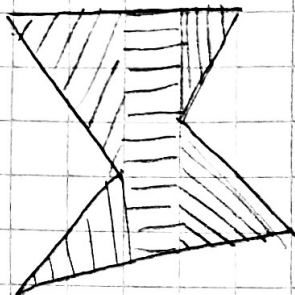


# Examen 3 parcial

1.  $\iint_R f(x,y) dy dx$



5 integrales con necesarias

2. Integral  $f(x,y) = 12x$  en  $y = 5x$ ,  $y = 7x - x^2$

$$\int_0^2 \int_{5x}^{7x-x^2} (12x) dy dx$$

$$5x = 7x - x^2$$

$$x^2 - 2x = 0$$

$$x \in (x-2) \rightarrow x=0$$

$$x=2$$

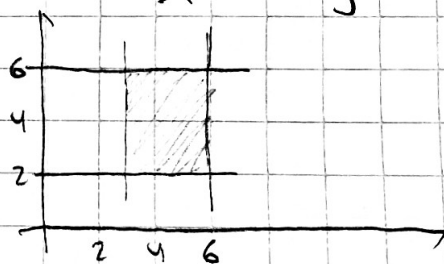
$$= \int_0^2 12xy \Big|_{5x}^{7x-x^2} dx = 12 \int_0^2 x(7x-x^2-5x) dx$$

$$= 12 \int_0^2 2x^2 - x^3 dx = 12 \left( \frac{2}{3} x^3 - \frac{x^4}{4} \right) \Big|_0^2 = 12 \left( \frac{16}{3} - \frac{16}{4} \right) = 12 \left( \frac{4}{3} \right) = 16$$

3. Promedio de  $f(x,y) = 4x + 9y^2$  de  $[3,6] \times [2,6]$

$$\frac{\int_3^6 \int_2^6 (4x + 9y^2) dy dx}{\int_3^6 \int_2^6 1 dx dy}$$

$$= \frac{2088}{12} = 174$$



4.  $\int_0^3 \int_{\frac{4x}{3}}^{\frac{4x}{3}} f dy dx + \int_3^5 \int_0^{\sqrt{25-x^2}} f dy dx \rightarrow \int_A^B \int_C^D f dx dy$

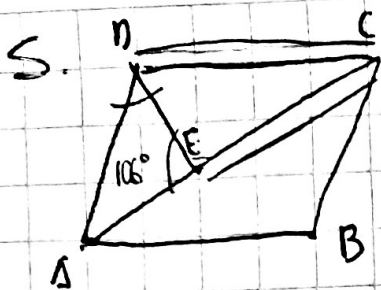
$y = \frac{4x}{3} \rightarrow x = \frac{3y}{4}$   $y = \sqrt{25-x^2} \rightarrow x = \sqrt{25-y^2}$

$$0 < y \leq 4$$

$$\int_0^4 \int_{\frac{3y}{4}}^{\sqrt{25-y^2}} f(x,y) dx dy$$

$$A=0, B=4$$

$$C = \frac{3y}{4}, D = \sqrt{25-y^2}$$

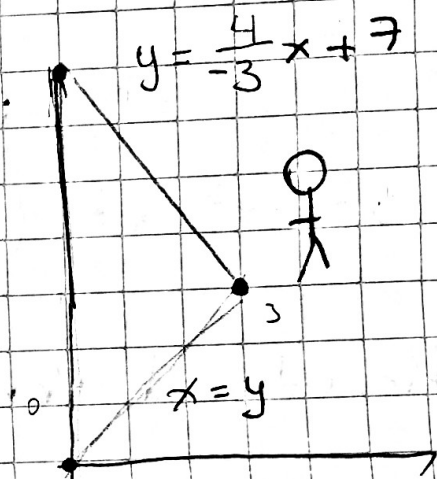


6.  $J_K = \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} 18 & 8 \\ 2 & 12 \end{vmatrix} = |216 - 16| = |200| = 200$

7.  $\rho(x,y) = 7x + 6y$

$$M_x = y_m = \frac{1}{M} \int_0^3 \int_x^{7-\frac{4}{3}x} y(7x+6y) dy dx$$

$$= \frac{1}{\left(\frac{567}{2}\right)}$$



$$M = \iint_R \rho(x,y) dA$$

$$= \iint_R (7x+6y) dy dx$$

$$= \int_0^3 \int_x^{7-\frac{4}{3}x} 7x+6y dy dx$$

$$= \frac{1}{\left(\frac{567}{2}\right)} \left( \frac{8597}{8} \right) = \frac{407}{108} \approx 3.76852$$

$$= \frac{567}{2}$$

$$\begin{aligned}
 9. \int_4^{10} \int_{12}^{19} \int_{23}^{29} \frac{f(x)}{g(y)h(z)} dz dy dx &= \\
 = \int_{23}^{29} \frac{1}{h(z)} dz \int_{12}^{19} \frac{1}{g(y)} dy \int_4^{10} f(x) dx &= \\
 = \int_{23}^{29} \frac{1}{4} dz \int_{12}^{19} \frac{1}{2} dy \int_4^{10} 7 dx &= \left[ \frac{1}{4}(29-23) \right] \left[ \frac{1}{2}(19-12) \right] [7(10-4)] \\
 = \frac{1}{4}(6) \frac{1}{2}(7) 7(6) &= \frac{6 \times 6 \times 7 \times 7}{2 \times 4} = \frac{441}{2} = 220.5
 \end{aligned}$$

10. Rect (x, y, z)  $x =$   
 Cil (r,  $\theta$ , z)  $r = 16, \theta = 1.3$   
 Esf ( $\rho, \theta, \phi$ )  $\rho = 44 \quad \phi = ?$

$$\begin{aligned}
 x &= r \cos \theta = 16 \cos(1.3) \approx 4.28 \\
 y &= r \sin \theta = 16 \sin(1.3)
 \end{aligned}$$

$$44^2 = r^2 \cos^2 \theta + r^2 \sin^2 \theta + z^2$$

$$44^2 = r^2 + z^2$$

$$z^2 = 44^2 - r^2$$

$$z = \pm \sqrt{44^2 - (16)^2}$$

$$z = \pm \sqrt{1936 - 256} = \pm 4\sqrt{105}$$

$$\begin{aligned}
 \phi &= \arccos\left(\frac{\pm 4\sqrt{105}}{44}\right) = \begin{aligned} &\rightarrow (+) 0.3721 \\ &\rightarrow (-) 2.7694 \end{aligned}
 \end{aligned}$$

$$\begin{aligned}
 100x + \phi &= 100(16 \cos(1.3)) + \arccos\left(\frac{\sqrt{105}}{11}\right) = 428.37029 \\
 &\quad + \arccos\left(\frac{-\sqrt{105}}{11}\right) = 430.76754
 \end{aligned}$$