Ejercicios para practicar

Punto fijo

1) **Considera la siguiente función** $f(x)=x^4+2x^2-x-3$**, obtén cuatro expresiones diferentes usando el manejo algebraico para el método del punto fijo**

$$f(x) = x^4 + 2x^2 - x - 3$$

A)	$\boldsymbol{x = x^4 + 2x^2 - 3}$
B)	$x^3 + 2x - 1 - \frac{3}{x} = 0$ $2x = -x^3 + \frac{3}{x} + 1$ $x = \frac{-x^3 + \frac{3}{x} + 1}{2}$
C)	$x + \frac{2}{x} - \frac{1}{x^2} - \frac{3}{x^3} = 0$ $x = \frac{3}{x^3} + \frac{1}{x^2} - \frac{2}{x}$
D)	$x^5 + 2x^3 - x^2 - 3x = 0$ $x = \frac{x^5 + 2x^3 - x^2}{3}$

2) **Aplica el método de iteración de punto fijo para determinar una aproximación de** $x^3 - x - 1 = 0$ en $[1,2]$ utiliza $p_0 = 1$

$$x^3 - x - 1 = 0$$
$$x^2 - 1 - \frac{1}{x} = 0$$
$$x^2 = 1 + \frac{1}{x}$$
$$x = \sqrt{1 + \frac{1}{x}}$$

In [76]:
```
FixedP(1, 0.1, 10)
```

```
        f(x) = sqrt(1 + 1/x)

1. P = 1.41421356237310 Er = 29.2893218813453
2. P = 1.30656296487638 Er = 8.23922002923940
3. P = 1.32867108974726 Er = 1.66392759212454
4. P = 1.32386997635227 Er = 0.362657472467466
5. P = 1.32490044510483 Er = 0.0777770704485051
```

Out[76]: 1.32490044510483

Interpolación

1) **Considera la siguiente tabla de datos, aproxima la función f(1.5) usando el polinomio de Lagrange de grado 2**

x	f(x)
1.0	0.7651977
1.3	0.6200860
1.6	0.4554022
1.9	0.281886
2.2	0.1103623

```
f, e = Lagrange(xInput, yInput), 1.5
print("\nf(", e, ") ≈ ", N(f.subs(x, e)), sep = "")
```

```
3 points:
        f(1.0) = 0.7651977
        f(1.3) = 0.620086
        f(1.6) = 0.4554022

Polynomial
0.7651977*(x - 1.3)/(1.0 - 1.3)*(x - 1.6)/(1.0 - 1.6) + 0.620086*(x - 1.0)/(1.3 - 1.0)*(x - 1.6)/(1.3 - 1.6) + 0.4554022*
(x - 1.0)/(1.6 - 1.0)*(x - 1.3)/(1.6 - 1.3)

Simplified
-0.10873388888889*x**2 - 0.23361772222222*x + 1.10754931111111

By Powers
-0.10873388888889*x**2 - 0.23361772222222*x + 1.10754931111111

f(1.5) ≈ 0.512471477777779
```

2) Para la función dada, $f(x) = \ln(x+1)$ sea $x_0 = 0,\ x_1 = 0.6,$ y $x_2 = 0.9$. Construye el polinomio de interpolación de Lagrange de grado uno y dos para aproximar f(0.45) y calcula el error.

```
2 points:
        f(0) = 0
        f(0.6) = 0.470003629245736

Polynomial
0*(x - 0.6)/(0 - 0.6) + 0.470003629245736*(x - 0)/(0.6 - 0)

Simplified
0.783339382076227*x

By Powers
0.783339382076227*x

f(0.45) ≈ 0.352502721934302
```

$$\frac{\log(1.45) - 0.352502721934302}{\log(1.45)} \times 100$$

Result:

