TAREA II.

1. Clasifique y resuelva. con separables

Misma integral

a) $\frac{dy}{dx} = \frac{(y-1)(x-2)(y+3)}{(x-1)(y-2)(x+3)} \Rightarrow \frac{(y-2)\,dy}{(y-1)(y+3)} = \frac{(x-2)\,dx}{(x-1)(x+3)}$

$\Rightarrow \frac{(y-2+1-1)dy}{(y-1)(y+3)} = \frac{(y-1)dy}{(y-1)(y+3)} - \frac{dy}{(y-1)(y+3)} = \frac{dy}{(y+3)} - \frac{dy}{(y-1)(y+3)}$

$\Rightarrow \int \frac{dy}{y+3} - \int \frac{dy}{(y-1)(y+3)} = \ln(y+3) - \int \frac{du}{(u-2)(u+2)} =$

$y = u-1$

$y-1 = u-2$ $\quad = \ln(y+3) - \int \frac{du}{u^2-4}$

$y+3 = u+2$

$dy = du$ $\quad = \ln(y+3) - \frac{\ln}{4}\left|\frac{y-1}{y+3}\right|$

$= \ln|y+3| - \frac{1}{4}(\ln|y-1| - \ln|y+3|) = \frac{(5\ln|y+3| - \ln|y-1|)}{4}$

$\rightarrow 5\ln|y+3| - \ln|y-1| = 5\ln|x+3| - \ln|x-1|$

$\frac{e^{5\ln|y+3|}}{e^{\ln|y-1|}} = \frac{e^{5\ln|x+3|}}{e^{\ln|x-1|}} \rightarrow \frac{(y+3)^5}{y-1} = \frac{(x+3)^5}{x-1}$

→ Ordinaria
Orden = 1
Grado =
No lineal
Variable dependiente (x, y)
Variable independiente (y, x)

b) $\dfrac{dU}{ds} = \dfrac{U+1}{\sqrt{s} + \sqrt{sU}}$

$$\frac{dU}{U+1} = \frac{ds}{\sqrt{s} + \sqrt{s}\sqrt{U}} \Rightarrow \frac{(\sqrt{U}+1)\,dU}{U+1} = \frac{ds}{\sqrt{s}}$$

$$\int \frac{(\sqrt{U}+1)\,du}{U+1} = \int \frac{\sqrt{U}\,du}{U+1} + \int \frac{du}{U+1} = \int \frac{2x^2\,dx}{x^2+1} + \ln|U+1| = 2\int \frac{x^2\,dx}{x^2+1} + \ln|U+1|$$

$x = \sqrt{U}$

$x^2 + 1 = U + 1$

$dx = \dfrac{dU}{2\sqrt{U}}$

$$= 2\left[\int \frac{x^2+1}{x^2+1}\,dx - \int \frac{dx}{x^2+1}\right] + \ln|U+1| = 2[x - \arctan x] + \ln|U+1| + c$$

