Avaliação 1 de Modelagem Computacional – PPGMO

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Questão 4:

Para esta questão especificamente, o Método de Runge Kutta se equipara ao Método Preditor Corretor de Adams, devido a escolha do h=0.01, i.e: a grandeza dos erros em ambos algoritmos são equivalentes. Entretanto é sabido que métodos multipasso como o Método de Adams são capazes de aproximar uma solução mais precisa e funcionam para EDO's de ordens superiores, limitações que o RK4 encontra.

Os gráficos abaixos demonstram como as soluções de ambos os métodos foram similares e satisfatórias quando comparadas a solução exata:
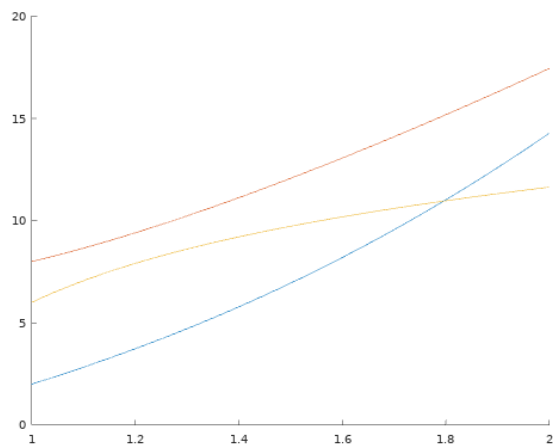

Figura 1: Séries de soluções aproximadas U1,U2,U3 obtido via Método de Adams, note que a série do Erro não é perceptível no gráfico, devido ao mesmo ser mínimo.
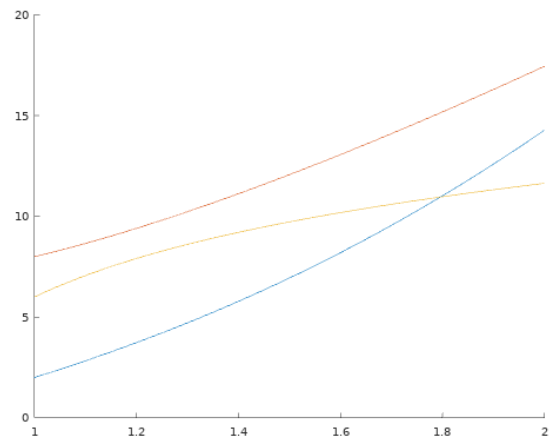

Figura 2: Séries de soluções aproximadas U1,U2,U3, obtido via RK4 note que a série do Erro não é perceptível no gráfico, devido ao mesmo ser mínimo.

Método Preditor Corretor de Adams (s é a solução exata):

t	w_1	erro(w_1-s_1)	w_2	erro(w_2-s_2)	w3	erro(w_3-s_3)
1.00	2.000000	0.000000	8.000000	0.000000	6.000000	0.000000
1.01	2.080302	0.000000	8.060595	0.000001	6.118424	0.000396
1.02	2.161216	0.000000	8.122358	0.000011	6.233787	0.001569
1.03	2.242753	0.000000	8.185261	0.000035	6.346221	0.003496
1.04	2.324925	0.000001	8.249273	0.000083	6.455852	0.006155
1.05	2.407742	0.000002	8.314369	0.000161	6.562797	0.009527
1.06	2.491216	0.000004	8.380521	0.000276	6.667169	0.013592
1.07	2.575356	0.000008	8.447704	0.000435	6.769073	0.018331
1.08	2.660173	0.000013	8.515894	0.000645	6.868610	0.023726
1.09	2.745677	0.000021	8.585069	0.000911	6.965873	0.029760
1.10	2.831878	0.000031	8.655204	0.001242	7.060955	0.036416
1.11	2.918784	0.000046	8.726281	0.001642	7.153939	0.043678
1.12	3.006406	0.000065	8.798277	0.002117	7.244908	0.051531
1.13	3.094753	0.000088	8.871172	0.002674	7.333938	0.059961
1.14	3.183833	0.000118	8.944949	0.003318	7.421103	0.068954
1.15	3.273655	0.000155	9.019588	0.004055	7.506472	0.078495
1.16	3.364227	0.000200	9.095073	0.004890	7.590112	0.088573
1.17	3.455559	0.000253	9.171385	0.005828	7.672086	0.099173
1.18	3.547658	0.000317	9.248509	0.006875	7.752453	0.110285
1.19	3.640532	0.000391	9.326429	0.008036	7.831271	0.121897

1.20	3.734189	0.000478	9.405130	0.009315	7.908596	0.133997
1.21	3.828637	0.000578	9.484596	0.010717	7.984478	0.146574
1.22	3.923883	0.000693	9.564814	0.012248	8.058967	0.159619
1.23	4.019936	0.000823	9.645771	0.013911	8.132111	0.173121
1.24	4.116801	0.000971	9.727452	0.015712	8.203956	0.187070
1.25	4.214487	0.001138	9.809846	0.017654	8.274544	0.201456
1.26	4.313000	0.001325	9.892939	0.019742	8.343916	0.216272
1.27	4.412348	0.001533	9.976720	0.021981	8.412113	0.231508
1.28	4.512537	0.001765	10.061178	0.024374	8.479171	0.247154
1.29	4.613574	0.002021	10.146300	0.026925	8.545128	0.263204
1.30	4.715465	0.002304	10.232077	0.029639	8.610018	0.279650
1.31	4.818218	0.002615	10.318497	0.032520	8.673874	0.296482
1.32	4.921837	0.002955	10.405551	0.035570	8.736728	0.313695
1.33	5.026331	0.003327	10.493228	0.038795	8.798610	0.331280
1.34	5.131704	0.003731	10.581520	0.042197	8.859550	0.349231
1.35	5.237963	0.004171	10.670416	0.045781	8.919576	0.367540
1.36	5.345114	0.004648	10.759908	0.049549	8.978714	0.386202
1.37	5.453163	0.005163	10.849988	0.053506	9.036990	0.405209
1.38	5.562116	0.005718	10.940645	0.057654	9.094430	0.424556
1.39	5.671978	0.006317	11.031874	0.061998	9.151056	0.444236
1.40	5.782755	0.006959	11.123664	0.066540	9.206893	0.464244
1.41	5.894453	0.007648	11.216009	0.071284	9.261962	0.484573

