CAP239 MFDFA

Detrended Fluctuation Analysis

Features

- proven method to detect long-range correlation in noisy, nonstationary and highly heterogeneous data [Peng et al., 1994]
- handle short length data [Coronado & Carpena, 2005]
- Identify correlation in discontinous data [Chen et al., 2002]

Computational Steps

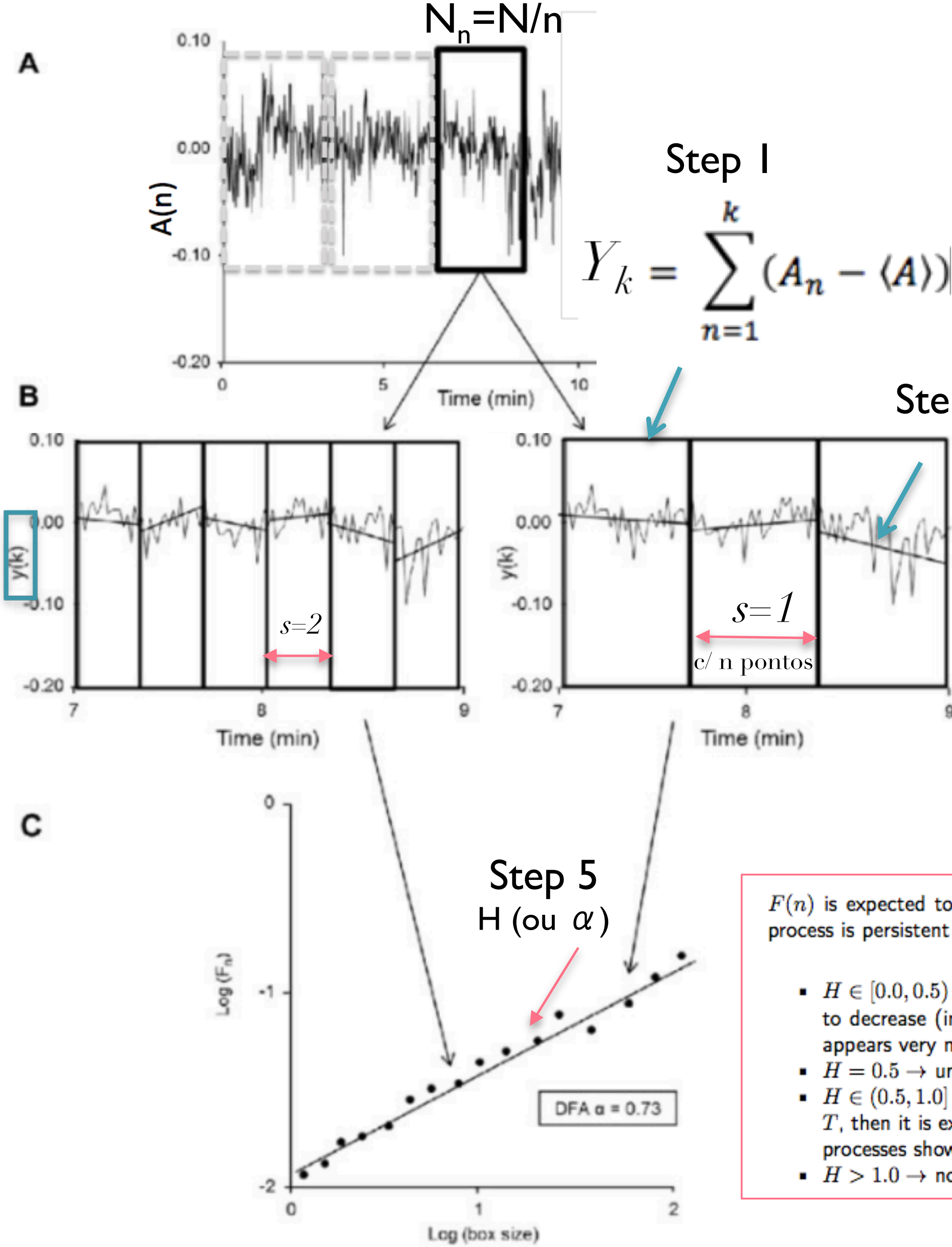
1. Obtain profile using cumulative sum

$$Y(i) \equiv \sum x_k - \bar{x}$$

2. Divide profile into various sized scales
3. Detrend all segments
4. Obtain Fluctuation profile $F(s)$ by computing local RMS variation

$$F(s) = \left(\frac{1}{s} \sum_{i=1}^{s} Y_i - y_{fit} \right)^{1/2}$$

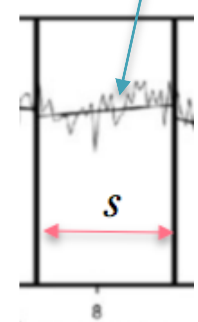
5. Linear fit to $F(s)$ on log-log scale yields exponent α



Step 1

$$Y_k = \sum_{n=1}^{k} (A_n - \langle A \rangle)$$

Polynomial of order q



Step 2

$Y_{k,q,s}$

Step 3

$$F^2(n,s) = \frac{1}{n} \sum_{i=1}^{n} \left(Y_k - Y_{k,q,s}\right)$$

Step 4

$$F(n) = \left[\frac{1}{2N_n} \sum_{s=1}^{2N_n} F^2(n,s)\right]^{1/2}$$

Step 5

H (ou α)

$F(n)$ is expected to scale as n^H. H is the Hurst exponent, and its value can indicate if a process is persistent or anti-persistent:

- $H \in [0.0, 0.5) \rightarrow$ anti-persistency. The process under study is anti-persistent and tends to decrease (increase) after a previous increase (decrease). An anti-persistent process appears very noisy;
- $H = 0.5 \rightarrow$ uncorrelated process;
- $H \in (0.5, 1.0] \rightarrow$ persistency. If a process has been increasing (decreasing) for a period T, then it is expected to continue to increase (decrease) for a similar period. Persistent processes show long-range correlations and exhibit relatively little noise;
- $H > 1.0 \rightarrow$ nonstationary process, stronger long-range correlations are present.

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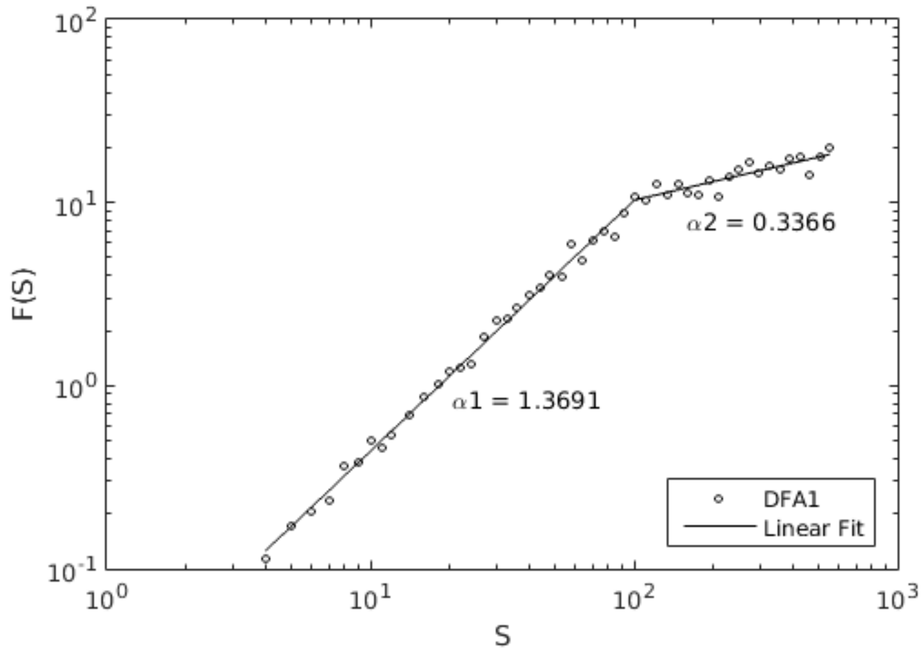
Detrended fluctuation analysis



- $\alpha = 0.5$
  uncorrelated data
- $0 < \alpha < 0.5$
  anti-correlated data
- $0.5 < \alpha$
  correlated data

