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Longevity - Linked Securities & Derivatives

Group 4

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1.Introduction

Demography has been one of humanity's hot topics up for questioning, writing and political policy making since the human history modern era, many prisms can be chosen to look at this subject, from the fear of over population investigated by Thomas Malthus at the very end of the 18th century, to the population growth boom of the second half of the 20th century, today the reality is of more than 8 billion human being populating the earth.

If we delve on the developed countries current demographic situation there are two topics of major concern, the low rate of newborns and the fast-growing ageing of populations. The reason for these issues is related with the exponential development of technology, access to good quality health care and quality of living mainly in the previously referred countries. All these improvements, which are undoubtedly positive, have contribute also for the growing of the life expectancy not only at birth but also for some groups of cohort ages. This is a major concern for the public sector and for the private insurance and pension fund sectors. With people living longer there is an enlargement of the period in which pensions must be paid which heavily impacts the social welfare state infrastructure, at the same time the insurance and pension fund sector also has to revise their policies and offers since they need to be adapted to this new reality where longer life expectancy means more paying periods for certain contracts and a higher probability of having to pay other types of contracts, this will have a impact on risk of higher costs. One of the ways that both the public and the private sectors can reduce their exposure to this risk is through the capital derivatives market where solutions as longevity bonds may hedge the new risk by betting on longevity of the populations.

In this project we were asked to pick up a country and run an analysis on its mortality rates. We have chosen Portugal, besides being our country it is one of Europe's leaders in terms of aging related indicators. In terms of life expectancy Portugal had an exponential growth in this indicator in the last 60 years, from 64 years old in 1960 to 82.4 years old in 2023, looking into the more immediate reality the life expectancy in Portugal at 65 it is now, for both sexes, at 21.1 which is higher than the one for newborns at 86.1. The most relevant indicator when comparing with the remaining European Union countries is the aging index where Portugal stands as having the second largest only elapsed by Italy, Portugal has a 185.3% aging index.

In our project we intend to analyse the mortality rate data for Portugal from 1980 to 2018 and forecast it until 2100. This will be done based on a Generalized Age-Period-Cohort (GAPC) stochastic mortality model, most specifically the Poisson-Lee-Carter model. Finally we will calculate the value of longevity bonds based on the mortality rate calculated previously and look into the several risk loadings.

2. Exploratory Analysis

After treating and importing Portuguese data from the Human Mortality Database, we explored the Portuguese Deaths and Exposure Risk values. In Exploratory analysis. The main goal was to produce the Portugal mortality rates according to the ages of the individuals between 60 to 90, and the years 1980 to 2018 of the respective genders. We decided to remove the years lower than 60 and higher than 90 from age, and we removed the years lower than 1980 according to years because Portugal in that specific time horizon does not have quality in their data and can negatively influence the final results from the next years. After that, we created the graphics of the mortality rates according to their ages and years, where we developed 3 matrixes for the deaths and then 3 matrixes for the exposure risk having a total of 6 matrixes where we had a different subgroup (Female, Male, and Total) in each matrix. After creating the matrixes, we did the calculations to have the final values of the mortality rates. To have the ***mortality rates per age***, we did the quotient of the total sum of the number of deaths for a given age (Dx) and the total sum of individuals that were exposed to death risk per age (Ex). Secondly, to have the ***mortality rates per year***, we did the quotient of the sum of a total number of deaths each year (Dy) and the sum of a number of individuals that were exposed to death risk per year (Ey). Before the analysis of the graphs, we defined into a log distribution each component because will fit better the values of the Portugal data.

In these two graphs, we calculated ***the mortality rates per age*** and sex in the first one, and in the second per year and sex. In these scenarios, x represents the age of reference and y the year of reference, both these values will make the mortality rate calculated to be conditional to either of these values. Observing the 2 graphics of the ***mortality rates per age*** and year, we can conclude that the mortality rates per age, according to the interval of ages of 60 to 90 years and the ***mortality rates per year*** following the interval of the years between 1980 to 2018, we could observe that Males have always higher mortality rates per year and ages in both graphics, compared to Female. Concluding they will have an always higher probability of being exposed to death when compared with Females, regardless of year or age. Regarding the ***mortality rates per year***, we can analyze that from all the subgroups (Male, Female, and Total) the slope of the mortality rates per year is majority negative passing the years. That is because of the lack of access to hospitals in past years, the emigration of young people, and the large population of elderly people in Portugal. All of this leads to an increase in the mortality rate in the country as the years go by.

3. GAPC Stochastic Models

Firstly, we would like to introduce the other stochastic models so that we may better comprehend their primary distinctions and goals before beginning our research by concentrating primarily on the Lee-Carter Stochastic Model. We will next display a table with the remaining models and their corresponding formulae.

Lee-Carter is one of the models in the field of actuarial science and demography, the main goal is to improve the goodness-of-fit, and we could do estimations about forecast mortality rates in the future

Model	Formula
M1 (Poisson-Lee-Carter)	$\ln(m_{x,t}) = \beta_x^{(1)} + \beta_x^{(2)} \cdot k_t^{(2)}$
M2 (Renshaw-Haberman Cohort-Lee-Carter)	$\ln(m_{x,t}) = \beta_x^{(1)} + \beta_x^{(2)} \cdot k_t^{(2)} + \beta_x^{(3)} \cdot \gamma_{t-x}^{(3)}$
M3 (APC, Currie 2006)	$\ln(m_{x,t}) = \beta_x^{(1)} + \frac{1}{n_x} \cdot k_t^{(2)} + \frac{1}{n_x} \cdot \gamma_{t-x}^{(3)}$
M4 (Plat, 2009)	$\ln(m_{x,t}) = \beta_x^{(1)} + k_t^{(1)} + k_t^{(2)}(x - \bar{x}) + \gamma_{t-x}^{(3)}$
M5 (CBD)	$\text{logit}(q_{x,t}) = k_t^{(1)} + k_t^{(2)}(x - \bar{x})$
M6 (CBD cohort)	$\text{logit}(q_{x,t}) = k_t^{(1)} + k_t^{(2)}(x - \bar{x}) + \gamma_{t-x}^{(3)}$
M7 (CBD quadratic age effect + cohort)	$\text{logit}(q_{x,t}) = k_t^{(1)} + k_t^{(2)}(x - \bar{x}) + k_t^{(3)}((x - \bar{x})^2 - \hat{\sigma}_x^2) + \gamma_{t-x}^{(4)}$

Table 1: GAPC Stochastic Models & Formulas

so we could see their behaviors, and the Lee-Carter has numerous variants and extensions. The Lee-Carter has 2 different types of the Lee-Carter Model. The first one is the **Poisson Lee-Carter M1** which follows the distribution of the Poisson and the second **Renshaw-Haberman Cohort-Lee-Carter M2**, is a Binomial distribution and incorporates the Cohorts effects. **The Age Period Cohort M3** is a mortality model that has a long-standing tradition in the fields of medicine and demography but was not widely used in the actuarial literature until it was considered by Currie (2006). APC model analyses health outcomes according to age groups, periods, and birth cohorts. The **Plat M4** integrates elements of the Lee-Carter model with the CBD model to create a model that is appropriate for all age groups and captures the cohort effects. The **Cairns-Blake-Dowd M5** is one of the most well-known Lee-Carter model adaptations. Focus on the linearity of the logit of one-year death probabilities at older ages and consider the slope and intercept parameters as random processes that vary over time. IN **CBD cohort M6**, we just include the cohort parameter in the formula, in the **CBD quadratic age plus the effect cohort M7**, we include the cohort to formula and the age in the quadratic form to obtain the final predictor.

