ANAEROBIC SUSPENDED AND ATTACHED GROWTH BIOLOGICAL TREATMENT PROCESSES

PROBLEM 10-1

Problem Statement – see text, page 1109

Solution

Define the conditions that determine the anaerobic reactor pH.
 It is safe to assume that the pH is below 8.0 and by referring to Fig. F-1 in Appendix F (page 1927) all of the alkalinity must be present as bicarbonate (HCO₃-). The corresponding carbonate equilibrium relationship is defined by Eq. (10-3):

$$\frac{[H^+][HCO_3^-]}{[H_2CO_3^*]} = K_{a1}$$

The value of first acid dissociation constant K_{a1} at 30°C is 4.677 x 10⁻⁷ mole/L. Rearrange Eq. (10-3) to solve for $[H^+]$.

$$[H^+] = \frac{K_{a1}[H_2CO_3^*]}{[HCO_3^-]}$$

- 2. Determine the carbonic acid concentration
 - a. Determine the concentration of H₂CO₃ using Eq. (2-46) in Chap. 2.

$$x_{H_2CO_3} = \frac{P_T}{H} p_g$$

Determine the value of the Henry's constant in atm at 30°C using Eq. (2-48) as indicated in Example 10-1.

$$\log_{10} H = \frac{-A}{T} + B$$

From Table 2-7, the values of A and B are 1012.40 and 6.606, respectively.

$$\log_{10} H = \frac{-A}{T} + B = \frac{-1012.40}{273.15 + 30} + 6.606 = 3.27$$

$$H = 10^{3.27} = 1862.1 atm$$

Note that the value given in Table F-1 of Appendix F is 1847, which is within 1 percent.

$$x_{H_2CO_3} = \frac{P_T}{H} p_g = \frac{(1 \text{ atm})(0.35)}{1862.1 \text{ atm}} = 1.88 \times 10^{-4}$$

Because one liter of water contains 55.6 mole [1000 g/(18 g/mole)], the mole fraction of H_2CO_3 is equal to:

$$\mathbf{x}_{\mathsf{H}_2\mathsf{CO}_3} = \frac{\mathsf{mole} \ \mathsf{gas}\,(\mathsf{n}_\mathsf{g})}{\mathsf{mole} \ \mathsf{gas}\,(\mathsf{n}_\mathsf{g}) + \mathsf{mole} \ \mathsf{water}\,(\mathsf{n}_\mathsf{w})}$$

1.88 x
$$10^{-4} = \frac{[H_2CO_3]}{[H_2CO_3] + (55.6 \text{ mole / L})}$$

Because the number of moles of dissolved gas in a liter of water is much less than the number of moles of water,

$$[H_2CO_3] \approx (1.88 \times 10^{-4})(55.6 \text{ mole / L}) \approx 10.45 \times 10^{-3} \text{ mole / L}$$

3. Solve for pH.

$$[H^{+}] = \frac{K_{a1}[H_{2}CO_{3}^{*}]}{[HCO_{3}^{-}]} = \frac{\left(4.677 \times 10^{-7} \text{ moles/L}\right)\left(10.45 \times 10^{-3} \text{ moles/L}\right)}{[HCO_{3}^{-}]}$$

- a. Determine moles/L of HCO_3 for alkalinity = 2200 mg/L as $CaCO_3$ meq/L of alkalinity = (2200 mg/L)/(50 mg/meq) = 44 meq/L Because valence of HCO_3 equal 1, the meq/L = moles/L Thus moles/L of HCO_3 = 44.0 x 10^{-3}
- b. Determine the H+ concentration and pH.

$$[H^{+}] = \frac{K_{a1}[H_{2}CO_{3}^{*}]}{[HCO_{3}^{-}]} = \frac{\left(4.677 \times 10^{-7} \text{ moles/L}\right)\left(10.45 \times 10^{-3} \text{ moles/L}\right)}{\left(44.0 \times 10^{-3} \text{ moles/L}\right)}$$

$$[H^{+}] = 1.11 \times 10^{-7} \text{ moles/L}$$

$$pH = -Log[H^{+}] = -Log\left(1.11 \times 10^{-7} \text{ moles/L}\right) = 7.0 - 0.05$$

pH = 6.95

PROBLEM 10-2

Problem Statement – see text, page 1109-1110

Solution: Part A Anaerobic Process

- 4. Determine the alkalinity required to maintain the pH of the anaerobic process at a value of 7.0, and the corresponding cost.
 - a. Determine the concentration of H₂CO₃ using Eq. (2-46) in Chap. 2.

$$x_{H_2CO_3} = \frac{P_T}{H} p_g$$

Determine the value of the Henry's constant in atm at 35°C using Eq. (2-48) as shown in Example 10-1.

$$log_{10} H = \frac{-A}{T} + B$$

From Table 2-7, the values of A and B are 1012.40 and 6.606, respectively.

$$\log_{10} H = \frac{-A}{T} + B = \frac{-1012.40}{273.15 + 35} + 6.606 = 3.32$$

$$H = 10^{3.32} = 2092 \text{ atm}$$

$$x_{H_2CO_3} = \frac{P_T}{H} p_g = \frac{(1 \text{ atm})(0.35)}{2092 \text{ atm}} = 1.67 \text{ x } 10^{-4}$$

Because one liter of water contains 55.6 mole [1000 g/(18 g/mole)], the mole fraction of H_2CO_3 is equal to:

$$x_{H_2CO_3} = \frac{\text{mole gas}(n_g)}{\text{mole gas}(n_g) + \text{mole water}(n_w)}$$

$$1.67 \times 10^{-4} = \frac{[H_2CO_3]}{[H_2CO_3] + (55.6 \text{ mole}/L)}$$

Because the number of moles of dissolved gas in a liter of water is much less than the number of moles of water,

$$[H_2CO_3] \approx (1.67 \text{ x } 10^{-4})(55.6 \text{ mole } / \text{L}) \approx 9.29 \text{ x } 10^{-3} \text{ mole } / \text{L}$$

b. Determine the concentration of HCO₃⁻ required to maintain the pH at a value of 7.0 using Eq. (10-3).

$$\frac{[H^+][HCO_3^-]}{[H_2CO_3]} = K_{a1}$$

where, $K_{a1} = 4.85 \times 10^{-7}$ (Table F-2) and $H^{+} = 10^{-7}$ mole/L

$$[HCO_3^-] = \frac{(4.85 \times 10^{-7})(9.29 \times 10^{-3} \text{ mole / L})}{(10^{-7} \text{ mole / L})}$$