Crossover (why???)
    change of correlation properties over two different scales

Intrinsic crossover
    Apply different detrending order
    For each order
        α should differ with crossover at different scale

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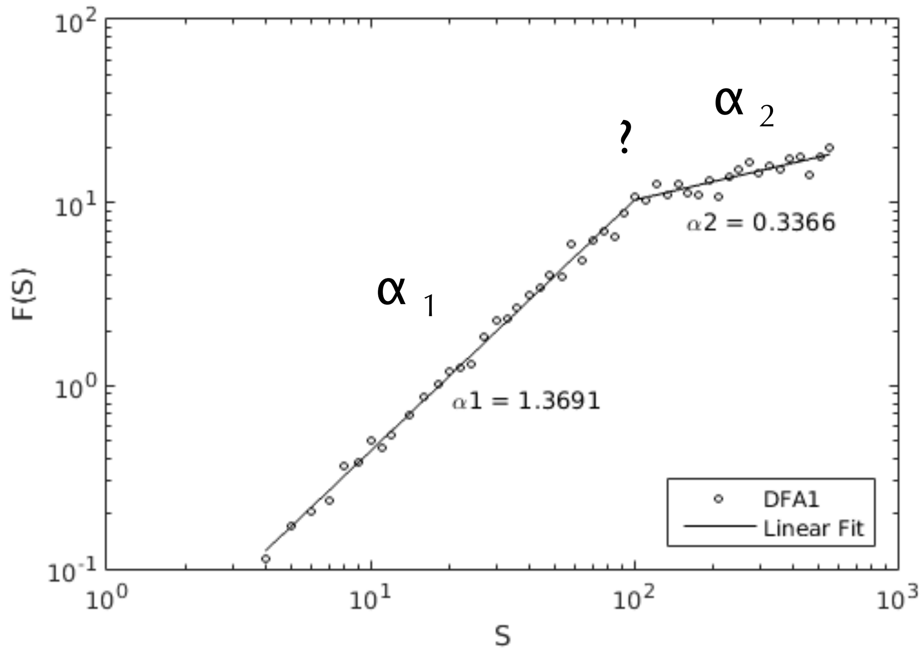
Detrended fluctuation analysis



- α = 0.5
  uncorrelated data
- 0 < α < 0.5
  anti-correlated data
- 0.5 < α
  correlated data

α = f(h) ??? ➔ but H=f(D)
Then ➔ more than one attractor?

Therefore must exist a structure function τ(s,q, α(h)) !!!!

Mono to Multifractal?

- change of correlation properties over various scales
- superposition of monofractals describe a multitude of scaling exponents
- Multifractal DFA [Kantelhardt et al., 2002] characterize multiple scaling behavior in data.

Multifractal DFA

Mono to Multi fractal

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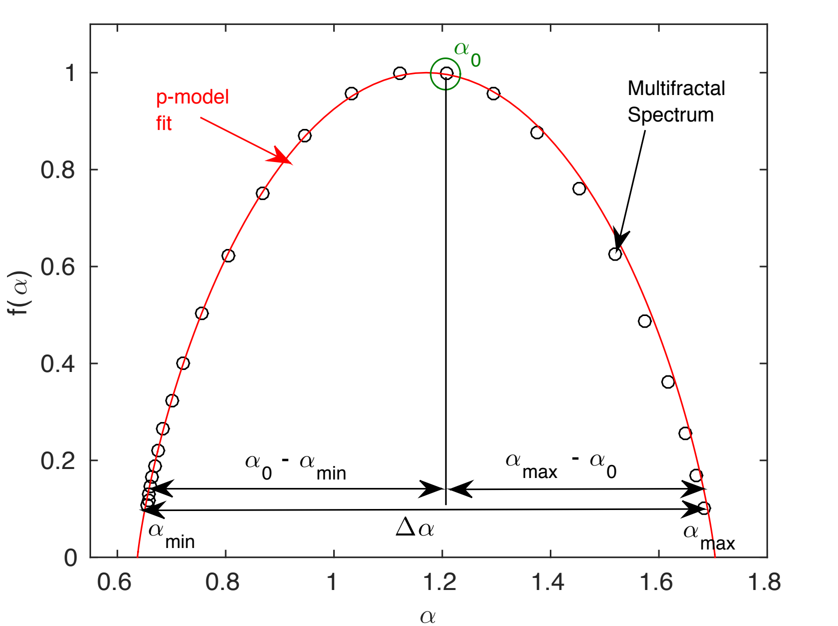
Computational Steps



ı variance over

ıe as DFA)

