Optimization and Decision Support Methodologies

Date: 12/02/2021 **Exam – Second Call** **Duration: 2h**

Note: *Present all the calculations that you perform and justify your answers.*

1. Consider the following Linear Programming problem:

Maximize $z = [-x_1 + 2x_2 ; 7x_1 + 21x_2]$

subject to

$7x_1 + 4x_2 \leq 28$

$-2x_1 + x_2 \leq 2$

$x_1 \geq 0 , x_2 \geq 0$

[5.00 valores] **a)** Determine the **set of efficient solutions and the set of non-dominated solutions** using the graphical representation of this problem in the decision space and in the objective space;

[0.75 valores] **b)** Build the **pay-off table** and determine the **ideal and anti-ideal solutions**.

2. Consider the following Pure Integer Linear Programming problem:

Maximize $z = x_1 + 2x_2 + 3x_3$

subject to

$x_1 + 2x_3 \leq 5$ **(1)**

$2x_1 + x_2 \leq 4$ **(2)**

$x_1 - x_2 - x_3 \leq 6$ **(3)**

$x_1 \geq 0 , x_2 \geq 0, x_3 \geq 0$

x_1, x_2 and x_3 integer

Assuming that $\mathbf{x_4}$, $\mathbf{x_5}$ and $\mathbf{x_6}$ are the slack variables associated with constraints **(1)**, **(2)** and **(3)**, respectively, suppose that the **Gomory algorithm** was applied to this same problem and that at the end of the 1st step, the following optimal tableau was obtained:

	c_i	1	2	3	0	0	0	
$\mathbf{x_B}$	$c_B \setminus x_i$	$\mathbf{x_1}$	$\mathbf{x_2}$	$\mathbf{x_3}$	$\mathbf{x_4}$	$\mathbf{x_5}$	$\mathbf{x_6}$	**b**
$\mathbf{x_3}$	3	1/2	0	1	1/2	0	0	5/2
$\mathbf{x_2}$	2	2	1	0	0	1	0	4
$\mathbf{X_6}$	0	7/2	0	0	1/2	1	1	25/2
zj-cj		9/2	0	0	3/2	2	0	31/2

[5.00 valores] **a)** Withdraw your conclusions and if necessary, proceed with the 2nd step of that algorithm;

[0.75 valores] **b)** Could the restriction $\mathbf{x_1 + x_2 + x_3 \geq 4}$ constitute an eventual cut constraint for this problem? Justify your answer.

3. Consider the following Goal Programming problem:

Minimize z = $\{ d_1^+, d_2^-, d_3^- \}$

subject to

$$x_1 - x_2 + d_1^- - d_1^+ = 1$$
$$x_1 + d_2^- - d_2^+ = 1$$
$$x_2 + d_3^- - d_3^+ = 2$$
$$5x_1 + 3x_2 + d_4^- = 15$$
$$x_1 \geq 0, \quad x_2 \geq 0, \quad d_i^- \geq 0, \quad d_i^+ \geq 0 \quad (i = 1,2,3,4)$$

[5.50 valores] Solve the problem using the **graphical method**.