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NON-LINEAR MULTIVARIABLE ADAPTIVE CONTROL OF AN ACTIVATED SLUDGE WASTEWATER TREATMENT PROCESS

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SUMMARY

In this paper a non-linear adaptive feedback-linearizing control is designed for a biological wastewater treatment model. The adaptive control structure is based on the non-linear model of the process and combined with a joint observer estimator which plays the role of the software sensor for the on-line estimation of biological states and parameter variables of interest of the bioprocess. The performances of both estimation and control algorithms are illustrated by simulation results. They demonstrate effectiveness and significant robustness against measurement noises and kinetic parameter jumps. Copyright © 1999 John Wiley & Sons, Ltd.

Key words: activated sludge process; bioprocess control; on-line estimation; non-linear adaptive control

1. INTRODUCTION

The activated sludge is one of the most widely used biological wastewater treatment processes. The interest in controlling this bioprocess grew due to the interest in high effluent quality and in reduction of energy consumption. The biological process is described by a set of non-linear equations obtained from mass-balance considerations. The dynamical model obtained is most often highly complex and high-order non-linear system.

The major difficulties for controlling these microbial processes lies in the lack of cheap and reliable sensors for on-line measurement of the key state variables, in particular, those involved in the model. In addition laboratory analysis, with delays of several days, cannot be used for on-line

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monitoring of this kind of biological process. A realistic method of process control design is to use estimators such as 'software sensors' to identify some key biological parameters. The estimators schemes are based on the exploitation of the non-linear structure of the model of the process in study and on the use of indirect measurements of the state, such as dissolved oxygen concentration, which can be readily obtained by a disolved oxygen electrode, or outlet gas flow rate.

Improved control can then be expected by exploiting the non-linear structure of the model and including an estimation procedure of the process states and kinetic parameters, in order to follow in real time the variations in the process dynamics, and correct the control actions, consequently. Such an idea has recently been used for the adaptive control of fermentation processes,[1–4] aerated lagoons wastewater treatment[5] and activated sludge process.[6–8] In a previous work of Dochain[6] the control was done using a reduced-order model of the plant obtained by a singular perturbation of the state-space model under some conditions. The idea pursued in the present paper is of somewhat different form for adaptive control of an activated sludge process. The purpose is to keep the pollutant substrate concentration and the dissolved oxygen concentration level close to prespecified values, in spite of disturbances and variations of process kinetics, by acting on the dilution rate and the aeration flow rate.

The aim of the first control action is to provide an acceptable pollutant level in the effluent while the second is to provide the biomass with the necessary and sufficient amount of energy to carry on the oxidation. As a fact, proper aeration is crucial to process efficiency, since an insufficient dissolved oxygen level would impair the oxidation process and eventually lead to biomass death, whereas a too high dissolved oxygen may cause the sludge to settle poorly. Excessive aeration is also undesirable from an economic point of view, since the oxygen in excess is simply lost to the atmosphere.

The key idea of control design in this work, which was briefly introduced in Reference 7, is to take advantage of what is well known about the dynamics of the bioprocess while taking into account the model uncertainty. Since the specific growth rate and some of the state variables in interest are not measured on-line, they are replaced in the control algorithm by on-line estimates provided by the joint observer estimator (JOE),[4,9,10] according to certainty equivalence principle. The resulting controller will be an adaptive linearizing controller. The JOE was chosen after a comparative study done in Nejjari *et al.*[10] between several estimation techniques which were developed to find the best one to apply to the process.

The joint observer/estimator consists of a procedure based on Narendra's[11] error model obtained by an extended linearization technique, analogous to the extended Luenberger observer,[12] using Kronecker's calculation. A preliminar useful change,[9] with a non-singular transformation, of co-ordinates is necessary to decouple the unmeasured states from the kinetic parameters. Instead of estimating the specific growth rate directly as a time-varying parameter, as it was done in a previous paper,[8] it is reconstructed on the basis of its analytical expression via the estimation of the kinetic parameters appearing in this expression. Consequently, we can estimate not only the specific growth rate but also, under some conditions, one of the kinetic parameters which we are not supposed to know practically.

The organization of the paper is as follows: in Section 2, the process model is briefly described. Section 3 deals with the construction and the stability study of the joint observer estimator. Section 4 is devoted to the non-linear control algorithm design. In Section 5 the efficiencies of the control and estimation schemes are demonstrated via simulation study. A general conclusion ends the paper.

2. BIOPROCESS MODELING

The activated sludge process is defined as a sysem in which a large number of biological organisms are maintained and continuously circulated so as to be in constant contact with the organic wastewater in the presence of air. The bioprocess is principally constituted by two sequential tanks, an aerator and a settler.

The bacteria and other microorganisms feed on the organic matter constituent of the incoming wastewater, thereby reducing the strength of the waste. The sludge is separated from the mixed liquor in solids separator (clarifier or settler). A portion of the settled sludge is recycled as return activated sludge from the clarifier to the aeration reactor so that the microorganisms content in the reactor is maintained at the reaction sustenance level. Excess sludge, not recycled, is extracted from the system as waste activated sludge and subsequently processed for different uses.

The aerator is considered to be perfectly mixed so that the concentration of each component is spatially homogeneous. We assume that no bioreaction takes place in the settler and that the sludge (biomass) is the only recycled component into the aerator; in other words, we suppose that the oxygen and substrate concentrations are neglected in the recycled stream.

The mass balance around the aerator and the settler gives the following set of non-linear differential equations:

$$\frac{\mathrm{d}X(t)}{\mathrm{d}t} = \mu(t)X(t) - D(t)(1+r)X(t) + rD(t)X_{\mathrm{r}}(t) \tag{1}$$

$$\frac{\mathrm{d}S(t)}{\mathrm{d}t} = -\frac{\mu(t)}{Y}X(t) - D(t)(1+r)S(t) + D(t)S_{\mathrm{in}} \tag{2}$$

$$\frac{\mathrm{d}C(t)}{\mathrm{d}t} = -\frac{K_0\mu(t)}{Y}X(t) - D(t)(1+r)C(t) + K_{\mathrm{La}}(C_{\mathrm{S}} - C(t)) + D(t)C_{\mathrm{in}} \tag{3}$$

$$\frac{\mathrm{d}X_{\mathrm{r}}(t)}{\mathrm{d}t} = D(t)(1+r)X(t) - D(t)(\beta + r)X_{\mathrm{r}}(t) \tag{4}$$

where $X(t)$, $S(t)$, $X_{\mathrm{r}}(t)$ and $C(t)$ are the state variables representing the biomass, the substrate, the recycled biomass and dissolved oxygen concentrations, respectively, $D(t)$ is the dilution rate, r and β represent, respectively, the ratio of recycled flow to influent flow and the ratio of waste flow to influent flow, S_{in} and C_{in} correspond to the substrate and dissolved oxygen concentrations in the feed stream, respectively. The kinetics of the cell mass production are defined in terms of the specific growth rate μ and the yield of cell mass Y; the term K_0 is a constant, C_{S} is the maximum dissolved oxygen concentration and K_{La} represents the oxygen mass transfer coefficient.

