Problemario

Serie de Taylor

1. *Encuentra el polinomio de Taylor de grado 3 alrededor de* $x = 2$*. Y úsala para aproximar* $2.1^3 * ln(2.1)$.

```
x**3*log(x)
3*x**2*log(x) + x**2
6*x*log(x) + 5*x
6*log(x) + 11

2.52648051389328*x**3 - 6.00000000000005*x**2 + 6.00000000000011*x - 2.66666666666671

f(x) ≈ 6.87106937249899
```

Operaciones binarias (C_2 números negativos)

2. Realiza la siguiente operación $76 - 79$*. Conviértelos primero a binario y realiza la operación usando complemento a 2. Comprueba tu resultado.*

$$\begin{aligned} &76 - 79 \\ &= 01001100_2 - 01001111_2 \\ &= 01001100_2 + C_2(01001111_2) \\ &= 01001100_2 + 10110001_2 \end{aligned}$$

$$\begin{array}{r} 01001100_2 \\ +10110001_2 \\ \hline (1)1111101_2 \end{array} \rightarrow C_2((1)1111101_2) = (1)0000011_2 = -3_{10}$$

Aproximación de raíces

3. *Aproxima la raíz de la siguiente función* $f(x) = 2x^3 - 11.7x^2 + 17.7x - 5$ *con* $x_0 = 3$.

   a. *Newton-Raphson.*

```
NewtonRaphson(3, 0.001, 50)
```

```
        f(x)  = 2*x**3 - 11.7*x**2 + 17.7*x - 5
        f'(x) = 6*x**2 - 23.4*x + 17.7

1. P = 5.13333333333332         Er = 41.5584415584414
2. P = 4.26975005653324         Er = 20.2256166137568
3. P = 3.79293448064323         Er = 12.5711524499930
4. P = 3.59981928839808         Er = 5.36458018510924
5. P = 3.56433803284734         Er = 0.995451475807357
6. P = 3.56316210032519         Er = 0.0330024985963792
7. P = 3.56316082486356         Er = 3.57957920041021e-5

3.56316082486356
```

   b. *Secante, aproximaciones 3 y 4.*

```
Secant(3, 4, 0.01, 100)
```

```
        f(x) = 2*x**3 - 11.7*x**2 + 17.7*x - 5

1. P = 3.32653061224490         Er = 20.2453987730062
2. P = 3.48127270941766         Er = 4.44498636243420
3. P = 3.58627538471173         Er = 2.92790329882931
4. P = 3.56134020948947         Er = 0.700162684705669
5. P = 3.56312261112788         Er = 0.0500235841685707
6. P = 3.56316088908759         Er = 0.00107426975376506

3.56316088908759
```

Interpolación

4. *Encuentra una aproximación para estimar* $ln(10)$ *mediante un polinomio de interpolación de Newton de segundo orden en* $x = \{8, 9, 11\}$.

   a. *Lagrange de primer y segundo orden.*

```
f, e = Lagrange(xInput, yInput), 10
print("\ng(", e, ") ≈ ", N(f.subs(x, e)), sep = "")
```

```
2 points:
        f(8)  = 2.07944154167984
        f(9)  = 2.19722457733622

Polynomial
2.07944154167984*(x - 9)/(8 - 9) + 2.19722457733622*(x - 8)/(9 - 8)

Simplified
0.11778303565638*x + 1.13717725642888

By Powers
0.11778303565638*x + 1.13717725642888

g(10) ≈ 2.31500761299260
```

```
f, e = Lagrange(xInput, yInput), 10
print("\ng(", e, ") ≈ ", N(f.subs(x, e)), sep = "")
```

```
3 points:
        f(8) = 2.07944154167984
        f(9) = 2.19722457733622
        f(11) = 2.39789527279837

Polynomial
2.07944154167984*(x - 9)/(8 - 9)*(x - 11)/(8 - 11) + 2.19722457733622*(x - 8)/(9 - 8)*(x - 11)/(9 - 11) + 2.39789527279837
*(x - 8)/(11 - 8)*(x - 9)/(11 - 9)

Simplified
-0.00581589597510164*x**2 + 0.216653267233108*x + 0.71843274622149

By Powers
-0.00581589597510168*x**2 + 0.216653267233109*x + 0.718432746221495

g(10) ≈ 2.30337582104242
```

