# PROBLEMARIO LAPLACE

0224969

| UNIVERSIDAD PANAMERICANA Campus Bonaterra | Escuela de Ingeniería | Problemario |
|---|---|---|
| | Área: Ciencias Básicas | Fecha: 16/05/2021 |
| | Materia: Ecuaciones Diferenciales. | Ciclo: 1212 |
| | Profesor: MCI Luis Alonso Romo Mercado | CALIFICACIÓN |
| | Carrera: | |
| | Alumno(a): | |

1. Encuentre las transformadas de Laplace de las siguientes funciones.

a) $f(t) = \begin{cases} t, 0 \leq t < 1 \\ 4 - 3t, t \geq 1 \end{cases}$

b) $e^{-5t}$

c) $te^{-5t}$

d) $Sen\ t\ U(t - \frac{\pi}{2})$

e) $e^{-t}\ Cos\ 2t$

2. Encuentre las transformadas inversas de las siguientes funciones.

a) $\frac{1}{9s-1}$

b) $\frac{4}{s^2-4}$

c) $\frac{s}{s^2-2s+1}$

3. Encuentre la solución a las siguientes ecuaciones diferenciales utilizando el método de Laplace.

a) $y''' - 3y' + 2y = 0, y(0) = 1, y'(0) = 0, y''(0)$

b) $y''' - y'' + \ y' - y = 0, Y(0) = 1, y'(0) = y''(0) = 0$

# TAREA V

## Transformadas de Laplace

a) $f(t) = \begin{cases} t & 0 \leq t < 1 \\ 4-3t & t \geq 1 \end{cases} = \begin{cases} t & t<1 \\ 4-3t & t \geq 1 \end{cases} = U(t-1)$

$= t + (4-3t-t)U(t-1)$

$= t + (4-4t)U(t-1)$

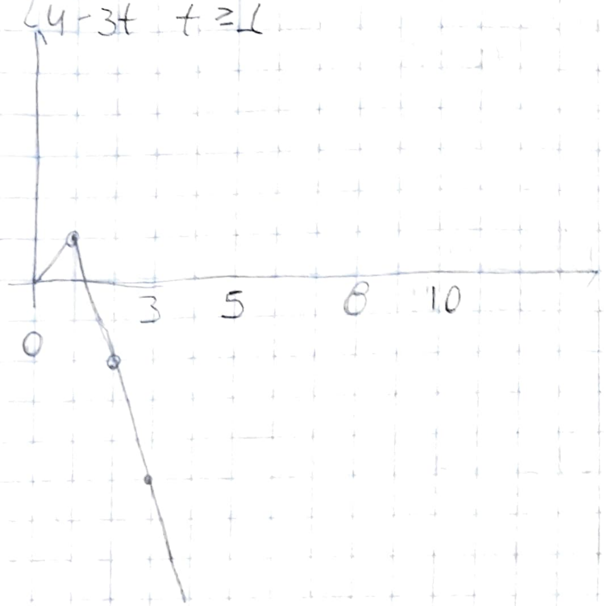
$= t + (4-4t)U_1(t)$

$= t - 4(-1+t)U_1(t)$

$\mathcal{L}\{f(t-a)U_a(t)\} = e^{-as}F(s)$

$\mathcal{L} = \frac{1}{s^2} + \frac{-4e^{-s}}{s^2}$

$= \frac{1-4e^{-s}}{s^2}$




b) $e^{-5t}$

4 $\mathcal{L}\{e^{-5t}\} = \frac{1}{s+5}$

c) $te^{-5t}$

5 $\mathcal{L}\{te^{-5t}\} = \frac{1}{(s+5)^2}$

e) $e^{-t}\cos(2t)$

11. $\mathcal{L}\{e^{-t}\cos(2t)\} = \frac{s+1}{(s+1)^2+4}$

d) $\operatorname{sen} t \, U\left(t - \frac{\pi}{2}\right)$

$= \operatorname{sen} t \, U_{\frac{\pi}{2}}(t)$

$= \operatorname{sen}\left(t - \frac{\pi}{2} + \frac{\pi}{2}\right) U_{\frac{\pi}{2}}(t)$

$\mathcal{L} = e^{-\frac{\pi s}{2}} \mathcal{L}\{\operatorname{sen}(t + \frac{\pi}{2})\}$

$\downarrow$

$\left\{ \begin{array}{l} = \operatorname{sen} t \cos\frac{\pi}{2} + \operatorname{sen}\frac{\pi}{2}\cos t \\ = \cos t \end{array} \right\}$