fluctuation function

$(s)]^{\frac{q}{2}}\Big)^{\frac{1}{q}}$

log scale yields the

$h(q)$

$\tau(q) = qh(q) - 1$

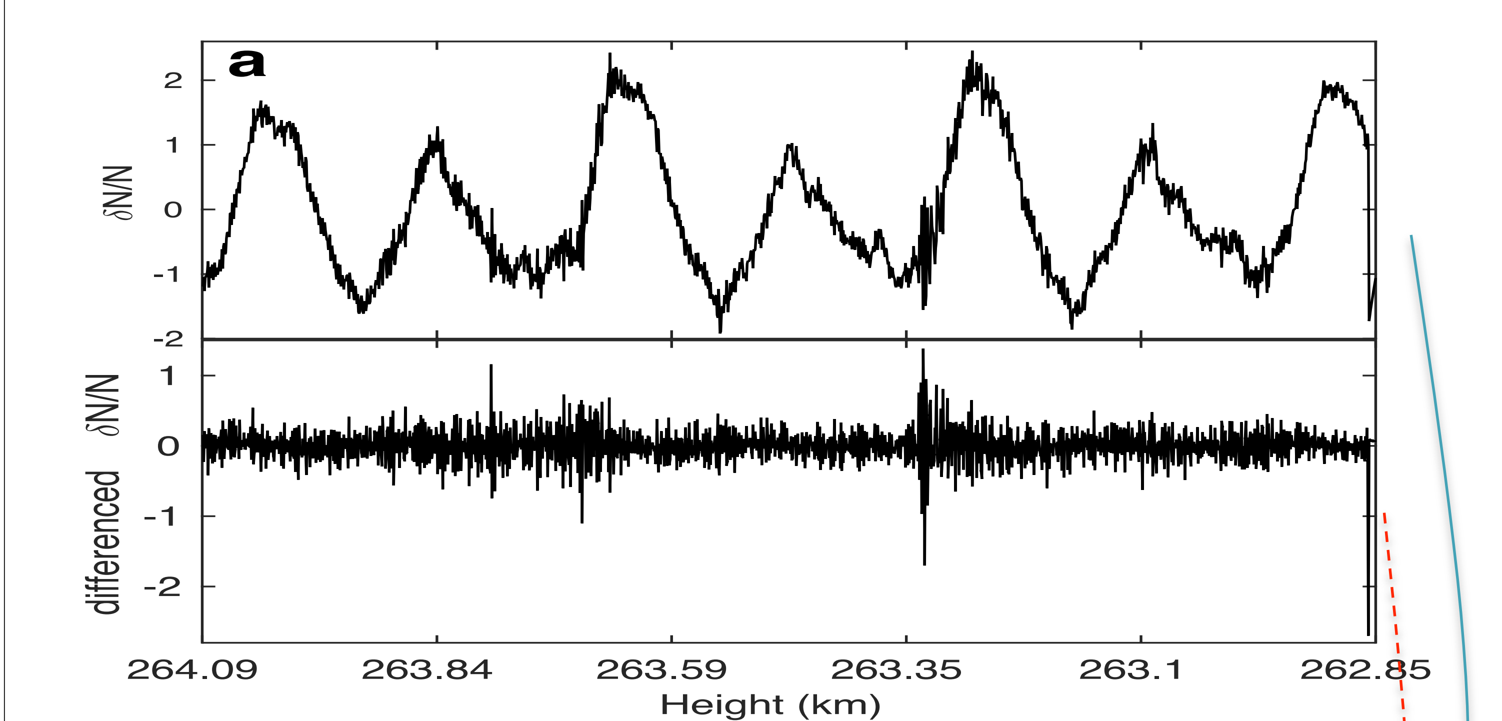
ponent $\tau(q)$

and multifractal spectrum α and $f(\alpha)$

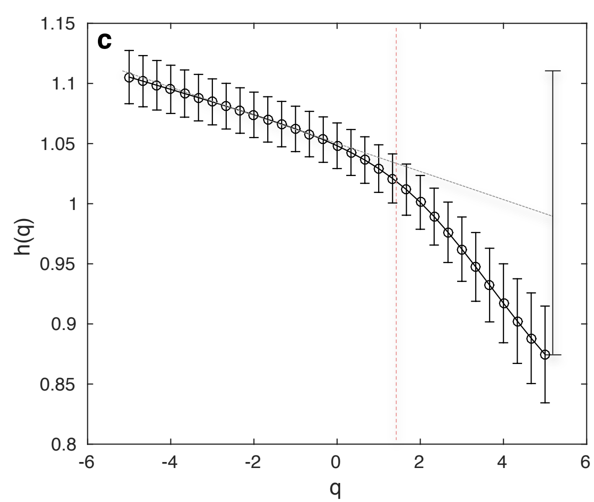
$\alpha = h(q) + qh'(q)$

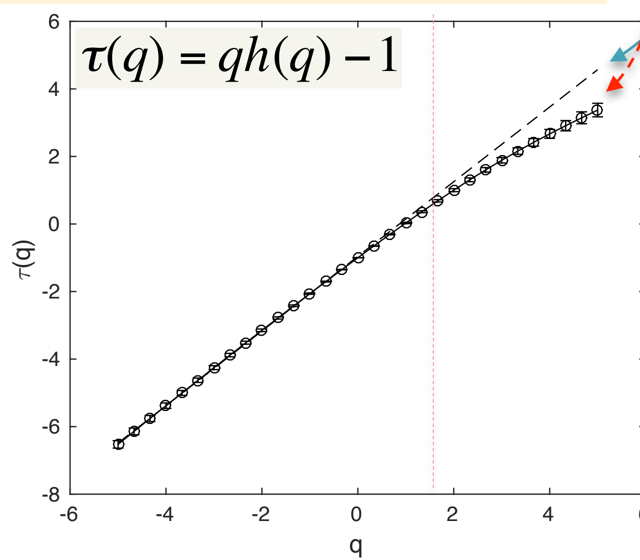
$f(\alpha) = q[\alpha - h(q)] + 1$

Multifractal DFA



The multifractal signature is inside the antipersistent component because different attractors are defined by the different families of amplitudes (structure function) which are frequency independent (hilen, 2012, Frontiers in Physiology 3:141)

