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Complex network structure of musical compositions: Algorithmic generation of appealing music

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ABSTRACT

In this paper we construct networks for music and attempt to compose music artificially. Networks are constructed with nodes and edges corresponding to musical notes and their co-occurring connections. We analyze classical music from Bach, Mozart, Chopin, as well as other types of music such as Chinese pop music. We observe remarkably similar properties in all networks constructed from the selected compositions. We conjecture that preserving the universal network properties is a necessary step in artificial composition of music. Power-law exponents of node degree, node strength and/or edge weight distributions, mean degrees, clustering coefficients, mean geodesic distances, etc. are reported. With the network constructed, music can be composed artificially using a controlled random walk algorithm, which begins with a randomly chosen note and selects the subsequent notes according to a simple set of rules that compares the weights of the edges, weights of the nodes, and/or the degrees of nodes. By generating a large number of compositions, we find that this algorithm generates music which has the necessary qualities to be subjectively judged as appealing.

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1. Introduction

A key finding in the research on complex networks is that most man-made networks involving couplings of different systems as well as networks involving connections of people are naturally connected in a scale-free manner, which means that the number of connections follows a power-law distribution [1]. In the past decade, the study of complex networks in physics has aroused a lot of interest across a multitude of application areas. Scale-free power-law distribution is a remarkable property that has been found across a variety of connected communities [2–6] and has been shown to be a key to optimal performance of networked systems [7]. Recently, some work has also been reported on the applications of network analysis in the areas of linguistics, literature and arts [8–10].

Music is a form of creative art which is often identified as a signature of a particular composer, a group of people, a country or a culture at different times in history. People from different parts of the world and in different eras have their own music. One fundamental question of interest is whether these different music types share similar properties, and the implication of this question is that we wonder whether a common process/rule exists in the human brain that is responsible for composing music. Obviously, the converse of the question is equally important, i.e., whether distinguishing properties can be identified for music of particular types, genres and/or composers.

In this paper we construct networks for music and exploit some network properties for composition. We specifically analyze classical music from Bach, Chopin and Mozart, as well as Chinese pop music, and conclude that similar network

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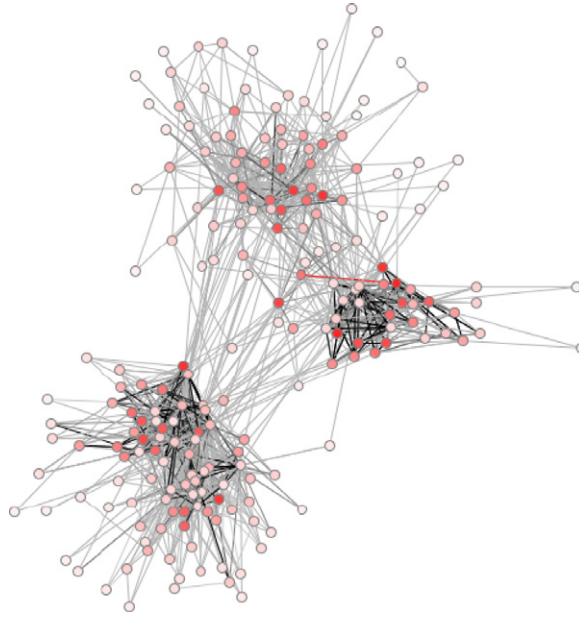


Fig. 1. A sample network from Bach's violin solos. Darkness of coloring of nodes indicates relative degrees, and darkness of coloring of edges indicates relative weights.

properties are shared by all of these different types of music. In particular, we show that the power-law degree distribution and small-world phenomenon are displayed universally in music. We conjecture that “good” music should necessarily display these universal features, and thus, preserving these properties forms the basis of artificial composition. We also propose a simple algorithm that preserves the universal network properties for re-composing music from networks.

2. Network construction for music

Music is basically a series of acoustic elements forming a constant stimulus meaningful or appealing to human brains. Our aim here is to model the acoustic elements and connections among them as a network and to study the properties of the network formed in order to find hidden properties of music. In this paper, we consider the musical note as the basic acoustic element.