4. Poisson Lee-carter Fitting & Age Period Cohort Goodness-of-Fit Residuals Analysis

4.1 Poisson Lee-carter Parameters Fitting

After having a brief idea of the main differences between the stochastic models, we can now start our analysis regarding the Poisson Lee-Carter Model for Female, Male, and Total subgroups, which we will use for the rest of the report. Firstly we apply the model of the Poisson Lee-Carter to our respective data on the exposure risk of death, and the number of deaths and we would like to see how the parameters are predicted, observe their average mortality by age overall period in the log scale, the sensitivity of the mortalities by age (the speed the decline is occurring in each age), and the time trend of the mortality rates of Portugal.

Observing the ***Poisson Lee-Carter values of mortality rates*** converted into a log scale of the graphs of the average mortality rates for ages from 60 to 90 years old, we can conclude that Female, Male, and Total subgroups of Portugal have the same continuous consistent upward trend in relation to age progression, meaning that going beyond the 60 to 90 years interval we will denote a higher average rate of mortality. Now, in the ***Poisson Lee-Carter sensitivity*** of the mortalities by age for the 60-90 age interval, we can conclude that the Female, Male, and Total subgroups of Portugal have always positive values. The higher values in Females, Males, and Total happened at the 60-78 age interval, which means that we had a faster mortality decline in that specific age. After 78 we had a slow mortality decline because we are decreasing the values of the sensitivity mortalities. The last parameter from Poisson Lee-Carter, the ***time trend of Mortalities Rates*** we can conclude that the Female, Male, and Total subgroups of Portugal have always a negative time trend, meaning passing the years is expected a decline in deaths and individuals are less exposed to the risk of dying

4.2 Poisson Lee-Carter & Age Period Cohort AIC, BIC & DEVIANCE Fitting

Total	AIC	BIC	DEV	Male	AIC	BIC	DEV	Female	AIC	BIC	DEV
LC	14479.88	14983.55	2814.5	LC	12791.07	13294.7	1899.1	LC	12756.21	13259.8	2043.2
APC	15590.32	16251.71	3559.1	APC	13405.54	14066.9	2119.8	APC	13470.05	14131.4	2567.9

Table 2: Poisson Lee-Carter & APC Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and Deviation for Total, Male and Female

Observing the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and Deviation (DEV) values for different gender subgroups provides insight into the model's fit. Lower AIC, BIC, and DEV values suggest a better model fit, with the lowest values observed in females at 12756.21 (AIC) regarding the Poisson Lee-Carter stochastic model and 13259.8 (BIC) as well in Poisson Lee-Carter, respectively. Similarly, lower dispersion in the Deviation metric, such as the value of 1899.1 for males in the Poisson Lee-Carter, indicates better model performance. It can be concluded from the three subdivisions in our calculations that the Lee-Carter model has better suitability for our data related to females since it presents the lowest values for AIC and BIC.

4.3 Poisson Lee-Carter Goodness-of-Fit Residuals

After analyzing the parameters of the Fitting Poisson Lee-Carter, now we will analyze the Goodness-of-fit of the standardized residuals, which will show us if we have good results or not efficient results by applying the normality tests.

Observing the *3 colormaps of the standardized residuals* of Portugal, we can conclude that the Total, Male, and Female subgroups both have a huge amount of residuals between the years 2007 to 2018, and the subgroup that we noted has more outliers from those 3 subgroups is the Males, and who have less will be the Females.

Upon examining the *scatter graphs of standardized residuals* for genders in Portugal, significant discrepancies are evident, with values exceeding ±2. In the Total subgroup, errors manifest at ages 80-90, and during the years 2010-2018. Among males, errors are notable at ages 80-90, and during the years 2016-2018. Similarly, female errors are observed at ages 87 and 90, also reported some errors in the year 2017, and for cohorts born in 1920 and 1923. These findings underscore potential areas for enhancing the model's accuracy.

5. Poisson Lee-carter Forecasting

5.1 Poisson Lee-Carter & Age Period Cohort AIC, BIC & DEVIANCE Fitting

Now, after observing the parameters of Poisson Lee-Carter, the residuals, and observing how well our model fits the data for Portugal according to the years 1980 to 2018 and the ages 60 to 90 years by using the Poisson Lee Carter Stochastic Model, we can now execute our analysis for forecasting methods. So, we will implement by adding the forecast from 2019 to 2100 for observe how will be the estimated values of the next years, regarding the behaviors from the mortality rates that we estimated. The *mortality rate forecast graphs for Portugal's total population, male and female* populations until 2100 demonstrate a decline in mortality rates, showing longer life expectancies for Portuguese people. Females are predicted to have lower mortality rates and longer life expectancies than males and the population in its entirety.

5.2 Poisson Lee-Carter & Age Period Cohort Forecasting Accuracies

Total	MSE	MAPE	RMSE	Male	MSE	MAPE	RMSE	Female	MSE	MAPE	RMSE
LC	125.9	200124.4	11224.9	LC	179.70	180907.5	13405.2	LC	111.16	246940.4	10543.6
APC	236.9	193100.6	15392.3	APC	330.27	173275	18173.5	APC	223.22	253577.17	14940.6

Table 3: Poisson Lee-Carter & Age Period Cohort Mean Squared Error (MSE), Mean Absolute Percentage Error (MAPE) and Root Mean Square Error (RMSE) for Total, Male and Female

Observing the Mean Squared Error (MSE), Mean Absolute Percentage Error (MAPE), and Root Mean Square Error (RMSE) values for different gender subgroups provides insight into the model's fit. Lower values indicate better performance. For females, the MSE is 111.16 of the Poisson Lee-Carter and the RMSE is 10543.6 of the Poisson Lee-Carter, both of which are the lowest among the subgroups, suggesting the model fits best for females. Males have a lower MAPE of 173275 for the Age Period Cohort, but the unusually high values of MAPE suggest a potential error or different scaling. Despite the lower MAPE for males in the Age Period Cohort, the significantly lower MSE and RMSE values for females indicate that the Lee-Carter model is more suitable for forecasting data related to females, demonstrating better predictive accuracy and consistency for this subgroup.

