

Optimization and Decision Support Methodologies

1st Assessment Test

Date: November 26, 2020

Duration: 1h 30m

Note: Present all the calculations you carry out, as well as any comments, justifications or conclusions you deem appropriate.

1. Consider the following linear programming problem:

Maximize $z = 2x_1 - x_2$

subject to

$$x_1 + 2x_2 \geq 4 \quad (1)$$

$$3x_1 + x_2 \leq 3 \quad (2)$$

$$x_1 \geq 0, x_2 \geq 0$$

Considering x_3 and x_4 the surplus and artificial variables of the functional constraint (1), and x_5 the slack variable of the functional constraint (2), the Simplex optimal tableau is:

	c_i	2	-1	0	-M	0	
x_B	$c_B \setminus x_i$	x_1	x_2	x_3	x_4	x_5	b
x2	-1	0	1	-3/5	3/5	-1/5	9/5
x1	2	1	0	1/5	-1/5	2/5	2/5
zj-cj		0	0	1	M-1	1	-1

a) For each of the following changes in the initial problem, determine, by carrying out a post-optimization study, what are the implications for the optimal solution presented (in the value of x^* , in the value of z^* and in the optimal basis), arising from:

[1.75 points]

1) Changing the objective function to Maximize $z = 3x_1 + x_2$;

[1.50 points]

2) Alteration in the vector of the coefficients of variable x_1 in the constraints from

$$\begin{bmatrix} 1 \\ 3 \end{bmatrix} \text{ to } \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

- [1.75 points]** b) Determine, by carrying out a sensitivity analysis study, for which interval of b_2 (independent term of the 2nd constraint) the optimal basis presented above will remain optimal.

2. Consider the following mixed integer linear programming problem:

$$\begin{aligned} & \text{Maximize } z = x_1 + 2x_2 + x_3 + x_4 \\ & \text{subject to} \\ & \quad x_1 + 2x_2 + x_3 - x_4 \leq 6 \quad (1) \\ & \quad 2x_1 - x_2 + 2x_3 + x_4 \leq 6 \quad (2) \\ & \quad x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0 \\ & \quad x_2 \text{ and } x_3 \text{ integers} \end{aligned}$$

Considering x_5 and x_6 the slack variables of constraints (1) and (2), respectively, suppose that the Gomory algorithm for MILP was applied to this same problem and that at the end of the 1st step, the following optimal tableau was obtained:

	c_i	1	2	1	1	0	0	
x_B	$c_B \setminus x_i$	x_1	x_2	x_3	x_4	x_5	x_6	b
x2	2	3	1	3	0	1	1	23/2
x4	1	5	0	5	1	1	2	17
zj-cj		10	0	10	0	3	4	40

- [3.00 points]** a) Withdraw your conclusions and if you find it necessary, proceed with the 2nd step of the referred algorithm, in order to solve the presented problem;

- [0.50 points]** b) Do you think the rounding method would work on this problem? Justify.