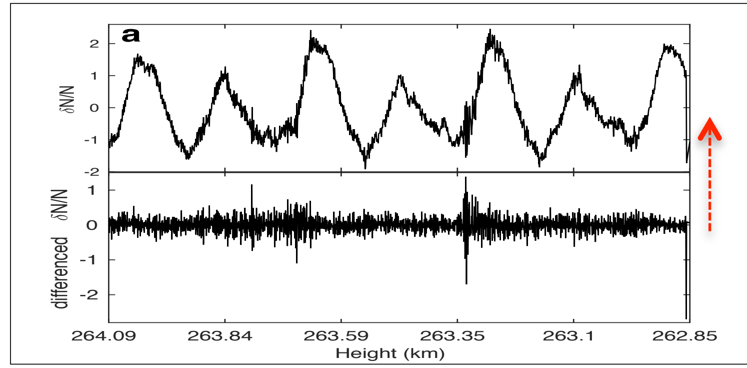


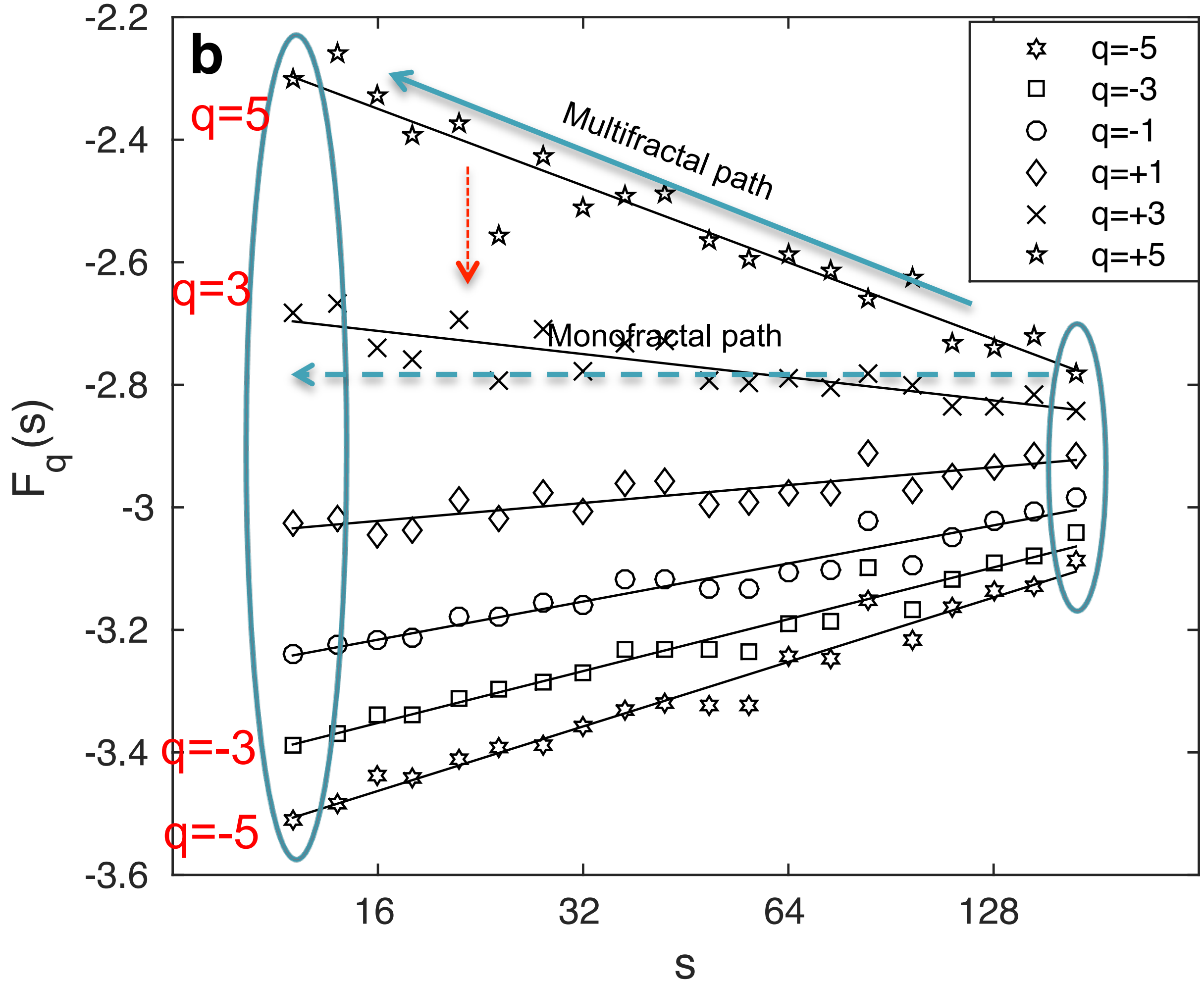


Multifractal DFA

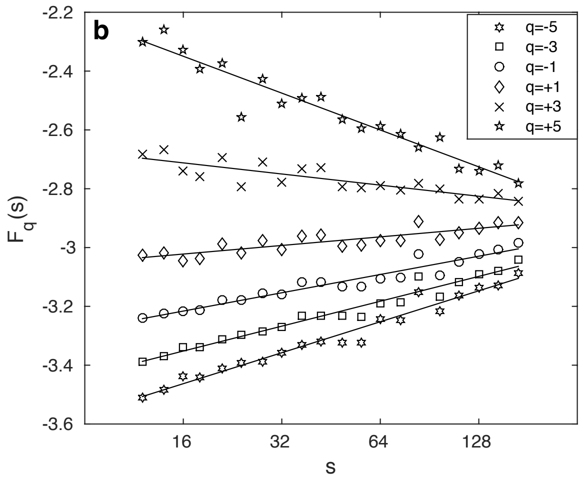
Understanding analysis

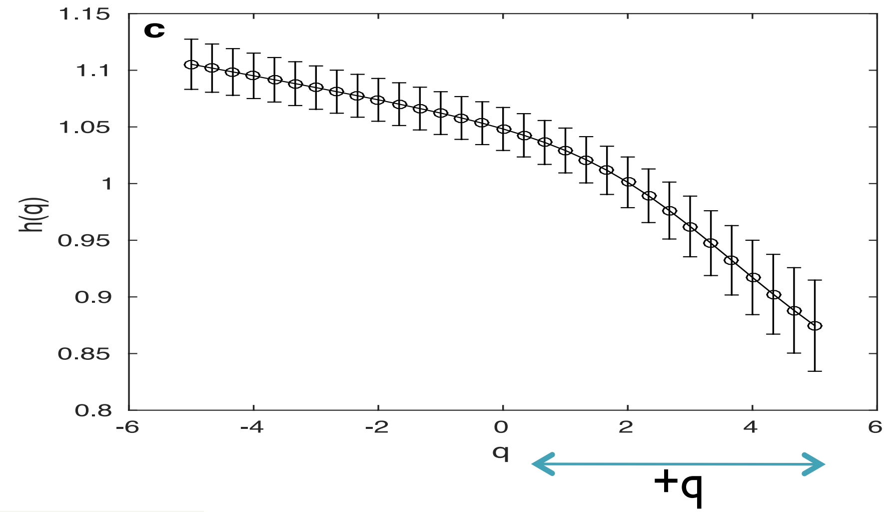
Fluctuation function



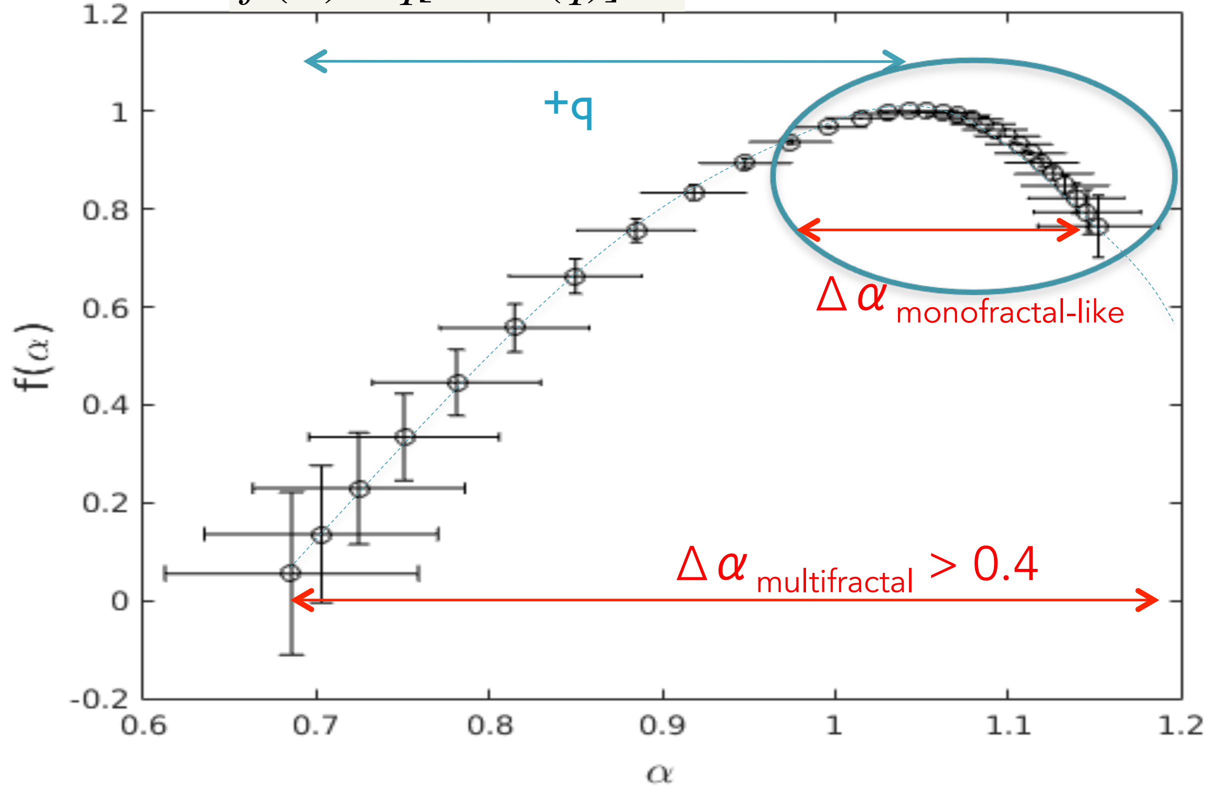


Multifractal DFA





$$f(\alpha) = q[\alpha - h(q)] + 1$$



Methods

Multifractal DFA

Multifractal measures

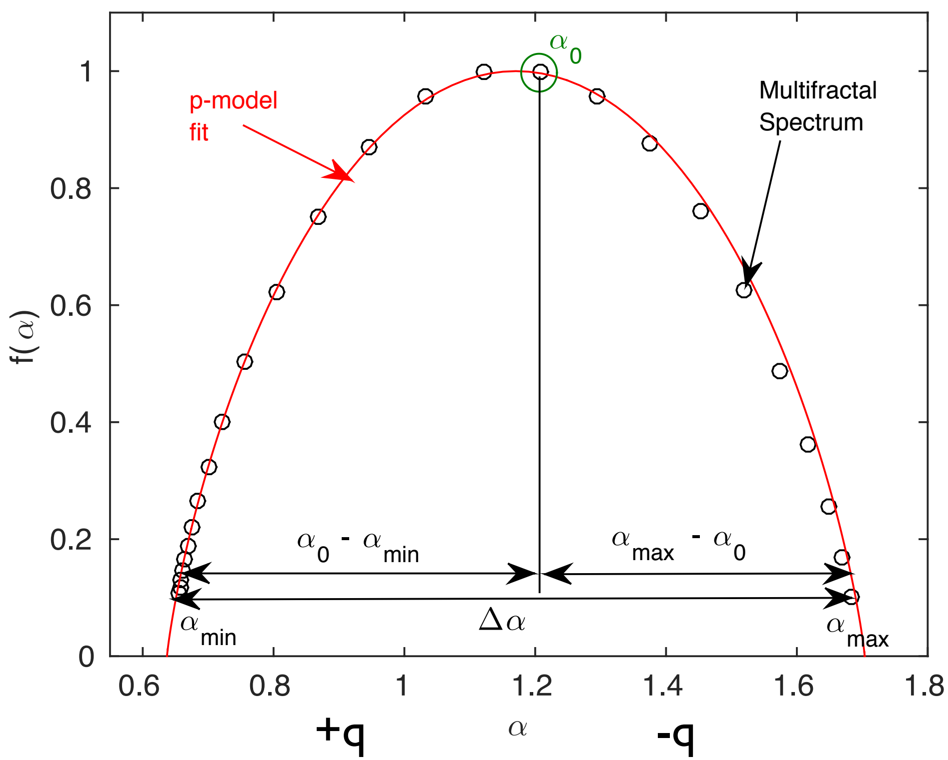
Degree of multifractality

$$\Delta\alpha = \alpha_{max} - \alpha_{min}$$

Higher the $\Delta\,\alpha$, stronger the multifractality

Measure of Asymmetry

$$A = \frac{(\alpha_0 - \alpha_{min})}{(\alpha_{max} - \alpha_0)}$$



$A < 1$; right skewed spectrum → dominance of smaller amplitude fluctuations

$A = 1$; symmetric spectrum

$A > 1$; left skewed spectrum → dominance of larger amplitude fluctuations

Methods

P model

- shows multifractal properties of one-dimensional sections of the dissipation field [Meneveau & Sreenivasan, 1987].
- Analytical formulation to determine the singularity spectrum [Halsey et al., 1986].