The specific growth rate μ is a key parameter for the description of biomass growth and is known to be a complex function of many physico-chemical and biological factors like the biomass concentration, the substrate concentration, the dissolved oxygen concentration, the pH, the temperature, and various others inhibitors. Many different analytical laws have been suggested for modelling this parameter. The most popular one is certainly the Monod law.

Here we assume that the specific growth rate depends on substrate, dissolved oxygen concentrations and several kinetic parameters. The kinetic model is then given by Olsson model[13]

$$\mu(t) = \mu_{\max}\frac{S(t)}{K_{\mathrm{S}} + S(t)}\frac{C(t)}{K_{\mathrm{c}} + C(t)} \tag{5}$$

where μ_{max} is the maximum specific growth rate, K_S is the so-called affinity constant, expressing the dependency of the degradation rate on the concentration of pollutant S, and K_c is the saturation constant.

The choice of Olsson model is partially motivated by its very large use in aerobic biotechnological applications, and particularly in activated sludge processes.

The objective is to control the pollutant substrate and the dissolved oxygen concentrations under the following assumptions:

A1. the dissolved oxygen concentration C is the only measurable state of the plant.
A2. the concentrations of the other state variables biomass X, substrate S and recycled biomass X_r are not available for on-line measurement.
A3. the constants C_S, K_0 and Y are known, the parameters r and β are known.
A4. the structure of the growth rate model is known.
A5. the kinetic parameters are unknown.
A6. the control variables are bounded.

3. ESTIMATION ALGORITHM

System (1)–(4) is rewritten in the following more general form

$$\begin{aligned} \dot{\xi} &= A(\theta, \xi)\xi + b(\xi)U + G \\ \xi_m &= h^T\xi \end{aligned} \tag{6}$$

where

$$\xi^T = [\xi_e^T \,|\, \xi_m^T] = [X\ S\ X_r \,|\, C], \quad h = [0\ 0\ 0\ 1]^T, \quad U = D$$

$$\theta = [\mu_{max}\ K_s\ K_c]^T, \quad G = [G_1 \,|\, G_2]^T = [0\ 0\ 0 \,|\, K_{La}C_S]^T$$

$$b(\xi) = \begin{bmatrix} b_1 \\ \hline b_2 \end{bmatrix} = \begin{bmatrix} -(1+r)X + rX_r \\ -(1+r)S + S_{in} \\ (1+r)X - (\beta+r)X_r \\ \hline -(1+r)C + C_{in} \end{bmatrix} \text{ and}$$

$$A(\theta, \xi) = \begin{bmatrix} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} \mu(\theta, \xi) & 0\ 0\ 0 \\ \dfrac{-\mu(\theta, \xi)}{Y} & 0\ 0\ 0 \\ 0 & 0\ 0\ 0 \\ \hline \dfrac{-K_0\mu(\theta, \xi)}{Y} & 0\ 0\ 0 \end{bmatrix}$$

ξ is the state vector, ξ_m represents the measurable state vector, ξ_e is the unmeasured state vector and θ corresponds to the unknown parameter vector.

The objective of the joint state and parameter estimator is to:

(i) Observe the unavailable state vector $\xi_e = [X\ SX_r]^T$
(ii) Estimate the constant parameter vector $\theta = [\mu_{max}\ K_s\ K_c]^T$

In order to decouple[1] the dynamics of the unmeasured states from the kinetic parameter vector θ, we introduce a non-singular linear transformation T defined by, $\xi^* = T\xi$:

$$T = \begin{bmatrix} 1 & 0 & 0 & \dfrac{Y}{K_0} \\ 0 & 1 & 0 & -\dfrac{1}{K_0} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and such that the block matrices $A^*_{1k}(k = 1, 2)$ of matrix A^* are independent of the parameter vector θ.

Consequently, we obtain a new auxiliary state vector

$$\xi^* = [\xi^*_e \mid \xi^*_m]^T = [\xi^*_1\ \xi^*_2\ \xi^*_3 \mid \xi^*_4]^T = \left[X + \frac{Y}{K_0} C\ S - \frac{C}{K_0} X_r \middle| C \right]^T$$

The model associated to the new state-space representation is given by

$$\begin{aligned} \dot{\xi}^* &= A^*(\theta, \xi^*)\xi^* + b^*(\xi^*)U + G^* \\ \xi^*_m &= h^T\xi^* \end{aligned} \tag{7}$$

with

$$A^*(\theta, \xi^*) = TA(\theta, \xi)T^{-1} = \begin{bmatrix} A^*_{11} & A^*_{12} \\ \text{----------} \\ A^*_{21} & A^*_{22} \end{bmatrix} = \left[\begin{array}{cccc} 0 & 0 & 0 & -\dfrac{Y}{K_0} K_{La} \\ 0 & 0 & 0 & \dfrac{1}{K_0} K_{La} \\ 0 & 0 & 0 & 0 \\ \hline -\dfrac{K_0}{Y}\mu & 0 & 0 & \mu - K_{La} \end{array}\right]$$

$$b^*(\xi^*) = Tb(\xi) = \left[\frac{b_1^*}{b_2^*}\right] = \left[\begin{array}{c} -(1+r)\xi_1^* + \xi_3^* + C_{\text{in}} \\ -(1+r)\xi_2^* + S_{\text{in}} - \dfrac{C_{\text{in}}}{K_0} \\ (1+r)\xi_1^* - (\beta+r)\xi_3^* - \dfrac{Y}{K_0}(1+r)\xi_4^* \\ \hline -(1+r)\xi_4^* + C_{\text{in}} \end{array}\right]$$

$$G^* = TG = \left[\frac{G_1^*}{G_2^*}\right] = \left[\begin{array}{c} \dfrac{Y}{K_0} K_{\text{La}} \\ -\dfrac{K_{\text{La}}}{K_0} C_{\text{s}} \\ 0 \\ \hline K_{\text{La}} C_{\text{s}} \end{array}\right]$$

The model of the proposed JOE is based on model (7) and described by the following non-linear equation:

$$\dot{\hat{\xi}}^* = A^*(\hat{\theta}, \hat{\xi}^*)\hat{\xi}^* + b^*(\hat{\xi}^*)U + G^* + K(h^{\text{T}}\hat{\xi}^* - \xi_{\text{m}}) \tag{8}$$

where the gain vector K has the components $K = [\Psi_1\ \Psi_2\ \Psi_3\ \Phi]^{\text{T}}$.