$$= 2\sqrt{U} - 2\arctan\sqrt{U} + \ln|U+1| + c$$

$$\int s^{-1/2}\,ds = 2\sqrt{s} + c$$

$$2\sqrt{s} = 2\sqrt{U} - 2\arctan\sqrt{U} + \ln|U+1| + c$$

→ Ordinaria
Orden = 1
Grado = 1
No lineal

d) $y\,dx + (x^3y^2 + x^3)\,dy = 0$

$y\,dx + (y^2+1)x^3\,dy = 0$

$(y^2+1)x^3\,dy = -y\,dx$

$\dfrac{(y^2+1)\,dy}{-y} = \dfrac{dx}{x^3}$

$-\dfrac{y^2}{2} - \ln y = -\dfrac{1}{2x^2} + c$

$\dfrac{y^2}{2} + \ln y = \dfrac{1}{2x^2} + c$

$y^2 + 2\ln y = \dfrac{1}{x^2} + c$

$$y^2 + \ln(y^2) = \frac{1}{x^2} + c$$

→ Ordinaria
Orden = 1
Grado = 1
No lineal

c) $x\dfrac{dy}{dx} - y = 2x^2y$

$\dfrac{x\,dy}{dx} = y(2x^2+1)$

$\dfrac{dy}{y} = \dfrac{dx(2x^2+1)}{x}$

$\ln y + c_1 = x^2 + \ln x + c_2$

$e^{\ln y} = e^{x^2} e^{\ln x} e^{c}$

$$y = e^{x^2} x C$$

→ Ordinaria
Orden = 1
Grado = 1
Lineal

TAREA II

2. Clasifique y resuelva con homogeneas

a) $(x+2y)dx + (2x+y)dy = 0$

$M(tx,ty) = tx + 2ty$
$= t(x+2y)$
$n=1$
$N(tx,ty) = 2tx + ty$
$= t(2x+y)$
$n=1$

$0 = (x+2(ux))dx + (2x+ux)(udx + xdu)$
$= xdx + 2uxdx + 2uxdx + 2x^2du + u^2xdx + ux^2du$
$= xdx + 4uxdx + x^2du(2+u) + u^2xdx$
$= xdx(1+u^2+4u) + x^2du(2+u)$
$-x^2du(2+u) = xdx(1+u^2+4u)$
$\frac{(2+u)du}{(u^2+4u+1)} = \frac{xdx}{-x^2} \rightarrow -\int \frac{dx}{x} = -\ln x + c$

$\int \frac{(2+u)du}{u^2+4u+1} = \frac{1}{2}\int \frac{dx}{x} = \frac{\ln|u^2+4u+1|}{2} + c$

$x = u^2+4u+1$
$dx = (2u+4)du$
$dx = 2(u+2)du$
$u+2 = \frac{dx}{2}$

$\ln|u^2+4u+1| = -2\ln x + c$
$\frac{1}{2}\ln|u^2+4u+1| = \ln x^{-1}$
$x = \sqrt{\ln|u^2+4u+1|}^{-1}$

$x = \frac{1}{\sqrt{\left(\frac{y}{x}\right)^2 + \frac{4y}{x} + 1}} + c$

→ Ordinaria
Orden = 1
Grado = 1
No lineal

b) $\frac{dy}{dx} = \frac{x}{y} + \frac{y}{x}$

$M(xt,yt) = t^2(x^2+y^2)$
$N(xt,yt) = -t^2xy$
$n=2$

$dy = \left(\frac{x^2+y^2}{yx}\right)dx$
$(x^2+y^2)dx + (-xy)dy = 0$
$(x^2+u^2x^2)dx + (-x(ux))(udx + xdu)$
$x^2dx + u^2x^2dx - u^2x^2dx - ux^3du$
$dx(x^2 + u^2x^2 - u^2x^2) - du(ux^3)$
$dx(x^2) = du(ux^3)$
$\frac{dx}{x} = udu$
$\ln x = \frac{u^2}{2} + C$
$\ln x = \frac{y^2}{2x^2} + C$

→ Ordinaria
Grado = 1
Orden = 1
No lineal

$y = ux$; $x = vy \rightarrow v = \frac{x}{y}$

$dy = u\,dx + x\,du$; $dx = v\,dy + y\,dv$

c) $(2xy + 2y^2 + x^2 + y^2)\,y' + (2x^2 + 2xy + x^2 + y^2) = 0$

$(2xy + 2y^2 + x^2 + y^2)\,dy + (2x^2 + 2xy + x^2 + y^2)\,dx = 0$

$n = 2$; $n = 2$

$(2xy + 3y^2 + x^2)\,dy + (3x^2 + 2xy + y^2)\,dx = 0$

$(2vy^2 + 3y^2 + v^2y^2)\,dy + (3v^2y^2 + 2vy^2 + y^2)(v\,dy + y\,dv) = 0 \rightarrow 1/y^2$

$(2v + 3 + v^2)\,dy + (3v^2 + 2v + 1)(v\,dy + y\,dv) = 0$

$(2v + 3 + v^2)\,dy + (3v^3dy + 3v^2y\,dv + 2v^2dy + 2vy\,dv + v\,dy + y\,dv) = 0$

$2v\,dy + 3dy + v^2dy + 3v^3dy + 3v^2y\,dv + 2v^2dy + 2vy\,dv + v\,dy + y\,dv = 0$