A musical note is a sign used in modern musical notation to represent the relative pitch and duration of a sound. *Pitch* represents the perceived fundamental frequency of a sound [11]. In the standardized pitch system (A440 system) [12] a pitch, p , is related to frequency, f , by

$$p = 69 + 12 \times \log_2 \left(\frac{f}{440 \text{ Hz}} \right), \quad p \in \mathbb{N}. \quad (1)$$

Assuming that the frequency perception range of human ears is from 20 Hz to 20 kHz, the total number of pitches perceived is 120, from $p = 16$ to 135. For each pitch, only about 20 time durations are commonly used (e.g., semibreve, dotted minim, minim, dotted crotchet, crotchet, dotted quaver, quaver, dotted semiquaver, semiquaver, demisemiquaver, etc. [13]), and hence there will be about 2400 music notes.

The two basic elements of a network are the *node* and the *edge*. To construct a network for a given piece of music, we need to define nodes and edges. A piece of music can be considered as a sequence of notes, and we may thus naturally take every individual note that appears in a piece of music as a node, and edges can be defined by connections from one note to another chronologically as the music is played. Suppose, in a piece of music, the notes start at t_0, t_1, t_2, \dots (the unit being a quarter-note). Suppose note- i and note- j start at time t_i and t_j , respectively, where $t_j > t_i$. If no other note starts within (t_i, t_j) , then note- j is said to co-occur with note- i and a co-occurring edge is defined from note- i to note- j . The duration of the co-occurring edge from note- i to note- j is $\delta t = t_j - t_i$. Every time when note- j co-occurs with note- i with duration δt , the weight of this co-occurring edge is increased by 1.

Using the aforementioned procedure, we are able to construct a weighted and directed network from a given piece of music. Fig. 1 shows a sample network modeled from Bach's violin solos.

3. Network analysis

Selected works of Bach, Chopin and Mozart, as well as collections of works by Chinese pop musicians Jay Chou and Teresa Teng, are analyzed. Our aim is to identify any common, or otherwise distinguishing, features in these works. In order to

Table 1

Selected compositions analyzed in this paper.

Composer	Work	Number of pieces	Genre	Collection name
Bach	Sonatas and partitas for violin solo	31	Classical	Bach Violin
	WTC Book I	48	Classical	Bach WTCI
	WTC Book II	24	Classical	Bach WTCII
Chopin	Op28	24	Classical	Chopin Op28
	Nocturne	19	Classical	Chopin Nocturne
Mozart	KV252	4	Classical	Mozart Collection
	KV457	3	Classical	
	KV487	12	Classical	
	KV525	4	Classical	
	KV545	3	Classical	
Chinese pop music	Jay Chou's Secret	13	Pop	Jay Secret
	Teresa Teng's collection	17	Pop	Teng's Collection

Table 2

Simplified MIDI file format.

	Time mark	Event	Note identity
	Tick 1	Start	Pitch name 1
	Tick 2	Start	Pitch name 2
	Tick 3	End	Pitch name 1
	Tick 4	End	Pitch name 2
	Tick 5	Start	Pitch name 3
	Tick 6	End	Pitch name 3

provide a sufficiently long piece of music for network construction, we concatenate a number of works of the same composer having the same style to form one piece. Specifically, we have considered Bach's Violin solos, Bach's Well-Tempered Clavier I, Bach's Well-Tempered Clavier II, Chopin's No. 28, Chopin's Nocturne, Mozart's Collection, Jay Chou's Secret and Teresa Teng's best hit songs. Table 1 summarizes the selected works to be analyzed.

Music data for the selected compositions are sourced from the internet and are provided in MIDI (Musical Instrument Digital Interface) format. The MIDI format file stores music information in digital forms that can facilitate repeated performance at later times [14]. Referring to Table 2, tick n is the time mark which indicates the time at which an event occurs. An event is either the start or end of a musical note. For instance, pitch 1 starts at tick 1 and ends at tick 3. In our study, MIDI files are created by direct conversion from the scores.

