Optimization and Decision Support Methodologies
1st Assessment Test

Date: **November 25, 2022** Duration: **1h 30m**

Note: Present all the calculations you carry out, as well as any comments, justifications or conclusions you deem appropriate.

Name: _____ No.: _____

1.

Consider the following **linear programming** problem:

Maximize $z = -x_1 + x_2 - 3x_3$

subject to

$2x_1 + x_2 + x_3 \geq 3$ **(1)**

$x_1 + 2x_2 - x_3 \leq 1$ **(2)**

$x_1 \geq 0 , x_2 \geq 0 , x_3 \geq 0$

Considering $\mathbf{x_4}$ and $\mathbf{x_5}$ the ***surplus*** and ***artificial*** variables of functional constraint **(1),** and $\mathbf{x_6}$ the ***slack*** variable of functional constraint **(2)**, the optimal *simplex* tableau is:

	c_i	-1	1	-3	0	-M	0	
$\mathbf{x_B}$	$c_B \setminus \mathbf{x_i}$	$\mathbf{x_1}$	$\mathbf{x_2}$	$\mathbf{x_3}$	$\mathbf{x_4}$	$\mathbf{x_5}$	$\mathbf{x_6}$	**b**
$\mathbf{x_3}$	-3	0	-1	1	-1/3	1/3	-2/3	1/3
$\mathbf{x_1}$	-1	1	1	0	-1/3	1/3	1/3	4/3
zj-cj		0	1	0	4/3	M-4/3	5/3	-7/3

a) Determine, by carrying out a post-optimization study, what are the implications for the optimal solution presented (in terms of the values of x* and z*), resulting from the following changes:

i) Alteration of the objective function to: ***Maximize*** $\mathbf{z = x_1 + 3x_2 - 3x_3}$;

ii) Introduction of a new constraint: $\mathbf{2x_1 + 2x_2 \leq 1}$.

b) Determine, by carrying out a sensitivity analysis study, for which **interval of $\mathbf{b_1}$** (independent term of the 1st constraint) the optimal basis presented above will remain optimal.

2.

Now consider the following **mixed integer linear programming** problem:

Maximize $z = -x_1 + 3x_2$
subject to

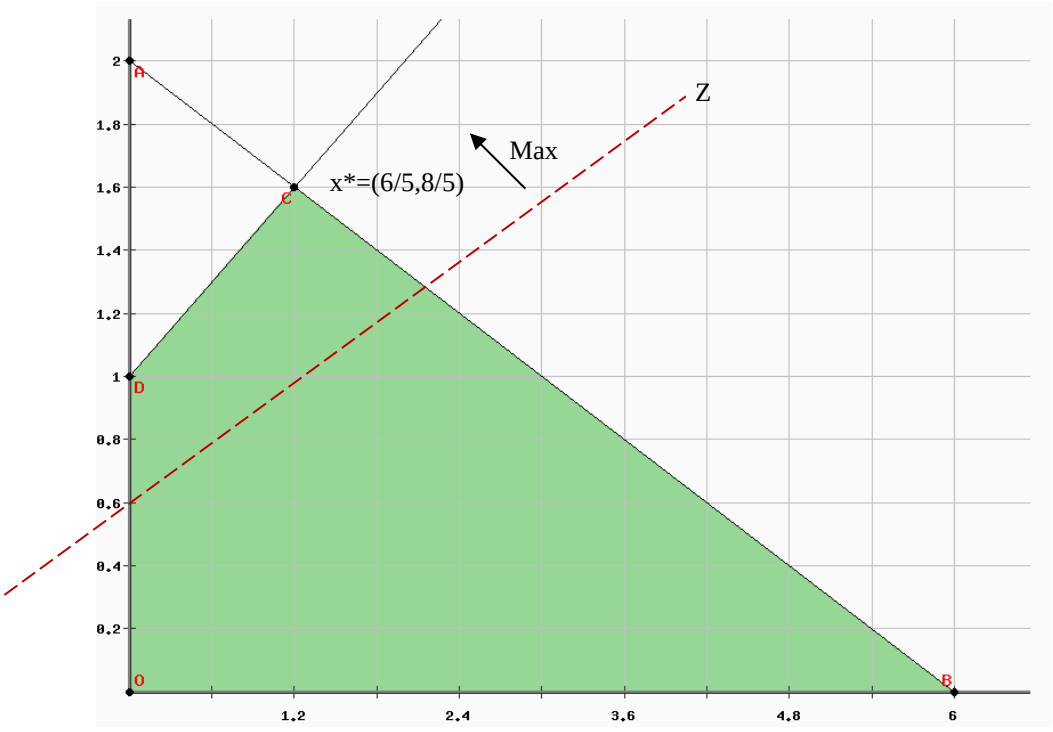
$x_1 + 3x_2 \leq 6$ **(1)**
$-2x_1 + 4x_2 \leq 4$ **(2)**
$x_1 \geq 0 , x_2 \geq 0$
x_2 *integer*

Considering $\mathbf{x_3}$ and $\mathbf{x_4}$ the ***slack*** variables of the functional constraints **(1)** and **(2)** respectively, suppose that the **Gomory algorithm** was applied to this same problem and that at the end of the 1st step, the following optimal tableau was obtained:

	$\mathbf{c_i}$	-1	3	0	0	
$\mathbf{x_B}$	$\mathbf{c_B \backslash x_i}$	$\mathbf{x_1}$	$\mathbf{x_2}$	$\mathbf{x_3}$	$\mathbf{x_4}$	**b**
$\mathbf{x_1}$	-1	1	0	2/5	-3/10	6/5
$\mathbf{x_2}$	3	0	1	1/5	1/10	8/5
zj-cj		0	0	1/5	3/5	18/5

a) Draw your **conclusions** and, if necessary, **proceed with the 2nd step** of the aforementioned algorithm, to solve the problem presented above;

b) Considering the graphic resolution of the associated LP problem presented below, **interpret the resolution of the previous paragraph**, completing the referred graphic.



c) Taking the example of the problem above, explain the reasons why you use Gomory's algorithm (or other alternatives), instead of simply solving the associated LP problem and rounding off the optimal values found for each one of the entire decision variables.