= 0.045 mole/L

 $HCO_3^- = 0.045 \text{ mole/L}$ (61 g/mole) (10³ mg/1g) = 2745 mg/L

c. Determine the amount of alkalinity required per day

Equivalents of
$$HCO_3^- = \frac{(2.745 g/L)}{(61 g/eq)} = 0.045 eq/L$$

1 eq.
$$CaCO_3 = \frac{m.w.}{2} = \frac{(100 \text{ g/mole})}{2} = 50 \text{ g CaCO}_3 / eq$$

Alkalinity as $CaCO_3 = (0.045 \text{ eq/L}) (50 \text{ g/eq}) 10^3 \text{ mg/g}$

Alkalinity needed = (2250 - 200) mg/L

Daily alkalinity addition = $(2050 \text{ g/m}^3) (4000 \text{ m}^3/\text{d}) (1 \text{ kg/}10^3 \text{ g})$ = $8200 \text{ kg CaCO}_3/\text{d}$

Alkalinity addition as HCO₃:

$$= \frac{(8200 \text{ kg CaCO}_3 / \text{d})(61 \text{ g HCO}_3 / \text{eq})}{(50 \text{ g CaCO}_3)} = 10,004 \text{ kg HCO}_3 / \text{d}$$

- d. Determine the daily cost for the needed alkalinity.Alkalinity cost = (10,004 kg/d)(\$0.90/kg) = \$9004/d
- 2. Determine the amount of energy, and the corresponding cost, required to raise the temperature of the sludge from 20 to 35°C using a specific heat value of 4200 J/kg•°C (see inside of back cover). Heat transfer efficiency = 80 percent (given).
 - a. Energy required

q = [(4000 m³/d) (10³ kg/m³) [(35 – 20)°C] (4200 J/kg
$$\cdot$$
°C)]/0.80 = 31.5 x 10¹⁰ J/d

b. Cost of the required energy at \$0.08/kWh (given)

$$1.0 \text{ kWh} = 3.6 \text{ MJ} \text{ (Table A-2)}$$

Cost =
$$\frac{(31.5 \times 10^{10} \text{ J/d})(\$0.08 / \text{kWh})}{(3.6 \times 10^6 \text{J/kWh})} = \$6776 / \text{d}$$

- 3. Determine the amount of methane produced per day at a COD concentration of **10,000** mg/L and the corresponding value of the methane.
 - a. Gas production. Ignoring biomass production, the methane production at 35° C = 0.40 m³/kg COD removed. The methane production rate at a COD removal efficiency 95 percent (given) is:

=
$$(0.40 \text{ m}^3/\text{kg}) (10,000 \text{ g COD/m}^3) (4000 \text{ m}^3/\text{d}) (1.0 \text{ kg}/10^3 \text{ g})(0.95)$$

 $= 15,200 \text{ m}^3/\text{d}$

At 0°C, the methane volume is:

Volume,
$$0^{\circ}C = \frac{(15,200 \text{ m}^3/\text{d})(273.25)}{(273.15 + 35)} = 13,473.6 \text{ m}^3/\text{d}$$

Energy content of methane at standard conditions = 38,846 kJ/m³ (Example problem 10-2)

b. Energy value produced

=
$$(13,473.6 \text{ m}^3/\text{d})(38,846 \text{ kJ/m}^3) = 523.4 \text{ x } 10^6 \text{ kJ/d}$$

c. Value of methane.

=
$$(523.4 \times 10^6 \text{ kJ/d})(\$5/10^6 \text{ kJ}) = \$2,615/d$$

d. Use of digester gas for heating water. To reduce the cost of electrical energy for heating the water, the energy contained in the digested gas will be used instead.

Energy required to heat water (from Step 2a) = $31.5 \times 10^{10} \text{ J/d}$ Energy available to heat water (Step 3b, above) = $52.34 \times 10^{10} \text{ J/d}$ Fraction of methane produced needed to heat the digester:

$$= \frac{(31.5 \times 10^{10} \text{ J/d})}{(52.34 \times 10^{10} \text{ J/d})} = 0.60$$

Excess methane at Std. conditions = $(1 - 0.60)(13,473.6 \text{ m}^3/\text{d})$

$$= 5389.4 \text{ m}^3/\text{d}$$

Value of excess methane =

$$(5389.4 \text{ m}^3/\text{d})(38,846 \text{ kJ/m}^3)(\$5/10^6 \text{ kJ}) = \$1046/\text{d}$$

Solution: Part B Aerobic Process

- Determine the oxygen required to treat the waste, and the corresponding energy cost.
 - a. O₂ required

=
$$(10,000 \text{ g COD/m}^3)$$
 $(4000 \text{ m}^3/\text{d})$ $(1.0 \text{ kg/}10^3 \text{ g})(0.99)(1.2 \text{ g/g O}_2)$
= $47,520 \text{ kg O}_2/\text{d}$

b. Cost of aeration

=
$$\frac{(47,520 \text{ kg O}_2/\text{d})(\$0.08/\text{kWh})}{(1.2 \text{ kg O}_2/\text{kWh})}$$
 = $\$3168/\text{d}$

- 5. Determine the daily net sludge production, and the corresponding processing cost.
 - a. Sludge production
 - = $(10,000 \text{ g COD/m}^3)(4000 \text{ m}^3/\text{d}) (1.0 \text{ kg}/10^3 \text{ g})(0.3 \text{ g TSS/g COD})$
 - = 12,000 kg TSS/d

b. Cost of sludge processing

= (12,000 kg TSS/d)(\$0.10/kg TSS) = \$1200/d

Solution: Comparison of Anaerobic and Aerobic Process

6. Prepare a summary table to compare the anaerobic and aerobic processes.

	Treatment process cost, \$/d		
Item	Anaerobic without internal use of digester gas	Anaerobic with internal use digester gas	Aerobic
Required alkalinity	< 9,004 >	< 9004 >	
Raise temperature	< 6,776 >	0	
Methane produced	2,615	1,046	
Net cost	< 13,165 >	< 7,958 >	
Aeration			< 3,168 >
Sludge processing			< 1,200 >
Net cost			< 4,368 >

Comment

In this problem, the importance of the organic concentration, alkalinity cost, and temperature of the wastewater is illustrated. The economic benefit of using of digester gas for heating is also illustrated. The need to add alkalinity is a major negative cost for the anaerobic process in this case.

