Optimization and Decision Support Methodologies

1st Assessment Test

Date: **November 19, 2021** Duration: **1h 30m**

Note: Present all the calculations you carry out, as well as any comments, justifications or conclusions you deem appropriate.

1.

Consider the following **single-objective linear programming problem**:

Maximize $z = 2x_1 + x_2 + 4x_3$
subject to

$$2x_1 + 3x_2 + x_3 \leq 6 \quad \textbf{(1)}$$
$$3x_1 + x_2 + 3x_3 \leq 10 \quad \textbf{(2)}$$
$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$$

Considering $\mathbf{x_4}$ and $\mathbf{x_5}$ the slack variables of the functional constraints **(1)** and **(2)** respectively, the *simplex* optimal tableau is:

	c_i	2	1	4	0	0	
$\mathbf{x_B}$	$c_B \setminus \mathbf{x_i}$	$\mathbf{x_1}$	$\mathbf{x_2}$	$\mathbf{x_3}$	$\mathbf{x_4}$	$\mathbf{x_5}$	**b**
$\mathbf{x_4}$	0	1	8/3	0	1	-1/3	8/3
$\mathbf{x_3}$	4	1	1/3	1	0	1/3	10/3
zj-cj		2	1/3	0	0	4/3	40/3

a) Determine, by carrying out a post-optimization study, what are the implications for the optimal solution presented (in terms of the values of x* and z*), resulting from the following changes:

i) Introduction of a new variable $\mathbf{x_{NEW}}$, with coefficients in the constraints equal to $\begin{bmatrix} -1 \\ -3 \end{bmatrix}$ and coefficient **3** in the objective function;

ii) Alteration of the coefficients of variable $\mathbf{x_3}$ in the constraints from $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ to $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$.

b) Determine, by carrying out a sensitivity analysis study, for which interval of $\mathbf{b_2}$ (independent term of the 2nd constraint) the optimal basis presented above will remain optimal.

c) Suppose that $\mathbf{x_1}$, $\mathbf{x_2}$ and $\mathbf{x_3}$ represent the number of tons of ration of **type 1**, **type 2** and **type 3**, respectively, to be produced monthly by a given animal ration factory. Moreover, suppose that **z** represents the monthly profit, expressed in thousands of euros, that results from the sale of these rations (knowing that everything the factory produces, sells). Assuming that the constraint parameters are not subject to changes, specify under what conditions would it be advantageous for the factory to produce ration of **type 2**.

2.

Now consider the following **pure integer linear programming** problem:

Maximize $z = x_1 + x_2$
subject to

$3x_1 + x_2 \leq 8$ **(1)**

$x_1 + 2x_2 \leq 9$ **(2)**

$x_1 \geq 0,\ x_2 \geq 0$

x_1 and x_2 integer

Considering x_3 and x_4 the slack variables of the functional constraints **(1)** and **(2)** respectively, suppose that the Gomory algorithm was applied to this same problem and that at the end of the 1st step, the following optimal tableau was obtained:

	c_i	1	1	0	0	
x_B	$c_B \setminus x_i$	x_1	x_2	x_3	x_4	**b**
x_1	1	1	0	2/5	-1/5	7/5
x_2	1	0	1	-1/5	3/5	19/5
	zj-cj	0	0	1/5	2/5	26/5

a) Draw your conclusions and, if necessary, proceed with the 2nd step of the aforementioned algorithm, to solve the problem presented above;

b) Graphically interpret the resolution of the previous paragraph.

Formulas

- Post-optimization and sensitivity analysis

$$X_B^* = B^{-1}b$$

$$\tilde{X}_B = B^{-1}\tilde{b}$$

$$\tilde{X}_f = B^{-1}\tilde{P}_f$$

$$\tilde{X}_f = B^{-1}\tilde{P}\,f$$

$$X_B{}^*{}_{\Delta_{b_k}} = B^{-1}b_{\Delta_{b_k}} \;/\; z^* = c'_B\, X_B{}^*{}_{\Delta_{b_k}}$$

- Pure integer linear programming

$$\sum_{j \notin I_B} f_{sj} x_j \geq f_{s0}$$