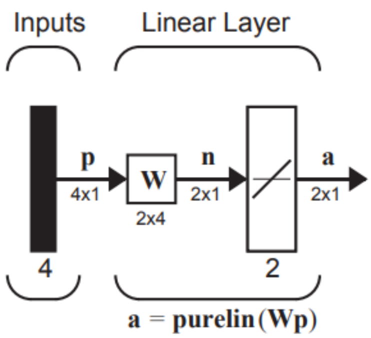
Exercícios Redes Neuronais

NN Design Cap. 7

Hebb Learning

Ex 01-

Considere a seguinte rede linear



Para o dataset de treino:

$$\left\{ \mathbf{p}_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \mathbf{t}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\} \quad \left\{ \mathbf{p}_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}, \mathbf{t}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}.$$

i) Determine os parâmetros pela regra de Hebb.
ii) Determine os parâmetros pela regra da pseudo-inversa.
iii) Teste i) para p1
iv) Teste ii) para p1

Resolução:

i)

$$\mathbf{P} = \begin{bmatrix} 1 & 1 \\ -1 & 1 \\ 1 & -1 \\ -1 & -1 \end{bmatrix}, \qquad \mathbf{T} = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}.$$

$$\mathbf{W}^h = \mathbf{T}\mathbf{P}^T = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 & -2 \\ 0 & 2 & -2 & 0 \end{bmatrix}.$$

Teste da rede:

$$\mathbf{W}^h \mathbf{p}_1 = \begin{bmatrix} 2 & 0 & 0 & -2 \\ 0 & 2 & -2 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 4 \\ -4 \end{bmatrix} \neq \mathbf{t}_1$$

ii)

$$\mathbf{W} = \mathbf{T}\mathbf{P}^+$$

$$\mathbf{P}^+ = (\mathbf{P}^T\mathbf{P})^{-1}\mathbf{P}^T.$$

$$\mathbf{P}^+ = \left(\begin{bmatrix} 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 1 \\ 1 & -1 \\ -1 & -1 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{bmatrix} = \left(\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \end{bmatrix}$$

$$\mathbf{W}^p = \mathbf{T}\mathbf{P}^+ = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 & -\frac{1}{2} \\ 0 & \frac{1}{2} & -\frac{1}{2} & 0 \end{bmatrix}$$

Os padrões de treino são perpendiculares mas não normalizados.

Teste da rede:

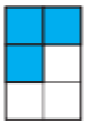
$$\mathbf{W}^p \mathbf{p}_1 = \begin{bmatrix} \frac{1}{2} & 0 & 0 & -\frac{1}{2} \\ 0 & \frac{1}{2} & -\frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \mathbf{t}_1$$

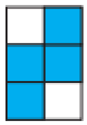
OK dado que garante que minimiza:

$$\sum_{q=1}^{2} \left\| \mathbf{t}_q - \mathbf{W}\mathbf{p}_q \right\|^2 ,$$

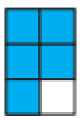
Ex2

Considere os padrões,





$\mathbf{p}_1$ $\mathbf{p}_2$



$\mathbf{p}_t$

i) Verifique se são ortogonais entre si

$\mathbf{p}_1 = \begin{bmatrix} 1 & 1 & -1 & 1 & -1 & -1 \end{bmatrix}^T \qquad \mathbf{p}_2 = \begin{bmatrix} -1 & 1 & 1 & 1 & 1 & -1 \end{bmatrix}^T$

$$\mathbf{p}_1^T \mathbf{p}_2 = \begin{bmatrix} 1 & 1 & -1 & 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 1 \\ 1 \\ 1 \\ -1 \end{bmatrix} = 0$$

