Departamento de Engenharia Informática e de Sistemas

Optimization and Decision Support Methodologies

Date: January 18, 2023 **Exam – First Call** **Duration: 2h**

Note: <u>Present all</u> the calculations you carry out, as well as any comments, justifications or conclusions you deem appropriate.

1. *(expected quotation: 7.5 points = 2.5 + 2.5 + 2.5)*

Consider the following **single-objective linear programming** problem:

Maximize $z = 2x_1 - x_2$
subject to

$$x_1 + 2x_2 \geq 4 \quad \textbf{(1)}$$
$$3x_1 + x_2 \leq 3 \quad \textbf{(2)}$$
$$x_1 \geq 0,\ x_2 \geq 0$$

Assuming that $\mathbf{x_3}$ and $\mathbf{x_4}$ are the surplus and artificial variables of the functional constraint **(1)**, and $\mathbf{x_5}$ is the slack variable of the functional constraint **(2)**, the *simplex* optimal tableau is:

	c_i	2	-1	0	-M	0	
$\mathbf{x_B}$	$c_B \setminus \mathbf{x_i}$	$\mathbf{x_1}$	$\mathbf{x_2}$	$\mathbf{x_3}$	$\mathbf{x_4}$	$\mathbf{x_5}$	**b**
$\mathbf{x_2}$	-1	0	1	-3/5	3/5	-1/5	9/5
$\mathbf{x_1}$	2	1	0	1/5	-1/5	2/5	2/5
	zj-cj	0	0	1	-1+M	1	-1

a) For each of the following alterations in the initial problem, determine, performing a post-optimization study, what are the implications in the optimal solution presented (in the value of x*, in the value of z* and in the optimal basis), resulting from the variation:

i) Changing the **coefficient of the variable x_1 in the objective function**, from **2** to **3**;

ii) Changing **independent terms of the constraints** from $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$ to $\begin{bmatrix} 5 \\ 3 \end{bmatrix}$.

b) Determine, carrying out a <u>sensitivity analysis</u> study, for which **interval of b_1** (independent term of the 1st constraint) the optimal basis presented above will remain optimal.

2. *(expected quotation: 6.5 points = 5.0 + 1.5)*

Now consider the following goal programming problem:

Minimize $Z = \left\{ d_3^+ ,\ d_4^- ,\ d_5^- + d_5^+ \right\}$
sujeito a

$$-x_1 + x_2 + d_1^- = 3 \quad \textit{\textbf{(1)}}$$
$$2x_1 + 3x_2 + d_2^- = 18 \quad \textit{\textbf{(2)}}$$
$$x_1 + 3x_2 + d_3^- - d_3^+ = 12 \quad \textit{\textbf{(3)}}$$
$$x_1 + 2x_2 + d_4^- - d_4^+ = 4 \quad \textit{\textbf{(4)}}$$
$$2x_1 + 3x_2 + d_5^- - d_5^+ = 24 \quad \textit{\textbf{(5)}}$$
$$x_1 \geq 0,\ x_2 \geq 0,\ d_i^- \geq 0,\ d_i^+ \geq 0\ (i = 1, 2, 3, 4, 5)$$

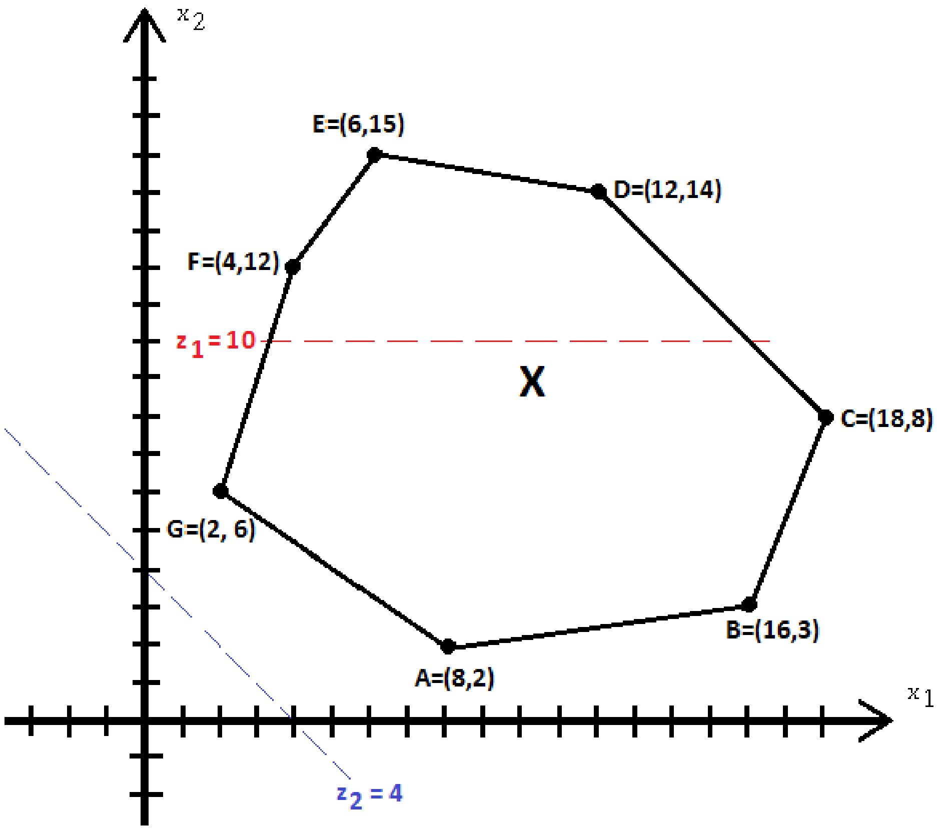
a) Solve this problem by the **graphical method**;

b) Tell what was the **intended objective for the goal with priority level 2** and if this was achieved or not. Justify your answer.

3. *(expected quotation: 6.0 = 4.5 + 1.5)*

Consider the following **linear programming** problem with **two objective functions**:

$$\begin{array}{ll} \textit{Min} & z_1 = x_2 \\ \textit{Max} & z_2 = x_1 + x_2 \\ \textit{subject to} & \\ & \underline{\mathbf{x}} = (x_1, x_2)^T \in X \end{array}$$



a) Determine the (strictly and/or weakly) **efficient region** of this problem and highlight it on the graph above. Justify your answer.

b) Obtain the **pay-off table** corresponding to this problem and identify the **ideal solution** and the **anti-ideal solution**.

Name:____No._________