For each network constructed for a collection of music, we are interested in computing the following parameters:

1. Length of composition, T .
2. Total number of nodes, N .
3. Total number of edges, $\sum k$.
4. Mean degree, \bar{k} .
5. Mean shortest distance between nodes, \bar{d} .
6. Network diameter, d_{\max} .
7. Clustering coefficient, C .
8. Centrality coefficient, β .
9. Power-law exponent of
 - edge weight distribution, γ_{ew} ,
 - node degree distribution, γ_{nd} ,
 - node strength distribution, γ_{nw} .

The length of composition is the total number of musical notes played in the music collection. The total number of nodes and edges can be found by simple counting. The mean degree is the average degree of a node and can be calculated from $\bar{k} = \frac{\sum k}{N}$. The calculation of the mean shortest distance between nodes requires some computational efforts, and in this work we have adopted the Floyd–Warshall algorithm [15]. One thing to be noted is that in our musical network a node is allowed to connect to itself. However, we do not count the distance from one node to itself when calculating the mean shortest distance. The network diameter is the largest value of the shortest distances between each pair of nodes.

To find the clustering coefficient, we use the following two formulas:

$$C_1 = \frac{3 \times \text{number of triangles in the network}}{\text{number of connected triples of nodes}}, \quad (2)$$

Table 3

Parameters of networks with co-occurring edges calculated from selected works.

Network	T	N	$\sum k$	\bar{k}	\bar{d}	d_{\max}	C_1	C_2	β
Bach Violin	18 000	301	4 424	14.7	2.9	10	0.35	0.30	1.3
Bach WTCI	18 000	589	11 857	20.1	2.9	8	0.26	0.18	1.2
Bach WTCII	18 000	726	12 215	16.8	2.8	9	0.48	0.29	1.1
Chopin Op28	18 000	721	12 552	17.4	3.0	12	0.36	0.27	1.2
Chopin Nocturne	18 000	899	14 283	15.9	3.5	15	0.15	0.15	1.1
Mozart Collection	18 000	509	7 673	15.1	3.3	13	0.24	0.16	1.1
Jay Secret	8 386	340	3 587	10.6	3.7	19	0.24	0.18	1.1
Teng's Collection	12 470	746	7 346	9.8	3.8	16	0.17	0.15	1.2

Table 4

Power-law exponents of networks with co-occurring edges from selected works.

Networks	γ_{ew} (KS statistics)	γ_{nd} (in/out/total) (KS statistics)	γ_{nw} (in/out/total) (KS statistics)
Bach Violin	2.0 (0.0126)	1.0/1.0/1.4 (0.0888/0.1733/0.1525)	1.0/1.1/0.5 (0.0907/0.0850/3.4513)
Bach WTCI	2.0 (0.0205)	1.5/1.9/1.3 (0.1486/0.0804/0.1263)	1.4/1.7/1.0 (0.1068/0.0707/0.5164)
Bach WTCII	2.0 (0.0163)	1.3/1.3/1.4 (0.0388/0.0553/0.0620)	1.5/1.4/1.5 (0.0413/0.0449/0.0770)
Chopin Op28	2.1 (0.0110)	1.0/1.2/1.0 (0.2527/0.1047/0.2589)	1.4/1.4/1.0 (0.1024/0.1853/0.6224)
Chopin Nocturne	2.2 (0.0105)	1.3/1.3/1.4 (0.0589/0.0471/0.0251)	1.4/1.3/1.6 (0.0440/0.0386/0.2431)
Mozart Collection	1.8 (0.0876)	1.6/1.4/1.3 (0.0456/0.309/0.0515)	1.1/0.9/1.3 (0.2047/0.6119/0.3489)
Jay Secret	2.1 (0.0473)	1.8/1.8/1.4 (0.1225/0.1140/0.0717)	2.3/1.4/0.8 (0.3062/0.759/1.1273)
Teng's Collection	2.1 (0.0109)	1.3/1.7/1.8 (0.0066/0.0117/0.0085)	1.2/1.8/1.5 (0.0151/0.0172/0.0193)

$$C_2 = \frac{1}{N} \sum_i \frac{\text{number of triangles connected to node } i}{\text{number of triples connected to node } i}. \quad (3)$$