PROBLEM 10-3

Problem Statement - See text, page 11110

Solution (flowrate of 1000 m³/d)

1. Determine the amount of substrate degradation that will occur.

$$C_{50}H_{75}O_{20}N_5S$$
 removed = $(0.95)(4000 \text{ mg/L}) = 3800 \text{ mg/L}$

2. Write a balanced equation for substrate degradation using Eq. (10-4).

$$\begin{split} &C_{50}H_{75}O_{20}N_{5}S \ + \left(50 \ - \ \frac{75}{4} \ + \ \frac{20}{2} \ + \ \frac{3(5)}{4} \ + \ \frac{1}{2}\right)H_{2}O \ \rightarrow \\ &\left(\frac{50}{2} \ + \ \frac{75}{8} \ + \ \frac{20}{4} \ + \ \frac{3(5)}{8} \ + \ \frac{1}{4}\right)CH_{4} \\ &+ \left(\frac{50}{2} \ - \ \frac{75}{8} \ + \ \frac{20}{4} \ + \ \frac{3(5)}{8} \ + \ \frac{1}{4}\right)CO_{2} \\ &+ \ (5)NH_{3} \ + \ (1)H_{2}S \end{split}$$

$$C_{50}H_{75}O_{20}N_5S + 45.5H_2O \rightarrow 41.5CH_4 + 22.75CO_2 + 5NH_3 + 1H_2S$$

3. Write the pertinent reaction for the formation of alkalinity as the substrate is degraded using Eq. (10-5).

$$C_{50}H_{75}O_{20}N_{5}S + 45.5H_{2}O \rightarrow 41.5CH_{4} + 22.75CO_{2} + 5NH_{3} + 1H_{2}S$$

$$+ 5(NH_{3} + H_{2}O + CO_{2}) \rightarrow 5NH_{4}^{+} + 5HCO_{3}^{-})$$

$$C_{50}H_{75}O_{20}N_{5}S + 50.5H_{2}O \rightarrow 41.5CH_{4} + 17.75CO_{2} + 5NH_{4}^{+} + 1H_{2}S + 5HCO_{3}^{-}$$

The amount of alkalinity (expressed as $CaCO_3$) produced per g/L of substrate degraded is 250/1097 = 0.23 g/L as $CaCO_3$.

 Determine the amount of alkalinity produced per day for an average flowrate of 1000 m³/d.

Alk prod. =
$$\left(\frac{3.8 \text{ g}}{\text{L}}\right) \left(\frac{0.23 \text{ g/L}}{\text{g/L}}\right) \left(\frac{10^3 \text{ L}}{\text{m}^3}\right) \left(\frac{10^3 \text{ m}^3}{\text{d}}\right) \left(\frac{1 \text{kg}}{10^3 \text{ g}}\right)$$

= 874 kg/d as CaCO₃

Summary of results for various flowrates

Flowrate, m ³ /d	Alkalinity produced, kg/d as CaCO ₃
1000	874
2000	1748
3000	2622

5. Determine the approximate mole fraction of CO₂, CH₄, and H₂S in the gas phase using Eqs. (10-6), (10-7), and (10-8), respectively.

$$f_{CO_2} = \frac{4(50) - 75 + 2(20) - 5(5) + 2(1)}{8(50 - 5 + 1)} = 0.386$$

$$f_{CH_4} = \frac{4(50) + 75 - 2(20) - 5(5) - 2(1)}{8(50 - 5 + 1)} = 0.565$$

$$f_{H_2S} = \frac{1}{8(50 - 5 + 1)} = 0.003$$

PROBLEM 10-4

Problem Statement - See text, page 1110

Instructors Note: The discussion in the text that accompanies Eq. (10-16) and the data presented in Fig. 10-7 are not correct. The percent H_2S should be on the right axis and the HS^- on the left axis. Thus, at a pH of 7, about 40% of the H_2S present is gaseous H_2S .

Solution

1. Determine the amount of COD that will be used for sulfate reduction at 98 percent degradation.

COD used =
$$0.98 \left(\frac{0.89 \text{ mg COD}}{\text{mg sulfate}} \right) \left(\frac{500 \text{ mg sulfate}}{\text{L}} \right) = 436 \text{ mg COD/L}$$

- 2. For the influent COD value of **4000 mg/L**, compute the amount of methane gas that will be generated (a) with and (b) without the presence of sulfate.
 - a. Determine the amount of methane produced accounting for the COD removed by sulfate reduction. The COD remaining after sulfate reduction and considering the 95 percent COD degradation is

COD remaining =
$$0.95(4000 \text{ mg/L}) - 436 \text{ mg/L} = 3364 \text{ mg/L}$$

Methane produced =
$$\left(\frac{3.364 \text{ g COD}}{\text{L}}\right) \left(\frac{0.40 \text{ L CH}_4}{\text{g COD}}\right) = 1.35 \text{ L CH}_4 / \text{L}$$

At the flowrate of 2000 m³/d, the total amount of methane produced per day is

Methane produced =
$$\left(\frac{1.35 \text{ m}^3 \text{ CH}_4}{\text{m}^3}\right) \left(\frac{2000 \text{ m}^3}{\text{d}}\right) = 2691 \text{ m}^3 / \text{d}$$

 Determine the amount of methane produced without accounting for the COD removed by sulfate reduction. The COD remaining assuming 95 percent COD degradation is

COD remaining =
$$0.95(4000 \text{ mg/L}) = 3800 \text{ mg/L}$$

Methane produced =
$$\left(\frac{3.8 \text{ g COD}}{\text{L}}\right) \left(\frac{0.40 \text{ L CH}_4}{\text{g COD}}\right) = 1.52 \text{ L CH}_4 / \text{L}$$

At the flowrate of 2000 m³/d, the total amount of methane produced per day is

Methane produced =
$$\left(\frac{1.52 \text{ m}^3 \text{ CH}_4}{\text{m}^3}\right) \left(\frac{2000 \text{ m}^3}{\text{d}}\right) = 3,040 \text{ m}^3 / \text{d}$$

- 3. Compute the amount of H₂S in the gas phase at a reactor pH value of 7.0.
 - a. Determine the total amount of H₂S produced (hydrogen sulfide gas and hydrogen sulfide ion) from sulfate reduction.

$$H_2S \text{ prod.} = \left(\frac{0.4 \text{ m}^3 \text{ H}_2S}{\text{kg COD}}\right) \left(\frac{0.436 \text{ kg COD}}{\text{m}^3}\right) \left(\frac{2000 \text{ m}^3}{\text{d}}\right) = 349 \text{ m}^3 / \text{d}$$

b. Compute the percent of H_2S in the gas phase at pH 7 and $35^{\circ}C$.