3.1. Error system

The structure of the error system is obtained by an extended linearization technique using Kronecker's calculation.[11]

If we define $\tilde{\xi}^* = \hat{\xi}^* - \xi^*$ as the state observation error and $\tilde{\theta} = \hat{\theta} - \theta$ as the parameter estimation error, then by using (7) and (8) error system can be written under the linear form

$$\dot{\tilde{\xi}}^* = F^*\tilde{\xi}^* + B^*\tilde{\theta} \tag{9}$$

where

$$F^* = \left[\frac{\hat{A}_1^*\ \Psi}{\hat{A}_2^*\ \Phi}\right] \text{ and } B^* = \bar{\nabla}_\theta(A^*) \otimes \hat{\xi}_{\text{s}}^* = [B_{\text{e}}^* \mid B_{\text{m}}^*]^{\text{T}}$$

with

$$\hat{A}_k^* = \hat{A}_{k1}^* + \bar{\nabla}_{\xi_{\text{e}}^*}(A_{k1}^*) \otimes \hat{\xi}_{\text{e}}^* + \bar{\nabla}_{\xi_{\text{e}}^*}(A_{k2}^*) \otimes \xi_{\text{m}}^* + \bar{\nabla}_{\xi_{\text{e}}^*}(G_k^*) \otimes U \quad \text{for } k = 1, 2$$

$$B_{\text{e}}^* = \bar{\nabla}_\theta([A_{11}^* A_{12}^*]) \otimes \hat{\xi}_{\text{s}}^*, \quad B_{\text{m}}^* = \bar{\nabla}_\theta([A_{21}^* A_{22}^*]) \otimes \hat{\xi}_{\text{s}}^* \quad \text{and} \quad \hat{\xi}_{\text{s}}^{*\text{T}} = [\hat{\xi}_{\text{e}}^{*\text{T}}\ \hat{\xi}_{\text{m}}^{*\text{T}}]$$

$\otimes$ is the Kronecker product and $\bar{\nabla}$ a differential operator.

We then obtain

$$F^* = \left[\begin{array}{cccc} -D(1+r) & 0 & rD & \Psi_1 \\ 0 & -D(1+r) & 0 & \Psi_2 \\ D(1+r) & 0 & -D(\beta+r) & \Psi_3 \\ \hline \dfrac{K_0}{Y}\left(\hat{\mu} - \hat{\xi}_2^* \dfrac{\partial \hat{\mu}}{\partial \hat{\xi}_2^*}\right) & \hat{\xi}_4^* \dfrac{\partial \hat{\mu}}{\xi \hat{\xi}_2^*} & 0 & \Phi \end{array}\right]$$

and

$$B^* = \left[\begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \hline -\hat{\xi}_1^* \dfrac{K_0}{Y}\dfrac{\partial \hat{\mu}}{\partial \hat{\theta}_1} + \hat{\xi}_4^* \dfrac{\partial \hat{\mu}}{\partial \hat{\theta}_1} & -\hat{\xi}_1^* \dfrac{K_0}{Y}\dfrac{\partial \hat{\mu}}{\partial \hat{\theta}_2} + \hat{\xi}_4^* \dfrac{\partial \hat{\mu}}{\partial \hat{\theta}_2} & -\hat{\xi}_1^* \dfrac{K_0}{Y}\dfrac{\partial \hat{\mu}}{\partial \hat{\theta}_3} + \hat{\xi}_4^* \dfrac{\partial \hat{\mu}}{\partial \hat{\theta}_3} \end{array}\right]$$

The observer design consists in choosing the gains $\{\Psi_i, i = 1, 2, 3\}$ and Φ such as F^* is an Hurwitz matrix.[9,14]

Note that the eigenvalues of F^* are the same as the eigenvalues of the matrix

$$F^{*\mathrm{T}} = \underbrace{\begin{bmatrix} \hat{A}_1^{*\mathrm{T}} & \hat{A}_2^{*\mathrm{T}} \\ 0 & \mathrm{I} \end{bmatrix}}_{A} - \underbrace{\begin{bmatrix} 0 \\ I \end{bmatrix}}_{B} \underbrace{[-\Psi^{\mathrm{T}} - \Phi^{\mathrm{T}} + I]}_{K}$$

where I is the identity matrix.

If the pair (A, B) is completely controllable[14] we can find a feedback matrix K which solve the observer design problem. The stabilization of matrix F^* can then be obtained by imposing via state feedback a desired characteristic polynomial $\delta(\lambda)$ with four roots located in the left-half of the complex plane

$$\delta(\lambda) = \det(F^* - \lambda I) = \prod_{i=1}^{4} (\lambda - \lambda_i), \{\lambda_i\}_{i=1,\ldots,4} < 0 \tag{10}$$

The gain matrix components are then computed in function of the eigenvalues λ_i, $i = 1, \ldots, 4$.

3.2. Parameter estimation

The estimator design deals with finding a suitable parameter adjustment law[11] of $\hat{\theta}(t)$, ensuring stability of (9).

The adjustment law has the form

$$\dot{\hat{\theta}} = -\Gamma B_{\mathrm{m}}^{*\mathrm{T}} P \tilde{\xi}_{\mathrm{m}}^* \tag{11}$$

which is clearly independent of the unmeasured states and where P and Γ are arbitrary symmetric positive-definite matrices and $\tilde{\xi}_{\mathrm{m}}^*$ represents the error between the estimated and the measured dissolved oxygen concentration.

For simulation purpose Γ and P are chosen such that $\Gamma = \mathrm{diag}(\gamma_{ii})$ where $\gamma_{ii} > 0$ for $i = 1, 2, 3$ and $P = I$ (Unity matrix).

Equation (11) is then given in the following form:

$$\begin{aligned}
\dot{\hat{\theta}}_1 &= -\gamma_{11}\left(\hat{\xi}_4 - \hat{\xi}_1^* \frac{K_0}{Y}\right)\frac{\hat{\mu}}{\hat{\theta}_1}(\hat{\xi}_4 - \xi_m) \\
\dot{\hat{\theta}}_2 &= -\gamma_{22}\left(\hat{\xi}_4 - \hat{\xi}_1^* \frac{K_0}{Y}\right)\left(\frac{-\hat{\mu}}{\hat{\theta}_2 + \hat{\xi}_2^* + \hat{\xi}_4/K_0}\right)(\hat{\xi}_4 - \xi_m) \\
\dot{\hat{\theta}}_3 &= -\gamma_{33}\left(\hat{\xi}_4 - \hat{\xi}_1^* \frac{K_0}{Y}\right)\left(\frac{-\hat{\mu}}{\hat{\theta}_3 + \hat{\xi}_4}\right)(\hat{\xi}_4 - \xi_m)
\end{aligned} \tag{12}$$

3.3. Stability proof

Consider the following Lyapunov quadratic function:

$$V(\tilde{\xi}^*, \tilde{\theta}) = \tilde{\xi}^{*\mathrm{T}} P \tilde{\xi}^* + \tilde{\theta}^{\mathrm{T}} \Gamma^{-1} \tilde{\theta} \tag{13}$$

where P and Γ are symmetric positive-definite matrices.

