Bose-Einstein Condensate

Super Solids

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Contents

1 Bose Einstein Condensate

1.1 Questions

- $\int \frac{1}{r^n} \mathrm{d}^3\mathrm{r}$ divergent just for $n \leq 3$?
- What is an s-wave?
- if $l < \frac{n-3}{2}$,
- and like k^{n-2} otherwise (Landau and Lishitz, 1977). For a van der Waals-like potential ($n = 6$), only $l = 0$ (s-wave) matters at low energies. Whats with $l = 0$? Lifschitz is a typo?
- In the notes: eq. 1.8 to 1.9 the commutator $[\psi, \psi^{\dagger}]$ was used, but from 1.9 to 1.10 Bogoliubov aproximation was used, why not directly on 1.8 ?
- What is the derivation of 1.11? Its a FFT, but what are the xact steps?
- What are the python packages to use operators like $\hat{\psi}$? There is some sympy implementation, but probably there is a better one?
- how to get from 1.11 to 1.12? Is it a commutator expansion? Why q is disappearing?
- In 1.17 ω_ρ part has a factor 2, but ω_z not, despite beeing symmetric in ψ. Why?
- What is variable a? Why should $a > 0$ as repulsive short-range interactions stabilize the BEC (p.10)?
- "When the atomic density grows due to the attractive interaction, three-body losses predominantly occur in the high-density region. " What does three-body losses mean?
- "As the collapse occurs mainly in the x-y direction due to anisotropy of the DDI (in the absence of inelastic losses, the condensate would indeed become an infinitely thin cigar-shaped cloud along z), and therefore the condensate explodes essentially radially, producing the anisotropic shape of the cloud." Why is the collaps not along z axis?
- How are the regions stable, metastable, unstable derived in Figure 1.5, here Figure 4?

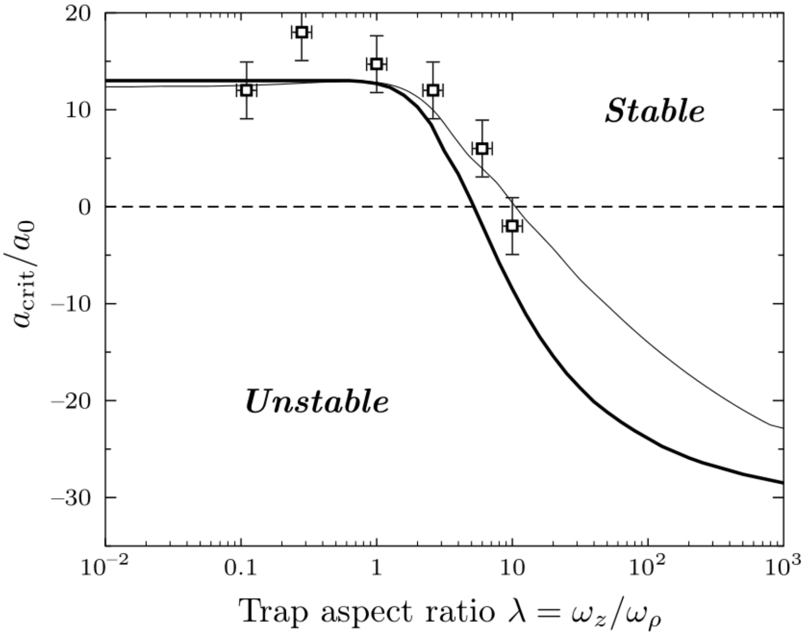


Figure 1: Logo
SANTOS, *title* (year)

- Typo in "we obtain a 1D equation similar to the a GP equation", just the or a
- "ground-state wave-function is independent of the in-plane coordinates " Why?
- 1.26 to 1.27, where does the U_{dd} go?
- Typo If: "roton momentum. if this were so"
- Why should a modulation with a finite wavelength allow superfluids?
- Typo repeatance: "the width of the width"
- What are the spin-F matrizes?
- Is the occurance of these spin textures in Figure 2 special?

