Optimization and Decision Support Methodologies		
Date: 02/10/2022	**Exam – 1st Call**	**Duration: 2 hours**

Note: Present all the calculations you make, and conveniently justify your answers.

1. *(Predicted quotation: 7.0 values)*

Consider the following **single-objective linear programming** problem:

Maximize $z = -x_1 + x_2 - 3x_3$

subject to

$2x_1 + x_2 + x_3 \geq 3$ **(1)**

$x_1 + 2x_2 - x_3 \leq 1$ **(2)**

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

Considering $\mathbf{x_4}$ and $\mathbf{x_5}$ respectively the **surplus** and **artificial** variables of the functional constraint **(1)**, and $\mathbf{x_6}$ the **slack** variable of the functional constraint **(2)**, the optimal simplex tableau is:

	c_i	-1	1	-3	0	-M	0	
$\mathbf{x_B}$	$\mathbf{c_B}$ \ x_i	$\mathbf{x_1}$	$\mathbf{x_2}$	$\mathbf{x_3}$	$\mathbf{x_4}$	$\mathbf{x_5}$	$\mathbf{x_6}$	**b**
$\mathbf{x_3}$	-3	0	-1	1	-1/3	1/3	-2/3	1/3
$\mathbf{x_1}$	-1	1	1	0	-1/3	1/3	1/3	4/3
zj-cj		0	1	0	4/3	M-4/3	5/3	-7/3

a) For each of the following changes in the initial problem, determine, by carrying out a post-optimization study, what are the **implications for the optimal solution presented** (in the value of x*, in the value of z* and in the optimal basis), arising from:

I. Changing the vector of **the independent terms** of the constraints from $\begin{bmatrix}3\\1\end{bmatrix}$ to $\begin{bmatrix}3\\3\end{bmatrix}$;

II. Changing the vector of **the coefficients of variable $\mathbf{x_2}$ in the constraints** from $\begin{bmatrix}1\\2\end{bmatrix}$ to $\begin{bmatrix}2\\4\end{bmatrix}$.

b) Determine, by carrying out a sensitivity analysis study, for **which interval of $\mathbf{c_2}$** (coefficient of x_2 in the objective function) the **optimal solution** presented above **will remain optimal**.

2. *(Predicted quotation: 5.0 values)*

Consider the following **goal programming** problem:

$$\text{Minimizar } Z = \left\{ d_1^-, d_2^- + d_2^+ \;, d_3^+ \right\}$$

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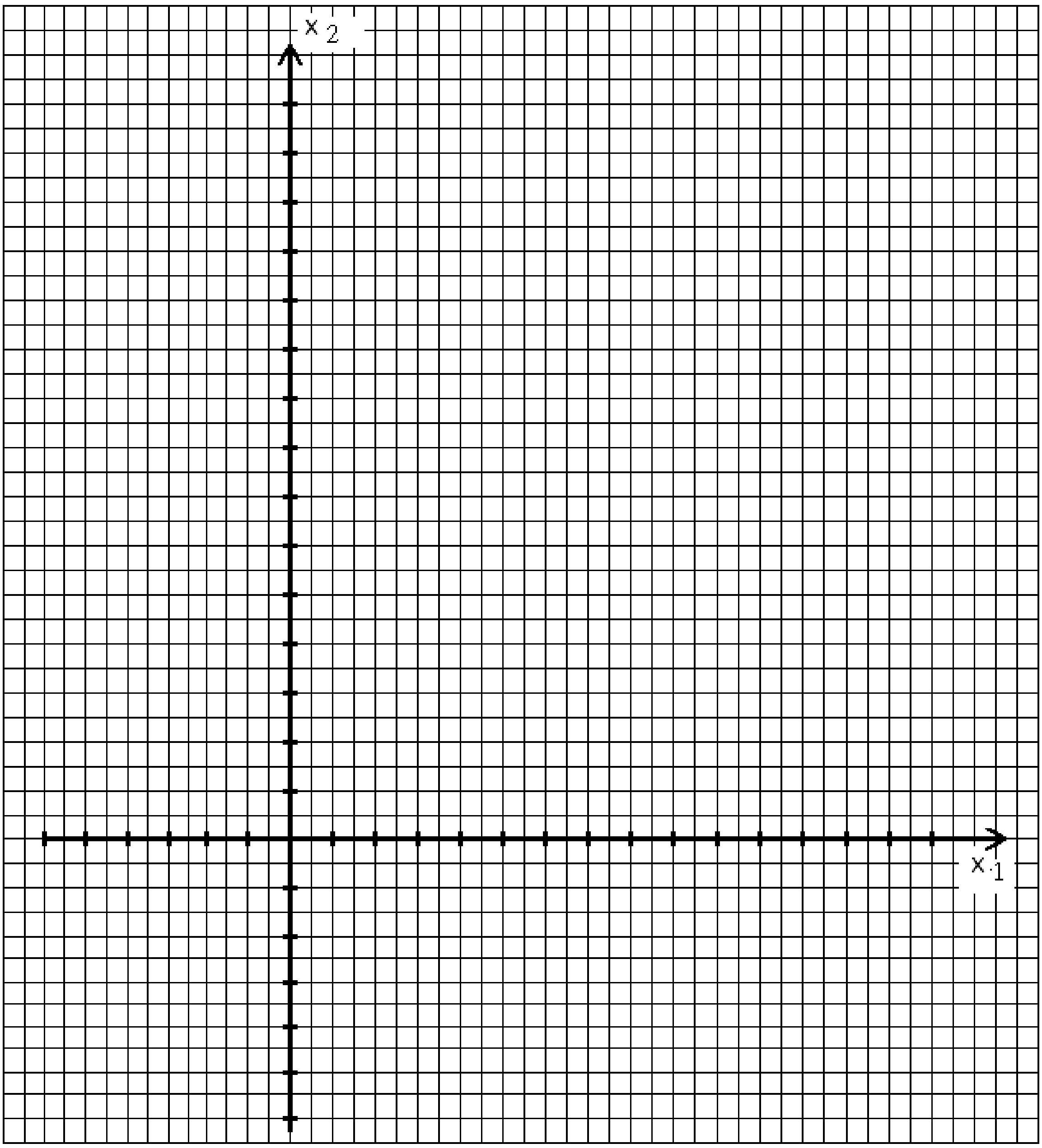
\begin{aligned}
x_1 - x_2 + d_1^- - d_1^+ &= 1 \\
x_1 + d_2^- - d_2^+ &= 2 \\
x_2 + d_3^- - d_3^+ &= 3 \\
5x_1 + 3x_2 + d_4^- &= 15
\end{aligned}
$$

