project's life would be about $2 million, the building about $1.4 million, and the equipment about $500,000. The annual operating and maintenance costs are estimated to be approximately 40% of the sales revenue each year. What is the IRR for this investment? If the company's MARR is 15%, determine whether the investment is a good one. (Assume that all figures represent the effect of the income tax.)

***7.35** You are considering purchasing a CNC machine which costs $150,000. This machine will have an estimated service life of 10 years with a net after-tax salvage value of $15,000. Its annual after-tax operating and maintenance costs are estimated to be $50,000. To expect an 18% rate of return on investment, what would be the required minimum annual after-tax revenues?

7.36 Recent technology has made possible a computerized vending machine that can grind coffee beans and brew fresh coffee on demand. The computer also makes possible such complicated functions as changing $5 and $10 bills, tracking the age of an item, and moving the oldest stock to the front of the line, thus cutting down on spoilage. With a price tag of $4,500 for each unit, Easy Snack has estimated the cash flows in millions of dollars over the product's six-year useful life, including the initial investment, as given in Table P7.36.

(a) On the basis of the IRR criterion, if the firm's MARR is 18%, is this product worth marketing?

(b) If the required investment remains unchanged, but the future cash flows are expected to be 10%

TABLE P7.36

n	Net Cash Flow
0	−$30
1	9
2	18
3	20
4	18
5	10
6	5

higher than the original estimates, how much of an increase in IRR do you expect?

(c) If the required investment has increased from $30 million to $35 million, but the expected future cash flows are projected to be 10% smaller than the original estimates, how much of a decrease in IRR do you expect?

Comparing Alternatives

*7.37 Consider two investments A and B with the sequences of cash flows given in Table P7.37.

TABLE P7.37

	Net Cash Flow	
n	Project A	Project B
0	−$125,000	−$110,000
1	30,000	20,000
2	30,000	20,000
3	120,000	130,000

(a) Compute the IRR for each investment.

(b) At MARR = 15%, determine the acceptability of each project.

(c) If A and B are mutually exclusive projects, which project would you select based on the rate of return on incremental investment?

7.38 With $10,000 available, you have two investment options. The first is to buy a certificate of deposit from a bank at an interest rate of 9% annually for five years. The second choice is to purchase a bond for $10,000 and invest the bond's interest in the bank at an interest rate of 5%. The bond pays 6.5% interest annually and will mature to its face value of $10,000 in five years. Which option is better? Assume that your MARR is 5% per year.

7.39 A manufacturing firm is considering the mutually exclusive alternatives given in Table P7.39.

Determine which project is a better choice at a MARR = 15% based on the IRR criterion.

7.40 Consider the two mutually exclusive alternatives given in Table P7.40.

TABLE P7.39

	Net Cash Flow	
n	Project A1	Project A2
0	−$2,500	−$3,600
1	1,600	2,600
2	1,840	2,200

TABLE P7.40

	Net Cash Flow	
n	Project A1	Project A2
0	−$11,000	−$13,000
1	5,000	6,200
2	5,000	6,200
3	5,000	6,200

(a) Determine the IRR on the incremental investment in the amount of $2,000.

(b) If the firm's MARR is 10%, which alternative is the better choice?

7.41 Consider the two mutually exclusive investment alternatives given in Table P7.41.

(a) Determine the IRR on the incremental investment in the amount of $5,000. (Assume that MARR = 10%.)

TABLE P7.41

	Net Cash Flow	
n	Project A1	Project A2
0	−$15,000	−$20,000
1	7,500	8,000
2	7,500	15,000
3	7,500	5,000
IRR	23.5%	20%