1.42	6.007077	0.008385	11.308901	0.076233	9.316284	0.505219
1.43	6.120633	0.009173	11.402332	0.081389	9.369879	0.526175
1.44	6.235126	0.010014	11.496296	0.086757	9.422768	0.547436
1.45	6.350561	0.010909	11.590785	0.092339	9.474969	0.568998
1.46	6.466943	0.011861	11.685793	0.098138	9.526500	0.590855
1.47	6.584278	0.012873	11.781313	0.104157	9.577378	0.613003
1.48	6.702571	0.013945	11.877338	0.110399	9.627622	0.635436
1.49	6.821827	0.015081	11.973863	0.116867	9.677246	0.658150
1.50	6.942050	0.016283	12.070881	0.123563	9.726267	0.681141
1.51	7.063246	0.017553	12.168387	0.130491	9.774699	0.704404
1.52	7.185419	0.018894	12.266373	0.137652	9.822558	0.727934
1.53	7.308575	0.020307	12.364836	0.145050	9.869858	0.751729
1.54	7.432718	0.021796	12.463769	0.152687	9.916611	0.775783
1.55	7.557852	0.023362	12.563166	0.160567	9.962832	0.800093
1.56	7.683982	0.025008	12.663024	0.168690	10.008533	0.824655
1.57	7.811114	0.026737	12.763335	0.177061	10.053725	0.849464
1.58	7.939251	0.028550	12.864097	0.185680	10.098422	0.874518
1.59	8.068397	0.030451	12.965302	0.194552	10.142635	0.899813
1.60	8.198558	0.032442	13.066948	0.203677	10.186374	0.925345
1.61	8.329738	0.034525	13.169028	0.213059	10.229650	0.951111
1.62	8.461940	0.036704	13.271539	0.222700	10.272474	0.977107
1.63	8.595170	0.038980	13.374476	0.232602	10.314856	1.003330

1.64	8.729431	0.041357	13.477835	0.242768	10.356805	1.029777
1.65	8.864728	0.043836	13.581611	0.253199	10.398331	1.056445
1.66	9.001065	0.046422	13.685800	0.263897	10.439443	1.083331
1.67	9.138445	0.049115	13.790398	0.274866	10.480150	1.110431
1.68	9.276874	0.051920	13.895402	0.286107	10.520461	1.137744
1.69	9.416355	0.054838	14.000806	0.297622	10.560383	1.165265
1.70	9.556891	0.057873	14.106608	0.309413	10.599925	1.192992
1.71	9.698488	0.061028	14.212804	0.321482	10.639095	1.220922
1.72	9.841149	0.064304	14.319389	0.333832	10.677900	1.249053
1.73	9.984877	0.067705	14.426360	0.346464	10.716347	1.277382
1.74	10.129677	0.071234	14.533715	0.359380	10.754444	1.305907
1.75	10.275553	0.074894	14.641448	0.372583	10.792198	1.334624
1.76	10.422507	0.078687	14.749557	0.386073	10.829615	1.363532
1.77	10.570545	0.082616	14.858039	0.399854	10.866702	1.392628
1.78	10.719669	0.086685	14.966890	0.413926	10.903465	1.421909
1.79	10.869884	0.090896	15.076108	0.428293	10.939911	1.451374
1.80	11.021193	0.095252	15.185688	0.442954	10.976045	1.481019
1.81	11.173599	0.099756	15.295627	0.457914	11.011873	1.510844
1.82	11.327107	0.104411	15.405924	0.473172	11.047402	1.540845
1.83	11.481719	0.109220	15.516574	0.488731	11.082635	1.571020
1.84	11.637439	0.114186	15.627576	0.504593	11.117580	1.601367
1.85	11.794271	0.119313	15.738925	0.520759	11.152241	1.631885

1.86	11.952219	0.124603	15.850620	0.537231	11.186622	1.662571
1.87	12.111285	0.130059	15.962657	0.554011	11.220730	1.693422
1.88	12.271473	0.135684	16.075033	0.571100	11.254568	1.724439
1.89	12.432787	0.141482	16.187747	0.588500	11.288143	1.755617
1.90	12.595229	0.147455	16.300795	0.606213	11.321457	1.786956
1.91	12.758804	0.153607	16.414175	0.624240	11.354516	1.818453
1.92	12.923514	0.159941	16.527885	0.642583	11.387323	1.850107
1.93	13.089363	0.166460	16.641921	0.661243	11.419884	1.881915
1.94	13.256353	0.173167	16.756282	0.680221	11.452202	1.913877
1.95	13.424489	0.180065	16.870964	0.699521	11.484281	1.945991
1.96	13.593774	0.187158	16.985967	0.719142	11.516126	1.978254
1.97	13.764210	0.194449	17.101286	0.739086	11.547739	2.010665
1.98	13.935800	0.201941	17.216921	0.759355	11.579125	2.043223
1.99	14.108549	0.209637	17.332868	0.779951	11.610287	2.075925
2.00	14.282459	0.217541	17.449126	0.800874	11.641230	2.108770

RK4:

t	w_1	erro(w_1-s_1)	w_2	erro(w_2-s_2)	w3	erro(w_3-s_3)
1.00	2.000000	0.000000	8.000000	0.000000	6.000000	0.000000
1.01	2.080302	0.000000	8.060595	0.000001	6.118424	0.000396
1.02	2.161216	0.000000	8.122358	0.000011	6.233787	0.001569
1.03	2.242753	0.000000	8.185261	0.000035	6.346221	0.003496
1.04	2.324925	0.000001	8.249273	0.000083	6.455852	0.006155
1.05	2.407742	0.000002	8.314369	0.000161	6.562797	0.009527
1.06	2.491216	0.000004	8.380521	0.000276	6.667169	0.013592
1.07	2.575356	0.000008	8.447704	0.000435	6.769073	0.018331
1.08	2.660173	0.000013	8.515894	0.000645	6.868609	0.023726
1.09	2.745677	0.000021	8.585069	0.000911	6.965873	0.029760
1.10	2.831878	0.000031	8.655204	0.001242	7.060955	0.036416
1.11	2.918784	0.000046	8.726281	0.001642	7.153939	0.043678
1.12	3.006406	0.000065	8.798277	0.002117	7.244908	0.051532
1.13	3.094753	0.000088	8.871172	0.002674	7.333938	0.059962
1.14	3.183833	0.000118	8.944949	0.003318	7.421103	0.068954
1.15	3.273655	0.000155	9.019588	0.004055	7.506472	0.078495
1.16	3.364227	0.000200	9.095073	0.004890	7.590112	0.088573
1.17	3.455559	0.000253	9.171385	0.005828	7.672085	0.099173
1.18	3.547658	0.000317	9.248509	0.006875	7.752453	0.110285
1.19	3.640532	0.000391	9.326429	0.008036	7.831271	0.121897

1.20	3.734189	0.000478	9.405130	0.009315	7.908596	0.133997
1.21	3.828637	0.000578	9.484596	0.010717	7.984478	0.146575
1.22	3.923883	0.000693	9.564814	0.012248	8.058967	0.159619
1.23	4.019936	0.000823	9.645771	0.013911	8.132111	0.173121
1.24	4.116801	0.000971	9.727452	0.015712	8.203956	0.187070
1.25	4.214487	0.001138	9.809846	0.017654	8.274543	0.201457
1.26	4.313000	0.001325	9.892939	0.019742	8.343916	0.216272
1.27	4.412348	0.001533	9.976720	0.021981	8.412112	0.231508
1.28	4.512537	0.001765	10.061178	0.024374	8.479171	0.247155
1.29	4.613574	0.002021	10.146300	0.026925	8.545128	0.263205
1.30	4.715465	0.002304	10.232077	0.029639	8.610018	0.279650
1.31	4.818218	0.002615	10.318497	0.032520	8.673874	0.296482
1.32	4.921837	0.002955	10.405551	0.035570	8.736728	0.313695
1.33	5.026331	0.003327	10.493228	0.038795	8.798610	0.331280
1.34	5.131704	0.003731	10.581520	0.042197	8.859550	0.349231
1.35	5.237963	0.004171	10.670416	0.045781	8.919575	0.367540
1.36	5.345114	0.004648	10.759908	0.049549	8.978713	0.386202
1.37	5.453163	0.005163	10.849988	0.053506	9.036990	0.405209
1.38	5.562116	0.005718	10.940645	0.057654	9.094430	0.424556
1.39	5.671978	0.006317	11.031874	0.061998	9.151056	0.444236
1.40	5.782755	0.006959	11.123664	0.066540	9.206893	0.464244
1.41	5.894453	0.007648	11.216009	0.071284	9.261962	0.484573