5.3 Lee-Carter Forecasting Life Expectancies

By observing the ***graph of life expectancies forecasted*** from 2019 to 2100, we can conclude that all genders will see an increase in life expectancy over the years. Females are projected to have the highest life expectancy, with significant improvements over time. By 2100, the life expectancy for females is expected to be higher than for males, with females generally increasing their life expectancy by 4 years every 20 years, while males increase theirs by 3 years every 20 years.

5.4 Poisson Lee-Carter Forecasting & Fitted 95% Prediction Intervals for Mortality Rates (60 & 75 & 90 years)

Observing the ***Poisson Lee-Carter model graphs*** for the probability of individuals aged 60, 75, and 90 dying before age x+1, we see that from 1980-2018 (fitted values) and forecast until 2100, the central forecast (dashed lines), 95% prediction intervals excluding parameter uncertainty (black dotted lines), and 95% confidence and prediction intervals including parameter uncertainty (dot-dashed green lines) for ***Total, Female, and Male*** indicate reliable predictions until around 2060, after which the predictions become less reliable due to increasing parameter uncertainty, illustrating significant impacts on mortality rate predictions at a 95% confidence level. For last, we can conclude the best age estimated is for the year 90, regarding the other two ages for all the gender

6. Poisson Lee-Carter & Age Period Cohort Analysis

After observing all the graphs of the parameters, residuals, and forecasting values of the Poisson Lee-Carter we can observe that the Age Period Cohort, this stochastic model has a worse performance compared to the Poisson Lee-Carter in general for analyzing the exposure to risk of the individuals and the deaths exposure per year, and pear ages. As we first saw regarding the AIC, BIC, and Deviance measures of fitting for all subgroups, the Period Cohort has the worst results compared to Poisson Lee-Carter, regarding the Forecasting Accuracies as we saw the Age Period Cohort has the worst results in Mean Squared Error, and in the Root Mean Square Error. Now doing an analysis of their ***parameters***, the average mortality rate of the Age Period Cohort has the same continuous consistent upward trend in relation to age progression, meaning that going beyond the 60 to 90 year interval we will denote a higher average rate of mortality, the time trend of Mortalities Rates we can conclude that the Female, Male, and Total subgroups of Portugal has a negative time trend, meaning passing the years is expected a decline in deaths and individuals are less exposed to the risk of dying, equal to Poisson Lee-Carter. Regarding ***residuals***, observing the graphs we conclude that the Age Period Cohort has more outliers than Poisson Lee Carter. For last, regarding the ***forecasts***, we have better values as well for Poisson Lee-Carter for all sub-genders because we have a higher range of negative values, meaning higher longevity, so less individuals dying passing the years.

7. Longevity Risk Loadings for Contracts on a Cohort Aged 65

Considering the different financial instruments available to manage longevity risk, we have chosen a Longevity Swap. To obtain the different values for Portugal's Nelson-Siegel-Svensson (NSS) *yield curve*, we've researched the European Central Bank's database, to apply the formula.

$$f(t) = \beta_0 + \beta_1 \left(\frac{1-\exp(-t/\tau_1)}{t/\tau_1}\right) + \beta_2 \left(\frac{1-\exp(-t/\tau_1)}{t/\tau_1} - \exp(-t/\tau_1)\right) + \beta_3 \left(\frac{1-\exp(-t/\tau_2)}{t/\tau_2} - \exp(-t/\tau_2)\right)$$

Image 1: Nelson-Siegel-Svensson Formula

Country	β0	β1	β2	β3	τ1	τ2
Portugal	0.047638	-0.050484	-0.219748	-0.125183	0.062279	1.106536

Table 4: Portugal's NSS Model Yield Curve

We conducted a simulation of 5 different contracts, with capital ranging between 50,000 and 1,000,000€. Although the data used in this analysis is hypothetical and for illustrative purposes only, the methods and principles applied are based on established actuarial and financial practices, ensuring the reliability of the conclusions drawn.

To continue with our project, based on the Portuguese investor, we assumed that the lambda value would be 0.3, which represents a person who is more risk averse.

As we need to quantify longevity risk, specifically how much premium an insurer needs to charge to cover the possibility of policyholders living longer than expected, we used 4 pricing principles to assure that by simulating future mortality rates we can generate the more adjusted scenarios.

Measure	Wang Tranform	Proportional Hazard	Exponential	Standard Deviation
Min.	0.00901	-8.424e-05	-9.489e-05	-0.8880
1st Qu.	0.02892	1.010e-02	8.323e-03	-0.8837
Median	0.03549	1.290e-02	1.049e-02	-0.8819
Mean	0.03510	1.2783-02	1.035e-02	-0.8819
3rd Qu.	0.04131	1.542e-02	1.242e-02	-0.8804
Max	0-06725	2.779e-02	2.112e-02	-0.8728
NA's	8			8

Table 5: Results obtained in all transformations

Wang Transform

We can highlight that the Wang Transform has the highest risk loading among the simulations. This can be explained by the fact that the ***Wang transform*** gives more importance on the possibility of extreme longevity, where individuals live significantly longer than average. As this is a more conversative approach, it leads to higher risk loading as a safeguard against the worst-case scenarios.

By observing the Wang transform distribution, we can highlight that the mean Wang of 3.51% indicates that, on average, insurers would need to add this percentage to their annuity premiums to consider the longevity risk. We can also conclude that, with the close values of the first quartile (0.02892), median (0.03549), mean (0.03510) and third quartile (0.04131), we have a consistent distribution without extreme outliers. Given that the values are relatively small and positive, which is consistent with the aim of this transformation, it suggests that the transformation yields consistent and relatively modest adjustments to the covariates.

Proportional Hazard

On this principle, the risk loadings are more moderate when compared to the Wang transform. By scaling the hazard rates by a constant factor, the results are more uniform across all ages, without placing emphasis on extreme longevity.

By observing the ***Proportional hazard results***, it´s important to highlight that the range of longevity risk loadings (between 1.01% and 1.542% at the quartile range) is narrower compared to the Wang Transform. This suggests less variability in the estimated risk loadings and a more predictable outcome for insurers to use this approach. We should also mention that the median (1.29e-02) and mean (1.278e-02) values are very identical, suggesting that there are no extreme outliers skewing the distribution significantly. The positive coefficients indicate that the covariates generally increase the hazard, which is associated with a higher risk of the event occurring.

Exponential Transform

It yields risk loadings like Proportional Hazard transform. It´s a principle that balances the need for protection against longevity risk with the objective of keeping premiums affordable.

By observing the ***Exponential Transform***, it shows that this approach is quite like the Proportional Hazard transform as they both indicate a more moderate assessment of longevity risk (smaller increase in annuity premiums compared to the Wang method).

It´s important to mention that the minimum Longevity Risk simulated in the Exponential Transform is -0.0094. This means that along being the smallest value in all simulations, it will have a minimum impact on the annuity premiums in the most optimistic longevity scenarios.