5.12990… $= E$

```
3 points:
        f(0) = 0
        f(0.6) = 0.470003629245736
        f(0.9) = 0.641853886172395

Polynomial
0*(x - 0.6)/(0 - 0.6)*(x - 0.9)/(0 - 0.9) + 0.470003629245736*(x - 0)/(0.6 - 0)*(x - 0.9)/(0.6 - 0.9) + 0.641853886172395*
(x - 0)/(0.9 - 0)*(x - 0.6)/(0.9 - 0.6)

Simplified
x*(0.923676176956691 - 0.233894658134107*x)

By Powers
-0.233894658134107*x**2 + 0.923676176956691*x

f(0.45) ≈ 0.368290611358354
```

$$\frac{\log(1.45) - 0.368290611358354}{\log(1.45)} \times 100$$

Result:

0.880857… $= E$

3) **Usa la fórmula de diferencias divididas interpolantes de Newton para construir polinomios interpolantes de grado uno, dos y tres con los siguientes datos y usa para aproximar el valor de f(8.4); si f(8.1)=16.94410, f(8.3)=17.56492, f(8.6)=18.50515 y f(8.7)=18.82091.**

```
f, e = Newton(xInput, yInput), 8.4
print("\nf(", e, ") ≈ ", N(f.subs(x, e)), sep = "")
```

```
2 points:
        f(8.1) = 16.9441
        f(8.3) = 17.56492

Polynomial
16.9441 + 3.104099999999993*(x-8.1)

Simplified
3.104099999999993*x - 8.199109999999943

By Powers
3.104099999999993*x - 8.199109999999943

f(8.4) ≈ 17.8753300000000
```

```
f, e = Newton(xInput, yInput), 8.4
print("\nf(", e, ") ≈ ", N(f.subs(x, e)), sep = "")
```

```
3 points:
        f(8.1) = 16.9441
        f(8.3) = 17.56492
        f(8.6) = 18.50515

Polynomial
16.9441 + 3.104099999999993*(x-8.1) + 0.06000000000003336*(x-8.1)*(x-8.3)

Simplified
0.06000000000003336*x**2 + 2.120099999999446*x - 4.1653099999977

By Powers
0.06000000000003336*x**2 + 2.120099999999446*x - 4.1653099999977

f(8.4) ≈ 17.8771300000000
```

```
f, e = Newton(xInput, yInput), 8.4
print("\nf(", e, ") ≈ ", N(f.subs(x, e)), sep = "")
```

```
4 points:
        f(8.1) = 16.9441
        f(8.3) = 17.56492
        f(8.6) = 18.50515
        f(8.7) = 18.82091

Polynomial
16.9441 + 3.104099999999993*(x-8.1) + 0.06000000000003336*(x-8.1)*(x-8.3) + -0.002083333333447855*(x-8.1)*(x-8.3)*(x-8.6)

Simplified
-0.002083333333447855*x**3 + 0.112083333333653*x**2 + 1.686204166663727 4*x - 2.960772499991079

By Powers
-0.002083333333447855*x**3 + 0.11208333333365299*x**2 + 1.6862041666637274*x - 2.9607724999910789

f(8.4) ≈ 17.8771425000000
```

4) Construye un polinomio de Hermite considerando los siguientes datos

x	f(x)	f'(x)
8.3	17.56492	3.116256
8.6	18.50515	3.151762

```
In [31]: f = Hermite(xInput, yInput, dInput)

2 points:
        f(8.3) = 17.56492
        f(8.6) = 18.50515

Polynomial
17.56492 + 3.116256*(x-8.3) + 0.05948000000003342*(x-8.3)*(x-8.3) + -0.002022222222440753*(x-8.3)*(x-8.3)*(x-8.6)

Simplified
-0.002022222222440753*x**3 + 0.1104400000055404*x**2 + 1.70088466661986*x - 3.004353955423783

By Powers
-0.002022222222440753*x**3 + 0.1104400000055404*x**2 + 1.70088466661986*x - 3.004353955423783
```