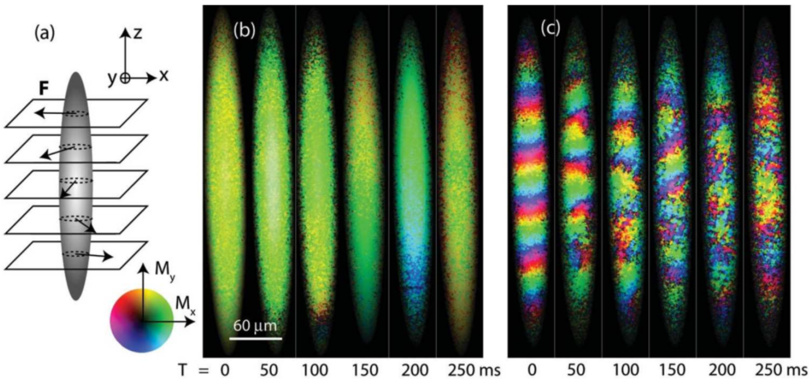


Figure 2: Is the occurance of these textures special?
SANTOS, *title* (year)

1.2 Summary

- dipol-dipol interaction (DDI):

$$U(r) = \underbrace{g\delta(r)}_{\frac{4\pi\hbar^2 a(d)\delta(r)}{m}} + \underbrace{U_{dd}(r)}_{\frac{C_{dd}}{4\pi}\frac{(e_1\cdot e_2)r^2-3(e_1\cdot r)(e_2\cdot r)}{r^5}} \tag{1}$$

- Use pseudo potential as dipol-dipol interaction is anisotropic and all partial wave (different l) mix
- coupling of different channels generates short-range contribution in the s-channel $s = 0 \Rightarrow$ by changing DDI strength a gets modified too $\Rightarrow$ shape resonances $\Rightarrow$ virtual state transform into a new ground state
- for fermions s-channel does not exists, so just long-range
- FFT of U_{dd} using sperical harmonics Y_{lm} gives:

$$\tilde{U_{dd}}(k) = \int \mathrm{d}^{\mathrm{r}}\mathrm{r} U_{dd}(r) e^{-ik\cdot r} = \frac{C_{dd}}{3}\left(3\cos^2(\theta_k) - 1\right) \tag{2}$$

- Use DDI in Gross-Pitajevski Equation, FFT, approximate to 2nd order, diagonalize with Bogoliubov transform

- As a result the square root can be imaginary, so the BEC gets dynamically unstable for long-wave length (phonon-instability):

$$\epsilon(p) = \sqrt{\frac{p^2}{2m}\left[\frac{p^2}{2m} + 2n_0\left(g + \tilde{U_{dd}}(p)\right)\right]} \quad (3)$$

$$= pc_s\sqrt{1 + \epsilon_{dd}\left(3\cos^2\theta_p - 1\right)} \quad (4)$$

$$\underset{p\to 0}{=} pc_s\sqrt{1 - \epsilon_{dd}} \quad (5)$$

- For dipolar BEC the trap geometry is crucial (for non-dipolar not)
- “pancake traps” can stabilize the phonon-instability
- qualitative features for $a_{crit}(\lambda)$ by gaussian ansatz, for exact numerical solution non-local Gross-Pitaevskii Equation needed