1.42	6.007077	0.008385	11.308901	0.076233	9.316284	0.505219
1.43	6.120633	0.009173	11.402332	0.081389	9.369879	0.526175
1.44	6.235126	0.010014	11.496296	0.086757	9.422768	0.547436
1.45	6.350561	0.010909	11.590785	0.092339	9.474968	0.568998
1.46	6.466943	0.011861	11.685793	0.098138	9.526499	0.590855
1.47	6.584278	0.012873	11.781313	0.104157	9.577378	0.613003
1.48	6.702571	0.013945	11.877338	0.110399	9.627621	0.635436
1.49	6.821827	0.015081	11.973863	0.116867	9.677246	0.658150
1.50	6.942050	0.016283	12.070881	0.123563	9.726267	0.681141
1.51	7.063246	0.017553	12.168387	0.130491	9.774699	0.704404
1.52	7.185419	0.018894	12.266373	0.137652	9.822558	0.727935
1.53	7.308575	0.020307	12.364836	0.145050	9.869858	0.751729
1.54	7.432718	0.021796	12.463769	0.152687	9.916611	0.775783
1.55	7.557852	0.023362	12.563166	0.160567	9.962832	0.800093
1.56	7.683982	0.025008	12.663024	0.168690	10.008533	0.824655
1.57	7.811114	0.026737	12.763335	0.177061	10.053725	0.849464
1.58	7.939251	0.028550	12.864097	0.185680	10.098422	0.874519
1.59	8.068397	0.030451	12.965302	0.194552	10.142635	0.899813
1.60	8.198558	0.032442	13.066948	0.203677	10.186374	0.925345
1.61	8.329738	0.034525	13.169028	0.213059	10.229650	0.951111
1.62	8.461940	0.036704	13.271539	0.222700	10.272474	0.977107
1.63	8.595170	0.038980	13.374476	0.232602	10.314856	1.003330

1.64	8.729431	0.041357	13.477835	0.242768	10.356805	1.029778
1.65	8.864728	0.043836	13.581611	0.253199	10.398331	1.056446
1.66	9.001065	0.046422	13.685800	0.263897	10.439443	1.083331
1.67	9.138445	0.049115	13.790398	0.274866	10.480150	1.110432
1.68	9.276874	0.051920	13.895402	0.286107	10.520461	1.137744
1.69	9.416355	0.054838	14.000806	0.297622	10.560383	1.165265
1.70	9.556891	0.057873	14.106608	0.309413	10.599925	1.192992
1.71	9.698488	0.061028	14.212804	0.321482	10.639095	1.220922
1.72	9.841149	0.064304	14.319389	0.333832	10.677899	1.249053
1.73	9.984877	0.067705	14.426360	0.346464	10.716347	1.277383
1.74	10.129677	0.071234	14.533715	0.359380	10.754444	1.305907
1.75	10.275553	0.074894	14.641448	0.372583	10.792198	1.334624
1.76	10.422507	0.078687	14.749557	0.386073	10.829615	1.363532
1.77	10.570545	0.082616	14.858039	0.399854	10.866702	1.392628
1.78	10.719669	0.086685	14.966890	0.413926	10.903465	1.421909
1.79	10.869884	0.090896	15.076107	0.428293	10.939911	1.451374
1.80	11.021193	0.095252	15.185688	0.442954	10.976045	1.481020
1.81	11.173599	0.099756	15.295627	0.457914	11.011873	1.510844
1.82	11.327107	0.104411	15.405924	0.473172	11.047402	1.540845
1.83	11.481719	0.109220	15.516574	0.488731	11.082635	1.571020
1.84	11.637439	0.114187	15.627576	0.504593	11.117580	1.601367
1.85	11.794271	0.119313	15.738925	0.520759	11.152240	1.631885

1.86	11.952219	0.124603	15.850620	0.537231	11.186622	1.662571
1.87	12.111285	0.130059	15.962657	0.554011	11.220730	1.693423
1.88	12.271473	0.135684	16.075033	0.571100	11.254568	1.724439
1.89	12.432787	0.141482	16.187747	0.588500	11.288142	1.755617
1.90	12.595229	0.147455	16.300795	0.606213	11.321457	1.786956
1.91	12.758804	0.153607	16.414175	0.624240	11.354515	1.818453
1.92	12.923514	0.159941	16.527885	0.642583	11.387323	1.850107
1.93	13.089363	0.166460	16.641921	0.661243	11.419884	1.881916
1.94	13.256353	0.173167	16.756282	0.680221	11.452202	1.913878
1.95	13.424489	0.180065	16.870964	0.699521	11.484281	1.945991
1.96	13.593774	0.187158	16.985966	0.719142	11.516125	1.978254
1.97	13.764210	0.194449	17.101286	0.739086	11.547739	2.010665
1.98	13.935800	0.201941	17.216920	0.759356	11.579125	2.043223
1.99	14.108549	0.209637	17.332868	0.779951	11.610287	2.075925
2.00	14.282459	0.217541	17.449125	0.800875	11.641229	2.108771

Questão 5:

Foi comparado o Método Trapezoidal Implícito ante aos métodos de Runge Kutta e o Método Preditor-Corretor de Adams. Como demonstrado pelas tabelas e gráficos abaixo, nota-se que os métodos tradicionais não são capazes de resolver EDOs rígidas, devido a n-ésima derivada da função ter magnitude $c^{(n)}*e^{(-ct)}$, de tal forma que o erro cresce de forma muito mais rápida que a aproximação da solução. O Método Trapezoidal Implícito por sua vez é capaz de resolver tais EDOs.