Evaluating the time derivative of V along trajectories (9) and (11) we obtain

$$\dot{V}(\tilde{\xi}^*, \tilde{\theta}) = \tilde{\xi}^{*\mathrm{T}}[F^{*\mathrm{T}} P + P F^*]\tilde{\xi}^* \tag{14}$$

Since F^* is an Hurwitz matrix, the solution of the matrix equation

$$F^{*\mathrm{T}} P + P F^* = -Q < 0 \tag{15}$$

where Q is any symmetric positive-definite matrix, yields a symmetric positive-definite matrix P.

Hence, we have

$$\dot{V}(\tilde{\xi}^*, \tilde{\theta}) = -\tilde{\xi}^{*\mathrm{T}} Q \tilde{\xi}^* \leqslant 0 \tag{16}$$

According to Narendra's theorem,[11] we conclude that the error system (9) with (11) is asymptotically locally stable.[9]

4. ADAPTIVE LINEARIZING CONTROL

4.1. Basic theory[14]

We give a brief review of the exact input–output linearization theory. We consider a square non-linear system described by the following non-linear differential equations:

$$\begin{aligned}
\dot{\xi} &= f(\xi) + g(\xi)U \\
Z &= h(\xi)
\end{aligned} \tag{17}$$

where $\xi \in \mathbb{R}^n$ is the state vector, $U \in \mathbb{R}^m$ is the control input, $Z \in \mathbb{R}^m$ is the output vector, and $f(.)$, $g(.)$ and $h(.)$ are smooth non-linear functions of states of appropriate dimension. Differentiating Z with respect to time until the input appears we obtain

$$\begin{aligned}
Z_i^{(k)} &= L_f^k h_i, \quad k = 1, \ldots, \gamma_i - 1 \\
Z_i^{(\gamma_i)} &= L_f^{\gamma_i} h_i + \sum_{j=1}^{m} L_{g_j} L_f^{\gamma_i - 1} h_i U_j, \quad \text{for } i = 1, \ldots, m
\end{aligned} \tag{18}$$

where $L_f h$ an $L_g h$ are the Lie derivatives of h with respect to f and g, respectively, and γ_i the relative degree of the output Z_i with respect to the manipulated vector U.

The relative degree γ_i is exactly equal to the number of times one has to differentiate the output $Z_i(t)$ in order to have the value of the input vector U explicitly appearing.

The result equation of system (18) can thus be rewritten in a matrix form as

$$\begin{bmatrix} Z_1^{(\gamma_1)} \\ \vdots \\ Z_{\mathrm{m}}^{(\gamma_{\mathrm{m}})} \end{bmatrix} = \underbrace{\begin{bmatrix} L_f^{\gamma_1} h_1(\xi) \\ \vdots \\ L_f^{\gamma_{\mathrm{m}}} h_{\mathrm{m}}(\xi) \end{bmatrix}}_{a(\xi)} + \underbrace{\begin{bmatrix} L_{g_1} L_f^{\gamma_1 - 1} h_1(\xi) & \cdots & L_{g_{\mathrm{m}}} L_f^{\gamma_1 - 1} h_1(\xi) \\ \vdots & \ddots & \vdots \\ L_{g_1} L_f^{\gamma_{\mathrm{m}} - 1} h_{\mathrm{m}}(\xi) & \cdots & L_{g_{\mathrm{m}}} L_f^{\gamma_{\mathrm{m}} - 1} h_{\mathrm{m}}(\xi) \end{bmatrix}}_{B(\xi)} \underbrace{\begin{bmatrix} U_1 \\ \vdots \\ U_{\mathrm{m}} \end{bmatrix}}_{U} \tag{19}$$

The matrix $B(\xi)$ is called the characteristic matrix of system (18). If this matrix is bounded away from singularity, the linearizing control law is then given by

$$U = -B(\xi)^{-1} a(\xi) + B(\xi)^{-1} \upsilon \tag{20}$$

which yields the linear closed-loop system

$$\begin{bmatrix} Z_1^{\gamma_1} \\ \vdots \\ Z_{\mathrm{m}}^{\gamma_{\mathrm{m}}} \end{bmatrix} = \begin{bmatrix} \upsilon_1 \\ \vdots \\ \upsilon_{\mathrm{m}} \end{bmatrix} \tag{21}$$

4.2. Activated sludge control design

The reduction of the organic matter concentration (pollutant substrate) in treated water is the basic aim of wastewater process control. Also, the control of dissolved oxygen concentration is important because this concentration must be kept above a critical level to maintain the microorganism activity, by acting on the air flow rate W. This quantity appears in equation (3) through the oxygen transfer rate coefficient K_{La}.

$$K_{\mathrm{La}} = \alpha W, \quad \alpha > 0 \tag{22}$$

The non-linear equations (1)–(4) which describe the evolution of the bioprocess variables, can be written in the following form:

$$\begin{aligned} \frac{\mathrm{d}\xi(t)}{\mathrm{d}t} &= f(\xi) + g_1 U_1(t) + g_2 U_2(t) \\ Z_1 &= h_1(\xi) \\ Z_2 &= h_2(\xi) \end{aligned} \tag{23}$$

where

$$\xi^{\mathrm{T}} = [\xi_1\ \xi_2\ \xi_3\ \xi_4] = [X\ S\ X_{\mathrm{r}}\ C], \qquad U^{\mathrm{T}} = [U_1\ U_2] = [D\ W]$$

and $Z^{\mathrm{T}} = [Z_1\ Z_2] = [S\ C]$.

The analytical non-linear functions $f(.)$, $g_1(.)$, and $g_2(.)$ are given by

$$f(\xi) = \begin{bmatrix} \mu X \\ -\dfrac{\mu X}{Y} \\ 0 \\ -\dfrac{K_0 \mu X}{Y} \end{bmatrix}, \qquad g_1(\xi) = \begin{bmatrix} -(1+r)X + rX_r \\ -(1+r)S + S_{in} \\ -(\beta+r)X_r + (1+r)X \\ -(1+r)C + C_{in} \end{bmatrix} \text{ and } \quad g_2(\xi) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \alpha(C_S - C) \end{bmatrix}$$

The relative degrees associated with the outputs Z_1 and Z_2 are equal to $\gamma_1 = 1 (L_{g_1} h_1 \neq 0)$ and $\gamma_2 = 1 (L_{g_2} h_2 \neq 0)$, respectively.

