Algorithm Cheat Sheet (Pageless)

Contents

1 QuickSelect

QuickSelect is an algorithm to find the **k-th smallest (or largest) element** in an unsorted array. It is related to QuickSort but only recurses on the side containing the k-th element.

- **Average time complexity:** $O(n)$
- **Worst-case time complexity:** $O(n^2)$
- **Space complexity:** $O(1)$ (in-place, recursive stack $O(\log n)$ on average)

1.1 Implementation

```cpp
// Partition function: rearranges elements around a pivot
// After partitioning:
// - elements <= pivot are on the left
// - elements > pivot are on the right
// Returns the final index of the pivot
int partition(vector<int>& arr, int left, int right) {
    // Randomly pick a pivot index to reduce worst-case
    int pivot_idx = left + rand() % (right - left + 1);
    int pivot = arr[pivot_idx];

    // Move pivot to the end temporarily
    swap(arr[pivot_idx], arr[right]);

    int i = left;  // i points to the next position for swapping smaller elements
    for (int j = left; j < right; j++) {
        if (arr[j] <= pivot) {    // If element <= pivot, move it to left
            swap(arr[i], arr[j]);
            i++;
        }
    }
    // Place pivot in its correct sorted position
    swap(arr[i], arr[right]);
    return i; // return the index of the pivot
}

// QuickSelect: finds the k-th smallest element (1-indexed)
int quickSelect(vector<int>& arr, int left, int right, int k) {
    if (left == right) return arr[left]; // only one element

    // Partition the array and get pivot index
    int pivot_idx = partition(arr, left, right);
    int count = pivot_idx - left + 1; // number of elements <= pivot

    if (count == k) {
        return arr[pivot_idx]; // pivot is the k-th smallest element
    } else if (k < count) {
        // k-th element lies in left partition
        return quickSelect(arr, left, pivot_idx - 1, k);
    } else {
        // k-th element lies in right partition
        // adjust k because we discard left partition
        return quickSelect(arr, pivot_idx + 1, right, k - count);
    }
}
```

1.2 Practice Problems

- K-th Largest Element in an Array (LeetCode)

2 Trie (Prefix Tree)

Efficient data structure for string retrieval problems (prefixes, alphabets, etc).

2.1 Implementation

```cpp
const int ALPHABET = 26; // number of lowercase letters

struct TrieNode {
    TrieNode *children[ALPHABET]; // pointers to child nodes
    int terminal;                 // number of words ending at this node
};

// Create and initialize a new Trie node
TrieNode *new_node() {
    TrieNode *node = new TrieNode;
    for(int i = 0; i < ALPHABET; ++i)
        node->children[i] = nullptr; // initialize all children to null
    node->terminal = 0;               // no word ends here yet
    return node;
}

// Insert a string into the trie
void insert(TrieNode *node, string s) {
    for (char c : s) {
        int idx = c - 'a';           // map char to index 0-25
        if (!node->children[idx])    // if child does not exist, create it
            node->children[idx] = new_node();
        node = node->children[idx]; // move to the child
    }
    node->terminal++;               // mark end of word
}

// Remove a string from the trie
void remove(TrieNode *node, string s) {
    for (char c : s) {
        int idx = c - 'a';
        if (!node->children[idx]) return; // word not found
        node = node->children[idx];
    }
    if (node->terminal > 0) node->terminal--; // unmark end of word
}

// Search for a string in the trie
bool search(TrieNode *node, string s) {
    for (char c : s) {
        int idx = c - 'a';
        if (!node->children[idx]) return false; // missing letter
        node = node->children[idx];
    }
    return node->terminal > 0; // true if word ends here
}
```

2.2 Practice Problems

- Codeforces 706D
- Word Search II (Leetcode)

3 Huffman Coding

Huffman Coding is a lossless data compression algorithm that assigns variable-length codes to characters based on their frequency. More frequent characters get shorter codes, while less frequent characters get longer codes.