For a reactor temperature of 35° C, the acid equilibrium constant for H_2S , K_{a1} , must be interpolated between the temperatures of 30 and 40° C, as given in Table 10-11. Plotting the K_{a1} values on log-paper as a function of temperature results in a value of 1.88 for K_{a1} at 35° C. The percent H_2S is then determined using Eq. (10-16).

$$H_2S$$
, % = $\frac{100}{1+K_{a1}/[H^+]} \frac{100}{1+(1.88 \times 10^{-7})/[10^{-7}]} = 35\%$

c. Determine the total amount of H_2S in the gas phase produced in the reactor.

 H_2S gas produced = 349 m³/d x 0.35 = 122 m³/d

4. Summary of values for all influent COD concentrations

		Influent COD		
Item	Unit	4000	6000	8000
COD remaining after sulfate reduction	mg/L	3364	5264	7164
Methane produced accounting sulfate reduction	m³/d	2691	4211	5731
Methane produced without accounting for sulfate reduction	m ³ /d	3040	4560	6080

PROBLEM 10-5

Problem Statement - See text, page 1110

Solution

1. Summarize the possible causes and mechanisms for a decrease in the methane gas production rate.

Cause	Mechanism
Nutrient limitation	The concentration of nitrogen, phosphorus, or sulfur may be insufficient to support anaerobic biomass growth.
Micronutrient limitation	Trace metals, such as iron, cobalt, nickel, and zinc, may be not present or bioavailable.
Alkalinity limitation	Because of the high concentration of dissolved CO ₂ , alkalinity is needed to buffer the pH in an anaerobic reactor. Alkalinity will need to be added if the wastewater does not contain sufficient alkalinity to maintain a neutral pH.
Ammonia toxicity	High concentrations of ammonia (toxicity threshold of 100 mg/L as NH ₃ -N), are inhibitory to methanogenic activity. Proteins and amino acids may be degraded to produce ammonium
Sulfide toxicity	High concentrations (50 to 250 mg/L) of H ₂ S has been shown to decrease methanogenic activity.
General toxicity	The presence of other toxic or inhibitory substances in wastewater may reduce methanogenic reaction rates.

PROBLEM 10-6

Problem Statement - See text, page 1110-1111

Instructors Note: Students should be instructed to assume a value for the percent COD removal. For the following example a value of 90 percent was selected.

Solution (wastewater 1):

 For a degradable COD concentration of 4000 mg/L and a temperature of 25°C, determine the SRT.

At 90 percent COD removal the effluent COD is:

$$= (1.0 - 0.9) (4000 \text{ mg/L}) = 400 \text{ mg/L}$$

The given effluent TSS concentration is 120 mg/L.

Effluent COD from TSS = (120 mg/L) 1.8 g COD/g TSS = 216 mg/L

Allowable effluent soluble COD = (400 - 216) mg/L = 184 mg/L

Rearranging Eq. (7-70) and substituting kY = μ [Eq. (7-16)]:

$$SRT = \left[\frac{\mu_{m} S_{o}}{K_{s} + S_{e}} - b \right]^{-1}$$

Use kinetic coefficients from Table 10-13,

$$\mu_{\rm m} = 0.20 \, {\rm g/g} \cdot {\rm d}$$

$$K_S = 120 \text{ mg/L}$$

$$b = 0.03 \text{ g/g} \cdot \text{d}$$

SRT =
$$\left\{ \frac{\left(0.20 \text{ g/g} \cdot \text{d}\right) \left(184 \text{ g/m}^3\right)}{\left[\left(120 + 184\right) \text{g/m}^3\right]} - 0.03 \text{ g/g} \cdot \text{d} \right\}^{-1}$$

$$SRT = 11.0 d$$

Use a factor of safety of 1.5

2. Determine the amount of sludge that will need to be wasted daily Use Eq. (8-21) in Table 8-10 to determine solids production:

$$P_{X,TSS} = \frac{QY(S_o - S)}{[1 + b(SRT)](0.85)} + \frac{f_d(b)QY(S_o - S)SRT}{(1 + bSRT)(0.85)} + Q(nbVSS)$$

 $S_o - S = degradable COD = 4000 mg/L - 184 mg/L = 3816 mg/L$

Use coefficients from Table 10-13 and assume $f_d = 0.15$

$$Y = 0.08 g VSS/g COD$$

$$b = 0.03 g/g \cdot d$$

$$P_{X,TSS} = \frac{(2000 \text{ m}^3 / \text{d})(0.08 \text{ g VSS/g COD})(3816 \text{ mg COD/L})}{[1 + 0.03 \text{ g/g} \cdot \text{d } (16.5 \text{ d})](0.85)} +$$

$$\frac{0.15 \,\, g \, / \, g(0.03 \,\, g \, / \, g \cdot d)(2000 \,\, m^3 \, / \, d)(0.08 \,\, g \, / \, g)(3816 \,\, mg \, / \, L)(16.5 \,\, d)}{[1 + 0.03(16.5 \,\, d)](0.85)} \,\, + \,\, 0 \,\, g \, / \, d$$

$$= 480,472 \text{ g/d} + 35,675 \text{ g/d} + 0 \text{ g/d}$$

$$P_{X,TSS} = 516,147 \text{ g/d} = 516.1 \text{ kg/d}$$

- 3. Determine reactor volume and τ
 - a. Determine the volume using Eq. (7-57)

$$Volume = \frac{(P_{X,TSS})(SRT)}{MLSS}$$

Assume MLSS = 5000 g/m^3

Volume =
$$\frac{(516,147 \text{ g/d})(16.5 \text{ d})}{(5,000 \text{ g/m}^3)}$$
 = 1703 m³

b. Determine the hydraulic detention time, τ

$$\tau = \frac{V}{Q} = \frac{1703 \text{ m}^3}{(2000 \text{ m}^3 / \text{d})} = 0.85 \text{ d}$$