Standard Deviation

It has the lowest risk loadings among the four methods, as it focuses on the volatility of annuity payments rather than the survival probabilities. It is the less conservative approach, as it primarily accounts for the average variation in longevity outcomes rather than the extreme scenarios.

We can observe that the ***Standard Deviation*** approach give us negative values, which may suggest that the data used have undergone a non-standard transformation or normalization process. Although, the values obtained in the quartile values suggest that we have a consistent distribution without any significant outliers.

After analyzing all transformations, we consider that the Wang transformation is the transformation that brings the most benefits for decision making when it comes to the longevity swap contract. It has the highest number of risk loadings compared to the other simulations. Given that the trend is towards an increase in average life expectancy, it makes sense to adopt the simulation that best predicts that individuals will live beyond the estimated average. It's a more conservative strategy, but Wang's transformation offers a robust, intuitive and versatile approach to dealing with a variety of analytical challenges, especially in situations where the order of the data is crucial.

8. Longevity Swap Contracts Issued on a Cohort Aged 65

One of the fundamental principles in longevity risk management: the longer the duration of the swap, the greater the uncertainty surrounding future mortality rates. After observing the graphs, they demonstrate that ***risk premiums rise*** as the maturity of swap contract increases. This happens because longer durations introduce more uncertainty about future mortality trends and, consequently, it makes the contract riskier. Implied risk premiums for longevity swap contracts vary across different pricing principles and maturities.

The Wang transform, being more conservative, shows the highest risk premiums, indicating a higher sensitivity to mortality risk. The Proportional Hazard (PH) and Exponential (Exp) transforms show moderate risk premiums. The Standard Deviation principle, being the least conservative, has relatively lower risk premiums. The risk premiums generally increase with maturity, reflecting the increased uncertainty and risk over a longer hor

9. Conclusion

In summary, this project analyzes the critical issue of longevity risk, a significant concern for the insurance and pension sectors due to rising life expectancies. The project applied a comprehensive approach, starting with an exploratory analysis of Portuguese mortality data, followed by the application of stochastic mortality models, namely the Poisson Lee-Carter and Age-Period-Cohort models. These models were fitted to historical data, and their goodness-of-fit was assessed using statistical measures like AIC, BIC, and deviance. The models were then used to forecast future mortality rates, providing valuable insights into potential trends and uncertainties.

The project further explored the estimation of longevity risk loadings (LRRs) for annuity contracts, considering various pricing principles such as the Wang transform, Proportional Hazard, Exponential transform, and Standard Deviation principle. The analysis showed that the choice of pricing principle significantly impacts the calculated LRRs, with the Wang transform being the most conservative and the Standard Deviation principle the least conservative.

Additionally, the project investigated the pricing of longevity swap contracts, financial instruments used to transfer longevity risk. The analysis demonstrated that risk premiums for these contracts increase with maturity, reflecting the heightened uncertainty associated with longer time horizons. The choice of pricing principle also significantly influences the risk premiums, with the Wang transform generally resulting in the highest premiums.

Overall, this project provides valuable insights into the complexities of longevity risk and its implications for the insurance and pension industries. The findings highlight the importance of selecting appropriate pricing principles and risk management strategies to ensure the financial sustainability of annuity providers and pension funds in the face of increasing life expectancies.

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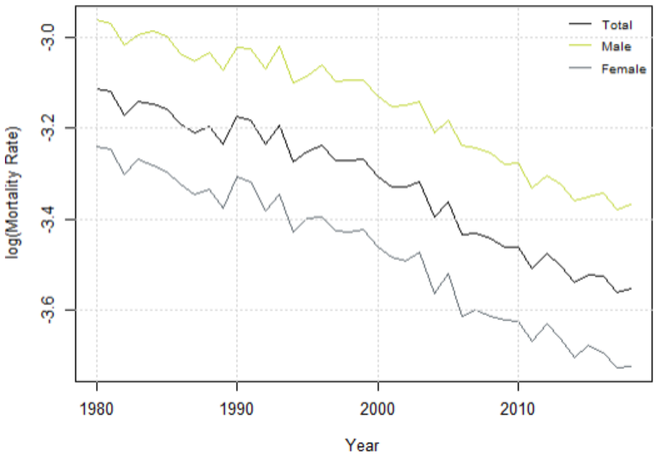
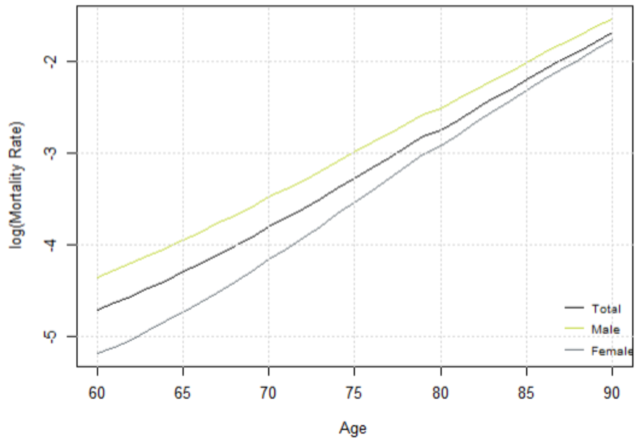
11. Annexes

