Materia:

Calculo Integral

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$a=0 \quad b=4 \quad c=4 \quad d=3$

intervalo $[1,3]$

1) $f(x) = 4x^2 + 5x + 4$

$$A = \int_1^3 4x^2+5x+4\,dx = 4\left(\frac{x^3}{3}\Big]_1^3\right) + 5\left(\frac{x^2}{2}\Big]_1^3\right) + 4x\Big]_1^3$$

$$= 4\left(\frac{3^3}{3} - \frac{1^3}{3}\right) + 5\left(\frac{3^2}{2} - \frac{1^2}{2}\right) - (4\cdot 3) - 4\cdot 1 = 62.666$$

2) $f(x) = \sqrt{16x^2 + 64}$

$$A = \int_1^3 \sqrt{16x^2+64}\,dx = \int_1^3 (16x^2+64)^{1/2}\,dx$$

$$= \int_1^3 \sqrt{16(2\tan^2(x)) + 64(2\sec^2(x))}\,dx$$

$$= 16\left(\frac{\tan x \sec x}{2}\Big]_1^3 + \frac{1}{2}\ln(\sec(x) + \tan(x))\Big]_1^3\right)$$

$$= 16\left(\frac{\tan(3-1)\sec(3-1)}{2} + \frac{1}{2}\ln(\sec(3-1) + \tan(3-1)\right)$$

$$= 22.869$$

$a=0, b=4, c=4, d=3$

3) $f(x) = a+b+c+d+1-x^2, g(x)=d$

$f(x) = 0+4+4+3+1-x^2, g(x)=3$

$f(x) = 12-x^2, g(x)=3$

$f(x)=g(x)$

$12-x^2=3$

$12-3=x^2$

$9=x^2$

$\pm\sqrt{9}=\sqrt{x^2}$

$\pm 3 = x$

$$\int_a^b (f(x)-g(x)dx = \int_{-3}^{3} (12-x^2-3)dx = \int_{-3}^{3} 9-x^2 dx =$$

$$9x-\frac{x^3}{3}\Big|_{-3}^{3} = 9(3)-\frac{3^3}{3}-\left(9(-3)-\left(\frac{-3^3}{3}\right)\right) = \underline{36}$$

$a=0, b=4, c=4, d=3$

4) $f(x) = -x^2 + 3x + a + b + c + d + 1, \; g(x) = x - d$

$f(x) = -x^2 + 3x + 0 + 4 + 4 + 3 + 1, \; g(x) = x - 3$

$f(x) = -x^2 + 3x + 12, \; g(x) = x - 3$

$f(x) = g(x)$

$-x^2 + 3x + 12 = x - 3$

$-x^2 + 3x - x + 12 + 3 = 0$

$-x^2 + 2x + 15 = 0$

$(-x-3)(x-5) = 0$

$x - 5 = 0 \Rightarrow x = 5, \; -x - 3 = 0 \Rightarrow x = -3$

$$\text{Área} = \int_a^b (f(x) - g(x))dx = \int_{-3}^{5} (-x^2 + 3x + 12 - x + 3)dx =$$

$$\int_{-3}^{5} (-x^2 + 2x + 15)dx = \frac{-x^3}{3} + \frac{2x^2}{2} + 15x \Big|_{-3}^{5} =$$

$$-\frac{5^3}{3} + \frac{2(5)^2}{2} + 15(5) - \left(-\frac{(-3)^3}{3} + \frac{2(-3)^2}{2} + 15(-3)\right) = 85.333\ldots$$

$a = 0, b = 4, c = 4, d = 3$

$$[d+1, d+3] = [3+1, 3+3]$$
$$[4, 6]$$

5) $f(x) = (a+1)x^2 + (b+1)x + c = (0+1)x^2 + (4+1)x + 4 =$

$f(x) = x^2 + 5x + 4 =$

$$\int_4^6 \sqrt{1+(y')^2}\,dx = \int_4^6 \sqrt{1+\left(\frac{d}{dx}(x^2+5x+4)\right)^2}\,dx =$$

$$\int_4^6 \sqrt{1+(2x+5)^2}\,dx =$$

$$\frac{1}{2}\int_4^6 2\sqrt{1+(2x+5)^2}\,dx = \frac{1}{2}\left(\frac{2x+5}{2}\sqrt{(2x+5)^2+1} + \frac{1}{2}\ln\left|2x+5+\sqrt{(2x+5)^2+1}\right|\right.$$

$$\frac{1}{2}\left(\frac{2x+5}{2}\sqrt{4x^2+20x+26} + \frac{1}{2}\ln\left|2x+5+\sqrt{4x^2+20x+26}\right|\right)\Bigg|_4^6 =$$

30,0669

$a=0,\ b=4,\ c=4,\ \delta=3$

$[4,6]$

6) $f(x)=(a+1)x^{\frac{3}{2}}+b+c=$

$f(x)=x^{\frac{3}{2}}+4+4$

$f(x)=x^{\frac{3}{2}}+8=$

$$\int_4^6 \sqrt{1+\left(\frac{d}{dx}\left(x^{\frac{3}{2}}+8\right)\right)^2}\,dx=\int_4^6\sqrt{1+\left(\frac{3}{2}x^{\frac{1}{2}}\right)^2}\,dx=\int_4^6\sqrt{1+\frac{9}{4}x}\,dx$$

$$\frac{4}{9}\int_4^6\sqrt{1+\frac{9}{4}x}\,dx=\frac{\frac{3}{2}x^{\frac{1}{2}}}{2}\sqrt{\frac{9}{4}x+1}+\frac{1}{2}\ln\left|\frac{3}{2}x^{\frac{1}{2}}+\sqrt{\frac{9}{4}x+1}\right|\Bigg|_4^6=$$

$$\frac{4}{9}\left(\frac{\frac{3}{2}(6)^{\frac{1}{2}}}{2}\sqrt{\frac{9}{4}(6)+1}+\frac{1}{2}\ln\left|\frac{3}{2}(6)^{\frac{1}{2}}+\sqrt{\frac{9}{4}(6)+1}\right|-\left(\frac{\frac{3}{2}(4)^{\frac{1}{2}}}{2}\right.\right.\ldots$$

$$\left.\left.\sqrt{\frac{9}{4}(4)+1}+\frac{1}{2}\ln\left|\frac{3}{2}(4)^{\frac{1}{2}}+\sqrt{\frac{9}{4}(4)+1}\right|\right)\right)=6.99009$$