$2v\,dy + 3dy + v^2dy + 3v^3dy + 2v^2dy + v\,dy = -3v^2y\,dv - 2vy\,dv - y\,dv$

$(3v^3 + 3v^2 + 3v + 3)\,dy = -(3v^2y + 2vy + y)\,dv$

$\frac{3\,dy}{y} = \frac{-(3v^2 + 2v + 1)\,dv}{v^3 + v^2 + v + 1}$

Ordinaria
Orden = 1
Grado = 1
No lineal

$3\ln y = -\ln|v^3 + v^2 + v + 1| + C$

$y^3 = \frac{C}{v^3 + v^2 + v + 1} \rightarrow y^3 = \frac{C}{\frac{x^3}{y^3} + \frac{x^2}{y^2} + \frac{x}{y} + 1}$

$y^3 = \frac{C}{\frac{x^3y^2 + y^3x^2}{y^5} + \frac{x + y}{y}} = \frac{C}{\frac{x^3y^3 + y^4x^2 + xy^3 + y^4}{y^6}} = \frac{C}{\frac{x^3 + yx^2 + x + y}{y^3}}$

$y^3 = \frac{y^3C}{x^3 + yx^2 + x + y} \rightarrow x^3 + yx^2 + x + y = C$

$x^3 + y(x^2 + 1) + x = C$

d) $\frac{dy}{dx} = \frac{3y - 4x}{2y - 3x}$ $y = ux$

Ordinaria
Grado = 1
Orden = 1
No lineal

$(3y - 4x)\,dx + (3x - 2y)\,dy = 0$

$n = 1$; $n = 1$

$(3ux - 4x)\,dx + (3x - 2ux)(u\,dx + x\,du) = 0$

$(3u - 4)\,dx + (3 - 2u)(u\,dx + x\,du) = 0$

$3u\,dx - 4dx + 3u\,dx + 3x\,du - 2u^2dx - 2ux\,du = 0$

$(3u - 4 + 3u - 2u^2)\,dx = (2ux - 3x)\,du$

$\frac{dx}{x} = \frac{(2u - 3)\,du}{(-2u^2 + 6u - 4)} = \frac{-(2u - 3\,du)}{(2u^2 - 6u + 4)}$

$\int \frac{dx}{x} = -1\int \frac{(2u - 3)\,du}{2u^2 - 6u + 4}$ $a = 2u^2 - 6u + 4 = 2(u^2 - 3u + 2)$
$da = 2(2u - 3)\,du$

$\ln x = -\frac{1}{2}\ln|2u^2 - 6u + 4| + C$

$x^2 = \frac{C}{2u^2 - 6u + 4}$

$x^2 = \frac{C}{2\left(\frac{y}{x}\right)^2 - 6\left(\frac{y}{x}\right) + 4} =$

$x^2 = \frac{C}{\left(\frac{y}{x}\right)^2 - \frac{3y}{x} + 2}$

$M(x,y)\,dx$
$N(x,y)\,dy$ $\qquad \dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$

TAREA ~~III~~ 2

3. Clasifique y resuelve por diferenciales exactas.

a) $\left(1 - \dfrac{y}{x} e^{\frac{y}{x}}\right) dx + e^{\frac{y}{x}} dy$

$$\frac{\partial M}{\partial y} = \left(\frac{-y}{x^2} e^{\frac{y}{x}} - \frac{e^{\frac{y}{x}}}{x}\right) = \frac{-xy\, e^{\frac{y}{x}} - x^2 e^{\frac{y}{x}}}{x^2}$$

$$\frac{\partial N}{\partial x} = \frac{-y e^{\frac{y}{x}}}{x^2}$$

No se puede resolver por ecuaciones exactas.

Ordinaria
Orden = 1
Grado = 1
No lineal

b) $(16xy - 3x^2)\,dx + (8x^2 + 2y)\,dy$

$\dfrac{\partial M}{\partial y} = 16x \qquad \dfrac{\partial N}{\partial x} = 16x$

$g'(y) = N(x,y) - \dfrac{\partial}{\partial y} \int M(x,y)\,dx$

$\int M(x,y)\,dx = (8x^2 y - x^3) + C$

$\dfrac{\partial}{\partial y} \int M(x,y)\,dy = 8x^2$

Ordinaria
Orden = 1
Grado = 1
No lineal

$g'(y) = 8x^2 + 2y - 8x^2 =$

$g(y) = y^2$

$F(x,y) = \int M(x,y)\,dx + g(y)$

$= 8x^2 y - x^3 + y^2 + C$

FIRST CLASS