Exploratory Analysis Per Age, and Per Year



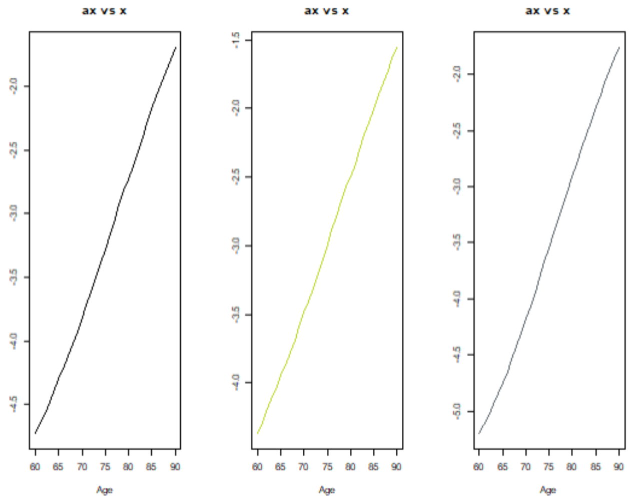
Graph 1: Mortality Rates per Year



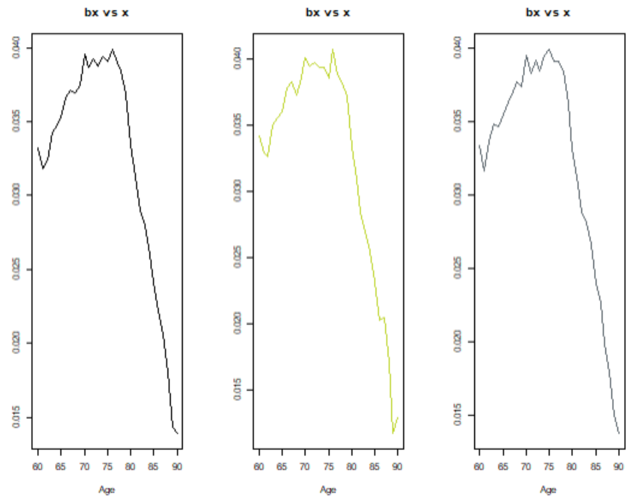
Graph 2: Mortality Rates per Age

Poisson Lee-Carter Parameters

Total ***Male*** ***Female*** ***Total*** ***Male*** ***Female***

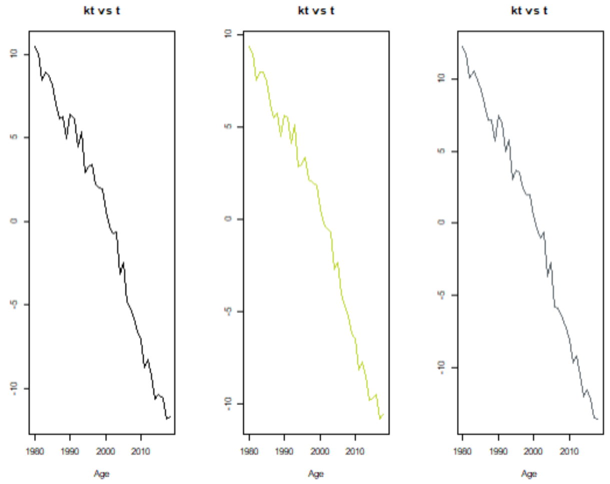


Graph 3: Poisson Lee-Carter Parameters ax



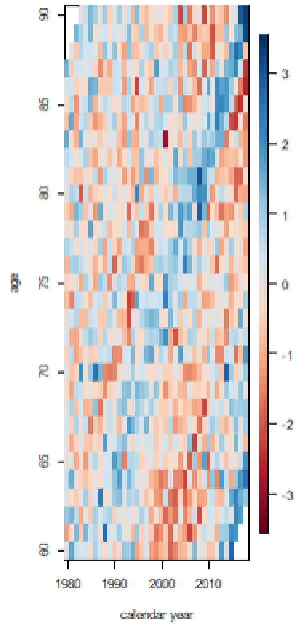
Graph 4: Poisson Lee-Carter Parameters bx

Total ***Male*** ***Female***

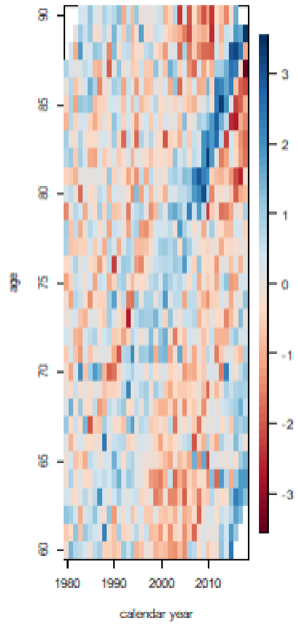


Graph 5: Poisson Lee-Carter Parameters kt

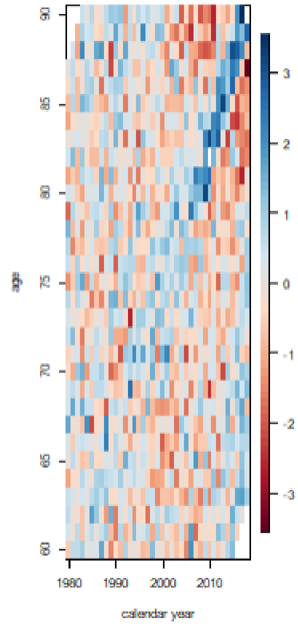
Poisson Lee-Carter Colourmap Residuals



Graph 6: Colourmap Total

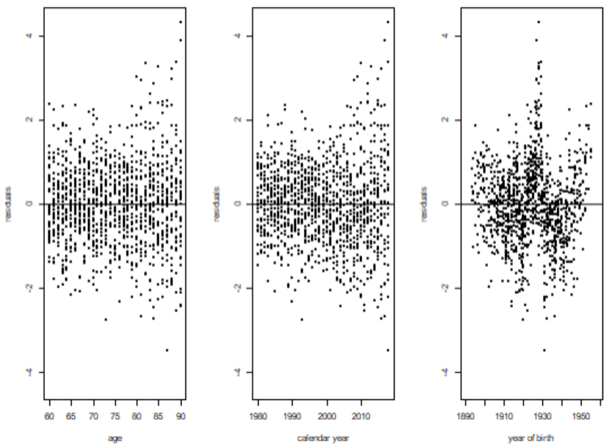


Graph 7: Colourmap Male

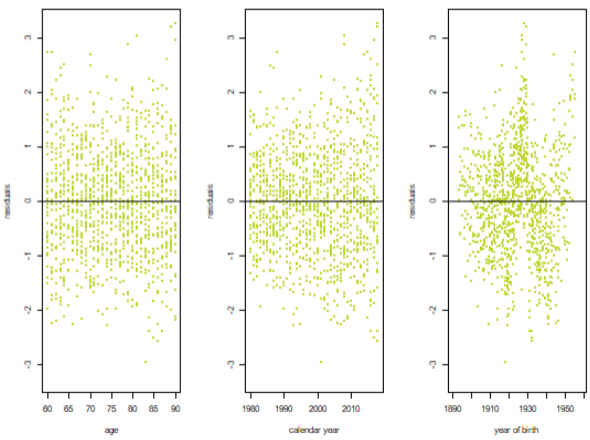


Graph 8: Colourmap Female

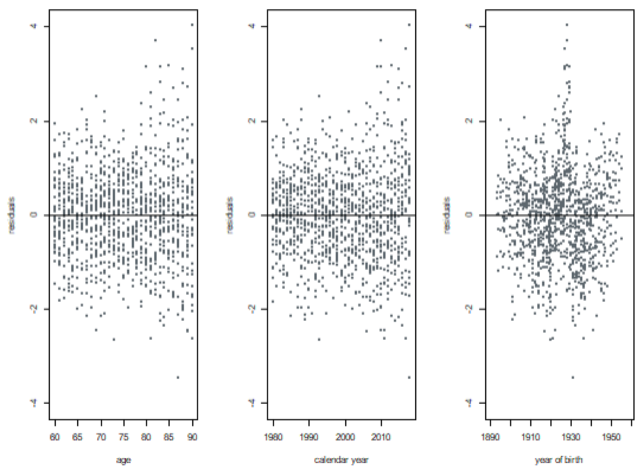
Poisson Lee-Carter Scatterplot Residuals



