isec Engenharia

Departamento de Engenharia Informática e de Sistemas

Optimization and Decision Support Methodologies

Date: 23/02/2022 **Exam – 2nd Call** **Duration: 2 hours**

Note: Present all the calculations you make, and conveniently justify your answers.

1. *(Expected quotation: 7.0 values)*

Consider the following single-objective linear programming problem:

$$
\begin{array}{lll}
\textit{Minimize} & z = 3x_1 + 2x_2 & \\
\text{subject to} & & \\
& 2x_1 + x_2 \geq 10 & \textbf{(1)} \\
& -3x_1 + 2x_2 \leq 6 & \textbf{(2)} \\
& x_1 + x_2 \geq 6 & \textbf{(3)} \\
& x_1 \geq 0,\ x_2 \geq 0 &
\end{array}
$$

Considering x_3 and x_5 the surplus and artificial variables of the functional constraint **(1)**, x_6 the slack variable of the functional constraint **(2)**, and x_4 and x_7 the surplus and artificial variables of the functional constraint **(3)**, the *simplex* optimal tableau (using the technique of the Big M) is:

	c_i	-3	-2	0	0	-M	0	-M	
$\mathbf{x_B}$	$c_B \setminus x_i$	$\mathbf{x_1}$	$\mathbf{x_2}$	$\mathbf{x_3}$	$\mathbf{x_4}$	$\mathbf{x_5}$	$\mathbf{x_6}$	$\mathbf{x_7}$	**b**
$\mathbf{x_1}$	-3	1	0	-1	1	1	0	-1	4
$\mathbf{x_6}$	0	0	0	-5	7	5	1	-7	14
$\mathbf{x_2}$	-2	0	1	1	-2	-1	0	2	2
zj-cj		0	0	1	1	M-1	0	M-1	-16

For each of the following changes in the initial problem, determine, by carrying out a **post-optimization study**, what are the implications for the presented optimal solution (in the value of x*, in the value of z* and in the optimal basis), resulting from the variation:

a) Changing the **vector of terms independent** of constraints, from $\begin{bmatrix} \mathbf{10} \\ \mathbf{6} \\ \mathbf{6} \end{bmatrix}$ to $\begin{bmatrix} \mathbf{15} \\ \mathbf{7} \\ \mathbf{8} \end{bmatrix}$;

b) Change in the **coefficient of the variable x_2 in the objective function**, from **2** to **4**;

c) Introducing a new functional constraint in the problem: $\mathbf{x_1 + 2x_2 \leq 16}$.

2. *(Expected quotation: 5.0 values)*

Consider now the following pure integer linear programming problem:

$$
\begin{array}{lll}
\textit{Maximize} & z = -x_1 + 3x_2 & \\
\text{subject to} & & \\
& -x_1 + 2x_2 \leq 4 & \textbf{(1)} \\
& x_1 + x_2 \leq 6 & \textbf{(2)} \\
& x_1 + 3x_2 \leq 9 & \textbf{(3)} \\
& x_1 \geq 0,\ x_2 \geq 0 & \\
& x_1 \text{ and } x_2 \text{ integers} &
\end{array}
$$

Considering x_3, x_4 and x_5 as the slack variables of the functional constraints **(1)**, **(2)** and **(3)**, respectively, suppose that the Gomory algorithm was applied to this same problem and at the end of the 1st step, the following optimal *simplex* tableau was obtained:

	c_i	-1	3	0	0	0	
x_B	$c_B \setminus x_i$	x_1	x_2	x_3	x_4	x_5	b
x_2	3	0	1	1/5	0	1/5	13/5
x_4	0	0	0	2/5	1	-3/5	11/5
x_1	-1	1	0	-3/5	0	2/5	6/5
	Zj-cj	0	0	6/5	0	1/5	33/5

a) Draw your **conclusions** and, if necessary, proceed with **the 2nd step of that algorithm** to solve the presented problem;

b) If the variable x_1 has no longer an integrality constraint, do you consider that we would still be faced with an integer programming problem? Justify your answer.