$$\alpha = \frac{\ln p_1 + (n/m - 1)\ln p_2}{\ln l_1 + (n/m - 1)\ln l_2}$$

$$f(\alpha) = \frac{(n/m - 1)\ln(n/m - 1) - (n/m)\ln(n/m)}{\ln l_1 + (n/m - 1)\ln l_2}$$

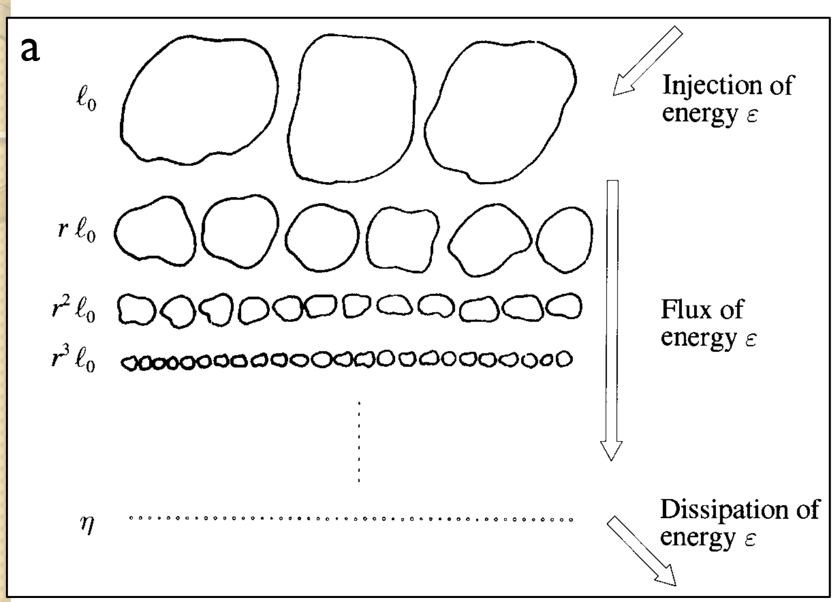
Probability parameters: p_1 and p_2 where $p_1 + p_2 = 1$

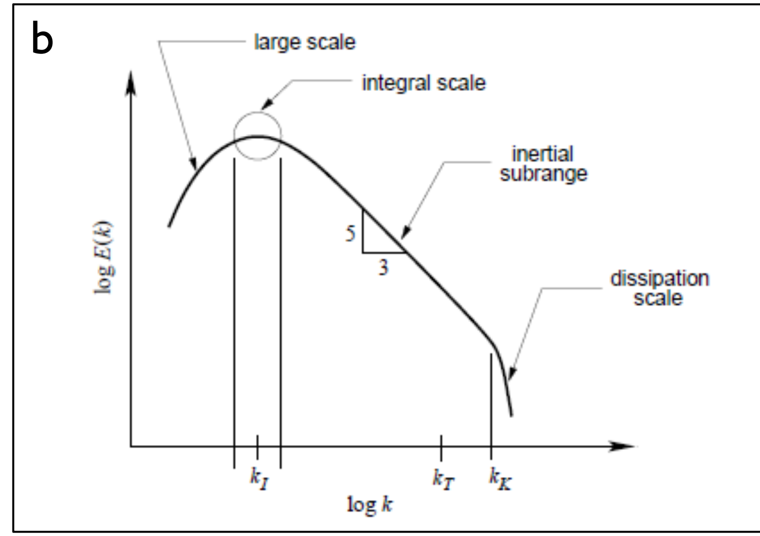
with dissipation parameter dp, $p_1 + p_2 + dp = 1$

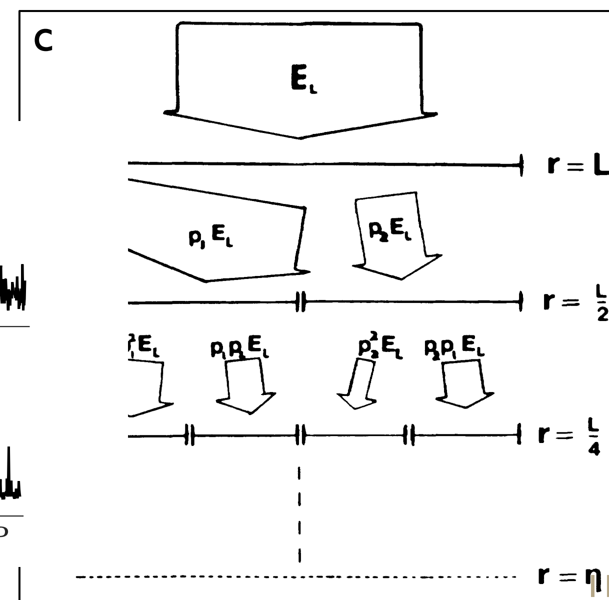
Length scales: l_1 and l_2 where $l_1 = l_2 = 0.5$

