Examen III

1) $\dfrac{dy}{dx} = \dfrac{3y-4x}{2y-3x}$ Homogenea

$(2y-3x)dy = (3y-4x)dx$

① $(-3y+4x)dx + (2y-3x)dy = 0$

Grado de Homogeneidad

1) $-3yt + 4xt$ 2) $2yt - 3xt$

$t(-3y+4x)$ $t(2y-3x)$

$n = 1$

Cambio: $y = ux \rightarrow u = \dfrac{y}{x}$

$dy = udx + xdu$

① $(-3ux+4x)dx + (2ux-3x)(udx+xdu) = 0$

$-3uxdx + 4xdx + 2u^2xdx + 2ux^2du - 3uxdx - 3x^2du = 0$

$-6uxdx + 4xdx + 2u^2xdx + 2ux^2du - 3x^2du = 0$

$dx(-6ux + 4x + 2u^2x) + du(2ux^2 - 3x^2) = 0$

$xdx(-6u + 4 + 2u^2) = -du(2ux^2 - 3x^2)$

$xdx(2u^2 - 6u + 4) = x^2du(3-2u)$

$\dfrac{xdx}{x^2} = \dfrac{(3-2u)du}{(2u^2-6u+4)} = \dfrac{(3-2u)du}{2(u^2-3u+2)}$

$\dfrac{dx}{x} = \dfrac{(3-2u)du}{2(u-1)(u-2)}$

Parciales

$\dfrac{3-2u}{(u-1)(u-2)} = \dfrac{A}{(u-1)} + \dfrac{B}{(u-2)}$

$3-2u = A(u-2) + B(u-1)$

$3-2u = Au - 2A + Bu - B$

$3-2u = u(A+B) + (-2A-B)$

$A+B = -2$

$-2A - B = 3$; $B = -2A-3$; $B = -1$

1) $A - 2A - 3 = -2$

$-A + 3 = -2$

$A = -1$

Integral

$\dfrac{dx}{x} = \left(\dfrac{-du}{(u-1)} + \dfrac{-du}{(u-2)}\right)\dfrac{1}{2}$

$\int \dfrac{dx}{x} = \dfrac{1}{2}\left(-\int \dfrac{du}{u-1} - \int \dfrac{du}{u-2}\right) \rightarrow \ln x = \dfrac{1}{2}(-\ln|u-1| - \ln|u-2|)$

$x = e^{-\frac{1}{2}(\ln|(u-1)(u-2)|)} e^c \rightarrow x = c\, e^{-\frac{1}{2}\ln|(u-1)(u-2)|}$

$x = \dfrac{c}{\sqrt{(u-1)(u-2)}} = \dfrac{c}{\sqrt{\left(\frac{y}{x}-1\right)\left(\frac{y}{x}-2\right)}}$

Laplace

(2) $y'' + 16y = 1, \quad y(0) = 1, \ y'(0) = 2$

$$\mathcal{L}\{f(y)\} = s^2 Y(s) - sY(0) - y'(0) + 16Y(s) = \frac{1}{s}$$

$$Y(s)(s^2+16) - s - 2 = \frac{1}{s}$$

$$Y(s)(s^2+16) = \frac{1}{s} + s + 2 = \frac{1+s^2+2s}{s}$$

$$Y(s) = \frac{s^2+2s+1}{(s^2+16)s}$$

Parciales con Wolfram:

$$\frac{s^2+2s+1}{(s^2+16)s} = \frac{15s+32}{16(s^2+16)} + \frac{1}{16s} = \frac{15}{16}\left(\frac{s}{s^2+16}\right) + \frac{9}{16}\left(\frac{4}{s^2+16}\right) + \frac{1}{16s}$$

$$\mathcal{L}^{-1}\{Y(s)\} = \frac{15}{16}\cos(4t) + \frac{9}{16}\sin(4t) + \frac{1}{16}$$

(3) $(16xy - 3x^2)dx + (8x^2 + 2y)dy = 0$ Exacta

$$\frac{\partial M}{\partial y} = 16x = \frac{\partial N}{\partial x} = 16x$$

$$\int M(x,y)dx = \int (16xy - 3x^2)dx = 8x^2y - x^3$$

$$\frac{\partial}{\partial y}\int M(x,y)dx = 8x^2$$

$$g'(y) = 8x^2 + 2y - 8x^2 = 2y \rightarrow g(y) = \int 2y\,dy = y^2$$

$$f(x,y) = \int M(x,y)dx + g(y) = 8x^2y - x^3 + y^2 + C$$

4) $m_1 y_1'' = -b_1 y_1' - K_1 y_1 - K_2(y_1 - y_2)$

$m_2 y_2'' = -K_2(y_2 - y_1)$

1) $b_1 y_1' + K_1 y_1 + K_2(y_1 - y_2) + m_1 y_1'' = 0$

2) $K_2(y_2 - y_1) + m_2 y_2'' = 0$

$m_1 = m_2 = 2$ | $x_1(0) = 0$
$K_1 = 5$ | $x_1'(0) = 0$
$K_2 = 3$ | $x_2(0) = 0$
$b_1 = 2$ | $x_2'(0) = 0$

