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Tarea $B = \{(1,1,1), (1,-1,0), (1,0,-1)\}$

Calcular las diferentes bases del vector

$e_1 = (1,1,1)$

$e_2 = (1,-1,0) - \frac{\langle U_2 \cdot e_1 \rangle}{\|e_1\|^2} e_1$

$e_3 = (1,0,-1) - \frac{\langle U_3 \cdot e_1 \rangle}{\|e_1\|^2} e_1 - \frac{\langle U_3 \cdot e_2 \rangle}{\|e_2\|^2} e_2$

$U_n = \frac{2\mathcal{S}_n}{\|2\omega_n\|^2}$

① Calcular U_1 $\quad U_1 = \frac{e_1}{\|e_1\|^2}$

$U_1 = \frac{(1,1,1)}{\|e_1\|^2} = \frac{1,1,1}{\sqrt{3}} = \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \therefore U_1 = \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

$\|e_1\|^2 = \|1,1,1\|^2 = \sqrt{(1)^2+(1)^2+(1)^2} = \sqrt{1+1+1} = \sqrt{3} \therefore$

$\|e_1\|^2 = \sqrt{3}$

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② U_2 Aplicar vector auxiliar $U_2 = \frac{w_2}{\|w_2\|^2}$

$w_2 = e_2 - [(e_2 \cdot U_1)] U_1$

$w_2 = (1,-1,0) - [(1,-1,0)\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)] \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

$w_2 = (1,-1,0) - \left[\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right] \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

$w_2 = (1,-1,0) - (0) \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

$w_2 = (1,-1,0) - (0,0,0)$

$w_2 = (1,-1,0)$

$\|w_2\|^2 = \|1,-1,0\| = \sqrt{(1)^2+(-1)^2+(0)^2} = \sqrt{1+1+0} = \sqrt{2}$ ∴

$\|w_2\|^2 = \sqrt{2}$

$U_2 = \frac{w_2}{\|w_2\|^2} = \frac{1,-1,0}{\|w_2\|^2} = \frac{1,-1,0}{\sqrt{2}} = \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, \frac{0}{\sqrt{2}}$ ∴

$U_2 = \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0$

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③ $U_3 = \frac{w_3}{\|w_3\|^2}$ Aplicar vector auxiliar

$w_3 = e_3 - [(e_3 \cdot U_1)]U_1 - [e_3 \cdot U_2]\,U_2$

$w_3 = (1,0,-1) - \left[(1,0,-1)\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)\right] - \left[(1,0,-1)\left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0\right)\right]\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0$

$w_3 = (1,0,-1) - \left[\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right]\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} - \left[(1,0,-1)\left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0\right)\right]\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0$

$w_3 = (1,0,-1) - [0]\,\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} - \left[(1,0,-1)\left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0\right)\right]\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0$

$w_3 = (1,0,-1) - \left[(1,0,-1)\left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0\right)\right]\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0$

$w_3 = (1,0,-1) - \left[\frac{1}{\sqrt{2}}\right]\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0$

$w_3 = (1,0,-1) - \left(\frac{1}{2}, \frac{-1}{2}, 0\right)$

$w_3 = \frac{1}{2}, \frac{1}{2}, -1$

$\|w_3\|^2 = \left\|\frac{1}{2}, \frac{1}{2}, -1\right\| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + (-1)^2} = \sqrt{\frac{1}{4} + \frac{1}{4} + 1} = \sqrt{\frac{6}{4}} = \sqrt{\frac{3}{2}}$

$\|w_3\|^2 = \sqrt{\frac{3}{2}}$ $U_3 = \frac{w_3}{\|w_3\|^2} = \frac{1/2, \frac{1}{2}, -1}{\sqrt{3/2}} = \frac{1}{2\sqrt{\frac{3}{2}}}, \frac{1}{2\sqrt{\frac{3}{2}}}, \frac{-1}{\sqrt{\frac{3}{2}}}$

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Por lo tanto, la base del espacio vectorial obtenida seran:

$U_1 = \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

$U_2 = \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0$

$U_3 = \frac{1}{2\sqrt{\frac{3}{2}}}, \frac{1}{2\sqrt{\frac{3}{2}}}, \frac{-1}{\sqrt{\frac{3}{2}}}$