4. Determine the methane gas production rate

Assume 0.4 m³ gas/kg COD at 35°C

At 25°C gas production rate =
$$(0.4)\frac{(273.15 + 25)}{(273.15 + 35)} = 0.39 \text{ m}^3 / \text{kg COD}$$

Amount of COD removed in waste sludge

$$= \frac{ \left(516,147 \ gTSS/d\right) \! \left(0.85 \ gVSS/gTSS\right) \! \left(1.42 \ gCOD/gVSS\right) }{ \left(2000 \ m^3/d\right) }$$

$$= 312 \text{ gCOD/m}^3$$

The methane gas production

=
$$(0.39 \text{ m}^3/\text{kg}) (3816 - 312 \text{ g COD/m}^3) (2000 \text{ m}^3/\text{d}) (1.0 \text{ kg}/10^3 \text{ g})$$

$$= 2733 \text{ m}^3/\text{d}$$

5. Determine the methane gas production rate

Methane production =
$$\frac{(2733 \text{ m}^3 \text{ CH}_4 / \text{d})}{(0.65 \text{ m}^3 \text{ CH}_4 / \text{m}^3 \text{ gas})} = 4,205 \text{ m}^3/\text{d}$$

6. Determine nutrient requirements

Biomass production =
$$P_{X,TSS}$$
 = 516,547 g TSS/d

N required =
$$(516,547)(0.12)(0.85) = 52,787 \text{ g/d}$$

P required =
$$(516,547)(0.02)(0.85) = 8,781 \text{ g/d}$$

Repeat the solution for 35°C

For a degradable COD concentration of **4000 mg/L** and a temperature of **35°C**, determine the SRT.

At 90 percent COD removal the effluent COD is:

$$= (1.0 - 0.9) (4000 \text{ mg/L}) = 400 \text{ mg/L}$$

The given effluent TSS concentration is 120 mg/L.

Effluent COD from TSS = (120 mg/L) 1.8 g COD/g TSS = 216 mg/L

Allowable effluent soluble COD = (400 - 216) mg/L = 184 mg/L

Rearranging Eq. (7-70) and substituting kY = μ [Eq. (7-16)]:

$$SRT = \left[\frac{\mu_{m}S_{o}}{K_{s} + S_{e}} - b\right]^{-1}$$

Use kinetic coefficients from Table 10-13,

$$\mu_{m} = 0.35 \text{ g/g} \cdot \text{d}$$

$$K_S = 120 \text{ mg/L}$$

$$b = 0.03 g/g \cdot d$$

SRT =
$$\left\{ \frac{\left(0.35 \text{ g/g} \cdot \text{d}\right) \left(184 \text{ g/m}^3\right)}{\left[\left(120 + 184\right) \text{g/m}^3\right]} - 0.03 \text{ g/g} \cdot \text{d} \right\}^{-1}$$

SRT = 5.5 d

Use a factor of safety of 1.5

Design SRT = 1.5 (5.5) = 8.3 d

2. Determine the amount of sludge that will need to be wasted daily Use Eq. (8-21) in Table 8-10 to determine solids production:

$$\mathsf{P}_{\mathsf{X},\mathsf{TSS}} = \frac{\mathsf{Q}\,\mathsf{Y}(\mathsf{S}_{\mathsf{o}} - \mathsf{S})}{[1 + \mathsf{b}(\mathsf{SRT})](0.85)} + \frac{\mathsf{f}_{\mathsf{d}}(\mathsf{b})\mathsf{Q}\,\mathsf{Y}(\mathsf{S}_{\mathsf{o}} - \mathsf{S})\mathsf{SRT}}{(1 + \mathsf{b}\mathsf{SRT})(0.85)} + \mathsf{Q}\big(\mathsf{nbVSS}\big)$$

 $S_o - S = degradable COD = 4000 mg/L - 184 mg/L = 3816 mg/L$

Use coefficients from Table 10-13 and assume $f_d = 0.15$

$$Y = 0.08 g VSS/g COD$$

$$b = 0.03 \, g/g \cdot d$$

$$P_{X,TSS} = \frac{(2000 \text{ m}^3 / \text{d})(0.08 \text{ g VSS/g COD})(3816 \text{ mg COD/L})}{[1 + 0.03 \text{ g/g} \cdot \text{d } (8.3 \text{ d})](0.85)} +$$

$$\frac{0.15 \text{ g/g}(0.03 \text{ g/g} \cdot \text{d})(2000 \text{ m}^3 \text{ / d})(0.08 \text{ g/g})(3816 \text{ mg/L})(8.3 \text{ d})}{[1+0.03(8.3 \text{ d})](0.85)} + 0 \text{ g/d}$$

$$= 575,104 \text{ g/d} + 21,480 \text{ g/d} + 0 \text{ g/d}$$

$$P_{X,TSS} = 596,584 \text{ g/d} = 596.6 \text{ kg/d}$$

- 3. Determine reactor volume and τ
 - a. Determine the volume using Eq. (7-57)

Volume =
$$\frac{(P_{X,TSS})(SRT)}{MLSS}$$

Assume MLSS = 5000 g/m^3

Volume =
$$\frac{(596,584 \text{ g/d})(8.3 \text{ d})}{(5,000 \text{ g/m}^3)}$$
 = 990 m³

b. Determine the hydraulic detention time, τ

$$\tau = \frac{V}{Q} = \frac{990 \text{ m}^3}{(2000 \text{ m}^3 / \text{d})} = 0.50 \text{ d}$$

4. Determine the methane gas production rate

Assume 0.4 m³ gas/kg COD at 35°C

Amount of COD removed in waste sludge =

$$= \frac{(596,584 \text{ gTSS/d})(0.85 \text{ gVSS/gTSS})(1.42 \text{ gCOD/gVSS})}{(2000 \text{ m}^3/\text{d})} = 360 \text{ gCOD/m}^3$$

The methane gas production =

=
$$(0.40 \text{ m}^3/\text{kg}) (3816 - 360 \text{ g COD/m}^3) (2000 \text{ m}^3/\text{d}) (1.0 \text{ kg}/10^3 \text{ g})$$

 $= 2765 \text{ m}^3/\text{d}$

5. Determine the methane gas production rate

Methane production =
$$\frac{(2765 \text{ m}^3 \text{ CH}_4 / \text{d})}{(0.65 \text{ m}^3 \text{ CH}_4 / \text{m}^3 \text{ gas})} = 4254 \text{ m}^3/\text{d}$$

6. Determine nutrient requirements

Biomass production =
$$P_{X,TSS}$$
 = 596,584 gTSS/d

$$N = 12\%, P = 2\% \text{ of VSS}$$

N required =
$$(596,584)(0.12)(0.85) = 60,851 \text{ g/d}$$

P required =
$$(596,584)(0.02)(0.85) = 10,142 \text{ g/d}$$

Summary

The solutions for 25 and 35°C for wastewater 1 is summarized in the following table.