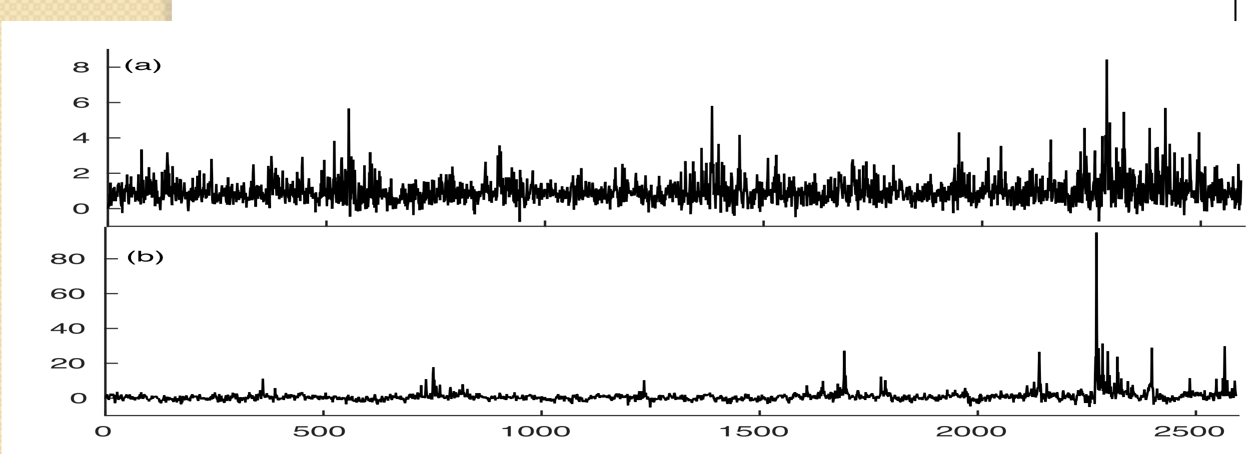
Methods

Multiplicative cascade model



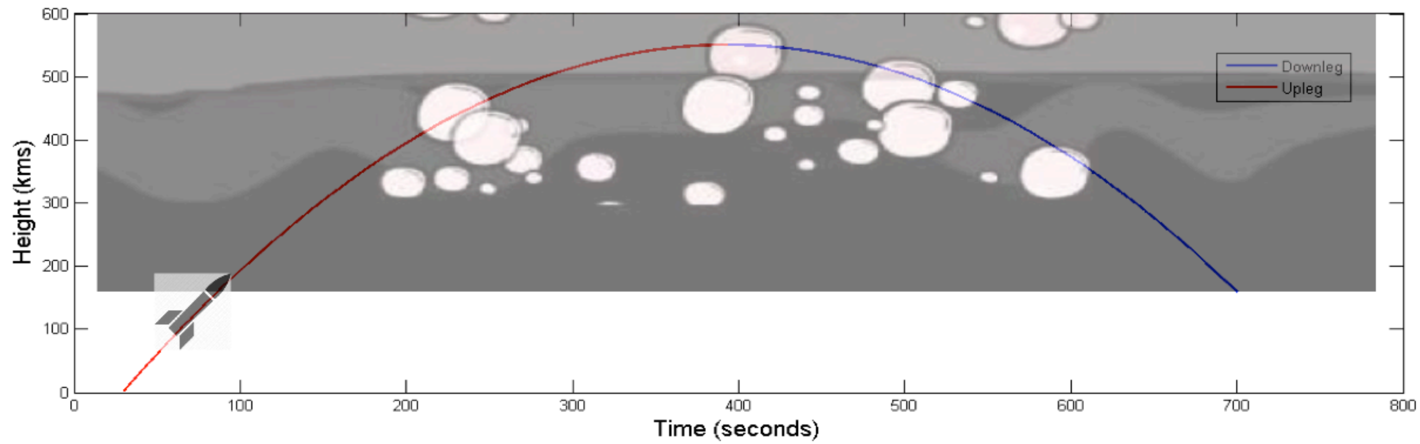






Case study 1

Ionospheric in situ data



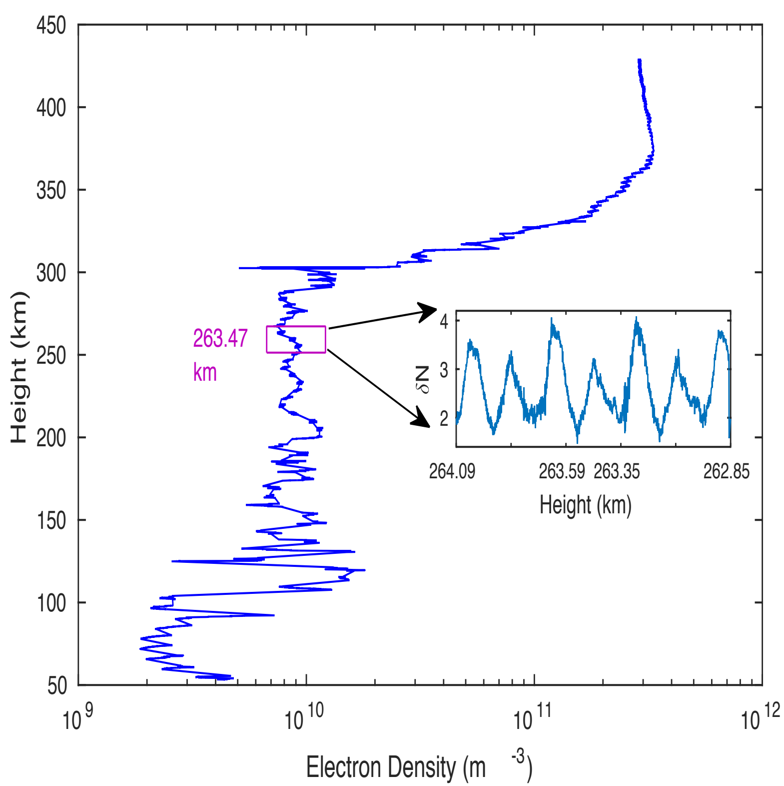
E-F valley region	F region
December 8, 2012 at 19:00 LT	December 18, 1995 at 21:17 LT
Alcântara (2.31° S; 44.4° W)	Alcântara (2.31° S; 44.4° W)
Apogee ~ 428 km; range ~ 384 km	Apogee ~ 557 km; range ~ 589 km
Langmuir probe: *electron density fluctuations*	Electric field probe: *electric field fluctuations*

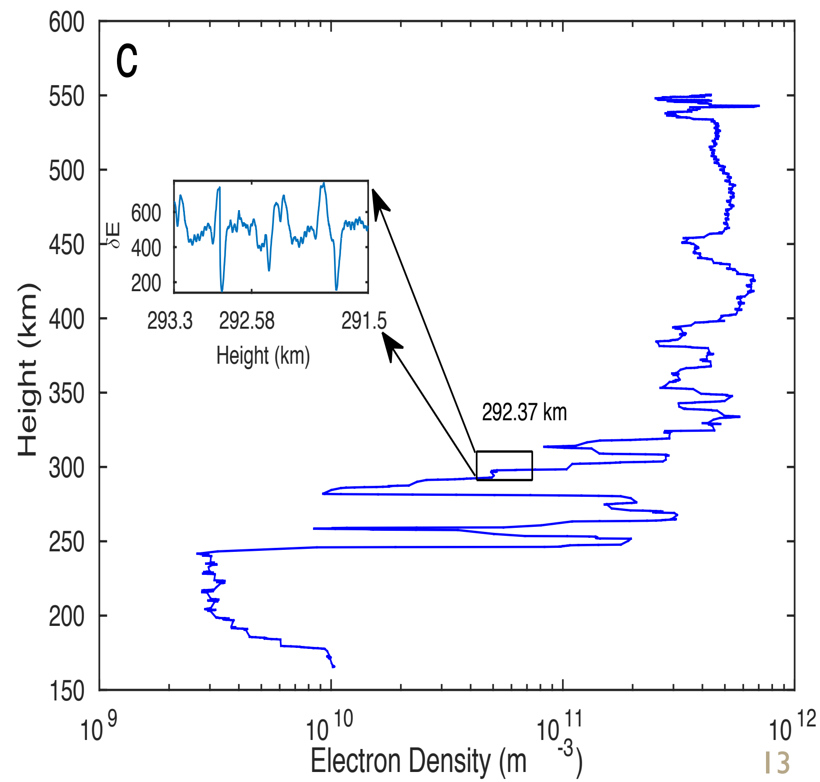
Case study 1

Ionospheric downleg profile

E-F valley region
December 8, 2012 at 19:00 LT
Valley region: 130 – 300 km
Small to medium scale irregularities

F region
December 18, 1995 at 21:17 LT
F region: 250 km onwards
medium to large scale irregularities



