

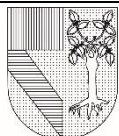


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COEFICIENTES
TAREA IV.

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 UNIVERSIDAD PANAMERICANA Campus Bonaterra	Escuela de Ingeniería		Problemario
	Área: Ciencias Básicas		Fecha: 26/03/2021
	Materia: Ecuaciones Diferenciales.		Ciclo: 1212
	Profesor: MCI Luis Alonso Romo Mercado		CALIFICACIÓN
	Carrera:		
	Alumno(a):		

1. Resuelva las siguientes ecuaciones diferenciales por el método de coeficientes constantes.

a) $y'' + 4y' - 5y = 0$

b) $16y'' - 8y' + y = 0$

c) $y''' + 5y'' + 2y = 0$

d) $y^{IV} - 20y'' + 25y = 0$

e) $y'' + 4y = 0$

f) $4y'' - 8y' + 7y = 0$

2. Resuelva las siguientes ecuaciones diferenciales por el método de coeficientes indeterminados.

a) $y'' + 16y = 5 \sin x$

b) $y'' + y = 2e^{3x}$

c) $(D^2 + 2D + 1)y = 4 \sin 2x$

d) $4l'' + l = t^2 + 2 \cos 3t$

TARE1 IV

I. Coeficientes constantes

a) $y'' + 4y' - 5y = 0$

$$m^2 + 4m - 5 = 0$$

$$(m-1)(m+5) = 0$$

$$m_1 = 1$$

$$m_2 = -5$$

$$y = C_1 e^x + C_2 e^{-5x}$$

$$y_k = e^{m_k x}$$

b) $16y'' - 8y' + y = 0$

$$16m^2 - 8m + 1 = 0$$

$$m = \frac{8 \pm \sqrt{64 - 64}}{32} = m_{1,2} = \frac{1}{4}$$

$$y = C_1 e^{\frac{x}{4}} + C_2 x e^{\frac{x}{4}}$$

c) $y''' + 5y'' + 2y = 0$

$$m^3 + 5m^2 + 2 = 0$$

$$m_1 = -5.0776$$

$$m_2 = 0.03879 + 0.6264i$$

$$m_3 = 0.03879 - 0.6264i$$

$$1502/m + 5.0776$$

$$y = C_1 e^{-5.0776x} + C_2 e^{0.03879x} \cos(0.6264x) + C_3 e^{0.03879x} \sin(0.6264x)$$

TAREA IV

a) $y^{IV} - 20y'' + 25y = 0$

$$m^4 - 20m^2 + 25 = 0$$

$$m^2 = \frac{20 \pm \sqrt{400 - 100}}{2} = 10 \pm 5\sqrt{3}$$

$$m = \pm \sqrt{10 \pm 5\sqrt{3}}$$

$$y = C_1 e^{-\sqrt{10-5\sqrt{3}}x} + C_2 e^{-\sqrt{10+5\sqrt{3}}x} + C_3 e^{\sqrt{10-5\sqrt{3}}x} + C_4 e^{\sqrt{10+5\sqrt{3}}x}$$

e) $y'' + 4y = 0$

$$m^2 + 4 = 0$$

$$m^2 = -4$$

$$m = \pm \sqrt{-4}$$

$$m_1 = 2i, m_2 = -2i$$

$$m_1 = \alpha + i\beta \Rightarrow C_1 e^{\alpha x} \cos \beta x$$

$$y = (C_1 \cos(2x) + C_2 \sin(2x))$$

f) $4y'' - 8y' + 7y = 0$

$$4m^2 - 8m + 7 = 0$$

$$m = \frac{8 \pm \sqrt{64 - 16(7)}}{2}$$

$$= \frac{8 \pm 4\sqrt{4-7}}{2}$$

$$= 1 \pm \frac{\sqrt{3}i}{2}$$

$$y = e^x (C_1 \cos(\frac{\sqrt{3}x}{2}) + C_2 \sin(\frac{\sqrt{3}x}{2}))$$

2. Resuelva por coeficientes indeterminados

a) $y'' + 16y = 5 \sin x$

$$y'' + 16y = 0$$

$$m^2 + 16 = 0$$

$$m = \pm 4i$$

$$y_c = C_1 \cos(4x) + C_2 \operatorname{sen}(4x)$$

$$D^2 + 16 = 5 \sin x$$

$$(D^2 + 16)(D^2 + 1) = (5 \sin x)(D^2 + 1)$$

$$(D^2 + 16)(D^2 + 1) = 0$$

$$\rightarrow (m^2 + 16)(m^2 + 1) = 0$$

$$\rightarrow m = \pm i$$

$$y_p = A \cos(x) + B \operatorname{sen}(x)$$

Se sustituye

$$y''_p = -A \cos x - B \operatorname{sen} x$$

$$-A \cos x - B \operatorname{sen} x + 16(A \cos x + B \operatorname{sen} x) = 5 \sin x$$