Parameter	Unit	25°C	35°C
Design SRT	d	16.5	8.3

Reactor volume	m³	1703	990
Detention time, $\boldsymbol{\tau}$	d	0.85	0.50
CH ₄ production rate	m³/d	2733	2765
Total gas production rate	m³/d	4205	4254
Solids to be wasted	kg/d	516	597
Nitrogen requirements	kg/d	52.6	60.8
Phosphorus requirements	kg/d	8.8	10.1

PROBLEM 10-7

Problem Statement - See text, page 1111

Solution:

- 1. It is stated in the problem that 95 percent of the influent soluble COD is degraded at a 30 d SRT and an **effluent VSS concentration = 100 mg/L**. Thus, effluent $S = 0.05 S_o$ and $S_o S = 0.95 S_o$.
- 2. Using Eq. (8-20) in Table 8-10 for biomass solids production, the yield and decay coefficients in Table 10-10, assuming $f_d = 0.15$ g/g and S = 0.95 S_o, the influent concentration S_O is calculated as follows.

$$\begin{split} P_{X,VSS} &= \frac{QY(S_o - S)}{1 + b(SRT)} + \frac{f_d(b)QY(S_o - S)SRT}{1 + b(SRT)} \\ \frac{P_{X,VSS}}{Q} &= X_e = \frac{Y(0.95 \, S_o)}{1 + b(SRT)} + \frac{f_d(b)Y(0.95 \, S_o)SRT}{1 + b(SRT)} \\ 100 \, g/m^3 &= \frac{(0.08 \, g/g)(0.95 \, S_o)}{[1 + (0.03 \, g/g \cdot d)(30 \, d)]} \\ &+ \frac{(0.15 \, g/g)(0.03 \, g/g \cdot d))(0.08 \, g/g)[(0.95 \, S_o)g/m^3](30d)}{[1 + (0.03 \, g/g \cdot d)(30 \, d]} \\ 100 &= 0.04S_o + 0.0054S_o \\ S &= 2203 \, g/m^3 \end{split}$$

3. The solution for effluent VSS concentrations of 100, 150, and 200 mg/L are summarized in the following table.

Effluent VSS,	Influent sCOD,
mg/L	mg/L
100	2203
150	3304
200	4405

PROBLEM 10-8

Problem Statement - See text, page 1111-1112 **Solution (Wastewater 1)**:

1. Define the wastewater components and amount of COD removed.

Particulate COD =
$$0.40 (6000 \text{ g/m}^3) = 2400 \text{ g/m}^3$$

Soluble COD =
$$0.60 (6000 \text{ g/m}^3) = 3600 \text{ g/m}^3$$

Soluble COD degraded =
$$0.97 (3600 \text{ g/m}^3) = 3492 \text{ g/m}^3$$

Total amount of COD degraded =
$$1440 + 3492 = 4932 \text{ g/m}^3$$

Non degraded particulate COD = $0.40 (2400 \text{ g/m}^3) = 960 \text{ g/m}^3$

Non degraded VSS =
$$\frac{(960 \text{ gCOD/m}^3)}{(1.8 \text{ gCOD/g VSS})} = 533.33 \text{ g/m}^3$$

- Determine the reactor process volume.
 - a. Determine the reactor volume based on the maximum upflow velocity Eq.

$$A = \frac{Q}{v} = \frac{(500 \text{ m}^3/\text{d})}{(0.50 \text{ m/h})(24 \text{ h/d})} = 41.67 \text{ m}^2$$

$$V_v = A(H) = 41.67 \text{ m}^2 (8 \text{ m}) = 333.3 \text{ m}^3$$

b. Determine the reactor volume based on the organic loading rate.

From Eq. (10-20)

$$V_{OLR} = \frac{Q S_0}{OLR} = \frac{(500 \text{ m}^3/\text{d})(6.0 \text{ kg COD/m}^3)}{(6.0 \text{ kg COD/m}^3 \circ \text{d})} = 500 \text{ m}^3$$

The organic loading rate controls the reactor volume design.

3. Determine the process hydraulic retention time.

$$\frac{V}{Q} = \frac{500 \text{ m}^3}{(500 \text{ m}^3/\text{d})} = 1.0 \text{ d}$$

Determine the reactor dimensions.

a. Reactor Area =
$$\frac{V}{H} = \frac{500 \text{ m}^3}{8 \text{ m}} = 62.5 \text{ m}^2$$

$$\frac{\pi D}{4}$$
 = 62.5 m², D = 8.92 m

b. Total reactor height

$$H_T$$
 = process hgt + clear zone hgt + separator hgt (see Example 10-3)

$$H_T = 8 \text{ m} + 0.5 \text{ m} + 2.5 \text{ m} = 11 \text{ m}$$

Reactor dimensions = 8.92 m dia. ×11m height

- 5. Determine the reactor SRT.
 - a. From Eq. (7-56), $(X)V = P_X SRT$
 - b. From Eq. (8-20),

$$P_{X} = \frac{Q(Y_{H})(S_{o} - S)}{1 + b_{H}(SRT)} + \frac{f_{d}b_{H}(Q)(Y_{H})(S_{o} - S)(SRT)}{1 + b(SRT)} + (nbVSS)Q$$

c. Substituting Eq. (8-20) into Eq. (7-56),

$$X_{VSS}(V) = \frac{Q(Y_H)(S_o - S)(SRT)[1 + f_db_H(SRT)]}{1 + b_H(SRT)} + (nbVSS)Q(SRT)$$