It has been found that in a weighted network, the average strength $s(k)$ of the nodes with degree k increases with the degree according to the following power-law relation [16]:

$$s(k) \sim k^\beta, \quad (4)$$

where the strength of a node is the total weight of the edges connected to it, and β is the centrality exponent. When the strength of a node is simply proportional to its degree, the network has $\beta = 1$.

Node degree distributions (in, out and total degrees), node strength distributions (in, out and total strengths) and edge weight distribution are studied in this paper. Assuming that they display scale-free degree distribution, we estimate the power-law exponent of degree distributions in a least squares sense. Also we will give the goodness of fit of each distribution by using Kolmogorov–Smirnov (KS) tests.

Tables 3 and 4 and Fig. 2 summarize the results for the selected music. The following points are worth noting.

1. To provide a fair comparison of the network properties, we have “normalized” the networks by limiting the length of each composition under study to 18 000 notes. However, the entire collection of works for Jay Chou's Secret and Teresa Teng's Collection have lengths shorter than 18000 and we simply keep their original lengths in our analysis.
2. The networks are found to follow a scale-free distribution in their node degree, node strength and edge weight distributions. Observing power-law distributions with exponent near 1 and distortions due to limited data sample sizes, we manually take the most significant interval in the distribution and measure their power-law exponent using the least squares method. We also test the goodness of fit using the KS test for the fitting interval. The intervals as well as KS statistics are presented in Table 4. The power-law exponents are surprisingly consistent and those of the node degree distribution and node strength distribution fall in the range from 1 to 1.8 while those of the edge weight distribution stay around 2.0.
3. With the length of the music fixed, all the networks show similar mean shortest distances around 3 and clustering coefficient around 0.3. They also demonstrate similar network diameters and clustering coefficients. We also find that the centralities of all the networks are approximately equal to 1, implying that their edge weight distribution and degree distribution display a similar network characteristic.
4. With the length of the music fixed, other parameters like the number of nodes, total number of edges, average degree and network diameter vary quite significantly for different music collections. This shows that the same scale-free structure of music can produce significantly different kinds of music, and other parameters may play some roles in characterizing them.