Método Trapezoidal Implícito:

t	w	y	erro
1.00	1.000000	1.000000	0.000000
1.25	0.497878	0.512000	0.014122
1.50	0.296431	0.296296	0.000134
1.75	0.185035	0.186589	0.001554
2.00	0.124836	0.125000	0.000164
2.25	0.087542	0.087791	0.000249
2.50	0.063923	0.064000	0.000077
2.75	0.048024	0.048084	0.000060
3.00	0.037007	0.037037	0.000030

Método de Runge-Kutta de 4th ordem:

t	w	y	erro
1.00	1.000000	1.000000	N/A
1.25	0.824063	0.512000	0.312063
1.50	1.568387	0.296296	1.272091
1.75	4.978672	0.186589	4.792083

2.00	18.029960	0.125000	17.904960
2.25	66.919013	0.087791	66.831222
2.50	249.479938	0.064000	249.415938
2.75	930.855541	0.048084	930.807457
3.00	3473.751002	0.037037	3473.713965

Método Preditor Corretor de Adams:

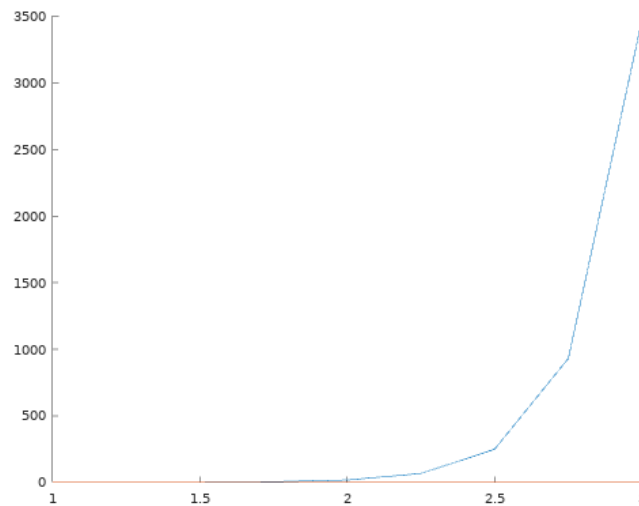| t	w	y	erro
1.00	1.000000	N/A	N/A
1.25	0.824063	0.512000	0.000000
1.50	1.568387	0.296296	0.312063
1.75	4.978672	0.186589	1.272091
2.00	28.782928	0.125000	4.792083
2.25	200.824274	0.087791	28.657928
2.50	1435.030947	0.064000	200.736482
2.75	10272.087411	0.048084	1434.966947
3.00	73531.371208	0.037037	10272.039327

Figura 3: Gráfico da solução aproximada e da solução exata, note que o Método de Runge-Kutta não foi capaz de aproximar a solução na EDO rígida, dado o grau do erro.
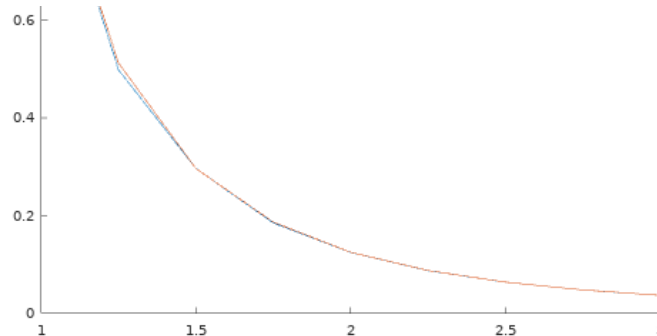

Figura 4: Gráfico da solução aproximada e da solução exata, note que o Método Trapezoidal Implícito foi capaz de aproximar a solução na EDO rígida

Questão 6:

As tabelas abaixo apresentam as soluções geradas pelo algoritmo de Runge-Kutta de 4ª ordem para a equação de Duffing, para h=0.01 e h=0.05 respectivamente, em um tempo t no intervalo de [0.0, 5.0], nas condições apresentadas pela questão.

Para h=0.01:

t	u1	u2
0.000000	1.500000	0.500000
0.010000	1.507262	0.757012
0.020000	1.511466	-0.086282
0.030000	1.502534	-1.831300
0.040000	1.472506	-4.265112
0.050000	1.415955	-7.078573
0.060000	1.330853	-9.916952
0.070000	1.218627	-12.456467
0.080000	1.083491	-14.472462
0.090000	0.931250	-15.873260
0.100000	0.768012	-16.684100
0.110000	0.599138	-17.024183
0.120000	0.428619	-17.036847
0.130000	0.259044	-16.855062
0.140000	0.091829	-16.577996
0.150000	-0.072383	-16.259132
0.160000	-0.233236	-15.902743
0.170000	-0.390174	-15.465272
0.180000	-0.541986	-14.860116
0.190000	-0.686421	-13.968248
0.200000	-0.819954	-12.658381

0.210000	*-0.937799*	*-10.815386*
0.220000	*-1.034275*	*-8.382261*
0.230000	*-1.103583*	*-5.397629*
0.240000	*-1.140910*	*-2.022009*
0.250000	*-1.143509*	*1.503252*
0.260000	*-1.111300*	*4.892695*
0.270000	*-1.046973*	*7.890909*
0.280000	*-0.955357*	*10.332406*
0.290000	*-0.842383*	*12.162285*
0.300000	*-0.713999*	*13.428658*
0.310000	*-0.575337*	*14.238636*
0.320000	*-0.430304*	*14.724869*
0.330000	*-0.281506*	*15.010278*
0.340000	*-0.130445*	*15.190307*
0.350000	*0.022150*	*15.323422*
0.360000	*0.175930*	*15.426697*
0.370000	*0.330503*	*15.473862*
0.380000	*0.484995*	*15.395503*
0.390000	*0.637630*	*15.081150*
0.400000	*0.785359*	*14.389256*
0.410000	*0.923646*	*13.168742*
0.420000	*1.046541*	*11.295089*

0.430000	*1.147177*	*8.717229*
0.440000	*1.218728*	*5.500520*
0.450000	*1.255726*	*1.850893*
0.460000	*1.255272*	*-1.936269*
0.470000	*1.217655*	*-5.527591*
0.480000	*1.146349*	*-8.636846*
0.490000	*1.047096*	*-11.099765*
0.500000	*0.926661*	*-12.876132*
0.510000	*0.791655*	*-14.031494*
0.520000	*0.647686*	*-14.692078*
0.530000	*0.498966*	*-15.006098*
0.540000	*0.348269*	*-15.107657*
0.550000	*0.197170*	*-15.100709*
0.560000	*0.046385*	*-15.053097*
0.570000	*-0.103863*	*-14.995206*
0.580000	*-0.253470*	*-14.921191*
0.590000	*-0.402104*	*-14.791037*
0.600000	*-0.548869*	*-14.533217*
0.610000	*-0.692019*	*-14.050059*
0.620000	*-0.828740*	*-13.228412*
0.630000	*-0.955085*	*-11.958084*
0.640000	*-1.066125*	*-10.158392*

0.650000	*-1.156402*	*-7.808611*
0.660000	*-1.220695*	*-4.982058*
0.670000	*-1.254922*	*-1.825385*
0.680000	*-1.256825*	*1.442433*
0.690000	*-1.226519*	*4.577478*
0.700000	*-1.166442*	*7.366133*
0.710000	*-1.080838*	*9.666671*
0.720000	*-0.974928*	*11.425851*
0.730000	*-0.854057*	*12.667745*
0.740000	*-0.723031*	*13.473302*
0.750000	*-0.585695*	*13.947676*
0.760000	*-0.444827*	*14.196735*
0.770000	*-0.302201*	*14.312724*
0.780000	*-0.158783*	*14.364535*
0.790000	*-0.014978*	*14.395119*
0.800000	*0.129107*	*14.420795*
0.810000	*0.273392*	*14.431531*
0.820000	*0.417572*	*14.391526*
0.830000	*0.560855*	*14.240441*
0.840000	*0.701739*	*13.896544*
0.850000	*0.837827*	*13.264845*
0.860000	*0.965745*	*12.247274*