3. *(Expected quotation: 5.0 values)*

Now consider the following goal programming problem:

Minimize Z = $\{ d_1^+,\ d_3^-,\ d_2^- \}$

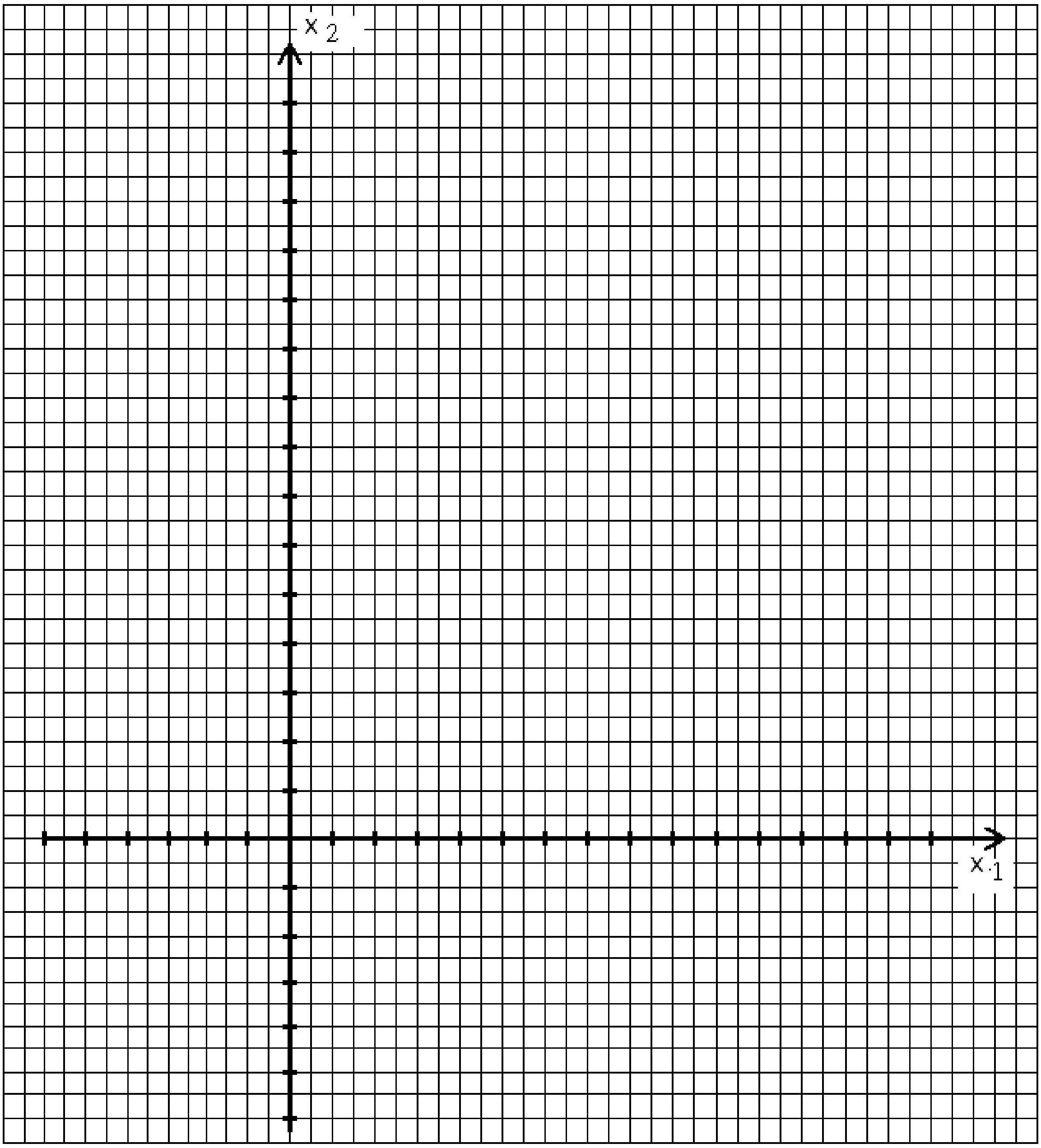
subject to

$$3x_1 + 4x_2 + d_1^- - d_1^+ = 24$$
$$2x_1 + x_2 + d_2^- - d_2^+ = 10$$
$$x_2 + d_3^- - d_3^+ = 5$$
$$2x_1 - 3x_2 + d_4^- = 6$$
$$x_1 \geq 0,\ x_2 \geq 0,\ d_i^- \geq 0,\ d_i^+ \geq 0\ \ (i = 1,2,3,4)$$

a) Solve this problem by the **graphical method**;

<u>Note</u>: You can use the grid on page 3, identifying yourself with student name and number.

b) If there was a need for a **new goal** of **priority level 4**, specifying that, as far as possible, $\mathbf{x_1+2x_2}$ should be greater than or equal to **3**, indicate what changes you would introduce in the model.



Name: ____ No. ______________