Graph 9: Residuals Scatterplot Total

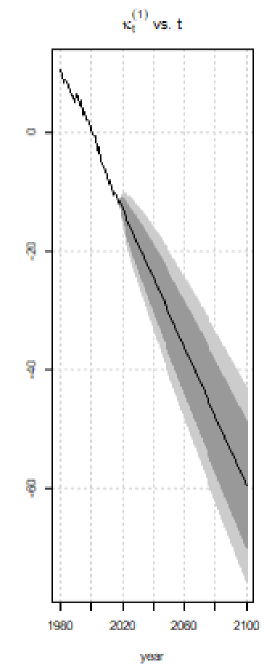


Graph 10: Residuals Scatterplot Male

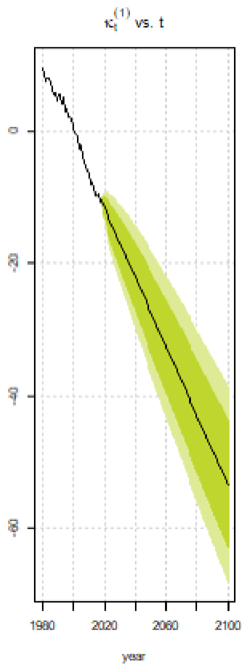


Graph 11: Poisson Lee-Carter Residuals Scatter Female

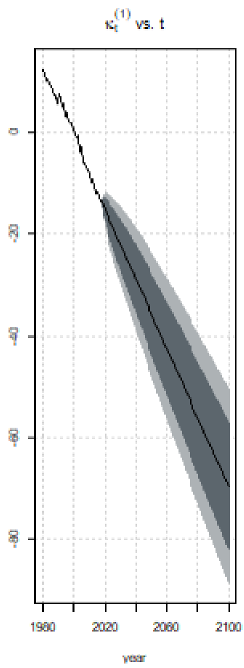
Poisson Lee-Carter Forecasting



Graph 12: Forecast Total



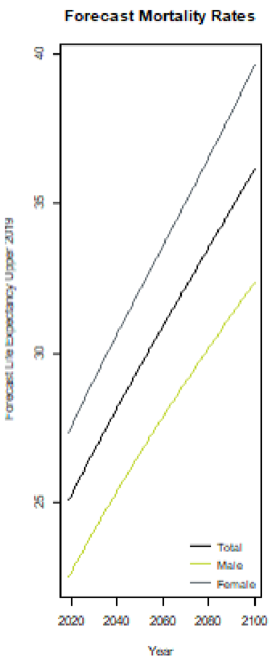
Graph 13: Forecast Male



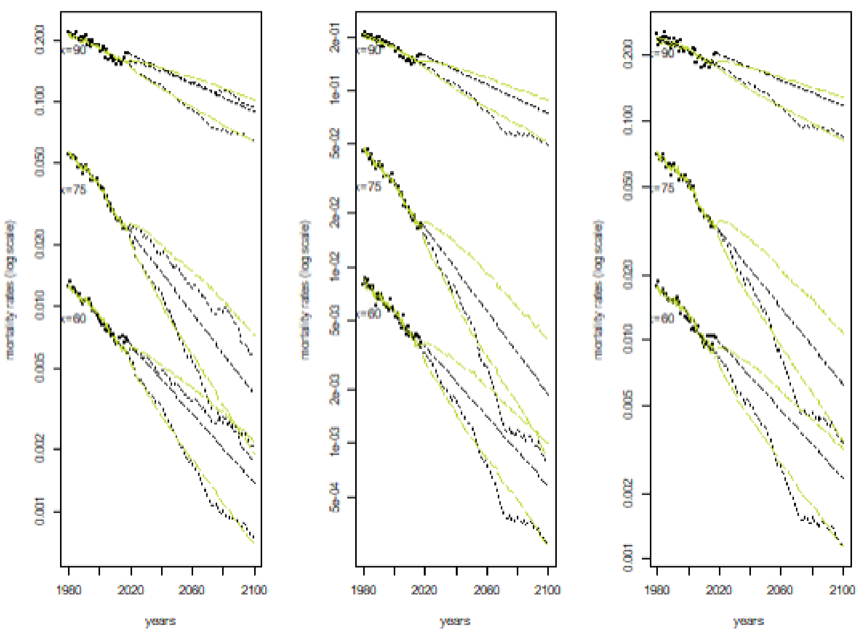
Graph 14: Forecast Female

Poisson Lee-Carter Forecast Life Expectancies & Semi-Parametric Bootstrapped

Total ***Female*** ***Male***



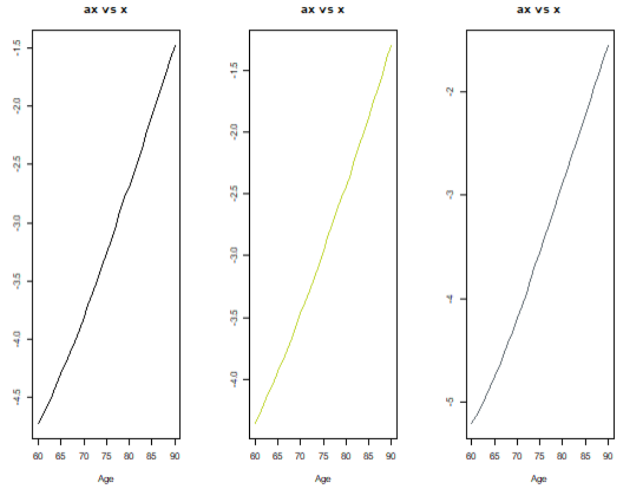
Graph 15: Forecast Life Expectancies



Graph 16: Semi-Parametric Bootstrapped (Total, Female, Male) Respectively

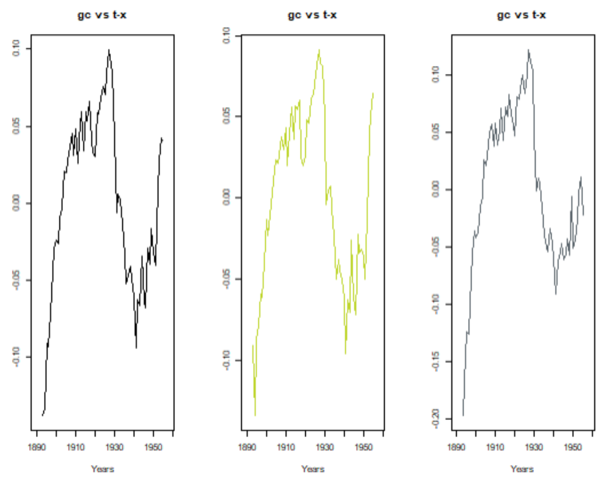
Age Period Cohort Parameters

Total **Male** **Female**



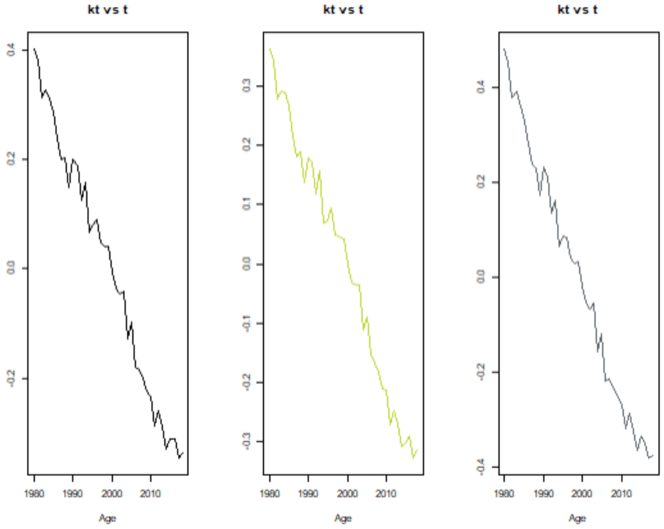
Graph 17: Age Period Cohort Parameters ax