The dynamics corresponding to the output Z is given by

$$\frac{dZ}{dt} = a(\xi) + B(\xi)U \tag{24}$$

where

$$a(\xi) = \begin{bmatrix} -\dfrac{\mu X}{Y} \\ -K_0 \dfrac{\mu X}{Y} \end{bmatrix} \text{ and } \quad B(\xi) = \begin{bmatrix} -(1+r)Z_1 + S_{in} & 0 \\ (1+r)Z_2 + C_{in} & \alpha(C_S - Z_2) \end{bmatrix}$$

It is obvious that in equation (24), the characteristic matrix $B(\xi)$ is non-singular if $(S_{in} - (1+r)Z_1)$ and $(C_S - Z_2)$ are different from zero.

(a) If $Z_1 = S_{in}/(1+r)$, then the bioprocess can be driven to a wash-out steady state ($X = 0$), i.e. to a state where the bacterial life has completely disappeared.
(b) If $Z_2 = C_S$, the process is driven to a steady state which is physically not realizable and not interesting ($X < 0$).

Recall that the objective of the control algorithm is to regulate the substrate and the dissolved oxygen concentrations at the setpoints Z_1^* and Z_2^*, respectively, by acting on the dilution rate D and on the aeration rate W.

For this purpose, we assume that the regulation error decreases according to the following stable linear time-varying first-order dynamics:

$$\frac{d(Z^* - Z)}{dt} = -\Lambda(Z^* - Z) \tag{25}$$

where $\Lambda = \text{diag}(\Lambda_1, \Lambda_2)$ with $\Lambda_i > 0$ and $Z^* = [Z_1^* \; Z_2^*]$.

By combining (24) and (25), we readily obtain the control law

$$U = B(\xi)^{-1} \left\{ \Lambda(Z^* - Z) + \frac{dZ^*}{dt} - a(\xi) \right\} \tag{26}$$

Since the biomass concentration, the substrate concentration and the specific growth rate are not measured on-line and the specific growth rate is unknown, they are replaced in the linearizing

control law (26) by their obtained estimates, according to certainty equivalence principle. The control law is then written as

$$U = \begin{bmatrix} U_1 = \dfrac{1}{S_{in} - (1+r)\hat{S}} \left(\dfrac{\hat{\mu}\hat{X}}{Y} + \Lambda_1 (S^* - \hat{S}) \right) \\ U_2 = \dfrac{(1+r)C - C_{in}}{\alpha(C_S - C)} U_1 + \dfrac{1}{\alpha(C_S - C)} \left(K_0 \dfrac{\hat{\mu}\hat{X}}{Y} + \Lambda_2 (C^* - C) \right) \end{bmatrix} \tag{27}$$

where $\hat{X}$, $\hat{S}$ and $\hat{\mu}$ denote on-line estimates of X, S and μ, respectively. They are calculated with the estimator described in Section 3.

The regulation is done under the following constraints:

$$\begin{aligned} D_{min} \leqslant U_1 \leqslant D_{max} \\ W_{min} \leqslant U_2 \leqslant W_{max} \end{aligned} \tag{28}$$

5. SIMULATION RESULTS

5.1. Numerical Values

Simulation results are obtained by using a fourth-order Runge–Kutta algorithm to integrate the non-linear process equations (1)–(4) with typical values of kinetic parameters and initial conditions given in Tables I–III. The sludge age of the system is about 2 days.

Table I. Process parameters

$Y = 0{\cdot}65$	$K_0 = 0{\cdot}5$
$r = 0{\cdot}6$	$C_S = 10\ \mathrm{mg\,l^{-1}}$
$\beta = 0{\cdot}2$	$S_{in} = 200\ \mathrm{mg\,l^{-1}}$
$\alpha = 0{\cdot}018\ \mathrm{m^{-3}}$	$C_{in} = 0{\cdot}5\ \mathrm{mg\,l^{-1}}$

Table II. Kinetic parameters

$\mu_{max} = 0{\cdot}15$ for $0\ \mathrm{h} < t \leqslant 50\ \mathrm{h}$	$\mu_{max} = 0{\cdot}21$ for $50\ \mathrm{h} < t \leqslant 300\ \mathrm{h}$
$K_s = 100\ \mathrm{mg\,l^{-1}}$ for $0\ \mathrm{h} < t \leqslant 150\ \mathrm{h}$	$K_s = 150\ \mathrm{mg\,l^{-1}}$ for $150\ \mathrm{h} < t \leqslant 300\ \mathrm{h}$
$K_c = 2\ \mathrm{mg\,l^{-1}}$ for $0\ \mathrm{h} < t \leqslant 250\ \mathrm{h}$	$K_c = 3\ \mathrm{mg\,l^{-1}}$ for $250\ \mathrm{h} < t \leqslant 300\ \mathrm{h}$

Table III. Initial conditions

$X(0) = 215\ \mathrm{mg\,l^{-1}}$	$C(0) = 6\ \mathrm{mg\,l^{-1}}$
$S(0) = 55\ \mathrm{mg\,l^{-1}}$	$S_{in} = 200\ \mathrm{mg\,l^{-1}}$
$X_r(0) = 400\ \mathrm{mg\,l^{-1}}$	$C_{in} = 0{\cdot}5\ \mathrm{mg\,l^{-1}}$

Table IV. JOE parameters

$\lambda_1 = 0{\cdot}02$	$\gamma_{11} = 0{\cdot}0016$
$\lambda_2 = -0{\cdot}15$	$\gamma_{22} = 150$
$\lambda_3 = -0{\cdot}061$	$\gamma_{33} = 0{\cdot}5$
$\lambda_4 = -0{\cdot}08$	

Table V. Controlled variables setpoints

Substrate setpoints	Dissolved oxygen setpoints
$S^* = 40\ \mathrm{mg\,l^{-1}}$ for $0\ \mathrm{h} < t \leqslant 100\ \mathrm{h}$ $S^* = 20\ \mathrm{mg\,l^{-1}}$ for $100\ \mathrm{h} < t \leqslant 300\ \mathrm{h}$	$C^* = 2\ \mathrm{mg\,l^{-1}}$ for $0\ \mathrm{h} < t \leqslant 300\ \mathrm{h}$

Table VI. Control law parameters

$D_{\min} = 0{\cdot}02\ \mathrm{h^{-1}}$	$W_{\min} = 0\ \mathrm{m^3\,h^{-1}}$
$D_{\max} = 0{\cdot}15\ \mathrm{h^{-1}}$	$\Lambda_1 = 0{\cdot}7$
$W_{\max} = 300\ \mathrm{m^2\,h^{-1}}$	$\Lambda_2 = 0{\cdot}85$

The output variable $C(t)$ is polluted with a 2% multiplicative signal noise in order to simulate more realistic measurements.