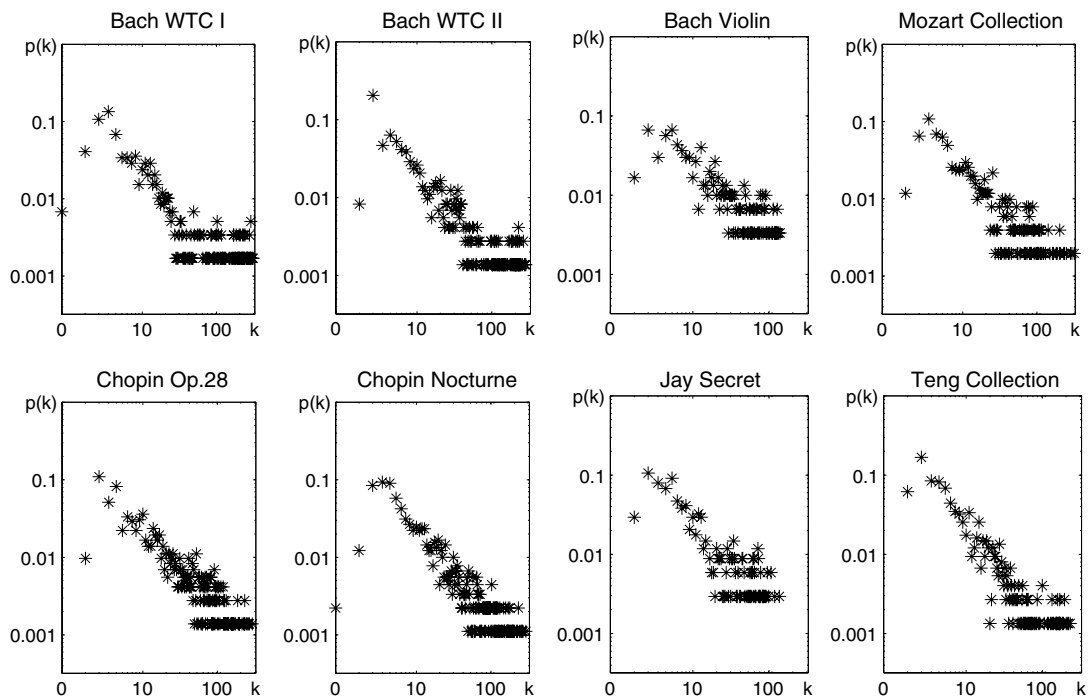


Fig. 2. Degree distributions (summation of in and out degrees) of networks with co-occurring edges. $p(k)$ versus k in log–log scale. Slopes are measured and reported as γ_{nd} in Table 4.



Fig. 3. A score from music composed by the proposed algorithm (Variant 2).

5. Community structure has been observed in most of the music networks. A few communities of nodes of similar time durations are found, consistent with the usual observation that sections of different tempos occur in a piece of work, e.g., *allegro*, *andante*, etc.

Remarks. From the above analysis, universality in terms of some network parameters is clearly demonstrated across music of different genres and composers. This result provides an important clue for understanding the fundamental rules of music construction, such as the scale-free distribution of notes from the viewpoint of co-occurrence networks and the small-world property of the connections of the co-occurring notes. This may also point to the way the human brain works in the process of composing music. A key observation from the scale-free distribution is that a relatively small number of notes are repeatedly used in the same piece of work while most other notes are only sparingly used. It is worth pointing out that while

Table 5

Parameters of networks with co-occurring edges calculated from selected works.

Network	T	N	$\sum k$	\bar{k}	\bar{d}	d_{\max}	C_1	C_2	β
Bach BWV 1002	6788	206	1618	7.9	3.3	7	0.36	0.31	1.2
Variant 1	6800	147	1104	7.5	3.2	7	0.35	0.34	1.3
Variant 2	6824	138	953	6.9	3.3	8	0.37	0.35	1.2
Variant 3	6800	117	932	8.0	3	7	0.41	0.38	1.3
Variant 4	6800	102	806	7.9	3	7	0.43	0.41	1.2

Table 6

Power-law exponents of networks with co-occurring edges from selected works.

Network	γ_{ew}	Fitting interval (KS statistics)	γ_{nd} (in/out/total)	Fitting interval (KS statistics)	γ_{ns} (in/out/total)	Fitting interval (KS statistics)
Bach BWV 1002	2.0	2–6 (0.09)	1.3/1.8/0.8	4–8/4–8/5–9 (0.05/0.16/0.02)	1.2/1.4/1.3	3–7/3–7/5–9 (0.05/0.05/0.04)
Variant 1	1.7	5–9 (0.07)	0.6/2.2/0.6	4–8/4–8/5–9 (0.02/0.21/0.04)	1.3/1.3/1.1	3–7/3–7/5–9 (0.06/0.02/0.03)
Variant 2	1.5	5–9 (0.00)	0.8/2.1/0.7	4–8/4–8/5–9 (0.03/0.21/0.03)	1.0/1.1/0.5	3–7/3–7/5–9 (0.06/0.02/0.02)
Variant 3	1.0	6–10 (0.01)	0.6/1.1/0.5	6–10/4–8/5–9 (0.04/0.09/0.07)	0.8/1.0/1.0	5–9/5–9/7–11 (0.02/0.02/0.03)
Variant 4	1.8	7–11 (0.03)	1.9/1.3/0.6	8–12/4–8/8–12 (0.05/0.14/0.02)	0.1/0.9/0.7	3–13/7–13/7–11 (0.02/0.04/0.02)

universality relates to the basic structure of music and hence gives the fundamental requirement for constructing “good” music, it does not provide the specific information needed to characterize music of particular genres or composers. The next question of interest, which certainly deserves further investigation, is how music of different styles, genres or composers can be distinguished from the specific contents of the corresponding networks.