- **Time complexity:** $O(n \log n)$ where n is the number of unique characters
- **Space complexity:** $O(n)$ for the tree structure

3.1 Key Properties

- **Prefix-free codes:** The tree structure ensures that no character's code is a prefix of another character's code. This eliminates the need for separators between encoded characters and allows unambiguous decoding.
- **Optimal compression:** For a given set of character frequencies, Huffman coding produces the minimum possible average code length.

3.2 Algorithm Steps

1. Count frequency of each character
2. Create leaf nodes for each character and build a min-heap
3. While heap has more than one node:
   - Extract two nodes with minimum frequency
   - Create new internal node with these as children
   - Set frequency as sum of children frequencies
   - Insert back into heap
4. Root of remaining tree is the Huffman tree
5. Assign codes: left edge = 0, right edge = 1

3.3 Implementation

```cpp
struct HuffmanNode {
    char data;
    int freq;
    HuffmanNode* left;
    HuffmanNode* right;

    HuffmanNode(char d, int f) : data(d), freq(f), left(nullptr), right(nullptr) {}
    HuffmanNode(int f) : data('\0'), freq(f), left(nullptr), right(nullptr) {}
};

// Comparator for priority queue (min-heap based on frequency)
struct Compare {
    bool operator()(HuffmanNode* a, HuffmanNode* b) {
        if (a->freq == b->freq) {
            return a->data > b->data; // tie-breaker for consistency
        }
        return a->freq > b->freq;
    }
};

// Build Huffman tree from character frequencies
HuffmanNode* buildHuffmanTree(unordered_map<char, int>& freq) {
    priority_queue<HuffmanNode*, vector<HuffmanNode*>, Compare> minHeap;

    // Create leaf nodes and add to heap
    for (auto& pair : freq) {
        minHeap.push(new HuffmanNode(pair.first, pair.second));
    }

    // Build tree bottom-up
    while (minHeap.size() > 1) {
        HuffmanNode* left = minHeap.top(); minHeap.pop();
        HuffmanNode* right = minHeap.top(); minHeap.pop();

        // Create internal node
        HuffmanNode* merged = new HuffmanNode(left->freq + right->freq);
        merged->left = left;
        merged->right = right;

        minHeap.push(merged);
    }

    return minHeap.top(); // root of Huffman tree
}

// Generate codes by traversing the tree
void generateCodes(HuffmanNode* root, string code, unordered_map<char, string>& codes) {
    if (!root) return;

    // Leaf node - store the code
    if (!root->left && !root->right) {
        codes[root->data] = code.empty() ? "0" : code; // handle single character case
        return;
    }

    generateCodes(root->left, code + "0", codes);
    generateCodes(root->right, code + "1", codes);
}

// Encode text using Huffman codes
string encode(string text, unordered_map<char, string>& codes) {
    string encoded = "";
    for (char c : text) {
        encoded += codes[c];
    }
    return encoded;
}

// Decode using Huffman tree
string decode(string encoded, HuffmanNode* root) {
    string decoded = "";
    HuffmanNode* current = root;

    for (char bit : encoded) {
        if (bit == '0') {
            current = current->left;
        } else {
            current = current->right;
        }

        // Reached leaf node
        if (!current->left && !current->right) {
            decoded += current->data;
            current = root; // reset to root
        }
    }

    return decoded;
}

// Complete Huffman coding example
void huffmanExample() {
    string text = "abracadabra";

    // Count frequencies
    unordered_map<char, int> freq;
    for (char c : text) {
        freq[c]++;
    }

    // Build tree and generate codes
    HuffmanNode* root = buildHuffmanTree(freq);
    unordered_map<char, string> codes;
    generateCodes(root, "", codes);

    // Print codes
    cout << "Huffman Codes:" << endl;
    for (auto& pair : codes) {
        cout << pair.first << ": " << pair.second << endl;
    }

    // Encode and decode
    string encoded = encode(text, codes);
    string decoded = decode(encoded, root);

    cout << "Original: " << text << endl;
    cout << "Encoded: " << encoded << endl;
    cout << "Decoded: " << decoded << endl;
}
```