From Step 1,
$$S_0 - S = 4932 \text{ g COD/m}^3$$

From Table 10-13,

$$Y_H = 0.08 g VSS/g COD$$

$$b_H = 0.03 \text{ g VSS/g VSS} \cdot \text{d}$$

$$\begin{split} &\left(50,000 \text{ g VSS/m}^3\right)\!\!\left(500 \text{ m}^3\right) = \\ &\frac{\left(500 \text{ m}^3/\text{d}\right)\!\!\left(0.08 \text{ g VSS/g COD}\right)\!\!\left(4932 \text{ g COD/m}^3\right)\!\!\left(\text{SRT}\right)\!\!\left(1 + 0.15\!\left(0.03 \text{ g/g•d}\right)\!\text{SRT}\right)}{1 + (0.03 \text{ g/g•d})\!\!\left(\text{SRT}\right)} \\ &+ 533.3 \text{ g VSS/m}^3\!\left(500 \text{ m}^3/\text{d}\right)\!\!\text{SRT} \end{split}$$

Solving: SRT = 71.5 d

6. Determine the daily sludge production rate from Eq. (7-56).

$$P_{X, VSS} = \frac{X_{VSS}(V)}{SRT}$$

$$= \frac{(50,000 \text{ g VSS/m}^3)(500 \text{ m}^3)(1 \text{ kg/10}^3 \text{ g})}{71.5 \text{ d}}$$

$$P_{X. VSS} = 349.65 \text{ kg VSS/d}$$

7. Determine the excess sludge daily waste volume.

$$\begin{split} P_{X, VSS} &= Q(X_e) + (X)Q_W \\ Q_W &= \frac{P_{X, VSS} - Q(X_e)}{X} \\ &= \frac{(349,650 \text{ g VSS/d}) - \left(500 \text{ m}^3/\text{d}\right) \left(200 \text{ g VSS/m}^3\right)}{(50,000 \text{ g VSS/m}^3)} \end{split}$$

$$Q_W = 5.0 \text{ m}^3/\text{d}$$

8. Determine the methane gas production rate by COD balance.

COD removal = methane COD + biomass COD

$$P_{X, bio} = P_{X, VSS} - nbVSS(Q)$$

$$P_{X, bio} = 349,650 \text{ g VSS/d} - 533.3 \text{ g VSS/m}^3 (500 \text{ m}^3/\text{d})$$

$$P_{X, bio}$$
 = 349,650 g VSS/d - 266,650 g VSS/d
= 83,000 g VSS/d

Methane COD = COD removed - biomass COD

$$= 500 \text{ m}^3/\text{d} (4932 \text{ g COD/m}^3) - 1.42 \text{ g COD/g VSS} (83,000 \text{ g VSS/d})$$

$$= (2,466,000 - 117,860) g COD / d$$

$$CH_4$$
 COD = 2,348,140 g CH_4 COD/d

At standard conditions, methane production rate =

$$(2,348,140 \text{ g CH}_4 \text{ COD/d})(0.35 \text{ L CH}_4/\text{g COD})(\text{m}^3/10^3 \text{ L})$$

= 821.8
$$\text{m}^3$$
 CH₄/d at 0°C

Methane production rate at 30°C =

$$\left(821.8 \text{ m}^3 \text{ CH}_4/\text{d}\right) \left[\frac{\left(273.15 + 30\right)^{\circ}\text{C}}{273.15^{\circ}\text{C}}\right] = 912 \text{ m}^3 \text{ CH}_4/\text{d}$$

8. Determine the total gas production rate; Percent methane = (100 - 35)% = 65%

Total gas production rate =
$$\frac{(912 \text{ m}^3 \text{ CH}_4/\text{d})}{(0.65 \text{ m}^3 \text{ CH}_4/\text{m}^3 \text{ gas})} = 1403 \text{ m}^3 \text{ gas/d}$$

9. Energy content of methane production

Energy =
$$(38,846 \text{ kJ/m}^3)(821.8 \text{ m}^3 \text{ CH}_4/\text{d}) = 31.9 \times 10^6 \text{ kJ/d}$$

10. Determine alkalinity requirements.

Assume
$$pH = 7.0$$

From Table 10-7 at pH =
$$7.0$$
, T = 30° C,

Alkalinity needed =
$$(2465-300)$$
g/m³ as CaCO₃
= 2165 g/m³ as CaCO₃

Alkalinity in kg/d =
$$\left(2165 \text{ g/m}^3\right)\left(500 \text{ m}^3/\text{d}\right)\left(\frac{1 \text{kg}}{10^3 \text{ g}}\right)$$

= 1083 kg/d

Summary of Results

Parameter	Unit	Value
Reactor process volume	m ³	500.0
Reactor total height	m	11.0
Reactor diameter	m	8.92
Hydraulic retention time	d	1.0
SRT	d	71.5
Excess waste sludge	m³/d	5.0
Total gas production rate	m³/d	1403
Methane production rate	m³/d	912
Energy production rate	kJ/d	31.9 x 10 ⁶
Alkalinity needed as CaCO ₃	kg/d	1083

PROBLEM 10-9

Problem Statement - See text, page 1112

Solution (Wastewater 1):

- 1. Determine the reactor dimensions.
 - a. Determine the reactor volume based on upflow velocity, Eq. (10-18) and Eq. (10-19).

Upflow velocity = 0.7 m/h from Table 10-17.

$$A = \frac{Q}{v} = \frac{(3000 \text{ m}^3/\text{d})}{(0.70 \text{ m/h})(24 \text{ h/d})} = 178.6 \text{ m}^2$$

Use reactor height = 5m from Table 10-17.

$$V = A(H) = 178.6 \text{ m}^2 (5 \text{ m}) = 892 \text{ m}^3$$

Hydraulic retention time =
$$\frac{892.9 \text{ m}^3}{(3000 \text{ m}^3/\text{d})}$$
 = 0.3 d = 7.1 h

b. Determine the organic loading rate.

$$OLR = \frac{QS_o}{V} = \frac{(3000 \text{ m}^3 / \text{d})(450 \text{ g COD} / \text{m}^3)}{892.9 \text{ m}^3} = 1.51 \text{ kg COD} / \text{m}^3 \cdot \text{d}$$

From Fig. 10-10, this is an acceptable organic loading rate at 25°C.

- c. Determine reactor dimensions
 - Reactor diameter

$$\frac{\pi D^2}{4}$$
 = 178.6 m², D = 15.1 m

ii. Reactor height

Assume per Example 10-3: Clear zone height = 0.5 m and gassolids separator = 2.5 m.

