Departamento de Engenharia Informática e de Sistemas

Optimization and Decision Support Methodologies

Date: 29/01/2021 **Exam – First call** **Duration: 2h**

Note: *Present all the calculations that you perform and justify your answers.*

1. Consider the following Linear Programming problem:

Maximize $z = 2x_1 - x_2$
subject to
$2x_1 + 4x_2 \geq 8$ **(1)**
$x_1 + 2x_2 \geq 4$ **(2)**
$2x_1 + 2x_2 \leq 6$ **(3)**
$x_1 \geq 0$, $x_2 \geq 0$

Considering x_3 and x_5 the surplus and artificial variables of the functional constraint **(1)**, x_4 and x_6 the surplus and artificial variables of the functional constraint **(2)**, and x_7 the slack variable of the functional restriction **(3)**, the optimal tableau of the Simplex is:

	c_i	2	-1	0	0	-M	-M	0	
x_B	$c_B \setminus x_i$	x_1	x_2	x_3	x_4	x_5	x_6	x_7	**b**
x_2	-1	0	1	0	-1	0	1	-1/2	1
x_3	0	0	0	1	-2	-1	2	0	0
x_1	2	1	0	0	1	0	-1	1	2
	zj-cj	0	0	0	3	M	M-3	5/2	3

[2.75 points] **a)** Determine, by carrying out a <u>post-optimization study</u>, what are the implications in the value of x *, in the value of z *, and in the optimal basis, arising from the introduction of a new decision variable x_8, with coefficients in constraints equal to $\begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}$, and coefficient in the objective function c_8=**7**.

[2.75 points] **b)** Determine for which interval of c_2 (coefficient of variable x_2 in the objective function), the table presented above will remain optimal.

2. Consider the following Pure Integer Linear Programming problem:

Maximize $z = 3x_1 + 4x_2$
subject to
$2x_1 + x_2 \leq 6$ **(1)**
$2x_1 + 3x_2 \leq 9$ **(2)**
$x_2 \leq 1$ **(3)**
$x_1 \geq 0$, $x_2 \geq 0$
x_1 and x_2 integer

Since x_3, x_4 and x_5 are the slack variables associated with constraints **(1)**, **(2)** and **(3)**, respectively, suppose that the **Gomory algorithm** was applied to this same problem and that at the end of the 1st step, the following optimal tableau was obtained:

	c_i	3	4	0	0	0	
x_B	$c_B \setminus x_i$	x_1	x_2	x_3	x_4	x_5	**b**
x_1	3	1	0	1/2	0	-1/2	5/2
x_4	0	0	0	-1	1	-2	1
x_2	4	0	1	0	0	1	1
zj-cj		0	0	3/2	0	5/2	23/2

[5.00 points] **a)** Withdraw your conclusions and if necessary, proceed with the 2nd step of that algorithm.

[0.75 points] **b)** Could the restriction $2x_1 + x_2 \geq 2$ constitute an eventual cut constraint for this problem? Justify your answer.

3. Consider the following Goal Programming problem:

Minimize z = $\{ d_1^+, d_2^- + d_2^+, d_3^+ \}$

subject to

$$2x_1 - x_2 + d_1^- - d_1^+ = 2$$
$$x_1 + d_2^- - d_2^+ = 1$$
$$x_2 + d_3^- - d_3^+ = 1$$
$$3x_1 + 3x_2 + d_4^- = 12$$
$$x_1 \geq 0,\ x_2 \geq 0,\ d_i^- \geq 0,\ d_i^+ \geq 0\ \ (i = 1,2,3,4)$$

[5.00 points] **a)** Solve the problem using the graphical method.

[0.75 points] **b)** If you wanted the value of $x_1 - x_2$ to be mandatorily greater than or equal to **2**, how would you represent it in the previous model?