Total **Male** **Female**



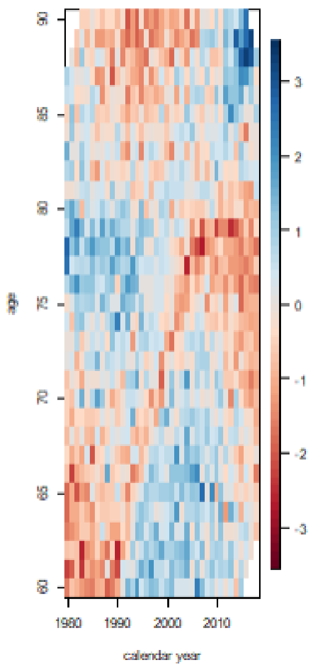
Graph 18: Age Period Cohort Parameters gc

Total **Male** **Female**

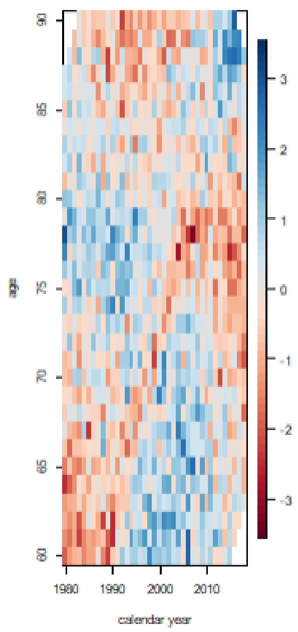


Graph 19: Age Period Cohort Parameters kt

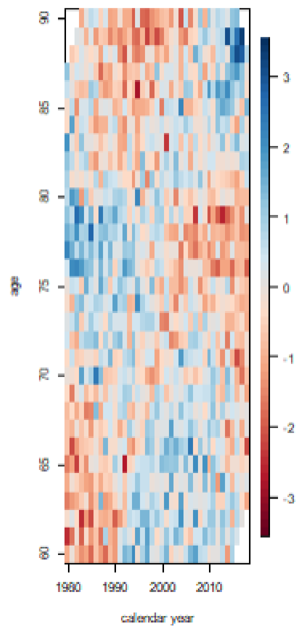
Age Period Cohort Colourmap Residuals



Graph 20: Colourmap Total

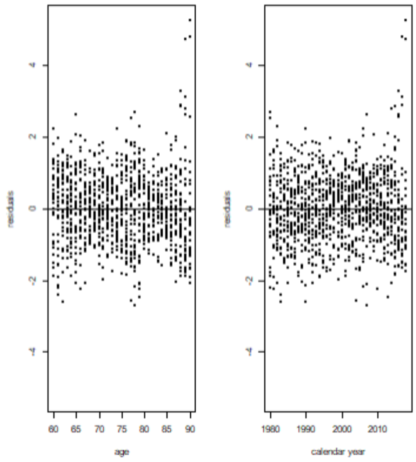


Graph 21: Colourmap Male

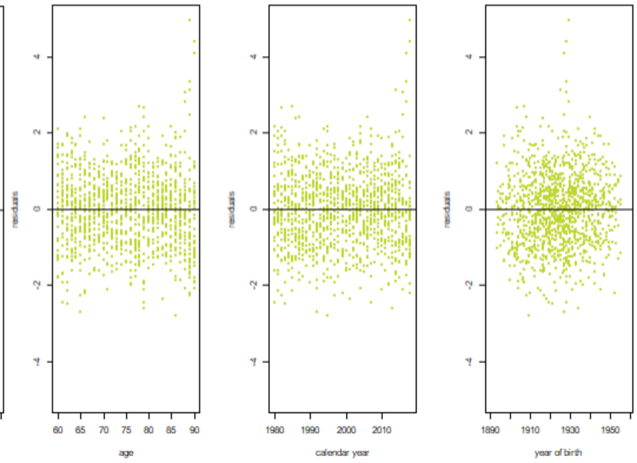


Graph 22: Colourmap Female

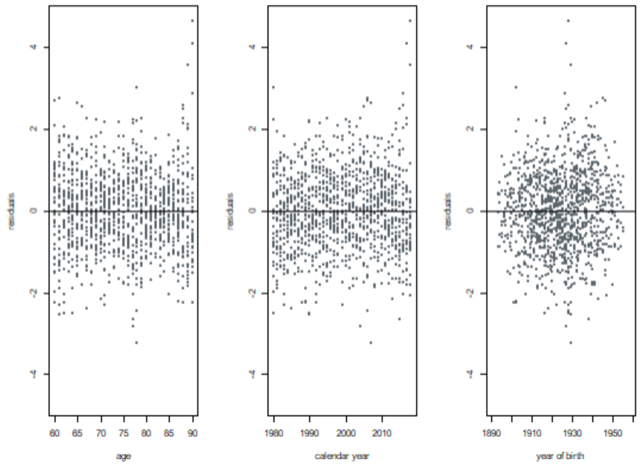
Age Period Cohort Scatterplot Residuals



Graph 23: Age Period Cohort Residuals Scatter Total

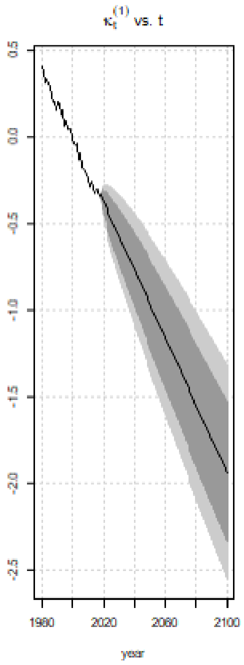


Graph 24: Age Period Cohort Residuals Scatter Male

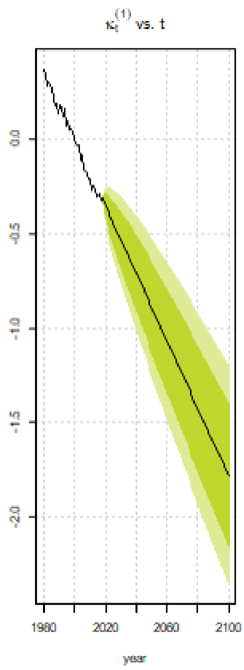


Graph 25: Age Period Cohort Residuals Scatter Female