Height = 5 m + 0.5 m + 2.5 m = 8.0 m

Reactor dimension = 15.1 m dia x 8.0 m height

2. Effluent BOD and TSS from UASB reactor. From p1091, effluent TSS concentration may range from 50 to 150 mg/L. At 85 percent VSS/TSS and 0.5 g BOD/gVSS, the effluent particulate BOD may range from 21 to 63 mg/L. The effluent soluble BOD is mainly from volatile fatty acids (VFA). VFA concentration from UASB reports may range from 20 to 50 mg/L at long SRT and thus the soluble BOD may range from

$$\left(1.07 \frac{\text{g COD}}{\text{g acetate}}\right) \left(\frac{1\text{g BOD}}{1.6 \text{ g COD}}\right) (20-50)$$

= 13 to 40 mg/L

Thus the total effluent BOD may range from 34 to 100 mg/L.

- 3. Post UASB treatment to obtain a secondary treatment effluent quality. The challenge is treating what could be a relatively weak wastewater with a possibility of a significant VSS concentration as dispersed solids. Conventional activated sludge process may have limited success due to the limitations on developing a flocculent suspension to capture the dispersed solids. A biological aerated filter (Sec. 9-6) or moving bed bioreactor (MBBR, Sec. 9-5) with chemical addition before the clarifier would be the better choice.
- 4. Based on review of the literature, alkalinity is typically not limited as alkalinity is produced and the high hydraulic throughput rate minimizes dissolved CO₂ concentrations.

PROBLEM 10-10

Problem Statement - See text, page 1112

Solution

- 1. Determine the reactor volume and dimensions.
 - a. A COD loading rate of 5 kg COD/m³•d is given.
 - b. Determine the reactor volume.

Reactor volume =
$$\frac{(4 \text{ kg COD/m}^3)(1000 \text{ m}^3/\text{d})}{(5 \text{ kg COD/m}^3 \cdot \text{d})} = 800 \text{ m}^3$$

c. Determine the reactor area given reaction depth of 4 m.

Area =
$$800 \text{ m}^3/4 \text{ m} = 200 \text{ m}^2$$

Assume 3 reactors in parallel to limit area needed for flow distribution,

Area each =
$$66.7 \text{ m}^2 = \pi D^2/4$$

Diameter =
$$[(4)(66.7 \text{ m}^2) / 3.14]^{0.5} = 9.2 \text{ m}$$

Assume at least 1 m above media for freeboard and gas collection

- 3 reactors at 9.2 m dia x 5.0 m height
- 2. Determine the methane gas production rate ignoring COD in biomass production.
 - Determine the biological COD consumption rate at 90 percent removal.
 COD consumption rate = (1000 m³/d)(4000 g/m³)(0.9) = 3,600,000 g/d
 - b. Determine the methane production rate.

Methano production rate =
$$(0.40 \text{ CH}_4 \text{ L/g COD})(3,600,000 \text{ g COD/d})$$

= $1,440,000 \text{ L/d}$

$$= 1440 \text{ m}^3/\text{d}$$

3. Determine the effluent TSS concentration using Eq. (8-20).

$$P_{X,TSS} = \frac{QY(S_o - S)}{[1 + b(SRT)](0.85)} + \frac{f_d(b)QY(S_o - S)SRT}{[1 + b(SRT)](0.85)} + Q(nbTSS)$$

Assume nbTSS = 0

$$P_{X,TSS} = \frac{(1000 \text{ m}^3 / \text{d})(0.08 \text{ g VSS/g COD})(3600 \text{ mg COD/L})}{[1 + 0.03 \text{ g/g·d } (30 \text{ d})](0.85)} +$$

$$\frac{(0.15~g\,/\,g)(0.03~g\,/\,g\cdot d)(1000~m^3\,/\,d)(0.08~g\,/\,g)(3600~mg\,/\,L)(30~d)}{[1+0.03(30~d)](0.85)}~+~0~g\,/\,d$$

 $P_{X,TSS} = 202,402 \text{ g/d}$

Effluent TSS = $(202,402 \text{ g/d}) / (1000 \text{ m}^3/\text{d}) = 202 \text{ mg/L}$

Effluent VSS = 0.85(202 mg/L) = 171.7 mg/L

- 4. Determine methane production rate when accounting for COD in biomass produced.
 - a. COD conversion rate to biomass = $(1000 \text{ m}^3/\text{d})(171.7 \text{ g VSS/m}^3)(1.42 \text{ g COD/g VSS}) = 243,814 \text{ g/d}$
 - b. COD consumption rate (step 2a) = 3,600,000 g/d
 - c. Determine the methane production rate.

Methane production rate = (0.40 L/d)[(3,600,000 - 243,8140) g COD/d]= 1,342,594 L/d= $1343 \text{ m}^3/\text{d}$

PROBLEM 10-11

Problem Statement - See text, page 1112

Problem Analysis

Process	Compatibility with high concentrations of COD and nbVSS	Potential impact of influent solids
UASB	The UASB process is compatible with high COD wastewaters, specific loading rate dependant on temperature	Influent solids can inhibit the formation of dense, granulated sludge. A prefermentation step may be helpful.
Anaerobic fluidized-bed reactor	High biomass concentration allows fluidized bed reactor to operate at high organic loading	Best suited for highly soluble wastewaters, does not capture solids well
Anaerobic baffled reactor	Lower COD loadings than some of the other processes, will result in a larger reactor.	Solids may inhibit the formation of sludge granules
Upflow packed- bed reactor	Compatible with high COD loadings, however, better suited for wastewaters with low suspended solids concentrations	Solids will eventually accumulate in the packing and cause clogging and flow short circuiting
Downflow attached growth reactor	Compatible with high COD loadings. Packing with high void volume recommended.	If high void volume packing used, solids less likely to cause clogging problems
Anaerobic covered lagoon	Low COD loading rate results in a large area needed for this process.	Solids not expected to be a problem.

PROBLEM 10-12

Problem Statement - See text, page 1113

The instructor should suggest reference sources for the students. Journal articles from Bioresource Technology, Journal of Bioscience and Bioengineering, Separation and Purification Technology, and Biomass and Bioenergy would be appropriate sources.

PROBLEM 10-13

Problem Statement - See text, page 1113

The instructor should suggest reference sources for the students. Journal articles from Bioresource Technology, Water Science and Technology, Biomass and Bioenergy and Water Environment Research would be appropriate sources.