$$15A \cos x + 15B \operatorname{sen} x \rightarrow A = 0, B = 1/3$$

$$y_p = \frac{\sin x}{3}$$

$$y = C_1 \cos(4x) + C_2 \operatorname{sen}(4x) + \frac{\sin x}{3}$$

$$b) y'' + y = 2e^{3x}$$

$$y'' + y = 0$$

$$m_{1,2} = \pm i$$

$$y_c = C_1 \cos x + C_2 \sin x$$

$$(D^2 + 1) = 2e^{3x} (\quad - \quad)$$

$$(D^2 + 1)(D - 3) = 2e^{3x}(D - 3)$$

↓

$$m = 3$$

$$y_p = Ae^{3x} \leftarrow \text{c sustituye } y_p' = 3Ae^{3x} \quad y_p'' = 9Ae^{3x}$$

$$9Ae^{3x} + Ae^{3x} = 2e^{3x}$$

$$A = \frac{1}{5}$$

$$y = C_1 \cos x + C_2 \sin x + \frac{e^{3x}}{5}$$

$$c) (D^2 + 2D + 1)y = 4 \sin 2x \quad \text{Dos interpretaciones...}$$

① $(y'' + 2y' + y)y = 4 \sin(2x) \rightarrow$ No tiene la forma $a_0 y^n + \dots + a_n y$

② $(D^2 + 2D + 1)[y] = 4 \sin 2x$

No se puede resolver por coef. indeterminados

$$m^2 + 2m + 1 = 0$$

$$(m+1)^2 = 0$$

$$m_{1,2} = -1 \rightarrow y_c = C_1 e^x + C_2 x e^x$$

$$(D^2 + 2D + 1) = 4 \sin(2x)$$

$$(D^2 + 2D + 1)(D^2 + 4) = 4 \sin(2x)(D^2 + 4)$$

$$(D^2 + 2D + 1)(D^2 + 4) = 0$$

$$\rightarrow D = \pm 2i$$

$$\rightarrow y_p = A \cos(2x) + B \sin(2x)$$

$$y_p' = -2A \sin(2x) + 2B \cos(2x)$$

$$y_p'' = -4A \cos(2x) - 4B \sin(2x)$$

$$-4A \cos(2x) - 4B \sin(2x) - 4A \sin(2x) + 4B \cos(2x) + A \cos(2x) + B \sin(2x) = 4 \sin(2x)$$

$$\sin(2x)(-3B - 4A) + \cos(2x)(-3A + 4B) = 4 \sin(2x)$$

$$-3B - 4A = 4 \rightarrow -12B - 16A = 16 \rightarrow -25A = 16 \rightarrow A = \frac{-16}{25}$$

$$+4B - 3A = 0 \quad 12B - 9A = 0$$

$$B = \frac{-12}{25}$$

$$y = C_1 e^x + C_2 x e^x - \frac{16 \cos(2x)}{25} - \frac{12 \sin(2x)}{25}$$

$$d) 4y'' + 1 = t^2 + 2\cos(3t)$$

$$4m^2 + 1 = 0$$

$$m^2 = -\frac{1}{4}$$

$$m = \pm \frac{i}{2} \rightarrow y_c = C_1 \cos\left(\frac{t}{2}\right) + C_2 \sin\left(\frac{t}{2}\right)$$

$$(4D^2 + 1)(D^3)(D^2 + 9) = t^2 + 2\cos(3t)$$

$$(4D^2 + 1)(D^3)(D^2 + 9) = 0$$

$$\downarrow$$

$$m=0$$

$$\downarrow$$

$$m = \pm 3i$$

$$\rightarrow y_p = A \cos(3t) + B \sin(3t) + C + Dt + Et^2$$

$$y'' = -9A \cos(3t) - 9B \sin(3t) + 2E$$

$$-36A \cos(3t) - 36B \sin(3t) + 2E + A \cos(3t) + B \sin(3t) + C + Dt + Et^2 = t^2 + 2\cos(3t)$$

$$-35A \cos(3t) - 35B \sin(3t) + C + Dt + 2Et^2 = t^2 + 2\cos(3t)$$

$$A = -\frac{2}{35}, B = 0, D = 0, E = 1$$

$$35$$

$$C + 2E = 0$$

$$C = -2$$

$$y = C_1 \cos\left(\frac{t}{2}\right) + C_2 \sin\left(\frac{t}{2}\right) - \frac{2\cos(3t)}{35} + t^2 - 2$$

$$35$$