$$x = 2u + v$$

$$y = u + 2v$$

$$J_F = \begin{vmatrix} x_u & y_u \\ x_v & y_v \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = |3| = 3$$

$$\int\int_R x - 3y\, dydx \to \int\int_R 3(2u + v - 3(u + 2v))dudv \ = \int\int_R 3(-u - 5v)dudv$$

$$y = \ -x + algo$$

$$1 = \ -2 + algo$$

$$algo = 3$$

Cambio de variable en las fronteras de mi área de integración.

$$y = 2x$$

$$u + 2v = 2(2u + v)$$

$$u + 2v = 4u + 2v \Rightarrow 3u = 0 \Rightarrow \boldsymbol{u = 0}$$

$$y = \frac{x}{2}$$

$$u + 2v = \frac{2u + v}{2}$$

$$2u + 4v = 2u + v \Rightarrow 3v = 0 \Rightarrow \boldsymbol{v = 0}$$

$$y = \ -x + 3$$

$$x = 2u + v$$

$$y = u + 2v$$

$$u + 2v = \ -2u - v + 3$$

$$3u = -3v + 3 \Rightarrow \boldsymbol{u = -v + 1}$$

$$x = \ -y + 1$$

$$\int\int \ 3(-u - 5v)dudv$$

$$\int_0^1 \int_0^{-u+1} 3(-u - 5v)dvdu = \int_0^1 \int_0^{-u+1} 3(-u - 5v)dvdu = -\ 3$$

19. $\iint_R xy\,dA$, donde R es la región en el primer cuadrante acotada por las rectas $y = x$ y $y = 3x$ y las hipérbolas $xy = 1$, $xy = 3$; $x = u/v$, $y = v$

Paréntesis: (

$$x = \frac{u}{v} \Rightarrow xv = u \Rightarrow xy = u \rightarrow [1,3]$$

$$y = v \rightarrow [x, 3x] \rightarrow \left[\frac{u}{v}, \frac{3u}{v}\right] \rightarrow v^2 \rightarrow [u, 3u] \rightarrow v \rightarrow [\sqrt{u}, \sqrt{3u}]$$

)

$$J_F = \begin{vmatrix} x_u & y_u \\ x_v & y_v \end{vmatrix} = \begin{vmatrix} \frac{1}{v} & 0 \\ -\frac{u}{v^2} & 1 \end{vmatrix} = \frac{1}{v} - 0 = \left|\frac{1}{v}\right|$$

$$\int\int_R xy dA = \int\int_R \frac{u}{v} v \left(\left|\frac{1}{v}\right|\right) dA = \int\int_R u \left(\left|\frac{1}{v}\right|\right) dA$$

Cambiar las fronteras del área de integración

$$y = 3x$$

$$v = 3\left(\frac{u}{v}\right)$$

$$v^2 = 3u \Rightarrow \boldsymbol{u} = \frac{\mathbf{1}}{\mathbf{3}}\boldsymbol{v}^{\mathbf{2}}$$

$$3u = v^2$$

$$v = \pm\sqrt{3u}$$

$$y = x$$

$$v = \frac{u}{v}$$

$$v^2 = u \Rightarrow \boldsymbol{u} = \boldsymbol{v}^{\mathbf{2}}$$

$$\boldsymbol{v} = \pm\sqrt{\boldsymbol{u}}$$

$$xy = 1$$

$$\left(\frac{u}{v}\right)(v) = 1 \Rightarrow \boldsymbol{u = 1}$$

$$xy = 3$$

$$\left(\frac{u}{v}\right)(v) = 3 \Rightarrow \boldsymbol{u = 3}$$

u es dependiente de v

y es dependiente de x

$$\int_{1}^{3}\int_{\sqrt{u}}^{\sqrt{3u}} \frac{u}{v}\,dv\,du = 2\ln 3 = \ln 9$$