0.870000	*1.081220*	*10.766252*
0.880000	*1.179376*	*8.781237*
0.890000	*1.255227*	*6.315604*
0.900000	*1.304400*	*3.468126*
0.910000	*1.323864*	*0.405400*
0.920000	*1.312474*	*-2.666428*
0.930000	*1.271213*	*-5.536680*
0.940000	*1.203016*	*-8.029597*
0.950000	*1.112190*	*-10.054015*
0.960000	*1.003590*	*-11.585551*
0.970000	*0.882014*	*-12.658953*
0.980000	*0.751694*	*-13.348623*
0.990000	*0.616015*	*-13.746293*
1.000000	*0.477436*	*-13.943121*
1.010000	*0.337557*	*-14.017882*
1.020000	*0.197280*	*-14.031296*
1.030000	*0.057000*	*-14.023128*
1.040000	*-0.083176*	*-14.012089*
1.050000	*-0.223229*	*-13.996424*
1.060000	*-0.363020*	*-13.954551*
1.070000	*-0.502100*	*-13.845995*
1.080000	*-0.639529*	*-13.613489*

1.090000	*-0.773720*	*-13.185807*
1.100000	*-0.902333*	*-12.484767*
1.110000	*-1.022250*	*-11.435678*
1.120000	*-1.129695*	*-9.983365*
1.130000	*-1.220508*	*-8.110649*
1.140000	*-1.290584*	*-5.844676*
1.150000	*-1.336378*	*-3.273800*
1.160000	*-1.355467*	*-0.528144*
1.170000	*-1.346895*	*2.230620*
1.180000	*-1.311361*	*4.838684*
1.190000	*-1.251087*	*7.159349*
1.200000	*-1.169440*	*9.102978*
1.210000	*-1.070412*	*10.633961*
1.220000	*-0.958115*	*11.761690*
1.230000	*-0.836348*	*12.538881*
1.240000	*-0.708278*	*13.034152*
1.250000	*-0.576360*	*13.320657*
1.260000	*-0.442332*	*13.466717*
1.270000	*-0.307301*	*13.529483*
1.280000	*-0.171875*	*13.551788*
1.290000	*-0.036307*	*13.561025*
1.300000	*0.099343*	*13.569009*

1.310000	*0.235059*	*13.572278*
1.320000	*0.370714*	*13.552532*
1.330000	*0.505926*	*13.477148*
1.340000	*0.639920*	*13.300267*
1.350000	*0.771405*	*12.965166*
1.360000	*0.898489*	*12.409698*
1.370000	*1.018651*	*11.570881*
1.380000	*1.128784*	*10.396537*
1.390000	*1.225358*	*8.856990*
1.400000	*1.304712*	*6.957191*
1.410000	*1.363458*	*4.746184*
1.420000	*1.398872*	*2.308230*
1.430000	*1.409281*	*-0.231886*
1.440000	*1.394333*	*-2.741557*
1.450000	*1.354987*	*-5.091394*
1.460000	*1.293394*	*-7.176257*
1.470000	*1.212576*	*-8.928855*
1.480000	*1.116017*	*-10.323631*
1.490000	*1.007261*	*-11.372461*
1.500000	*0.889611*	*-12.110120*
1.510000	*0.765904*	*-12.593878*
1.520000	*0.638379*	*-12.883869*

1.530000	*0.508682*	*-13.037124*
1.540000	*0.377926*	*-13.103071*
1.550000	*0.246780*	*-13.120792*
1.560000	*0.115577*	*-13.117863*
1.570000	*-0.015563*	*-13.110136*
1.580000	*-0.146625*	*-13.101954*
1.590000	*-0.277580*	*-13.086515*
1.600000	*-0.408274*	*-13.046260*
1.610000	*-0.538331*	*-12.953414*
1.620000	*-0.667045*	*-12.770929*
1.630000	*-0.793303*	*-12.454331*
1.640000	*-0.915525*	*-11.955717*
1.650000	*-1.031647*	*-11.226617*
1.660000	*-1.139148*	*-10.225904*
1.670000	*-1.235168*	*-8.927257*
1.680000	*-1.316679*	*-7.326843*
1.690000	*-1.380793*	*-5.454033*
1.700000	*-1.425022*	*-3.361679*
1.710000	*-1.447580*	*-1.136033*
1.720000	*-1.447618*	*1.125750*
1.730000	*-1.425291*	*3.320257*
1.740000	*-1.381756*	*5.353138*

1.750000	*-1.319015*	*7.151483*
1.760000	*-1.239662*	*8.670607*
1.770000	*-1.146589*	*9.894409*
1.780000	*-1.042738*	*10.830808*
1.790000	*-0.930821*	*11.513521*
1.800000	*-0.813178*	*11.983265*
1.810000	*-0.691715*	*12.285256*
1.820000	*-0.567884*	*12.463790*
1.830000	*-0.442718*	*12.558513*
1.840000	*-0.316883*	*12.602200*
1.850000	*-0.190760*	*12.619674*
1.860000	*-0.064520*	*12.627489*
1.870000	*0.061786*	*12.633849*
1.880000	*0.188153*	*12.638856*
1.890000	*0.314534*	*12.634739*
1.900000	*0.440767*	*12.606098*
1.910000	*0.566499*	*12.530263*
1.920000	*0.691119*	*12.378208*
1.930000	*0.813694*	*12.115296*
1.940000	*0.932929*	*11.703762*
1.950000	*1.047147*	*11.105778*
1.960000	*1.154316*	*10.289377*

1.970000	*1.252119*	*9.229358*
1.980000	*1.338054*	*7.916181*
1.990000	*1.409634*	*6.361467*
2.000000	*1.464570*	*4.594263*
2.010000	*1.500983*	*2.667019*
2.020000	*1.517607*	*0.649449*
2.030000	*1.513935*	*-1.378787*
2.040000	*1.490265*	*-3.337398*
2.050000	*1.447661*	*-5.153823*
2.060000	*1.387865*	*-6.769196*
2.070000	*1.313051*	*-8.153582*
2.080000	*1.225613*	*-9.293195*
2.090000	*1.127989*	*-10.193051*
2.100000	*1.022488*	*-10.872751*
2.110000	*0.911171*	*-11.361520*
2.120000	*0.795781*	*-11.693250*
2.130000	*0.677714*	*-11.902728*
2.140000	*0.558022*	*-12.023205*
2.150000	*0.437451*	*-12.083161*
2.160000	*0.316481*	*-12.106220*
2.170000	*0.195390*	*-12.110067*
2.180000	*0.074304*	*-12.106591*

2.190000	*-0.046739*	*-12.101963*
2.200000	*-0.167735*	*-12.096800*
2.210000	*-0.288658*	*-12.086398*
2.220000	*-0.409413*	*-12.060908*
2.230000	*-0.529778*	*-12.005585*
2.240000	*-0.649362*	*-11.901389*
2.250000	*-0.767565*	*-11.724984*
2.260000	*-0.883537*	*-11.450929*
2.270000	*-0.996166*	*-11.051855*
2.280000	*-1.104068*	*-10.501278*
2.290000	*-1.205609*	*-9.776255*
2.300000	*-1.298955*	*-8.860356*
2.310000	*-1.382155*	*-7.746823*
2.320000	*-1.453249*	*-6.441252*
2.330000	*-1.510424*	*-4.968755*
2.340000	*-1.552160*	*-3.358870*
2.350000	*-1.577289*	*-1.655842*
2.360000	*-1.585144*	*0.086664*
2.370000	*-1.575615*	*1.811631*
2.380000	*-1.549156*	*3.463626*
2.390000	*-1.506744*	*4.995557*
2.400000	*-1.449778*	*6.368827*