3.4 Example

For text "abracadabra":

- Frequencies: a(5), b(2), r(2), c(1), d(1)
- Possible codes: a(0), b(10), r(110), c(1110), d(1111)
- Original: 88 bits (8 bits per char $\times$ 11 chars)
- Compressed: 27 bits ($5\times1 + 2\times2 + 2\times3 + 1\times4 + 1\times4$)
- Compression ratio: 69% reduction

3.5 Practice Problems

- Huffman Tree Construction
- Text Compression Problems

4 Unit Testing

How to write unit tests for your code.

4.1 Pytest

Simple Python program:

```python
# file: math_utils.py
def add(a, b):
    return a + b

def divide(a, b):
    if b == 0:
        raise ValueError("Cannot divide by zero")
    return a / b
```

```python
# file: test_math_utils.py
import pytest
from math_utils import add, divide

def test_add():
    assert add(2, 3) == 5
    assert add(-1, 1) == 0
    assert add(0, 0) == 0

def test_divide():
    assert divide(6, 3) == 2
    assert divide(5, 2) == 2.5

    # Check that dividing by zero raises an exception
    with pytest.raises(ValueError):
        divide(1, 0)
```

How to run the tests:

```
pytest test_math_utils.py
```

4.2 Catch2

```cpp
// file: math_utils.hpp
int add(int a, int b) {
    return a + b;
}

double divide(double a, double b) {
    if (b == 0) throw std::runtime_error("Cannot divide by zero");
    return a / b;
}
```

5 Disjoint Set Union (DSU)

6 Monotonic Stack

6.1 What is it and when to use it

A monotonic stack is a standard stack (LIFO), but the elements mantain a monotonic order at every time.

- **Increasing stack:** elements are in increasing order from bottom to top.
- **Decreasing stack:** elements are in decreasing order from bottom to top.

This allows us to efficiently answer the following questions:

- What is the next **greater/smaller element**?
- Who **blocks visibility**? (think about the problem about heights and how many people can every person see on leetcode)
- How many elements are visibiel until I hit a blocker?

6.2 Principle

Whenever we want to push an element x, while the top of the stack violates the monotone property, pop it (and posibly update something related to the answer). After this, push x. This has $O(n)$ time complexity, because every element is pushed and popped at most once.

6.3 Problems

- Next Greater Element I (LeetCode)
- Next Greater Element II (LeetCode)
- Daily Temperatures (LeetCode)
- Largest Rectangle in Histogram (LeetCode)
- Maximal Rectangle (LeetCode)
- Number of Visible People in a Queue (LeetCode)

7 Dijkstra's Algorithm

7.1 Implementation

```cpp
class Dijk {
public:
    vector<int> fat;
    vector<ll> d;
    vector<vector<pair<int, int>>> adj;

    Dijk(const vector<vector<pair<int, int>>> &adjacency_list) {
        adj = adjacency_list;
        d.assign(adj.size(), INF);
        fat.assign(adj.size(), -1);
    }

    void search(int source) {
        priority_queue<pair<ll, int>, vector<pair<ll, int>>, greater<pair<ll, int>>> q;
        q.push({0, source});
        d[source] = 0;

        while (!q.empty()) {
            pair<ll, int> curr = q.top();
            q.pop();

            for (pair<int, int> nbr : adj[curr.ss]) {
                if (d[nbr.ff] > d[curr.ss] + nbr.ss) {
                    fat[nbr.ff] = curr.ss;
                    d[nbr.ff] = d[curr.ss] + (ll) nbr.ss;
                    q.push({d[nbr.ff], nbr.ff});
                }
            }
        }
    }
};
```

7.2 Practice Problems

- CSES 1671 - Shortest Routes I
- CSES 1194 - Flight Discount
- CSES 1202 - Message Route
- CSES 1203 - Labyrinth
- Codeforces 20C - Dijkstra?