TAREA IV

1. Coeficientes constantes

a) $y'' + 4y' - 5y = 0$

$m^2 + 4m - 5 = 0$

$(m-1)(m+5) = 0$

$m_1 = 1$

$m_2 = -5$

$y_K = Ce^{m_1 x}$

$y = C_1 e^{x} + C_2 e^{-5x}$

b) $16y'' - 8y' + y = 0$

$16m^2 - 8m + 1$

$m = \frac{8 \pm \sqrt{0}}{32} = m_{1,2} = \frac{1}{4}$

$y = C_1 e^{\frac{x}{4}} + C_2 x e^{\frac{x}{4}}$

c) $y''' + 5y'' + 2y = 0$

$1 \quad 5 \quad 0 \quad 2 \,|\, m + 5.0776$

$m^3 + 5m^2 + 2 = 0$

$m_1 = -5.0776$

$m_2 = 0.03879 + 0.6264i$

$m_3 = 0.03879 - 0.6264i$

$y = C_1 e^{-5.0776x} + C_2 e^{0.03879x} \cos(0.6264x) + C_3 e^{0.03879x} \operatorname{sen}(0.664x)$

TAREA IV

d) $y^{IV} - 20y'' + 25y = 0$

$m^4 - 20m^2 + 25 = 0$

$m^2 = \frac{20 \pm \sqrt{400-100}}{2} = 10 \pm 5\sqrt{3}$

$m = \pm\sqrt{10 \pm 5\sqrt{3}}$

$y = C_1 e^{-\sqrt{10-5\sqrt{3}}\,x} + C_2 e^{-\sqrt{10+5\sqrt{3}}\,x} + C_3 e^{\sqrt{10-5\sqrt{3}}\,x} + C_4 e^{\sqrt{10+5\sqrt{3}}\,x}$

e) $y'' + 4y = 0$

$m^2 + 4 = 0$

$m^2 = -4$

$m = \pm\sqrt{-4}$

$m_1 = 2i, \quad m_2 = -2i$

$m_1 = \alpha + i\beta \rightarrow C_1 e^{\alpha x} \cos \beta x$

$y = (C_1 \cos(2x) + C_2 \operatorname{sen}(2x))$

f) $4y'' - 8y' + 7y = 0$

$4m^2 - 8m + 7 = 0$

$m = \frac{8 \pm \sqrt{64 - 16(7)}}{2(4)}$

$= \frac{8 \pm 4\sqrt{4-7}}{8}$

$= 1 \pm \frac{\sqrt{3}\,i}{2}$

$y = e^{x}\left(C_1 \cos\left(\frac{\sqrt{3}x}{2}\right) + C_2 \operatorname{sen}\left(\frac{\sqrt{3}x}{2}\right)\right)$

2. Resuelva por coeficientes indeterminados

a) $y'' + 16y = 5\sin x$

$y'' + 16y = 0$

$m^2 + 16 = 0$
$m = \pm 4i$

$y_c = C_1\cos(4x) + C_2\,\text{sen}(4x)$

$D^2 + 16 = 5\sin x$
$(D^2+16)(D^2+1) = (5\sin x)(D^2+1)$
$(D^2+16)(D^2+1) = 0$
$\rightarrow \underline{(m^2+16)(m^2+1) = 0}$
$\hookrightarrow m = \pm i$