The design parameters of the JOE and control algorithms are given in Tables IV–VI.

5.2. Simulation results

The evolution of the control variables which are the dilution rate and the air flow rate are shown in Figures 1 and 2, respectively.

The profiles show the reaction to changes due to abrupt jumps of the kinetic parameters of the plant (20% change after 50 h for the maximum specific growth rate value, 50% change after 150 h

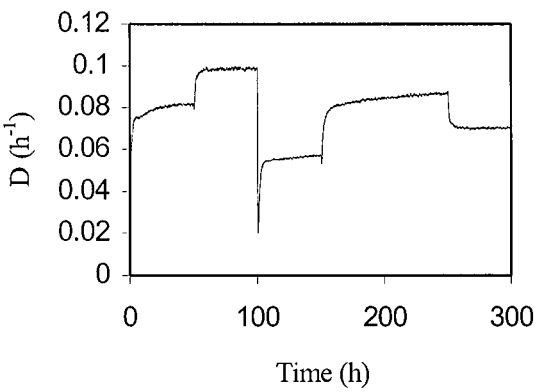

Figure 1. Behaviour of the dilution rate

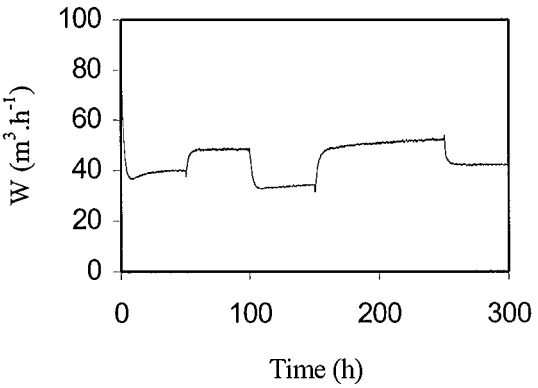

Figure 2. Behaviour of the air flow rate

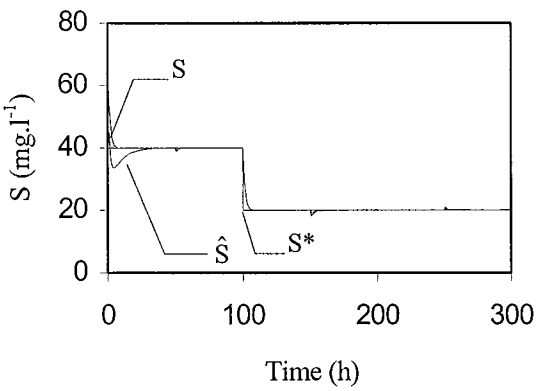

Figure 3. Behaviour of the substrate concentration

for the affinity constant value and 50% change after 250 h for the saturation constant) and to step changes of the substrate setpoints (from 40 to 20 mg^{-1} at $t = 100$ h).

The output variables evolution, that are the estimated substrate and the measured dissolved oxygen concentrations, and their corresponding reference trajectories are given in Figures 3 and 4, respectively. The figures show the performance and the effectiveness of the regulator. In particular, one can appreciate the ability of the controller to track the desired values of the controlled variables in response to the step change of the substrate setpoints and its robustness to reject the disturbances due to kinetic parameters changes.

The evolution of the reconstructed biomass conentration is depicted in Figure 5. The estimation algorithm tracks suitably the biomass concentration to its true simulated value with short transient response about 20 h. The JOE also provides good estimate of the recycled biomass concentration (Figure 6). One can see that this concentration is not in exact proportionality with the biomass concentration as it was considered in Reference[6]. The estimated specific growth rate evolution Figure 7 is in good agreement with the simulated one in spite of the kinetic parameters