2.410000	*-1.379967*	*7.562336*
2.420000	*-1.299172*	*8.564728*
2.430000	*-1.209295*	*9.380382*
2.440000	*-1.112147*	*10.021841*
2.450000	*-1.009376*	*10.508272*
2.460000	*-0.902425*	*10.861807*
2.470000	*-0.792503*	*11.106390*
2.480000	*-0.680578*	*11.266364*
2.490000	*-0.567384*	*11.363796*
2.500000	*-0.453447*	*11.417841*
2.510000	*-0.339119*	*11.444316*
2.520000	*-0.224611*	*11.455435*
2.530000	*-0.110032*	*11.459839*
2.540000	*0.004581*	*11.462557*
2.550000	*0.119219*	*11.465142*
2.560000	*0.233877*	*11.465805*
2.570000	*0.348513*	*11.459533*
2.580000	*0.463018*	*11.438240*
2.590000	*0.577191*	*11.390929*
2.600000	*0.690705*	*11.304017*
2.610000	*0.803089*	*11.162042*
2.620000	*0.913703*	*10.947134*

2.630000	*1.021727*	*10.640866*
2.640000	*1.126154*	*10.224863*
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2.660000	*1.319346*	*9.001681*
2.670000	*1.405344*	*8.173355*
2.680000	*1.482321*	*7.197588*
2.690000	*1.548819*	*6.079121*
2.700000	*1.603485*	*4.835119*
2.710000	*1.645169*	*3.486528*
2.720000	*1.672964*	*2.062836*
2.730000	*1.686293*	*0.599341*
2.740000	*1.684945*	*-0.865461*
2.750000	*1.669114*	*-2.291845*
2.760000	*1.639344*	*-3.647736*
2.770000	*1.596492*	*-4.904046*
2.780000	*1.541669*	*-6.038848*
2.790000	*1.476176*	*-7.036144*
2.800000	*1.401419*	*-7.891802*
2.810000	*1.318813*	*-8.606136*
2.820000	*1.229740*	*-9.187325*
2.830000	*1.135470*	*-9.647522*
2.840000	*1.037148*	*-10.000389*

2.850000	*0.935768*	*-10.261542*
2.860000	*0.832169*	*-10.447039*
2.870000	*0.727028*	*-10.572370*
2.880000	*0.620876*	*-10.651503*
2.890000	*0.514108*	*-10.697370*
2.900000	*0.407003*	*-10.720622*
2.910000	*0.299742*	*-10.729819*
2.920000	*0.192432*	*-10.731443*
2.930000	*0.085124*	*-10.729928*
2.940000	*-0.022165*	*-10.727737*
2.950000	*-0.129431*	*-10.725428*
2.960000	*-0.236669*	*-10.721731*
2.970000	*-0.343851*	*-10.713608*
2.980000	*-0.450911*	*-10.696342*
2.990000	*-0.557727*	*-10.663670*
3.000000	*-0.664107*	*-10.607853*
3.010000	*-0.769777*	*-10.519848*
3.020000	*-0.874364*	*-10.389687*
3.030000	*-0.977392*	*-10.206226*
3.040000	*-1.078274*	*-9.958399*
3.050000	*-1.176309*	*-9.635214*
3.060000	*-1.270693*	*-9.226637*

3.070000	*-1.360531*	*-8.724703*
3.080000	*-1.444851*	*-8.122637*
3.090000	*-1.522642*	*-7.418193*
3.100000	*-1.592871*	*-6.610773*
3.110000	*-1.654529*	*-5.704992*
3.120000	*-1.706673*	*-4.709891*
3.130000	*-1.748473*	*-3.638892*
3.140000	*-1.779254*	*-2.509315*
3.150000	*-1.798531*	*-1.342420*
3.160000	*-1.806067*	*-0.163986*
3.170000	*-1.801835*	*1.007319*
3.180000	*-1.786019*	*2.149207*
3.190000	*-1.759017*	*3.241238*
3.200000	*-1.721419*	*4.265885*
3.210000	*-1.673971*	*5.209298*
3.220000	*-1.617533*	*6.062553*
3.230000	*-1.553049*	*6.817968*
3.240000	*-1.481502*	*7.474968*
3.250000	*-1.403874*	*8.034831*
3.260000	*-1.321111*	*8.503023*
3.270000	*-1.234097*	*8.886402*
3.280000	*-1.143638*	*9.193358*

3.290000	*-1.050455*	*9.432800*
3.300000	*-0.955173*	*9.614779*
3.310000	*-0.858319*	*9.748894*
3.320000	*-0.760325*	*9.844203*
3.330000	*-0.661536*	*9.909066*
3.340000	*-0.562220*	*9.950915*
3.350000	*-0.462573*	*9.976236*
3.360000	*-0.362732*	*9.990413*
3.370000	*-0.262787*	*9.997791*
3.380000	*-0.162788*	*10.001616*
3.390000	*-0.062759*	*10.004044*
3.400000	*0.037292*	*10.006191*
3.410000	*0.137364*	*10.008168*
3.420000	*0.237452*	*10.009176*
3.430000	*0.337538*	*10.007441*
3.440000	*0.437583*	*10.000159*
3.450000	*0.537511*	*9.983586*
3.460000	*0.637208*	*9.953107*
3.470000	*0.736509*	*9.903338*
3.480000	*0.835190*	*9.828110*
3.490000	*0.932963*	*9.720642*
3.500000	*1.029472*	*9.573776*

3.510000	*1.124281*	*9.379685*
3.520000	*1.216882*	*9.130645*
3.530000	*1.306686*	*8.819205*
3.540000	*1.393035*	*8.438539*
3.550000	*1.475207*	*7.982891*
3.560000	*1.552428*	*7.448150*
3.570000	*1.623901*	*6.832710*
3.580000	*1.688810*	*6.135639*
3.590000	*1.746349*	*5.359471*
3.600000	*1.795753*	*4.509589*
3.610000	*1.836320*	*3.593353*
3.620000	*1.867432*	*2.621033*
3.630000	*1.888603*	*1.607836*
3.640000	*1.899509*	*0.570412*
3.650000	*1.899980*	*-0.476274*
3.660000	*1.890009*	*-1.515292*
3.670000	*1.869755*	*-2.530069*
3.680000	*1.839541*	*-3.505068*
3.690000	*1.799832*	*-4.426818*
3.700000	*1.751221*	*-5.283810*
3.710000	*1.694399*	*-6.067747*
3.720000	*1.630129*	*-6.773044*