$$x_1 \geq 0,\ x_2 \geq 0,\ d_i^- \geq 0,\ d_i^+ \geq 0\ \ (i = 1,2,3,4)$$

a) Solve this problem by the **graphical method**;

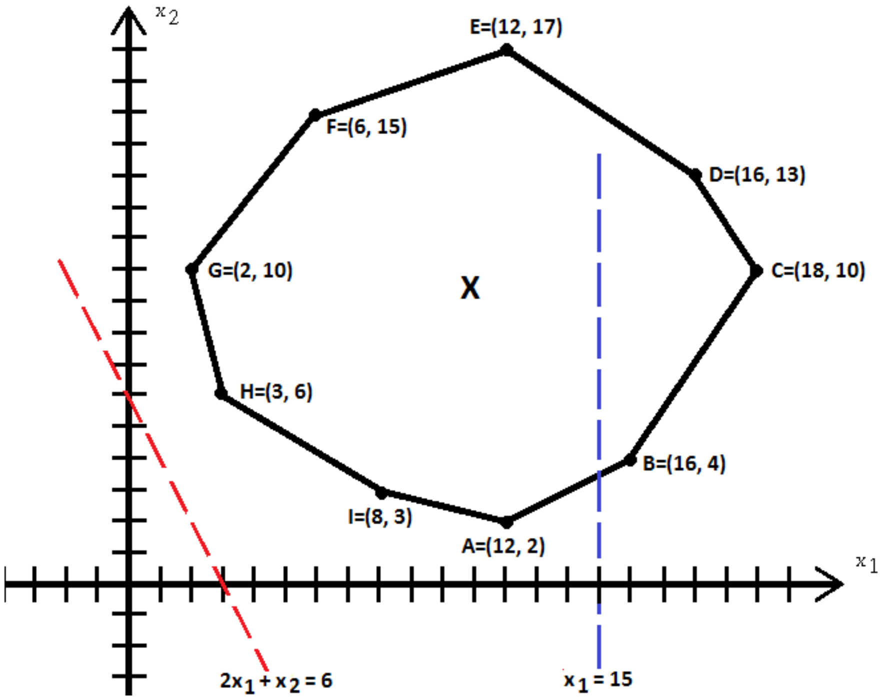


b) If there was a need for a new functional constraint that specified that $\mathbf{x_1 + 2x_2}$ should be greater than, or equal to **3**, indicate what change you would introduce to the model.

3. (*Predicted quotation: 5.0 values*)

Consider the following **linear programming** problem with **two objective functions**:

Min $z_1 = 2x_1 + x_2$
Max $z_2 = x_1$
subject to $\underline{\mathbf{x}} = (x_1, x_2)^T \in X$



a) Identify the **efficient region** of this problem by marking it on the graph on the proof sheet. Justify your answer;

b) Obtain the **pay-off table** corresponding to this problem, and identify the **ideal solution** and the **anti-ideal solution**.

Name: ______________________________ No. ____________