$y_p = A\cos(x) + B\,\text{sen}(x) \Leftarrow$

Se sustituye

$y''_p = -A\cos x - B\,\text{sen}x \leftarrow$

$-A\cos x - B\,\text{sen}x + 16(A\cos x + B\,\text{sen}x) = 5\sin x$

$15A\cos x + 15B\,\text{sen}x \rightarrow A = 0,\ B = 1/3$

$y_p = \frac{\sin x}{3}$

$$y = C_1\cos(4x) + C_2\,\text{sen}(4x) + \frac{\sin x}{3}$$

b) $y'' + y = 2e^{3x}$

$y'' + y = 0$

$m_{1,2} = \pm i$

$y_c = C_1 \cos x + C_2 \operatorname{sen} x$

$(D^2 + 1) = 2e^{3x}$

$(D^2+1)(D-3) = \cancel{2e^{3x}(D-3)}$

$\downarrow$

$m = 3$

$y_p = Ae^{3x} \leftarrow$ c sustituye $y_p' = 3Ae^{3x}$ $y_p'' = 9Ae^{3x}$

$9Ae^{3x} + Ae^{3x} = 2e^{3x}$

$A = \frac{1}{5}$

$y = C_1 \cos x + C_2 \operatorname{sen} x + \frac{e^{3x}}{5}$

c) $(D^2 + 2D + 1)y = 4\sin 2x$ Dos interpretaciones...

① $(y'' + 2y' + y)y = 4\sin(2x) \Rightarrow$ No tiene la forma $a_0 y^n + \dots + a_n y$

No se puede resolver por coef. indeterminado

② $(D^2 + 2D + 1)[y] = 4\sin 2x$

$m^2 + 2m + m = 0$

$(m+1)^2 = 0$

$m_{1,2} = -1 \Rightarrow y_c = C_1 e^x + C_2 x e^x$

$(D^2 + 2D + 1) = 4\sin(2x)$

$(D^2 + 2D + 1)(D^2 + 4) = 4\sin(2x)(D^2 + 4)$

$(D^2 + 2D + 1)(D^2 + 4) = 0$

$\hookrightarrow D = \pm 2i$

$\Rightarrow y_p = A\cos(2x) + B\operatorname{sen}(2x)$

$y_p' = -2A\operatorname{sen}(2x) + 2B\cos(2x)$

$y_p'' = -4A\cos(2x) - 4B\operatorname{sen}(2x)$

$-4A\cos(2x) - 4B\operatorname{sen}(2x) - 4A\operatorname{sen}(2x) + 4B\cos(2x) + A\cos(2x) + B\operatorname{sen}(2x) = 4\sin(2x)$

$\operatorname{sen}(2x)(-3B - 4A) + \cos(2x)(-3A + 4B) = 4\sin(2x)$

$-3B - 4A = 4 \rightarrow -12B - 16A = 16 \Rightarrow -25A = 16 \Rightarrow A = \frac{-16}{25}$

$+4B - 3A = 0 \quad 12B - 9A = 0 \quad B = \frac{-12}{25}$

$y = C_1 e^x + C_2 x e^x - \frac{16\cos(2x)}{25} - \frac{12\sin(2x)}{25}$

d) $4y'' + 1 = t^2 + 2\cos(3t)$

$4m^2 + 1 = 0$

$m^2 = \frac{-1}{4}$

$m = \pm \frac{i}{2} \rightarrow y_c = C_1 \cos\left(\frac{t}{2}\right) + C_2\, sen\left(\frac{t}{2}\right)$

$(4D^2 + 1)(D^3)(D^2 + 9) = t^2 + 2\cos(3t)$

$(4D^2 + 1)(D^3)(D^2 + 9) = 0$

$m = 0 \qquad m = \pm 3i \rightarrow y_p = A\cos(3t) + B\,sen(3t) + C + Dx + Et^2$

$y'' = -9A\cos(3t) - 9B\,sen(3t) + 2E$

$-36A\cos(3t) - 36B\,sen(3t) + 8E + A\cos(3t) + B\,sen(3t) + C + Dt + Et^2 = t^2 + 2\cos(3t)$

$-35A\cos(3t) - 35B\,sen(3t) + C + Dt + 8E + Et^2 = t^2 + 2\cos(3t)$

$A = \frac{-2}{35}, \quad B = 0, \quad D = 0, \quad E = 1$

$C + 8E = 0$

$C = -8$

$y = C_1 \cos\left(\frac{t}{2}\right) + C_2\, sen\left(\frac{t}{2}\right) - \frac{2\cos(3t)}{35} + t^2 - 8$