3.730000	*1.559211*	*-7.396913*
3.740000	*1.482463*	*-7.939205*
3.750000	*1.400691*	*-8.402109*
3.760000	*1.314672*	*-8.789741*
3.770000	*1.225131*	*-9.107307*
3.780000	*1.132735*	*-9.361907*
3.790000	*1.038076*	*-9.561325*
3.800000	*0.941669*	*-9.712892*
3.810000	*0.843953*	*-9.824209*
3.820000	*0.745294*	*-9.902681*
3.830000	*0.645986*	*-9.955020*
3.840000	*0.546260*	*-9.987434*
3.850000	*0.446285*	*-10.005478*
3.860000	*0.346182*	*-10.013840*
3.870000	*0.246028*	*-10.016316*
3.880000	*0.145866*	*-10.015744*
3.890000	*0.045717*	*-10.014064*
3.900000	*-0.054415*	*-10.012245*
3.910000	*-0.154528*	*-10.010207*
3.920000	*-0.254615*	*-10.006873*
3.930000	*-0.354654*	*-10.000141*
3.940000	*-0.454596*	*-9.986824*

3.950000	*-0.554354*	*-9.962613*
3.960000	*-0.653794*	*-9.922093*
3.970000	*-0.752720*	*-9.858703*
3.980000	*-0.850866*	*-9.764711*
3.990000	*-0.947884*	*-9.631622*
4.000000	*-1.043337*	*-9.450076*
4.010000	*-1.136690*	*-9.209844*
4.020000	*-1.227305*	*-8.901044*
4.030000	*-1.314450*	*-8.513983*
4.040000	*-1.397296*	*-8.040319*
4.050000	*-1.474941*	*-7.472349*
4.060000	*-1.546411*	*-6.804403*
4.070000	*-1.610680*	*-6.032483*
4.080000	*-1.666724*	*-5.159820*
4.090000	*-1.713559*	*-4.192122*
4.100000	*-1.750283*	*-3.139569*
4.110000	*-1.776120*	*-2.017906*
4.120000	*-1.790468*	*-0.844722*
4.130000	*-1.792913*	*0.358648*
4.140000	*-1.783272*	*1.568618*
4.150000	*-1.761600*	*2.761588*
4.160000	*-1.728165*	*3.916667*

4.170000	*-1.683477*	*5.008657*
4.180000	*-1.628267*	*6.018680*
4.190000	*-1.563420*	*6.933999*
4.200000	*-1.489934*	*7.745458*
4.210000	*-1.408868*	*8.449674*
4.220000	*-1.321302*	*9.045809*
4.230000	*-1.228300*	*9.537748*
4.240000	*-1.130870*	*9.933022*
4.250000	*-1.029928*	*10.241761*
4.260000	*-0.926289*	*10.474325*
4.270000	*-0.820656*	*10.642650*
4.280000	*-0.713610*	*10.758805*
4.290000	*-0.605615*	*10.834348*
4.300000	*-0.497023*	*10.879853*
4.310000	*-0.388087*	*10.904550*
4.320000	*-0.278975*	*10.916211*
4.330000	*-0.169786*	*10.920841*
4.340000	*-0.060567*	*10.922743*
4.350000	*0.048668*	*10.924283*
4.360000	*0.157918*	*10.925548*
4.370000	*0.267171*	*10.924378*
4.380000	*0.376383*	*10.916243*

4.390000	*0.485450*	*10.894173*
4.400000	*0.594189*	*10.848729*
4.410000	*0.702307*	*10.767909*
4.420000	*0.809381*	*10.637263*
4.430000	*0.914829*	*10.439588*
4.440000	*1.017889*	*10.156851*
4.450000	*1.117619*	*9.770011*
4.460000	*1.212878*	*9.259907*
4.470000	*1.302349*	*8.609543*
4.480000	*1.384556*	*7.805497*
4.490000	*1.457920*	*6.840414*
4.500000	*1.520826*	*5.714443*
4.510000	*1.571702*	*4.436457*
4.520000	*1.609102*	*3.023770*
4.530000	*1.631835*	*1.508848*
4.540000	*1.639065*	*-0.069856*
4.550000	*1.630373*	*-1.667980*
4.560000	*1.605796*	*-3.238882*
4.570000	*1.565836*	*-4.737656*
4.580000	*1.511421*	*-6.123927*
4.590000	*1.443832*	*-7.368110*
4.600000	*1.364600*	*-8.450817*

4.610000	1.275374	-9.365794
4.620000	1.177856	-10.110040
4.630000	1.073712	-10.693118
4.640000	0.964477	-11.131389
4.650000	0.851500	-11.445185
4.660000	0.735912	-11.657331
4.670000	0.618616	-11.790527
4.680000	0.500294	-11.866056
4.690000	0.381424	-11.902868
4.700000	0.262312	-11.916663
4.710000	0.143128	-11.919080
4.720000	0.023943	-11.917765
4.730000	-0.095226	-11.915964
4.740000	-0.214371	-11.912289
4.750000	-0.333445	-11.900311
4.760000	-0.452312	-11.868528
4.770000	-0.570695	-11.800160
4.780000	-0.688120	-11.673013
4.790000	-0.803868	-11.459761
4.800000	-0.916926	-11.129639
4.810000	-1.025959	-10.649021
4.820000	-1.129296	-9.985584

4.830000	*-1.224964*	*-9.110767*
4.840000	*-1.310737*	*-8.004435*
4.850000	*-1.384258*	*-6.660757*
4.860000	*-1.443190*	*-5.089845*
4.870000	*-1.485396*	*-3.321537*
4.880000	*-1.509114*	*-1.402942*
4.890000	*-1.513191*	*0.594250*
4.900000	*-1.497224*	*2.592629*
4.910000	*-1.461597*	*4.513640*
4.920000	*-1.407452*	*6.285614*
4.930000	*-1.336563*	*7.855073*
4.940000	*-1.251143*	*9.187943*
4.950000	*-1.153639*	*10.271282*
4.960000	*-1.046530*	*11.112021*
4.970000	*-0.932132*	*11.733172*
4.980000	*-0.812499*	*12.164858*
4.990000	*-0.689343*	*12.444211*
5.000000	*-0.563994*	*12.609552*

Para h=0.05:

t	u1	u2
0.000000	1.500000	0.500000

0.050000	1.403086	-6.903270
0.100000	0.775109	-16.283331
0.150000	-0.048083	-15.899925
0.200000	-0.783307	-12.644313
0.250000	-1.116880	0.263454
0.300000	-0.755761	12.261280
0.350000	-0.065513	14.492811
0.400000	0.666250	14.219380
0.450000	1.190881	5.000947
0.500000	1.021516	-10.229302
0.550000	0.377174	-13.991037
0.600000	-0.320234	-13.762771
0.650000	-0.957619	-10.643264
0.700000	-1.202453	1.454790
0.750000	-0.843862	11.279051
0.800000	-0.218325	12.988781
0.850000	0.434763	13.018674
0.900000	1.033280	9.845775
0.950000	1.256429	-1.455586
1.000000	0.917606	-10.697766
1.050000	0.319097	-12.475180
1.100000	-0.305043	-12.432580