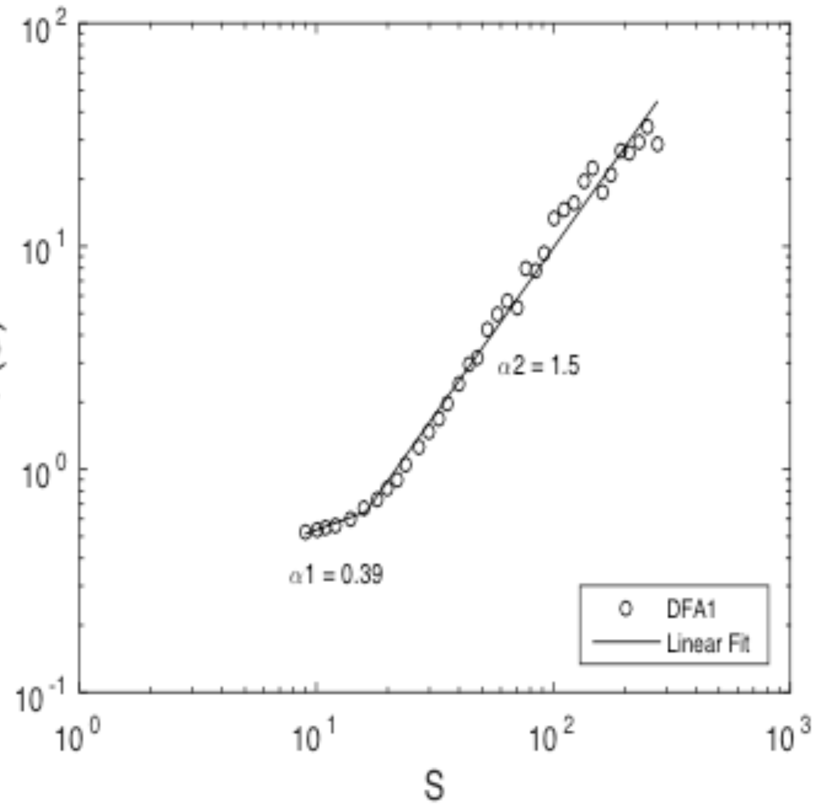


Case study 1

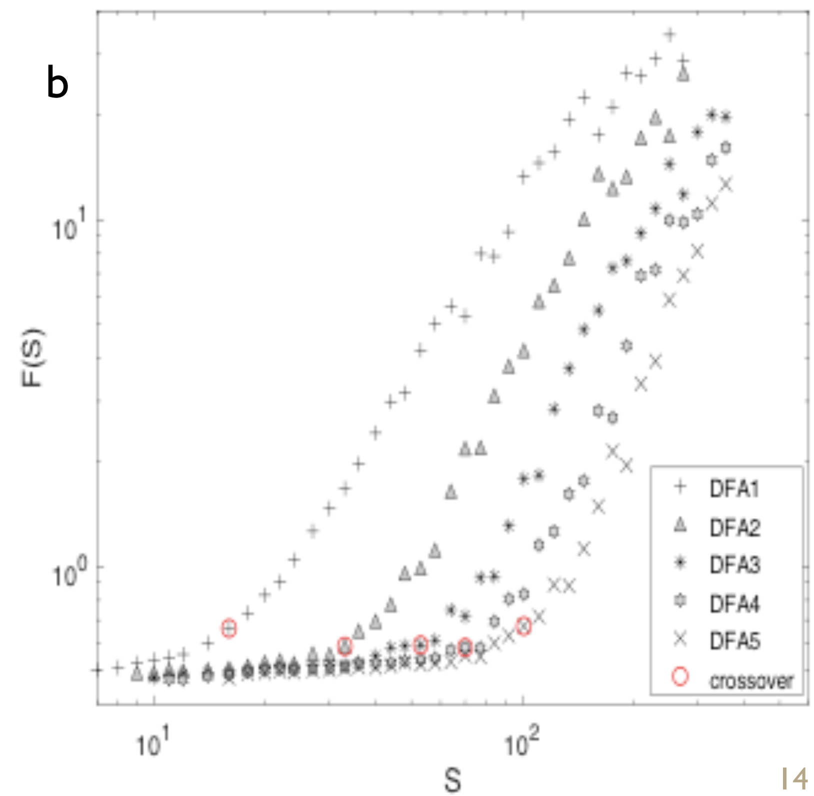
DFA: Valley region

<height> (km)	exponent	DFA1	DFA2	DFA3	DFA4	DFA5
143.03	α 1	0.39	0.15	0.12	0.10	0.13
	α 2	1.50	1.93	2.08	2.26	2.53
	scale	16	33	53	70	101

a



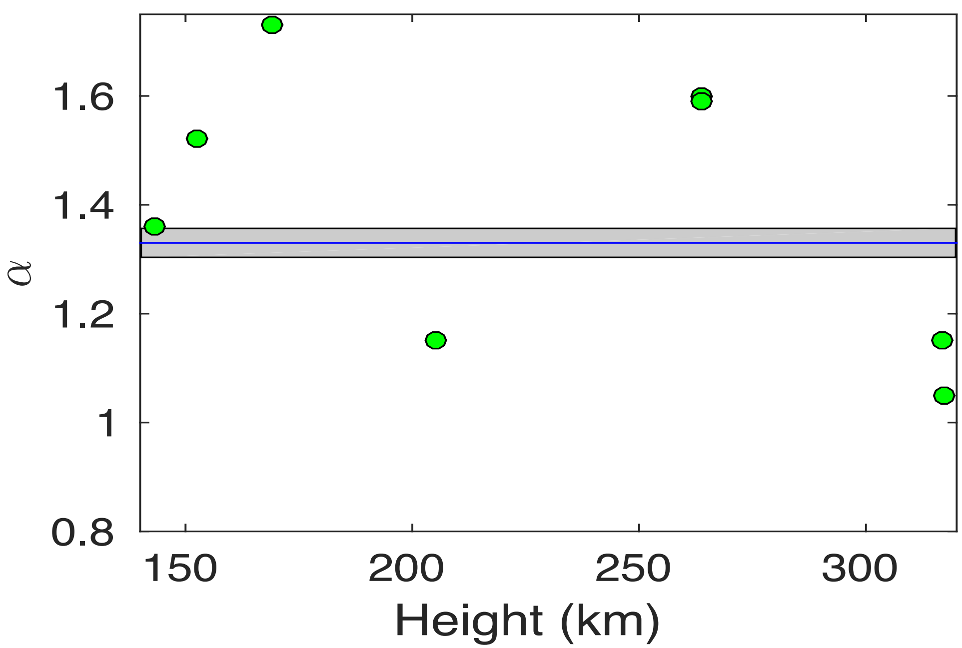
b



Case study 1

DFA: Valley region

Date	Altitude (km)	β	$<\beta>$	σ m	References
17/07/1979	250 to 370	-1.20 to -3.4	-2.3	110%	Rino et al., 1981
17/07/1979	250 to 485	-2.00 to -3.4	-2.7	70%	Kelley et al., 1982
31/10/1986	100 to 220	-1.54 to -3.30	-2.42	88%	Muralikrishna et al., 2007
15/01/2007	- to 127	-1.60 to -2.70	-2.15	55%	Sinha et al., 2010
08/12/2012	100 to 317	-0.98 to -2.14	-1.56	58%	This work



Kolmogorov's spectral exponent
β = -1.66 ± 2%
α = 1.33 ± 2%

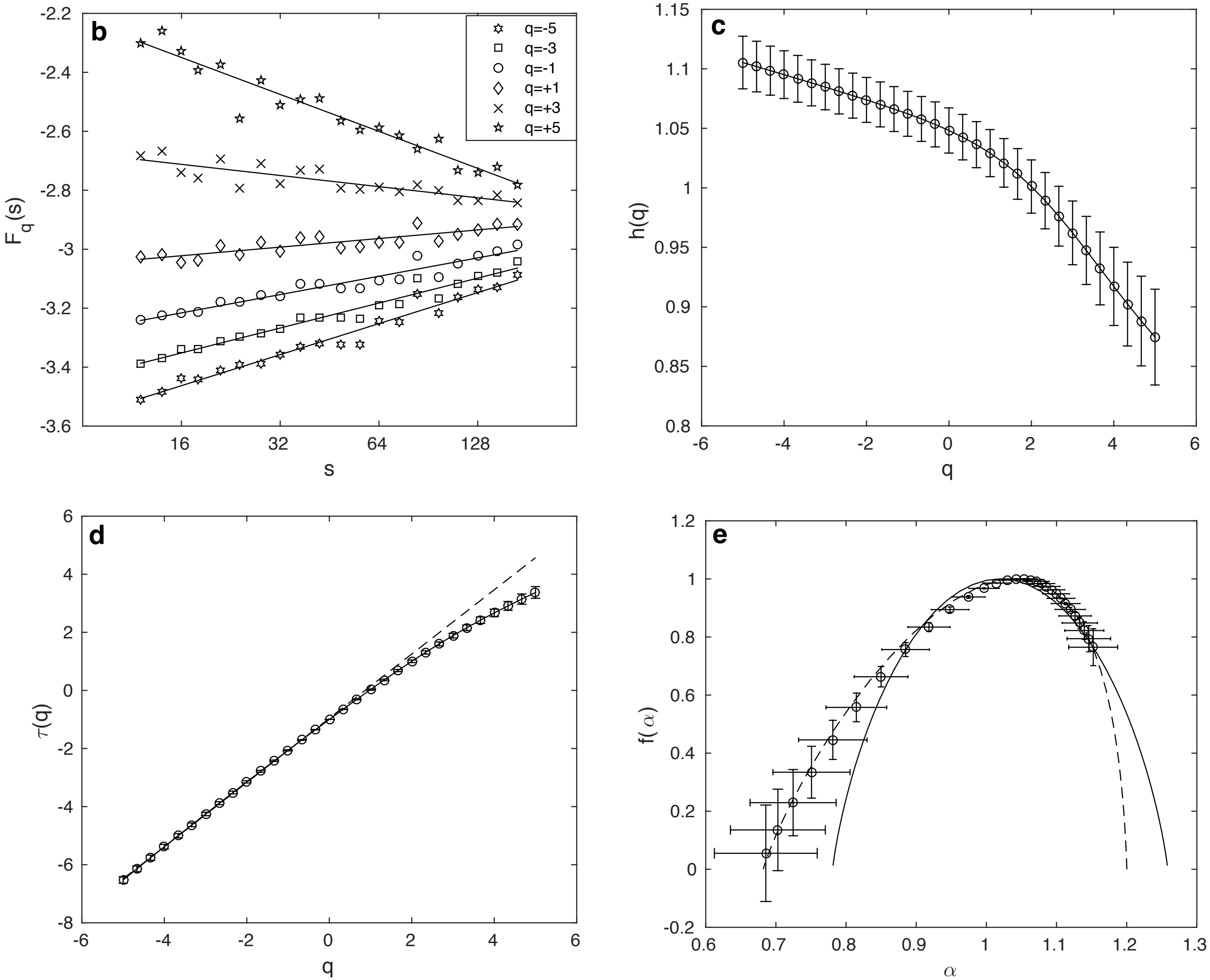
$\beta = 2\alpha - 1$

[Neelakshi et al. 2019]