$a=0 \quad b=4 \quad c=4 \quad d=3 \quad [4,6]$

7) $f(x)=(a+1)x^2+(b+1)x+c = (0+1)x^2+(4+1)x+4$

$f(x) = x^2+5x+4$

$$\pi\int_4^6 f^2(x)\,dx = \pi\int_4^6 (x^2+5x+4)^2\,dx$$

$$=\pi\int_4^6 x^4+10x^3+33x^2+40x+16\,dx$$

$$\frac{x^5}{5}+\frac{10x^4}{4}+\frac{33x^3}{3}+\frac{40x^2}{2}+16x\Big|_4^6 =$$

$$\frac{x^5}{5}+\frac{5x^4}{2}+11x^3+20x^2+16x\Big|_4^6 =$$

$$\frac{6^5}{5}+\frac{5(6)^4}{2}+11(6)^3+20(6)^2+16(6) -$$

$$\left(\frac{4^5}{5}+\frac{5(4)^4}{2}+11(4)^3+20(4)^2+16(4)\right) = 19{,}020.458$$

$a=0,\ b=4,\ c=4,\ d=3$

$[4,6]$

8) $f(x) = (a+1)\,sen([b+1]x) + c = sen(5x)+4$

$f(x) = sen(5x)+4$

$\pi\int_4^6 f^2(x)\,dx = \pi\int_4^6 (sen(5x)+4)^2 dx = \pi\int_4^6 sen^2(5x) + 8sen(5x) + 16\,dx =$

$8\pi\int_4^6 \frac{1}{2} - \frac{1}{2}\cos(10x) + 8sen(5x) + 16\,dx =$

$\frac{\pi}{2}\int_4^6 dx - \frac{1}{2}\int_4^6 \cos(10x)\,dx + 8\pi\int_4^6 sen(5x)\,dx + \pi\int_4^6 16\,dx =$

$\frac{\pi x}{2}\Big|_4^6 - \frac{1}{20} sen(10x)\Big|_4^6 + \left(-\frac{1}{5} - \cos(5x)\right)\Big|_4^6 + 16\pi x\Big|_4^6 = 105.113$

$a=0 \quad b=4 \quad c=4 \quad d=3$

V. Calcula el área del sólido de revolución en el intervalo $[0, a+1]$ $[0,1]$

9) $f(x) = \overset{x+8}{x+4+4}$, Área$=$ $\underline{\underline{75.5284}}$//

$f(x) = x+8 \ [0,1]$, $\frac{d}{dx}(x+8) = 1$

$$A = 2\pi \int_a^b f(x)\sqrt{1+\left(\frac{d}{dx}f(x)\right)^2}\,dx$$

$$= 2\pi \int_0^1 (x+8)\sqrt{1+1^2}\,dx$$

$$= 2\pi\sqrt{2}\int_0^1 (x+8)\,dx = 2\pi\sqrt{2}\left(\frac{x^2}{2}+8x\right)\Big|_0^1$$

$$= 2\pi\sqrt{2}\left(\frac{1^2}{2}+8(1)-\left(\frac{0^2}{2}+8(0)\right)\right) = 75.5284$$

10) $f(x) = \frac{5x^2}{8}$, Área $= \underline{1.8187}$

$f(x) = \frac{5}{8}x^2 \ [0,1]$, $\frac{d}{dx}\left(\frac{5}{8}x^2\right) = \frac{5}{4}x$

$A = 2\pi\int_a^b f(x)\sqrt{1+\left(\frac{d}{dx}f(x)\right)^2}\,dx = 2\pi\int_0^1 \frac{5}{8}x^2\sqrt{1+\left(\frac{5}{4}x\right)^2}\,dx$

Área bajo la curva $= \int_a^b f(x)\,dx \approx \frac{h}{3}\left(f(a) + 4f(\bar{x}) + f(b)\right)$

$= 2\pi\int_0^1 \frac{5}{8}x^2\sqrt{1+\left(\frac{5}{4}x\right)^2}\,dx \approx 2\pi\,\frac{0.5}{3}\left(0 + 4(0.1841) + 1.0004\right)$

$= 1.8187$

$h = \frac{b-a}{2} = \frac{1-0}{2} = \frac{1}{2}$ ó 0.5

$\bar{x} = \frac{a+b}{2} = \frac{0+1}{2} = \frac{1}{2}$ ó 0.5

$f(x) = \frac{5}{8}x^2\sqrt{1+\left(\frac{5}{4}x\right)^2}$

$f(a) = f(0) = \frac{5}{8}(0)^2\sqrt{1+\left(\frac{5}{4}(0)\right)^2} = 0$

$f(\bar{x}) = f(0.5) = \frac{5}{8}(0.5)^2\sqrt{1+\left(\frac{5}{4}(0.5)\right)^2} = 0.1841$

$f(b) = f(1) = \frac{5}{8}(1)^2\sqrt{1+\left(\frac{5}{4}(1)\right)^2} = 1.0004$