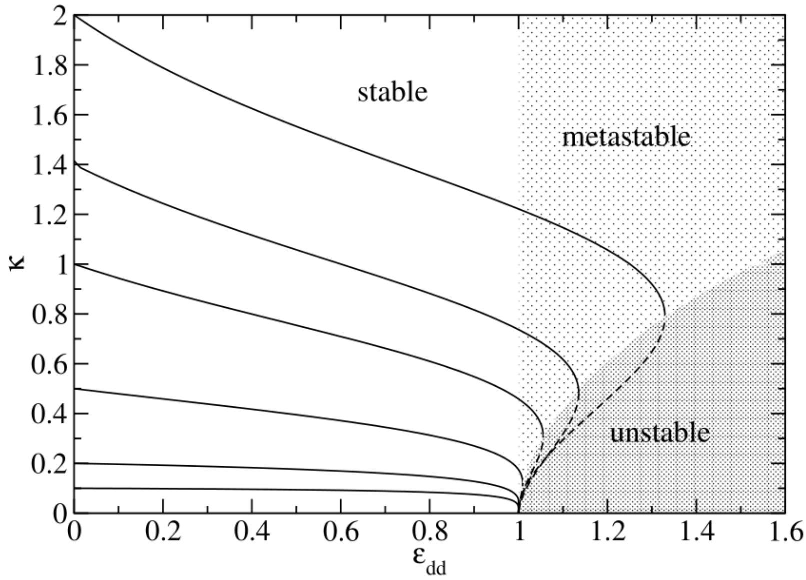
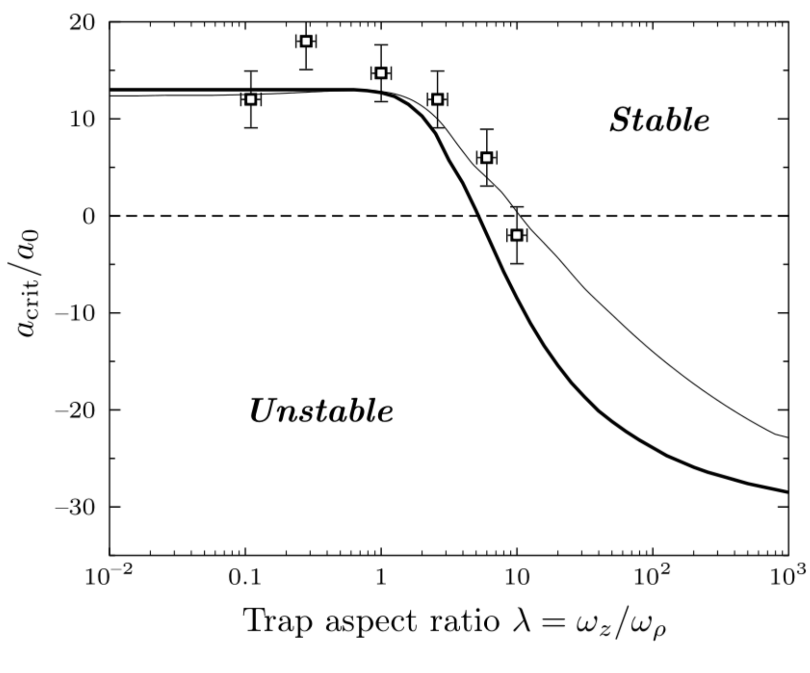


Figure 3: Logo
Santos, *title* (year)

- for sufficiently strong interactions, we may neglect quantum pressure, and consider the Thomas-Fermi (TF) regime
- TF solution for the trapped BEC has the same inverted parabola shape (as in non-dipolar case)
- BEC is prolat for $0 < \kappa < 1$ and $1 < \kappa$ oblat

- Bogoliubov-de Gennes Equation shows that the nonlocal character of the DDI causes a momentum dependend coupling constant, leading to a roton-like dispersion law, leading to dynamically instability, when the roton $\beta = \frac{g_d}{g}$ touches zero (experimetally not obeserved yet)
- by varying the density, the frequency of the confinement, and the short-range coupling, one can control the spectrum (roton minimum deeper/shallower)
- sequence of the non-local non-linearity 2D bright solitary waves may become stable under appropriate conditions (Pedri and Santos, 2005)
- two instability regions for 2D solitons (against col- lapse and against unlimited expansion)
- $\tilde{g}_{cr}(\beta) \equiv \frac{gN_{cr}}{2\pi l_z}$, so stable 2D anisotropic self-localised solitons exists just for $N < N_{cr}$
- non-dipolar BECs scatter elastically, the scattering of dipolar solitons is inelastic due to the lack integrability
- The solitons may transfer centre-of-mass energy into internal vibrational modes, resulting in intriguing scattering properties:
  - including soliton fusion (Fig. 1.8)
  - appearance of strong inelastic resonances
  - possibility of observing 2D- soliton spiraling as that already observed in photo-refractive materials
- Dipolar effects in spinor condensates
  - spinor BECs: we focus on an effect which resembles the Einstein-de Haas effect
  - Because of Zeeman sub-levels short-range interactions may occur in different s-wave scattering channels with different total angular momentum (for bosons even number) (spin-1 bosons we have just F = 0 and F = 2)
  - Each scattering channel has an associated s-wave scattering length a_F
  - short-range interactions necessarily preserve the spin projection Sz
  - DDI does not necessarily conserve the spin projection along the quantisation axis as DDI is anisotropic
  - for initially maximally stretched state ($m_F = -F$)
  - short-range interactions cannot induce any spinor dynamics (due to conservation of total magnetisation S_z

