Optimization and Decision Support Methodologies

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Post-Optimization

1st Case: Changing the coefficient of a variable in the objective function

⇨ The altered coefficient is directly replaced in the optimal tableau.

If the variable in question **is not in the basis**, the only value affected and having to be recalculated is the corresponding zj-cj.

- If this is ≥ 0, the tableau remains optimal, the basis remains optimal, as well as the values of x* and z*.
- If it is < 0, the tableau is no longer optimal and the *simplex* method has to be used to calculate the new optimal tableau, the new optimal basis, and the new values of x* and z*.

If the variable in question **is in the basis**, the entire zj-cj row and the value of z have to be recalculated.

- If all the values of the zj-cj row are ≥ 0, the tableau remains optimal, the basis remains optimal and so do the values of x*. The z* value changes to the recalculated value.
- If any of the values is less than 0, the tableau is no longer optimal and the *simplex* method has to be used to calculate the new optimal tableau, the new optimal basis, and the new values of x* and z*.

2nd Case: Changing the terms independent of constraints

⇨ The formula to apply: $\tilde{\mathbf{x}}_B = B^{-1}\tilde{\mathbf{b}}$

Column b of the current optimal tableau is replaced by the values of the vector $\tilde{\mathbf{x}}_B$ and the value of z is recalculated.

- If the values of the new column b are all ≥ 0, the tableau remains optimal and the basis also remains optimal. The x* and z* values change according to the new values in column b and z in the changed tableau. (NOTE: The solution x* can be **degenerate** if one of the values in the new column b is zero).
- If a negative value appears in column b, then the tableau is no longer optimal (the solution is now unfeasible). Procedure: The dual *simplex* method is applied to determine the new values of x* and z*.

3rd Case: Changing the coefficients of a variable in the constraints

⇨ The formula to apply: $\tilde{\mathbf{X}}_f = B^{-1}\tilde{\mathbf{P}}_f$

If the variable in question **is not in the basis**, the column of that variable in the optimal tableau is replaced by the values of the vector $\tilde{\mathbf{X}}_f$ and the corresponding value in the zj-cj row is recalculated.

Then proceed as in the **1st case**:

- If this value is ≥ 0, the tableau remains optimal, the basis remains optimal, the values of x* and z* also remain optimal.
- If it is < 0, the tableau is no longer optimal and the *simplex* method has to be used to calculate the new optimal tableau, the new optimal basis, and the new values of x* and z*.

On the contrary, if the **variable is in the basis**, the column of that variable in the optimal tableau is replaced by the values of the vector $\tilde{\mathbf{X}}_f$ and the identity matrix has to be reconstructed because it will be affected. Only after resetting this matrix, the zj-cj row is recalculated.

In the new tableau obtained 3 situations can occur:

- Column b and row zj-cj only contain values ≥ 0.
    - The tableau is optimal, but the optimal solution x* and the value of z* change according to the new values in column b.
- Column b only contains values ≥ 0, but negative values appear in row zj-cj.
    - Apply the *simplex* method to calculate the new optimal tableau, the new optimal basis, and the new values of x* and z*.
- Row zj-cj only contains values ≥ 0, but negative values appear in column b.
    - Apply the dual *simplex* method to calculate the new optimal tableau, the new optimal basis, and the new values of x* and z*.
- Column b and row zj-cj both contain values < 0.
    - Complicated case where it becomes necessary to remove $\mathbf{x}_f$ from the basis.

4th Case: Introduction of a new decision variable

⇨ The formula to apply: $X_{new} = B^{-1}P_{new}$

Then add a column in the optimal tableau for the new variable x_{new} and fill it with the values of the X_{new} vector. After inserting the coefficient of the new variable in the objective function, c_{new}, into the tableau, the corresponding value is calculated in the row zj-cj.

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Then proceed as in the **1st case**:

- If it is ≥ 0, the tableau remains optimal, the basis remains optimal, as well as the values of x* and z*.
- If it is < 0, the tableau is no longer optimal and the *simplex* method must be used to calculate the new optimal tableau, the new optimal basis, and the new values of x* and z*.

5th Case: Introduction of a new constraint

First, it should be verified whether the current optimal solution satisfies the new constraint.

- If true, then it follows that the current solution is still optimal for the problem with the new constraint and that the value of z* is also the optimal for the new problem.
- If it is not true, the new constraint is introduced in the tableau and the identity matrix is reconstructed.
- If the values of the new column b are all ≥ 0, and the values of the zj-cj row are also ≥ 0, the tableau remains optimal and the basis also remains optimal. The x* and z* values change according to the new values in column b and z in the changed tableau.
- If a negative value appears in column b, then the tableau is no longer optimal (the solution is now unfeasible). Procedure: The dual *simplex* method is applied to determine the new values of x* and z*.
- If any of the values in the zj-cj row are less than 0, the tableau is no longer optimal. Procedure: the *simplex* method is applied to determine the new values of x* and z*.

Particular cases

In different situations, it is necessary to pay attention to particular cases. For example:

- ⇨ If, after the changes, a 0 value appears on the zj-cj row corresponding to a non-basic variable, it is because the problem has an **alternative optimal solution** that must be calculated using the *simplex* method.
- ⇨ If the table is no longer optimal because a negative value appeared in the zj-cj row and there is a need to iterate through the *simplex* method, but in the "pivot" column there are only values ≤0, then it is concluded that, after the changes, the problem has **an unbounded optimal solution**.

Sensitivity analysis

To the independent terms of the constraints:

$$X_B{}^*{}_{\Delta_{b_k}} = B^{-1} b_{\Delta_{b_k}} \;/\; z^* = c'_B \, X_B{}^*{}_{\Delta_{b_k}}$$

Integer programming

Gomory algorithm

Step 1 - Solve the associated PL problem (or relaxed problem). In case the optimal solution satisfies the integrality constraints, then it is also an optimal solution of the PILP/MILP problem; otherwise, the process continues.

Step 2 - Introduce a new constraint in the problem, a cut constraint, and solve the associated PL problem again. If the obtained solution satisfies the integrality constraints, then it is also an optimal solution of the PILP/MILP problem. Otherwise, repeat the procedure until obtaining a complete solution, or concluding that the problem is impossible.

Cut constraint for PILP:

$$\sum_{j \notin I_B} f_{sj} x_j \geq f_{s0}$$

Cut constraint for MILP:

$$\sum_{j \notin N} d_{sj} x_j \geq f_{s0}$$

where,

$$d_{sj} = \begin{cases} x_{sj} & j \in \mathcal{N}_+^C \\ \dfrac{f_{s0}}{1-f_{s0}} \left| x_{sj} \right| & j \in \mathcal{N}_-^C \\ f_{sj} & j \in \mathcal{N}^I \quad se \;\; f_{sj} \leq f_{s0} \\ \dfrac{f_{s0}}{1-f_{s0}} (1-f_{sj}) & j \in \mathcal{N}^I \quad se \;\; f_{sj} > f_{s0} \end{cases}$$