Case study 1

<263.47> km

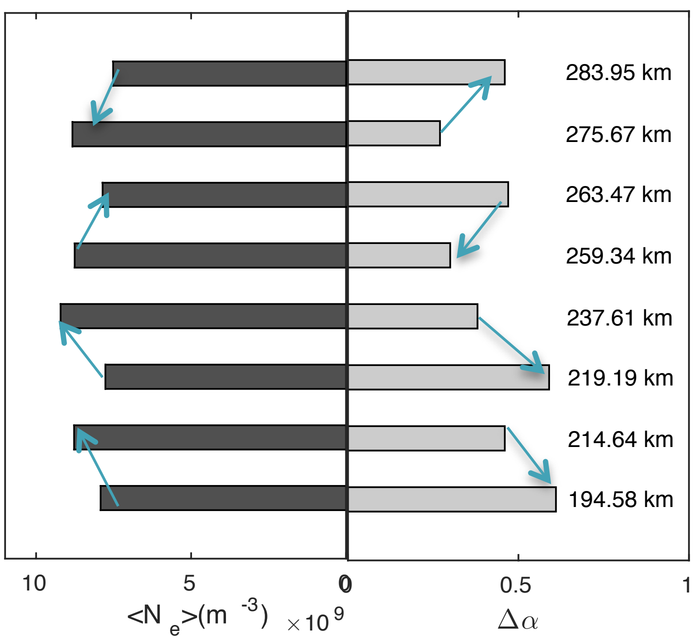
MFDFA: valley region

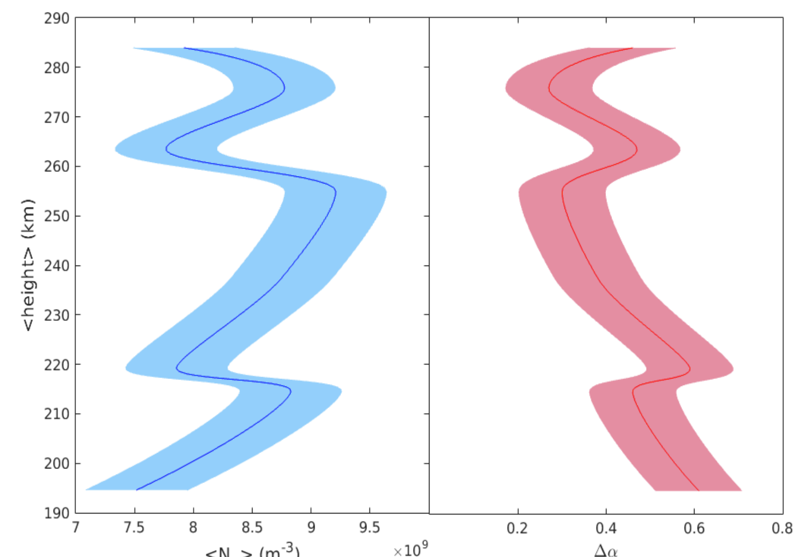


Case study 1

MFDFA: valley region data

$<height>$	degree of multifractality	measure of asymmetry	p model fit parameters		
(km)	$\Delta\alpha$	A	p_1	l_1	dp
194.58	0.61	0.54	0.3650	0.5	0.0500
214.64	0.46	0.91	0.4010	0.5	0.0150
219.19	0.59	1.05	0.3780	0.5	0.0180
237.61	0.38	0.94	0.4190	0.5	0.0080
259.34	0.47	5.51	0.4300	0.5	0.0000
263.47	0.47	3.25	0.4180	0.5	0.0000
			0.0785	0.12	0.005
275.67	0.27	2.98	0.4050	0.5	0.0100
283.95	0.46	4.46	0.4250	0.5	0.0000



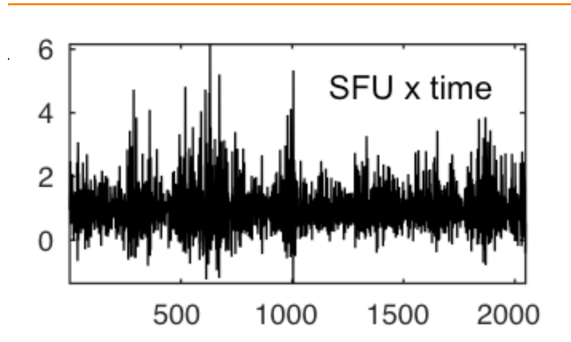


Case study 2

Type I solar noise storm

- non-thermal solar radio emissions
- last for several hours to days – distinguished characteristic
- well known for their dissipative and intermittent nature
- PSD indices are found to deviating from -5/3 [*Veronese et al., 2011; Sodré et al., 2015*]

Figure 2. The hardware of CALLISTO shown in the foreground consists of the main board for data acquisition and interface with a RISC processor ATmega16 (left) and of two synchronous receivers (right). The complete spectrometer is shown in the background. Its physical size (width) is 24 cm.

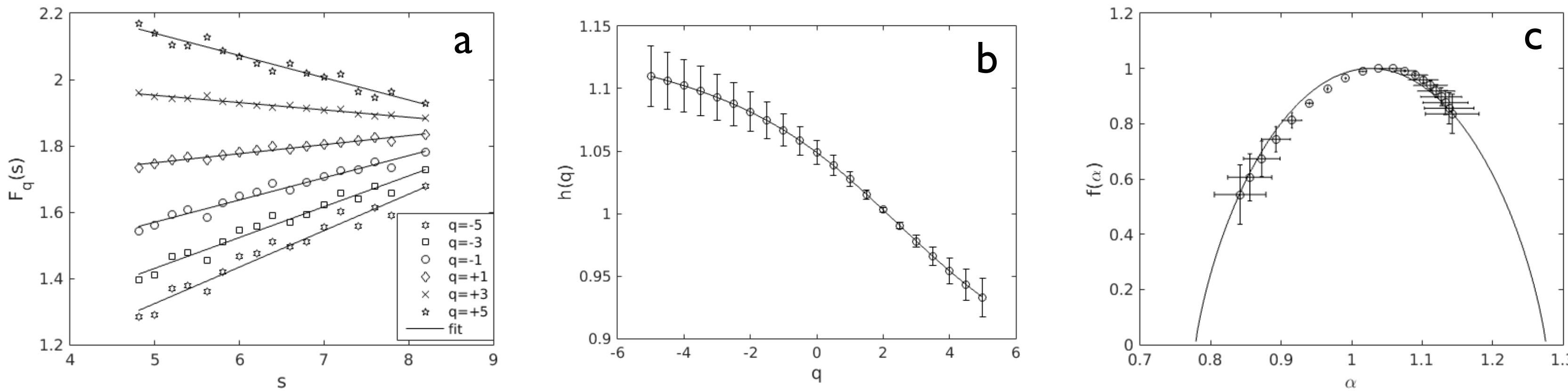


E-Callisto

- worldwide network of the *Compact Astronomical Low-cost Low-frequency Instrument for Spectroscopy and Transportable Observatory (CALLISTO)* type radio spectrometers
- Eight time series of frequency 263.3MHz recorded by BLEN7M spectrogram (Switzerland) on July 30, 2011

Preliminary Results

Data5 (6:45 UT)



Data	start time (UT)	p_1	dp	$\Delta\alpha$	A
data1	05:45	0.445	0.0	0.18	1.00
data2	06:00	0.405	0.009	0.38	1.31
data3	06:15	0.437	0.0	0.21	1.33
data4	06:30	0.412	0.001	0.28	1.70
data5	06:45	0.413	0.004	0.30	2.00
data6	10:00	0.434	0.0	0.33	1.75
data7	11:00	0.422	0.018	0.25	0.50
data8	11:45	0.375	0.038	$\boxed{0.53}$	0.41