1.150000	-0.902229	-10.819908
1.200000	-1.269375	-2.912085
1.250000	-1.138000	7.456317
1.300000	-0.636933	11.540537
1.350000	-0.045016	11.921212
1.400000	0.550858	11.785834
1.450000	1.089531	8.965828
1.500000	1.336226	0.326820
1.550000	1.117230	-8.276974
1.600000	0.609883	-11.208303
1.650000	0.040713	-11.407711
1.700000	-0.527888	-11.246202
1.750000	-1.052917	-9.168328
1.800000	-1.358783	-2.424724
1.850000	-1.261614	5.910098
1.900000	-0.842071	10.084657
1.950000	-0.311126	10.850672
2.000000	0.232870	10.888553
2.050000	0.771569	10.450647
2.100000	1.233965	7.437804
2.150000	1.444146	0.589322
2.200000	1.288116	-6.382925

2.250000	0.870813	-9.688955
2.300000	0.364397	-10.323091
2.350000	-0.152261	-10.324340
2.400000	-0.665604	-10.108485
2.450000	-1.139423	-8.462372
2.500000	-1.459935	-3.895146
2.550000	-1.494586	2.495354
2.600000	-1.237668	7.313343
2.650000	-0.812861	9.282670
2.700000	-0.335900	9.657413
2.750000	0.147760	9.678395
2.800000	0.630490	9.572227
2.850000	1.091242	8.626482
2.900000	1.460102	5.767892
2.950000	1.632454	0.928036
3.000000	1.549359	-4.076977
3.050000	1.255102	-7.339686
3.100000	0.848661	-8.661301
3.150000	0.406253	-8.938773
3.200000	-0.041051	-8.942433
3.250000	-0.487623	-8.902713
3.300000	-0.926231	-8.542753

3.350000	-1.326013	-7.237486
3.400000	-1.624299	-4.444659
3.450000	-1.748421	-0.404853
3.500000	-1.663697	3.676485
3.550000	-1.401164	6.578484
3.600000	-1.030835	8.021503
3.650000	-0.615395	8.486654
3.700000	-0.188518	8.559645
3.750000	0.239746	8.568296
3.800000	0.667086	8.488036
3.850000	1.080729	7.927411
3.900000	1.440503	6.221604
3.950000	1.674661	2.891775
4.000000	1.709412	-1.567298
4.050000	1.524130	-5.647799
4.100000	1.172050	-8.144915
4.150000	0.736104	-9.093727
4.200000	0.275659	-9.256587
4.250000	-0.187161	-9.250879
4.300000	-0.647884	-9.120533
4.350000	-1.085819	-8.162069
4.400000	-1.428580	-5.140121

4.450000	-1.556571	0.269174
4.500000	-1.396916	5.875727
4.550000	-1.009971	9.144533
4.600000	-0.522303	10.095370
4.650000	-0.014345	10.175853
4.700000	0.494140	10.118405
4.750000	0.981883	9.062952
4.800000	1.345416	4.858567
4.850000	1.406565	-2.565436
4.900000	1.111369	-8.629526
4.950000	0.613171	-10.754068
5.000000	0.067559	-10.951017

Para um h=0.01, a função de Duffing foi avaliada 2000 vezes, enquanto que para um h=0.05, a função foi avaliada apenas 400 vezes. Um passo maior resulta em menos iterações do algoritmo, (visto que a quantidade de iterações é dada por (b-a)/h) e consequentemente um tempo de execução menor.

Os gráficos abaixo demonstram as aproximações geradas pelo algoritmo RK4 para y(t) (em azul) e y'(t) (em amarelo) para h=0.01 e h=0.05 respectivamente:
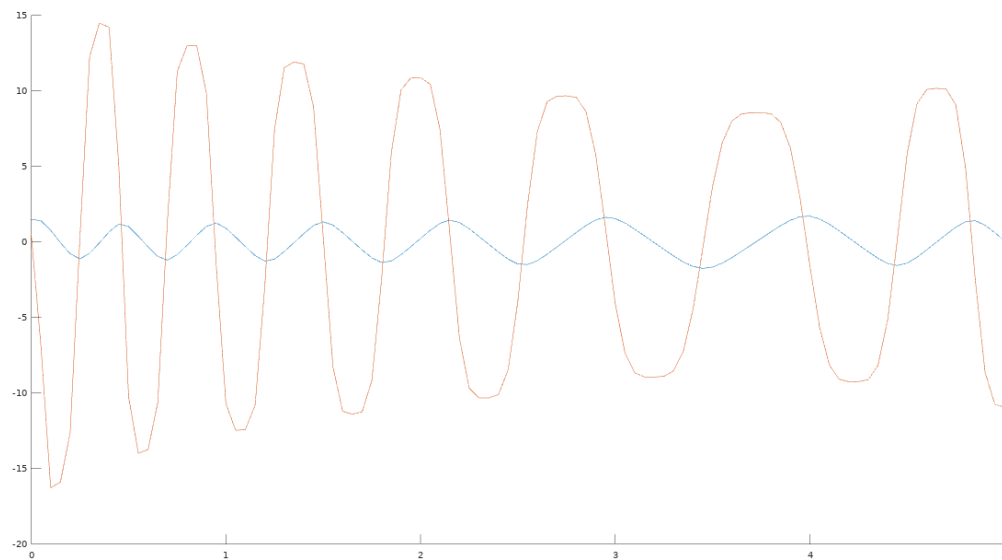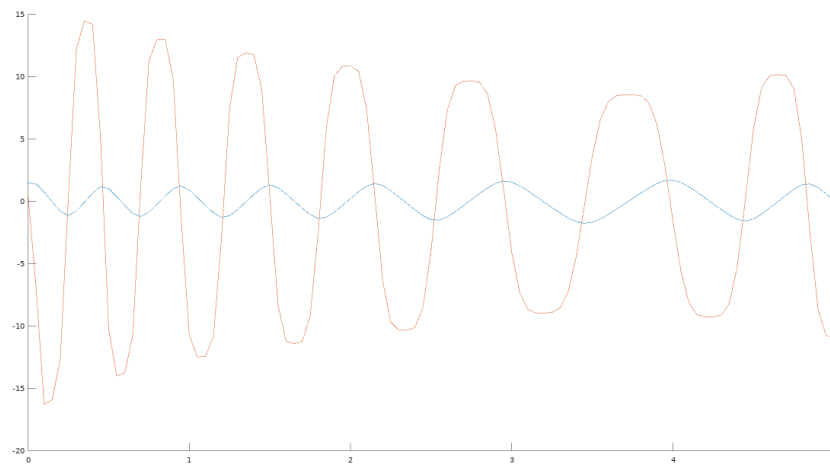
Figura 5: Gráfico das séries de y(t) e y'(t) para h=0.01

Figura 6: Gráfico das séries de y(t) e y'(t) para h=0.05

Note que um h menor possibilita a obtenção de uma solução mais "suave",i.e: existe uma melhor aproximação para a mudança do sistema de um (x,f(x)) até um (x+h,f(x+h)). Este fato é notado ao comparar as tabelas das soluções aproximadas para cada um dos passos de tempo, de tal forma que a diferença entre um y(t) e/ou y'(t) de um valor aproximado via h=0.01 e h=0.05 chegam a ser maiores que (10^-1), representando um ganho significativo de precisão (e custo computacional) dado o decremento do tamanho do passo no algoritmo.