4. Music composition from networks

The foregoing section has introduced a procedure for constructing networks for music and reported some properties in terms of network parameters. In this section, we are interested in the process of generating music from networks. As shown earlier, networks constructed from music have shown universalities such as scale-free degree distribution and the small-world phenomenon. We therefore conjecture that the necessary condition for composing “good” music is to preserve such universal features. In the following we propose a set of algorithms for generating music from networks based on a controlled random walk approach.

A simple interpretation of composition is a melody which contains a stream of musical notes. We first construct a seed network from a given collection of music. From a seed network (which is directed and weighted), we can compose music by randomly choosing a starting node and picking the next node among the nodes connected to it. Music is generated as nodes continue to be selected one after another. A few possible sets of criteria can be used to select the next connecting node. The process stops until a sufficient number of music notes have been chosen or when a node with no outgoing edge has been reached. Of course there are many possible variations on this theme: different ways to select nodes, for example. We consider in more detail the following four possible versions.

Variant 1: We begin with a randomly selected node. Then, the next node is selected from among the nodes connected to it. Each of the connecting nodes has equal probability of being selected.

Variant 2: Like in Variant 1, we start from a randomly selected node and move to another node via one of the connecting edges. The probability of each edge being selected is proportional to its weight. The succeeding node is the ending node of the edge being selected.

Variant 3: From a randomly selected node, we move to another node via one of the connecting edges. The probability of each edge being selected is proportional to the degree of its ending node. The succeeding node is the ending node of the edge being picked.

Variant 4: From a randomly selected node, we move to another node via one of the connecting edges. The probability of each edge being chosen is proportional to the strength of its ending node. The succeeding node is the ending node of the edge being picked.

It should be clear that the music generated from the above algorithms depends on the seed network which has been constructed from a collection of works, such as those given in Table 1. Thus, to make the composed music sound harmonious, the collection of music for which the seed network is constructed should be in the same musical scale. In our experiments, Bach's violin works No. BWV1002 in B minor are used for constructing the seed network. We apply each algorithm to generate one piece of music with approximately the same length as the original composition. Fig. 3 shows a sample score

from the music composed by the Variant 2 algorithm. Some samples of generated music can be downloaded from the following website: <http://ckts.eie.polyu.edu.hk/MUSIC/>.

Tables 5 and 6 show the properties of original seed network and the networks of our composed music. We can see that all four variants of the algorithm are able to approximate a similar musical network structure to the original BWV 1002 network. Among them Version 1 (based on a pure random walk) has the closest resemblance to the original network, having the number of nodes, mean degree and clustering coefficient closer than the others to the original seed network. The reason why Version 1 reproduces its original music is that the other three “controlled random walks” keep picking edges and/or nodes with heavier weights and/or higher degrees and/or higher strengths, so they tend to stay in the central part of the network and iterate only a portion of the original network.

To the perception of the authors, some of the generated pieces of music are quite appealing but not all of them are acceptable. For instance, the score shown in Fig. 3 generally lacks a fixed rhythm and theme. Thus, our future research will be focused on how to identify better music from the large pool of music that preserve the same network properties.

5. Conclusion

We have studied music by constructing complex networks that connect co-occurring musical notes. Selected classical pieces and some Chinese pop music have been analyzed. In particular, we have examined the constructed networks in terms of network properties such as node degree, node strength and edge weight distributions, mean shortest distances, etc. Universal characteristics have been found among all the music collections studied, such as scale-free degree distribution with power-law exponents ranging from 1 to 2, mean shortest distances around 3 and clustering coefficients around 0.3. We conjecture that preserving these universal features is a necessary step in artificial composition of music, and we have proposed a simple algorithm for composing music from complex networks based on a “controlled random walk” approach.

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