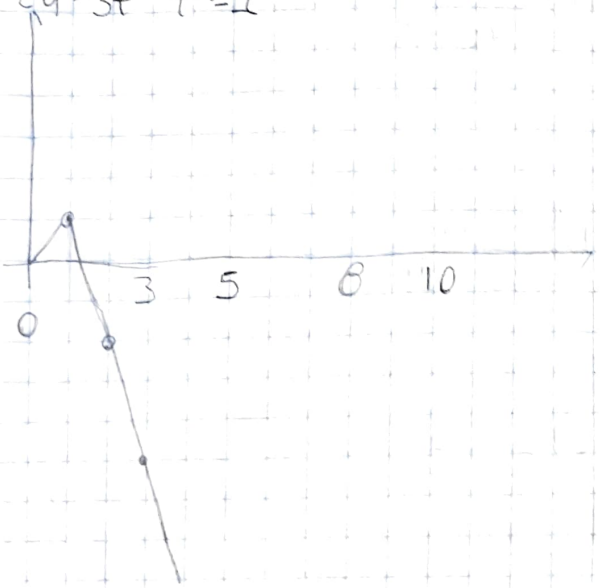
TAREA V

Transformadas de Laplace

a) $f(t) = \begin{cases} t & 0 \le t < 1 \\ 4 - 3t & t \ge 1 \end{cases} = \begin{cases} t & t < 1 \\ 4 - 3t & t \ge 1 \end{cases} = u(t-1)$



$= t + (4 - 3t - t)u(t-1)$

$= t + (4 - 4t)u(t-1)$

$= t + (4 - 4t)U_1(t)$

$= t - 4(-1 + t)U_1(t)$

$\mathcal{L}\{f(t-a)U_a(t)\} = e^{-as}F(s)$

$\mathcal{L} = \frac{1}{s^2} + \frac{-4e^{-s}}{s^2}$

$= \frac{1 - 4e^{-s}}{s^2}$

b) e^{-5t}

$4\ \mathcal{L}\{e^{-5t}\} = \frac{1}{s+5}$

c) te^{-5t}

$5\ \mathcal{L}\{te^{-5t}\} = \frac{1}{(s+5)^2}$

d) $\text{sen}\,t\ u\left(t - \frac{\pi}{2}\right)$

$= \text{sen}\,t\ U_{\frac{\pi}{2}}(t)$

$= \text{sen}\left(t - \frac{\pi}{2} + \frac{\pi}{2}\right) U_{\frac{\pi}{2}}(t)$

$\mathcal{L} = e^{\frac{-\pi s}{2}}\mathcal{L}\{\text{sen}(t + \frac{\pi}{2})\}$

$\{= \text{sen}\,t\cos\frac{\pi}{2} + \text{sen}\frac{\pi}{2}\cos t$
$\{= \cos t$

$\mathcal{L} = \frac{e^{\frac{-\pi s}{2}}\, s}{s^2+1}$

e) $e^{-t}\cos(2t)$

11. $\mathcal{L}\{e^{-t}\cos(2t)\} = \frac{s+1}{(s+1)^2 + 4}$

2. Inversas

a) $\frac{1}{9s-1} = \frac{\frac{1}{9}}{s-\frac{1}{9}} = \frac{1}{9}\left(\frac{1}{s-\frac{1}{9}}\right)$

$\mathcal{L}^{-1}\left\{\frac{1}{9}\left(\frac{1}{s-\frac{1}{9}}\right)\right\} = \frac{\mathcal{L}^{-1}\left\{\frac{1}{s-\frac{1}{9}}\right\}}{9} = \frac{e^{\frac{t}{9}}}{9}$

b) $\frac{4}{s^2-4} = 2\left(\frac{2}{s^2-2^2}\right)$

$\mathcal{L}^{-1}\left\{2\left(\frac{2}{s^2-2^2}\right)\right\} = 2\mathcal{L}^{-1}\left\{\frac{2}{s^2-2^2}\right\} = 2\sinh(2t)$

c) $\frac{s}{s^2-2s+1} = \frac{s}{(s-1)^2}$

Fracciones

$\frac{s}{(s-1)^2} = \frac{A}{s-1} + \frac{B}{(s-1)^2}$

$s = (s-1)A + B$

$= sA - A + B = s(A) + (B-A)$

$A = 1$

$B - A = 0 \Rightarrow A = B = 1$

$\mathcal{L}^{-1}\left\{\frac{1}{s-1} + \frac{1}{(s-1)^2}\right\} = e^t + te^t = e^t(t+1)$

3. Solve

a) $y''' - 3y' + 2y = 0; \quad y(0) = 1,\ y'(0) = 0,\ y''(0) = 0$

$\mathcal{L} = s^3y(s) - s^2y(0) - sy'(0) - y''(0) - 3(sy(s) - y(0)) + 2y(s) = 0$

$= s^3y(s) - s^2 - 3sy(s) + 3 + 2y(s) = 0$

$y(s)(s^3 - 3s + 2) = s^2 - 3$

$y(s) = \frac{s^2-3}{s^3-3s+2} = \frac{s^2-3}{(s-1)^2(s+2)}$

$\frac{s^2-3}{(s-1)^2(s+2)} = \frac{A}{(s-1)} + \frac{B}{(s-1)^2} + \frac{C}{(s+2)} \Rightarrow s^2 - 3 = A(s-1)(s+2) + B(s+2) + C(s-1)^2$

$s^2 - 3 = As^2 + As - 2A + Bs + 2B + Cs^2 - 2Cs + C$

$= s^2(A+C) + s(A+B-2C) + (-2A+2B+C)$

$A + C = 1 \qquad A = 6/9$

$A + B - 2C = 0 \qquad B = -3/3$

$-2A + 2B + C = -3 \qquad C = 1/9$

$y(s) = \frac{6}{9(s-1)} - \frac{6}{9(s-1)^2} + \frac{1}{9(s+2)} \Rightarrow y(t) = \frac{6e^t}{9} - \frac{6te^t}{9} + \frac{e^{-2t}}{9}$

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b) $y''' - y'' + y' - y = 0$; $y(0) = 1$, $y'(0) = y''(0) = 0$

$$\mathcal{L} = s^3 Y(s) - s^2 y(0) - s y'(0) - y''(0) - s^2 Y(s) + s y(0) + y'(0) + s Y(s) - y(0) - Y(s) = 0$$

$$Y(s)(s^3 - s^2 + s - 1) - s^2 + s - 1 = 0$$

$$Y(s) = \frac{s^2 - s + 1}{s^3 - s^2 + s - 1} = \frac{s^2 - s + 1}{(s-1)(s^2+1)}$$

$$\rightarrow \frac{s^2 - s + 1}{(s-1)(s^2+1)} = \frac{A}{(s-1)} + \frac{Bs + C}{(s^2+1)}$$

$$s^2 - s + 1 = A(s^2+1) + B(s-1)$$
$$= As^2 + A + Bs^2 - Bs + Cs - C$$
$$= s^2(A+B) + s(-B+C) + (A-C)$$

$$A = \frac{1}{2} \qquad B = \frac{1}{2} \qquad C = \frac{-1}{2}$$

$$\mathcal{L}^{-1}\left\{ \frac{1}{2(s-1)} + \frac{s}{2(s^2+1)} - \frac{1}{2(s^2+1)} \right\} = \frac{e^t + \cos t - \operatorname{sen} t}{2}$$