$\mathcal{L} = \frac{e^{-\frac{\pi s}{2}} s}{s^2+1}$

## 2. Inversas

a) $\dfrac{1}{9s-1} = \dfrac{\frac{1}{9}}{s-\frac{1}{9}} = \dfrac{1}{9}\left(\dfrac{1}{s-\frac{1}{9}}\right)$

$\mathcal{L}^{-1}\left\{\dfrac{1}{9}\left(\dfrac{1}{s-\frac{1}{9}}\right)\right\} = \dfrac{\mathcal{L}^{-1}\left\{\frac{1}{s-\frac{1}{9}}\right\}}{9} = \underline{\dfrac{e^{\frac{t}{9}}}{9}}$

b) $\dfrac{4}{s^2-4} = 2\left(\dfrac{2}{s^2-2^2}\right)$

$\mathcal{L}^{-1}\left\{2\left(\dfrac{2}{s^2-2^2}\right)\right\} = 2\mathcal{L}^{-1}\left\{\dfrac{2}{s^2-2^2}\right\} = \underline{2\sinh(2t)}$

c) $\dfrac{s}{s^2-2s+1} = \underline{\dfrac{s}{(s-1)^2}} = \dfrac{s}{(s-1)(s-1)}$

Fracciones

$\dfrac{s}{(s-1)^2} = \dfrac{A}{s-1} + \dfrac{B}{(s-1)^2}$

$s = (s-1)A + B$

$= sA - A + B = s(A) + (B-A)$

$A = 1$

$B - A = 0 \rightarrow A = B = 1$

$\mathcal{L}^{-1}\left\{\dfrac{1}{s-1} + \dfrac{1}{(s-1)^2}\right\} = e^t + te^t = e^t(t+1)$

## 3. Solve

a) $y''' - 3y' + 2y = 0; \quad y(0) = 1,\ y'(0) = 0,\ y''(0) = 0$

$\mathcal{L} = s^3 y(s) - s^2 y(0) - sy'(0) - y''(0) - 3(sy(s) - y(0)) + 2y(s) = 0$

$= s^3 y(s) - s^2 - 3sy(s) + 3 + 2y(s) = 0$

$y(s)(s^3 - 3s + 2) = s^2 - 3$

$y(s) = \dfrac{s^2-3}{s^3-3s+2} = \dfrac{s^2-3}{(s-1)^2(s+2)}$

$\dfrac{s^2-3}{(s-1)^2(s+2)} = \dfrac{A}{(s-1)} + \dfrac{B}{(s-1)^2} + \dfrac{C}{(s+2)} \rightarrow s^2 - 3 = A(s-1)(s+2) + B(s+2) + C(s-1$

$s^2 - 3 = As^2 + As - 2A + Bs + 2B + Cs^2 - 2Cs + C$

$= s^2(A+C) + s(A+B-2C) + (-2A+2B+C)$

$A + C = 1 \qquad A = 8/9$

$A + B - 2C = 0 \qquad B = -2/3$

$-2A + 2B + C = -3 \qquad C = 1/9$

$y(s) = \dfrac{8}{9(s-1)} - \dfrac{6}{9(s-1)^2} + \dfrac{1}{9(s+2)} \rightarrow \underline{y(t) = \dfrac{8e^t}{9} - \dfrac{6te^t}{9} + \dfrac{e^{-2t}}{9}}$

# TAREA V

b) $y''' - y'' + y' - y = 0; \quad y(0) = 1, \quad y'(0) = y''(0) = 0$

$\mathcal{L} = s^3 y(s) - s^2 y(0) - sy'(0) - y''(0) - s^2 y(s) + sy(0)$
$+ \quad y'(0) + sy(s) - y(0) - y(s) = 0$

$y(s)(s^3 - s^2 + s - 1) - s^2 + s - 1 = 0$

$$y(s) = \frac{s^2 - s + 1}{s^3 - s^2 + s - 1} = \frac{s^2 - s + 1}{(s-1)(s^2+1)}$$

$$\rightarrow \frac{s^2 - s + 1}{(s-1)(s^2+1)} = \frac{A}{(s-1)} + \frac{Bs + C}{(s^2+1)}$$

$$s^2 - s + 1 = A(s^2+1) + B(s-1)$$
$$= As^2 + A + Bs^2 - Bs + Cs - C$$
$$= s^2(A+B) + s(-B+C) + (A-C)$$

$$A = \frac{1}{2} \qquad B = \frac{1}{2} \qquad C = \frac{-1}{2}$$

$$\mathcal{L}^{-1}\left\{ \frac{1}{2(s-1)} + \frac{s}{2(s^2+1)} - \frac{1}{2(s^2+1)} \right\} = \frac{e^t + \cos t - \operatorname{sen} t}{2}$$