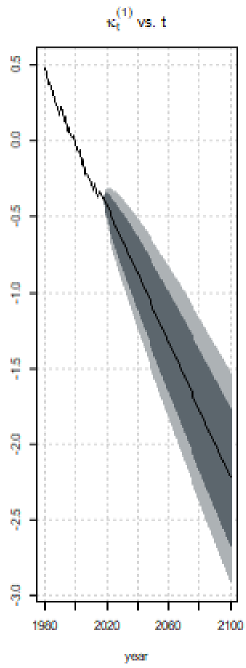
Age Period Cohort Forecasting



Graph 26: Forecast Total

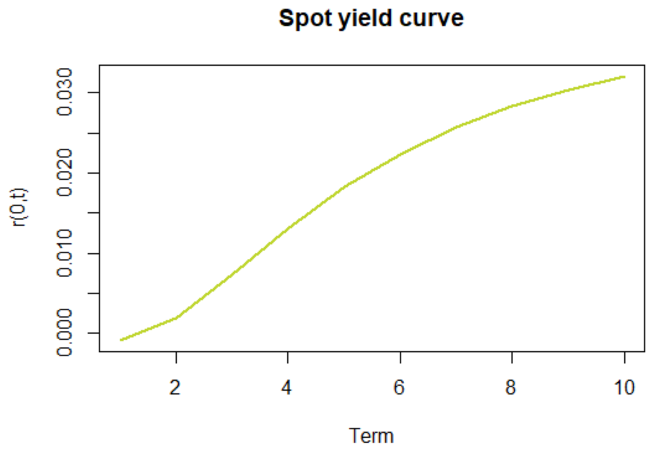


Graph 27: Forecast Male



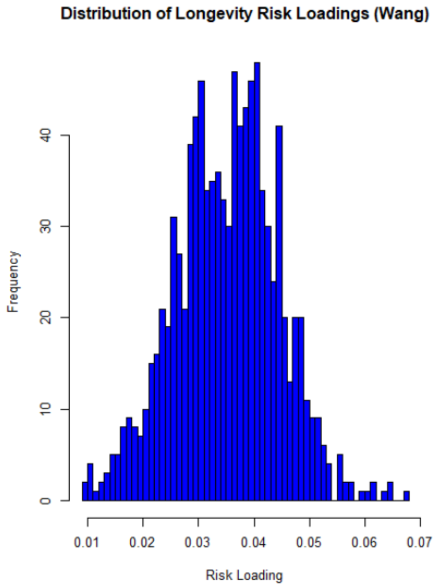
Graph 28: Forecast Female

Spot Yield Curve Forecast

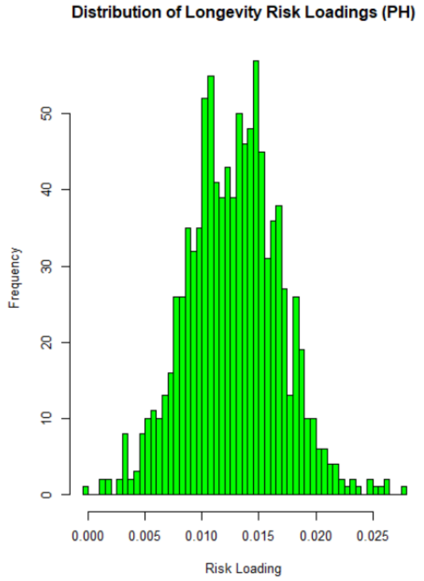


Graph 29: Spot Yield Curve Forecast

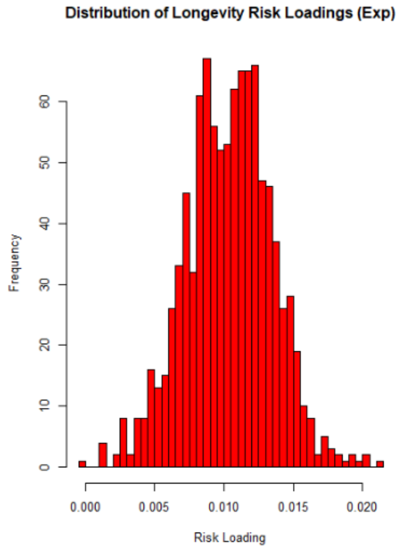
Risk Loadings Transformations



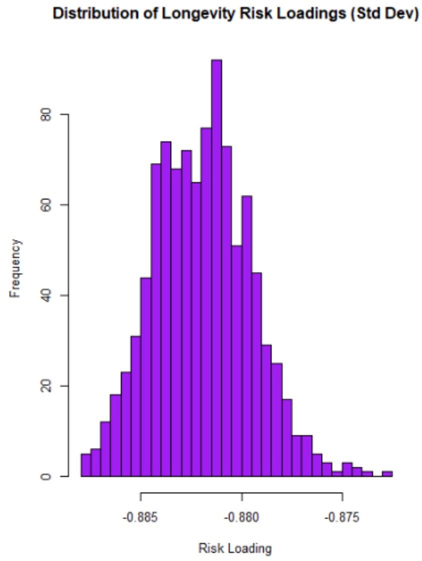
Graph 30: Wang Risk Loading Transformation



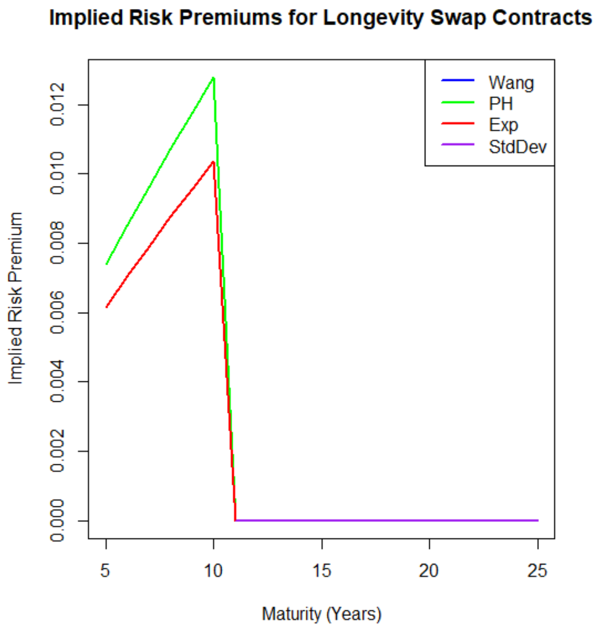
Graph 31: Proportional Hazard Risk Loading Transformation



Graph 32: Exponential Risk Loading Transformation



Graph 33: Standard Deviation Risk Loading Transformation



Graph 34: Implied Risk Premiums for Longevity Swap Contracts