- DDI may induce a transfer to $m_F + 1$
- for cylindrical symmetry around the quantisation axis, this violation of the spin projection is accompanied by a transfer of angular momentum to the centre of mass, resembling the well known Einstein-de Haas effect ⇒ initially spin-polarised dipolar condensate can generate dynamically vorticity
- Einstein-de Haas effect is destroyed by weak magnetic fields (1 mG)
- the dominant Larmor precession, and invoking rotating-wave-approximation arguments, the physics must be constrained to manifolds of preserved magnetisation (2D optical lattices could help)
- Effect of DDI could be even observable under conserved S_z (alkali spinor condensates)
- spin-changing collisions: collisions that conserve S_z, but do not conserve the relative population of the different Zeeman components
- Spin-changing collisions are characterised by an energy scale proportional to the difference between scattering lengths at different channels
- this difference is very small, so can be significantly modified by the presence of other small energy scales (DDI) ⇒ helical spin textures



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2 Supersolids

- supersolid: features both the crystalline structure of a solid and the frictionless flow of a superfluid In this state, every constituent atom is part of the solid and the superfluid simultaneously
- direct observation was limited to systems where the structure formation was mediated by external light fields
- beyond mean-field approximation leads to corrections to the ground state energy stemming from quantum fluctuations of the collective modes in a BEC (LHY-correction)
- In 2018 quantum droplets in a Bose-Bose mixture were observed
- mean- field energy depends on the difference of the two coupling constants $\delta(g) = |g_{rep}| - |g_{att}|$
- LHY-correction depends on the individual coupling constants
- For weakly attractive combination of interactions, a repulsive beyond mean-field correction can stabilize the BEC
- after a peak density increasing the number of particles only leads to an increase in the size of the droplet
- eGPE: kinetic energy, external trapping, and two-body interactions, LHY
- beyond mean-field correction has only been calculated for a homogeneous system and can therefore only be included within a local-density approximation

- QMC calculations in full many-body system verified the formation
- intra-species scattering lengths a_{11} and a_{22} lead to different equilibrium densities $n_0^{(i)}$ for the two components of the mixture.
- droplet forms an intrinsic imbalance in the atom numbers of the two components ($\frac{N1}{N2} = \sqrt{\frac{a_{22}}{a_{11}}}$)
- larger density than in original BEC increases the rate of three-body loss $\Rightarrow$ extra term in eGPE

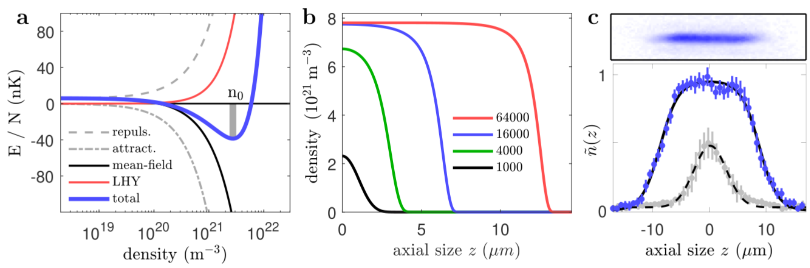

Figure 5: Peak density of the droplet saturates in z-driection
Santos, *title* (year)