São perpendiculars embora não normalizados:

$$\mathbf{p}_1^T \mathbf{p}_1 = \mathbf{p}_2^T \mathbf{p}_2 = 6$$

Desenhe uma rede auto-associativa. Determine os pesos pela regra de Hebb:

$$\mathbf{W} = \mathbf{T}\mathbf{P}^T,$$

where

$$\mathbf{P} = \mathbf{T} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \\ -1 & 1 \\ 1 & 1 \\ -1 & 1 \\ -1 & -1 \end{bmatrix}$$

$$\mathbf{W} = \mathbf{T}\mathbf{P}^T = \begin{bmatrix} 1 & -1 \\ 1 & 1 \\ -1 & 1 \\ 1 & 1 \\ -1 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & -1 & 1 & -1 & -1 \\ -1 & 1 & 1 & 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & -2 & 0 & -2 & 0 \\ 0 & 2 & 0 & 2 & 0 & -2 \\ -2 & 0 & 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 2 & 0 & -2 \\ -2 & 0 & 2 & 0 & 2 & 0 \\ 0 & -2 & 0 & -2 & 0 & 2 \end{bmatrix}$$

ii) Calcule a resposta para o padrão de teste

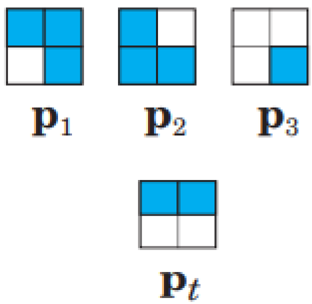
$$\mathbf{p}_t = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & -1 \end{bmatrix}^T$$

$$\mathbf{a} = \mathbf{hardlims}(\mathbf{W}\mathbf{p}_t) = \mathbf{hardlims}\left(\begin{bmatrix} 2 & 0 & -2 & 0 & -2 & 0 \\ 0 & 2 & 0 & 2 & 0 & -2 \\ -2 & 0 & 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 2 & 0 & -2 \\ -2 & 0 & 2 & 0 & 2 & 0 \\ 0 & -2 & 0 & -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ -1 \end{bmatrix} \right)$$

$$\mathbf{a} = \mathbf{hardlims}\left(\begin{bmatrix} -2 \\ 6 \\ 2 \\ 6 \\ 2 \\ -6 \end{bmatrix} \right) = \begin{bmatrix} -1 \\ 1 \\ 1 \\ 1 \\ 1 \\ -1 \end{bmatrix} = \mathbf{p}_2 .$$

Ex 03

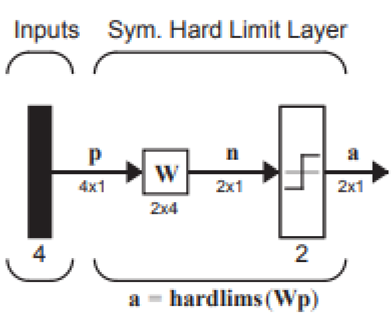
Considere os padrões,



i) Calcule os pesos de uma rede perceptron para reconhecer os padrões p1,p2,p3
ii) Determine a resposta para pt

Resolução:

i)



$$\mathbf{p}_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix} \qquad \mathbf{p}_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix} \qquad \mathbf{p}_3 = \begin{bmatrix} -1 \\ -1 \\ -1 \\ 1 \end{bmatrix} \qquad \mathbf{p}_t = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$$

$$\mathbf{t}_1 = \begin{bmatrix} -1 \\ -1 \end{bmatrix} \qquad \mathbf{t}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \qquad \mathbf{t}_3 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\mathbf{W} = \mathbf{TP}^T = \begin{bmatrix} -1 & -1 & 1 \\ -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ -1 & -1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} -3 & -1 & -1 & -1 \\ 1 & 3 & -1 & -1 \end{bmatrix}$$

ii)

$$\mathbf{a} = \mathbf{hardlims}(\mathbf{Wp}_t) = \mathbf{hardlims}\left(\begin{bmatrix} -3 & -1 & -1 & -1 \\ 1 & 3 & -1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} \right)$$

$$= \mathbf{hardlims}\left(\begin{bmatrix} -2 \\ -2 \end{bmatrix} \right) = \begin{bmatrix} -1 \\ -1 \end{bmatrix} \rightarrow \mathbf{p}_1 \, .$$