b. Diferencias divididas de segundo orden.

```
f, e = Newton(xInput, yInput), 10
print("\nf(", e, ") ≈ ", N(f.subs(x, e)), sep = "")
```

```
3 points:
	f(8) = 2.07944154167984
	f(9) = 2.19722457733622
	f(11) = 2.39789527279837

Polynomial
2.07944154167984 + 0.117783035656384*(x-8) + -0.00581589597510275*(x-8)*(x-9)

Simplified
-0.00581589597510275*x**2 + 0.216653267233131*x + 0.71843274622137

By Powers
-0.00581589597510275*x**2 + 0.216653267233131*x + 0.71843274622137

f(10) ≈ 2.30337582104240
```

c. Hermite.

```
f, e = Hermite(xInput, yInput, dInput), 10
print("\nf(", e, ") ≈ ", N(f.subs(x, e)), sep = "")
```

```
3 points:
	f(8) = 2.07944154167984 f'(8) = 0.125000000000000
	f(9) = 2.19722457733622 f'(9) = 0.111111111111111
	f(11) = 2.39789527279837	f'(11) = 0.0909090909090909

Polynomial
2.07944154167984 + 0.125000000000000*(x-8) + -0.00721696434361618*(x-8)*(x-8) + 0.000545039798343472*(x-8)*(x-8)*(x-9) + -
3.90085044194978e-5*(x-8)*(x-8)*(x-9)*(x-9) + 2.93198529847141e-6*(x-8)*(x-8)*(x-9)*(x-9)*(x-11)

Simplified
2.93198529847141e-6*x**5 - 0.000170947842850711*x**4 + 0.00423744108447283*x**3 - 0.0588761877031229*x**2 + 0.543484438276
746*x - 0.0658007167281455

By Powers
2.93198529847141e-6*x**5 - 0.000170947842850711*x**4 + 0.00423744108447283*x**3 - 0.0588761877031229*x**2 + 0.543484438276
746*x - 0.0658007167281455

f(10) ≈ 2.30258608153988
```

Polinomio Característico

5. *Considera la siguiente matriz.*

$$\begin{pmatrix} \mathbf{1} & \mathbf{-1} & \mathbf{4} \\ \mathbf{3} & \mathbf{2} & \mathbf{-1} \\ \mathbf{2} & \mathbf{1} & \mathbf{-1} \end{pmatrix}$$

   a. *Usa el método de Krilov para encontrar el polinomio característico.*

```
Krilov(matrix, np.array([1, 0, 0]))
```

```
Matrix:

 [[ 1 -1  4]
 [ 3  2 -1]
 [ 2  1 -1]]

Using vector:

 [1 0 0]

Vectors calculated:

[[1. 0. 0.]
 [1. 3. 2.]
 [6. 7. 3.]]

'λ^3 + -2.0λ^2 + -5.0λ^1 + 6.0λ^0'
```

   b. *Usa el método de Leverrier Faddeev para encontrar el polinomio característico.*

```
Leverrier_Faddeev(matrix)
```

```
Matrix:

 [[ 1 -1  4]
 [ 3  2 -1]
 [ 2  1 -1]]



'1.0*λ^3 + -2.0*λ^2 + -5.0*λ^1 + 6.0*λ^0'
```

Ecuaciones diferenciales

6. *Aproxima las soluciones del siguiente PVI:*

$$y = -5y + 5t^2 + 2t \text{ sujeto a } y(0) = \frac{1}{3} \; en \; 0 \le t \le 1 \; con \; n = 10.$$

   a. *Usa el método de Euler.*

```
: Euler(yp, a, b, n, c)

          f(x) = 5*t**2 + 2*t - 5*y

  0        0.3333333333333333
  0.1      0.166666666666667
  0.2      0.108333333333333
  0.30000000000000004      0.114166666666667
  0.4      0.162083333333333
  0.5      0.241041666666667
  0.6      0.345520833333333
  0.7      0.472760416666667
  0.7999999999999999       0.621380208333333
  0.8999999999999999       0.790690104166667

: (0.9999999999999999, 0.980345052083333)
```

   b. *Usa el método de Runge-Kutta.*

```
: RunggeKutta(yp, a, b, n, c)

          f(x) = 5*t**2 + 2*t - 5*y

  0.1      0.212282986111111
  0.2      0.162765457718461
  0.30000000000000004      0.164516540751045
  0.4      0.205240505195296
  0.5      0.277476660704437
  0.6      0.376698077979515
  0.7      0.500157948357362
  0.7999999999999999       0.646189588456420
  0.8999999999999999       0.813781703412359
  0.9999999999999999       1.00232066899760

: (0.9999999999999999, 1.00232066899760)
```