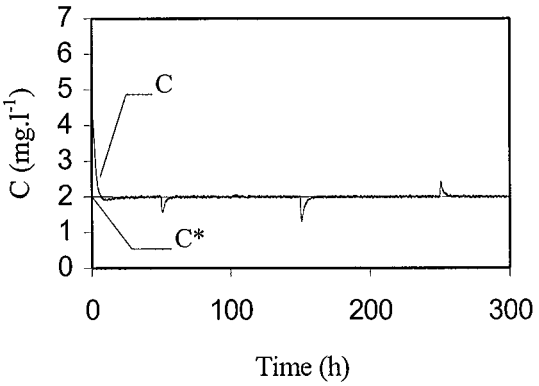

Figure 4. Behaviour of the dissolved oxygen concentration

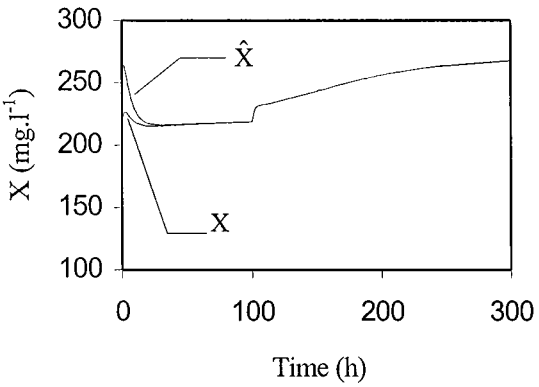

Figure 5. Process and estimate of the biomass concentration

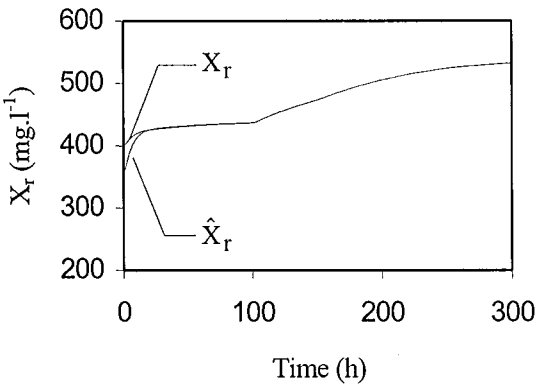

Figure 6. Process and estimate of the recycled biomass concentration

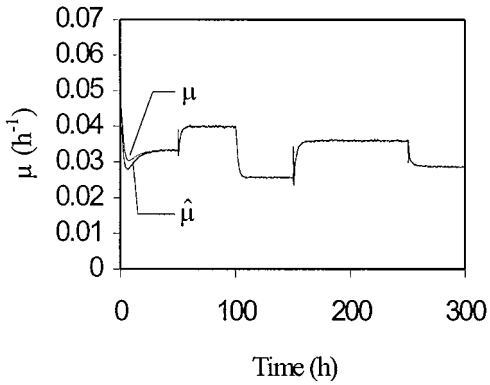

Figure 7. Process and estimate of the specific growth rate

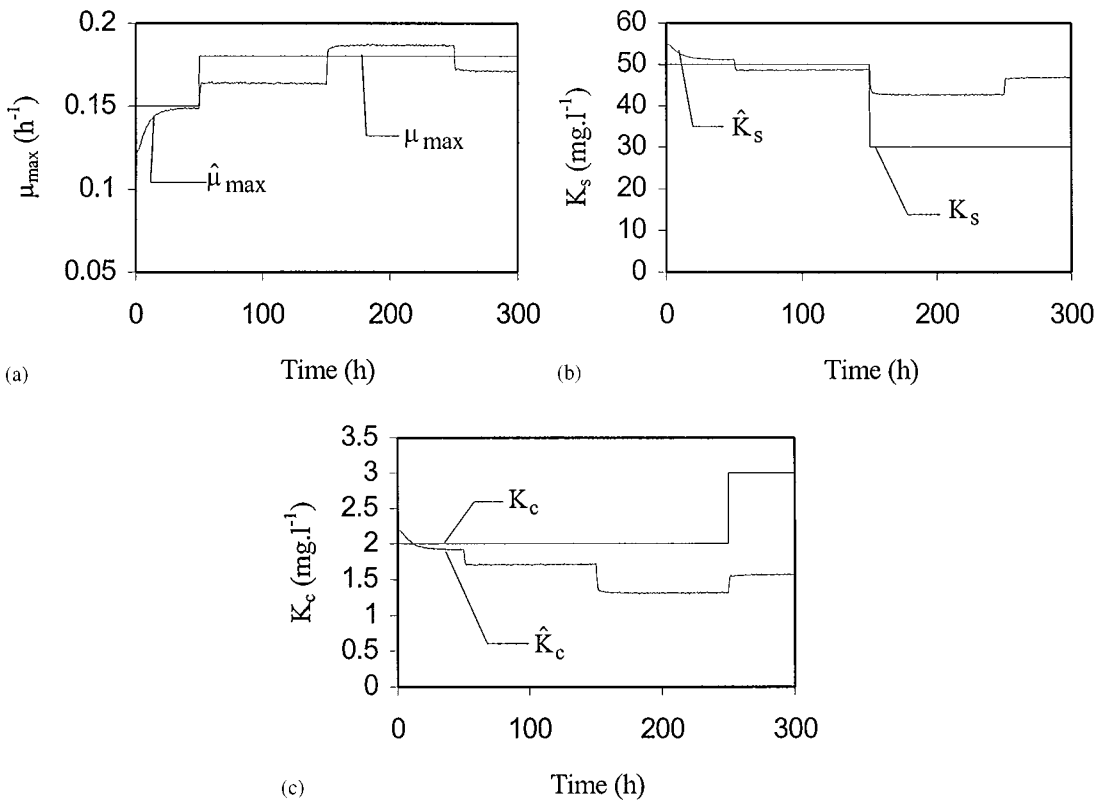

Figure 8. Process and estimate of the: (a) maximum specific growth rate; (b) affinity constant; (c) saturation constant

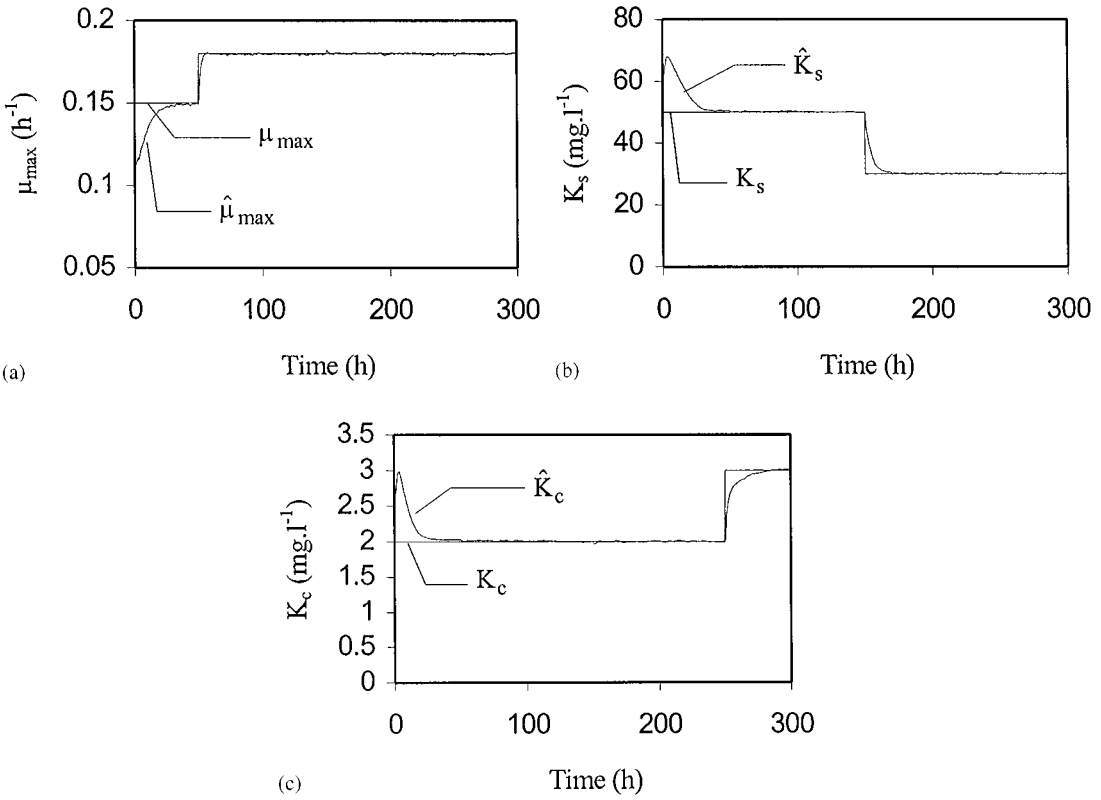

Figure 9. Process and estimate of: (a) μ_{max} when K_s and K_c are known; (b) K_s when μ_{max} and K_c are known; (c) when μ_{max} and k_s are known

convergence to biased values (Figures 8(a)–8(c)). This is due to non-uniqueness of the identification solutions of Olsson model non-linearities or, in other words, to the insufficient persistent excitation of matrix B^* of the error system[2] (equation (9)). But as it is shown in Figures 8(a)–8(c) if two of the parameters are known and applied directly in the state estimation scheme, for example, the affinity constant and the saturation constant, the third parameter, the maximum specific growth rate can be tracked to the real value. So if only one of the kinetic parameters of the bioprocess model is unknown, there exists a unique identification solution and convergence of the estimate to the correct value is ensured. This can provide real-time information on the culture physiology.