Transformar

1) $b_1(s y_1(s) - y(0)) + K_1(y(s)) + K_2(y_1(s) - y_2(s)) + m_1(s^2 y(s) - s y(0) - y'(0)) = 0$

$2s y_1(s) + 5y_1(s) + 3y_1(s) - 3y_2(s) + 2s^2 y(s) = 0$

$y_1(s)(2s + 5 + 3 + 2s^2) + y_2(s)(-3) = 0$

$y_1(s)(2s^2 + 2s + 8) + y_2(s)(-3) = 0$

2) $K_2(y_2(s) - y_1(s)) + m_2(s^2 y_2(s) - s y_2(0) - y'(0)) = 0$

$3y_2(s) - 3y_1(s) + 2s^2 y_2(s) = 0$

$y_1(s)(-3) + y_2(s)(3 + 2s^2) = 0 \Rightarrow y_1 = \frac{(3 + 2s^2)}{3} y_2(s)$ 3)

Sistema

3 en 1

$y_2\left(1 + \frac{2}{3}s^2\right)(2s^2 + 2s + 8) + y_2(-3) = 0$

$\left[\left(1 + \frac{2}{3}s^2\right)(2s^2 + 2s + 8) - 3\right] y_2 = 0$

$\left(2s^2 + 2s + 8 + \frac{4}{3}s^4 + \frac{4s^3}{3} + \frac{16}{3}s^2 - 3\right) y_2 = 0$

↑ Si se divide todo entre esto: $y_2 = 0$

$y_1 = \left(1 + \frac{2s^2}{3}\right) y_2 = \left(1 + \frac{2s^2}{3}\right)(\phi) = 0$

Sistema indeterminado

⑤ Inversa

$$\frac{(s+2)^2}{s^3} = \frac{s^2+4s+4}{s^3} = \frac{1}{s} + \frac{4}{s^2} + \frac{4}{s^3}$$

$$= \frac{1}{s} + 4\left(\frac{1}{s^2}\right) + 2\left(\frac{2}{s^3}\right)$$

$$\mathcal{L}^{-1} = 1 + 4t + 2t^2$$

⑥ $$\frac{1}{s^4-9} = \frac{1}{(s^4-3^2)} = \frac{1}{(s^2-3)(s^2+3)}$$

Parciales con wolfram

$$\frac{1}{(s^2-3)(s^2+3)} = \frac{1}{6(s^2-3)} - \frac{1}{6(s^2+3)} = \frac{1}{\sqrt{3}\,6}\left(\frac{\sqrt{3}}{s^2-3}\right) - \frac{1}{6\sqrt{3}}\left(\frac{\sqrt{3}}{s^2+3}\right)$$

$$\mathcal{L}^{-1} = \frac{\sinh(\sqrt{3}t)}{\sqrt{3}\,6} - \frac{\sin(\sqrt{3}t)}{\sqrt{3}\,6}$$

⑦ Laplace transform

$\cos(5t) + \operatorname{sen}(2t)$

$$\mathcal{L}\{\cos(5t)+\operatorname{sen}(2t)\} = \frac{5}{s^2+25} + \frac{2}{s^2+4}$$

⑧ $e^{-t}\operatorname{sen}t$

$$\mathcal{L}\{e^{-t}\operatorname{sen}t\} = \frac{1}{(s+1)^2+1}$$

$$\mathcal{L}\{\operatorname{sen}t\} = \frac{1}{s^2+1}$$

9 Ecuación dif parcial

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$$

$$U = x(t)y(t)$$

$$\frac{\partial u}{\partial x} = x'(t)y(t)$$

$$\frac{\partial u}{\partial y} = x(t)y'(t)$$

$$x x' y + y x y' = 0$$

$$x x' y = -x y y'$$

$$x' y = -y y'$$

$$x' = \frac{-y y'}{y}$$

$$x' = -y'$$

>0

$$x' = \lambda^2 \qquad -y' = \lambda^2$$

$$x = \lambda^2 x + C_1 \qquad -y = \lambda^2 y$$

$$y = -\lambda^2 y + C_2$$

$$\underline{U_1 = (\lambda^2 x + C_1)(-\lambda^2 y + C_2)}$$

<0

$$x' = -\lambda^2 \qquad -y' = -\lambda^2$$

$$x = -\lambda^2 x + C_3 \qquad y = \lambda^2 y + C_4$$

$$\underline{U_2 = (-\lambda^2 x + C_3)(\lambda^2 y + C_4)}$$

$=0$

$$x' = 0 \qquad -y' = 0$$

$$x = C_5 \qquad y' = 0$$

$$y = C_6$$

$$U_3 = \underline{C_5 C_6}$$