Robustness of the estimator is demonstrated by efficiency of rejecting abrupt variations of the kinetic parameters and tracking setpoint changes. Also, a structural mismatch of the specific growth rate has no effect on the technique developed. To demonstrate that, we considered that the structure of the specific growth rate of the plant μ_p follow the Haldane model and is written under the following expression:

$$\mu_p(t) = \mu_{max} \frac{S(t)}{K_s + S(t) + K_I/S(t)^2}$$

with $\mu_{max} = 0.15\ h^{-1}$, $K_s =$ $mg\,l^{-1}$ and $K_I = 100\ mg\,l^{-1}$.

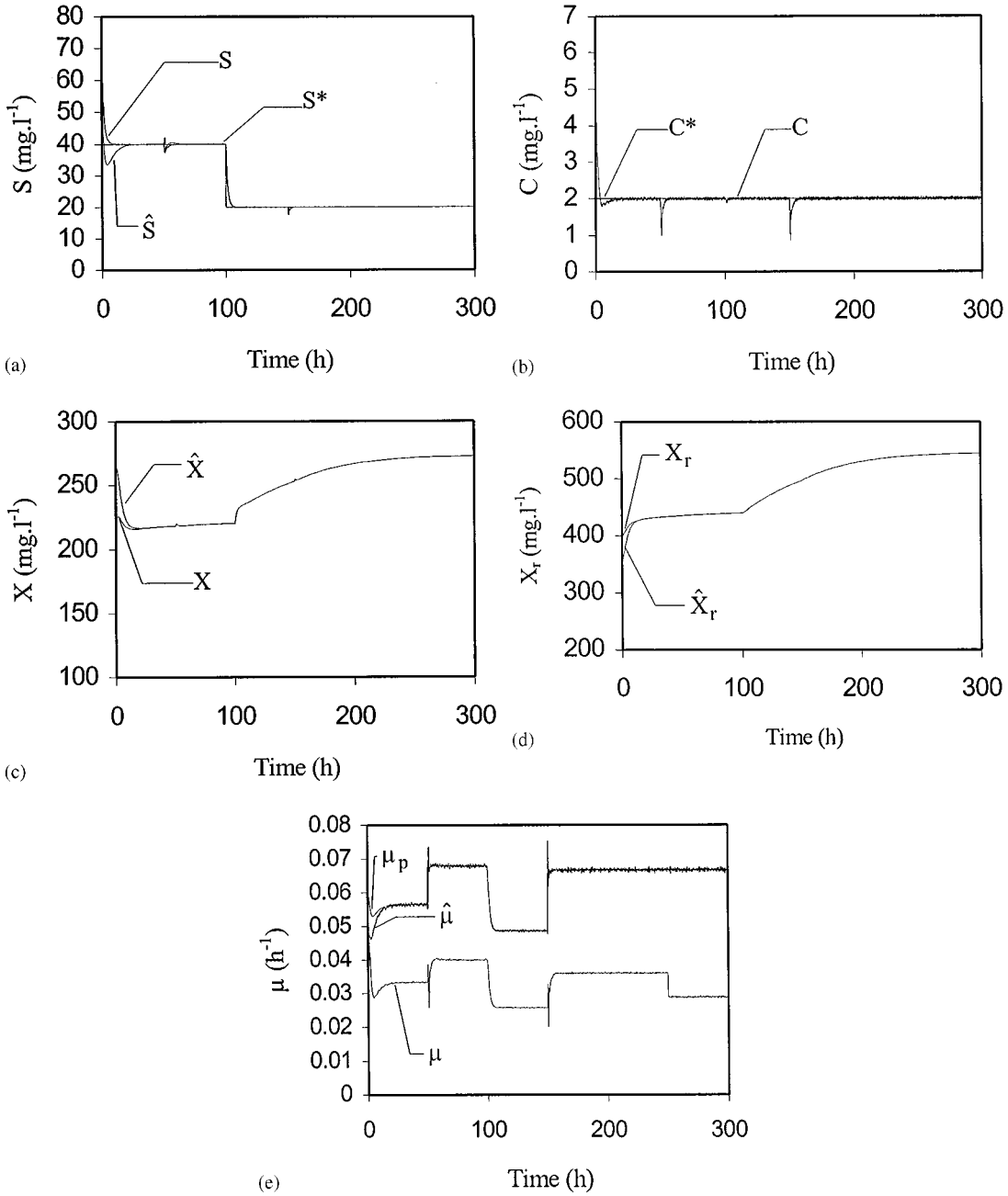

Figure 10. Evolution of the: (a) substrate concentration in case of Haldane model; (b) oxygen concentration in case of Haldane model; (c) biomass concentration in case of Haldane model; (d) recycled biomass concentration in case of Haldane model; (e) specific growth rate in case of Haldane model

Simulation results given in Figures 10(a)–10(e) show that this mismatch does not influence the efficiency of the proposed procedure. One can see clearly (Figure 10(e)) that the estimated specific growth rate tracks suitably the real value of the specific growth of the plant and not the modelled one. So although one can be mistaken in the structure of the kinetic parameter, this does not influence the results and the efficiency of the estimation procedure.

6. CONCLUSION

The aim of this paper was to present a design of an adaptive non-linear multivariable controller algorithm for an activated sludge process.

The controller is based on direct exploitation of the non-linear wastewater treatment dynamical model, obtained from mass balance equations, and is coupled with a joint observer estimator for on-line tracking of the unavailable states and parameters. The study starts with the assumption that the dissolved oxygen concentration is the only measurable state variable of the process.

The adaptive linearizing regulator is used to maintain the pollutant substrate and the dissolved oxygen concentrations at prespecified levels by acting on the dilution rate and the air flow rate, respectively.

Our attention has been to present an approach for the design of a controller based on dynamical mass balance equations of the bioprocess while including the on-line estimation of the unmeasured state variables (the biomass, the substrate and the recycled biomass concentrations) and the uncertain parameter (the specific growth rate).

The effectiveness and robustness of both controller and estimator schemes have been illustrated by simulation studies. They were able to reject disturbances (measurement noises and abrupt jumps of the kinetic parameters), track setpoints step changes and ensure convergence with relatively short transient responses. It has also been shown that a structural mismatch of the kinetic parameter, the specific growth rate, has no effect on the proposed techniques. Stability of the estimation scheme is proved using Lyapunov method.

As a concluding remark, it can be stated that the application of estimators such as 'intelligent sensors' to identify important biological variables and parameters with physical meaning would be able to constitute an interesting alternative to the lack of sophisticated instrumentation and would provide real-time information on the culture physiology appreciated by the process users.

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