(8) Min $f(x_1, x_2) = (x_1 - 3)^2 + (x_2 - 2)^2$

Sujeito a:

$h_1(x_1, x_2): 2x_1 + x_2 = 8$

$h_2(x_1, x_2): (x_1 - 1)^2 + (x_2 - 4)^2 = 4$

$g_1(x_1, x_2): x_1 + x_2 \leq 7$

$g_2(x_1, x_2): x_1 - 0.25 x_2^2 \leq 0$

$0 \leq x_1 \leq 10$

$0 \leq x_2 \leq 10$

$$\nabla f = \begin{bmatrix} 2x_1 - 6 \\ 2x_2 - 4 \end{bmatrix} = 0 \Rightarrow \begin{cases} 2x_1 - 6 = 0 \\ 2x_2 - 4 = 0 \end{cases} \quad \times \tfrac{1}{2} \qquad \begin{matrix} x_1 = 3 \\ x_2 = 2 \end{matrix}$$

Avaliando as restrições p/ ponto (3,2)

$h_1 = 6 + 2 - 8 = 0$ (ok)

$h_2 =$ ~~[illegible]~~ $8 \neq 4$ (ñ ok)

$g_1 = 3 + 2 - 7 = -2$ (ok)

$g_2 = 3 - 0.25 \cdot 4 = 2$ (ñ ok)

$2x_1 + x_2 = 8 - x_2$

$(x_1 - 1)^2 + (x_2 - 4)^2 = 4$

$x_1^2 - 2x_1 + 1 + ((8 - 2x_1) - 4)^2 = 4$

$x_1^2 - 2x_1 + 1 + 16 - 16x_1$

$5x_1^2 - 18x_1 + 13 = 0$

Ponto 1: $x_1 = 2.6$
2.8

Ponto 2: $x_1 = 1$
$x_2 = 6$

P/ ponto 1 - viola restrição:

$h_1 = 5$

$h_2 = (2.56)$

$g_1 = 5.4 - 7 = -1.6$, Ok => restrição não ativa.

$g_2 = 2.6 - 0.25 \cdot (2.8^2) = 0.6$ (violação)

P/ ponto 2:

$h_1 = 2 \cdot 1 + 6 - 8 = 0$ Ok

$h_2 = 4 = 4$ Ok

$g_1 = 7 \leq 7$

$g_2 = 1 - 0.25 \cdot (6)^2 = -8$ Ok => restrição não ativa.

Possível ótimo => (1, 6)

Usando:

$$\begin{cases} 2x_1 + x_2 = 8 \\ x_1 + x_2 - 7 = 0 \end{cases} \Rightarrow \begin{matrix} x_2 = 8 - 2x_1 \\ x_1 + 8 - 2x_2 - 7 = 0 \end{matrix} \Bigg] \begin{matrix} x_1 = 1 \\ x_2 = 6 \end{matrix}$$

Soluções:

Ponto 1: $\begin{cases} x_1 = 1 \\ x_2 = 6 \end{cases}$

Ponto 2: $\begin{cases} x_1'' = 3 \\ x_2'' = 4 \end{cases}$

$h_1 = 2\cancel{\ldots}$ ñ Ok

$h_2 = (3-1)^2 + (4-4)^2 - 4 = 0$ Ok

$g_1 = 3 + 4 - 7 = 0$ Ok

$g_2 = 3 - 0.25 \cdot 4^2 = -1$ Ok mas ñ ativa.

$x_1 - 0.25x_2^2$ e $(x_1-1)^2 - (x_2-4)^2 - (x_2-4)^2 - 4 = 0 \Rightarrow$ usando essas restrições:

$$(0.25x_2^2 - 1)^2 - (x_2-4)^2 - 4 = 0$$

$$0.0625x_2^4 + 0.5x_2^2 - 8x_2 + 13 = 0$$

$$x_2^7 + 8x_2^2 - 128x_2 - 208 = 0$$

$x_2' = 2$

$x_2'' = 3.4128$

$x_2''' = -2.706 + 4.816$

$x_2''' = -2.706 + 4.816$ } válidas

p/ $\begin{cases} x_1' = 1 \\ x_2' = 2 \end{cases} \Rightarrow \begin{cases} h_1 = 2 + 2 - 8 = 4 & \text{ñ ok} \\ h_2 = (1-1)^2 + (2-4)^2 - 4 = 0, & \text{ok} \\ g_1 = 1 + 2 - 7 = -4 & \text{ok, mas ñ ativa.} \\ g_2 = 1 - 0.25 \cdot 2^2 = 0 & \text{ok} \end{cases}$

$\Downarrow$

viável.

Encontrado os valores ótimos:

$$x_2 - 0.12 + \frac{0.07}{0.06}x_1 = 0 \qquad x_2 = 0.12 - 0.11667x_1$$

$\Rightarrow$

$$\begin{cases} -\pi \left[3x_1 + 3x_2 - 2\sqrt{x_1 + x_2}\right] + 1.75 = 0 \\ -\pi \left[3x_1 + 0.36 - 0.35x_1 - 2\sqrt{0.88333x_1 + 0.12}\right] + 1.75 = 0 \\ -\pi \left[3x_1 + 0.36 - 0.35x_1 - 2\sqrt{0.88333x_1 + 0.12}\right] + 1.75 = 0 \\ -\pi \left[2.65x_1 + 0.36 - 2\sqrt{0.88333x_1 + 0.12}\right] - 0.557 = 0 \\ 2.65x_1 + 0.36 - 2\sqrt{0.88333x_1 + 0.12} - 0.557 = 0 \\ 2.65x_1 - 0.197 = \left[2\sqrt{(0.88333x_1 + 0.12)}\right]^2 = 0 \end{cases}$$

$$7.0225x_1^2 - 1.0773x_1 + 0.038 = 4 \cdot (0.88333x_1 + 0.12)$$

$$7.0225x_1^2 - 4.577654x_1 - 0.77114 = 0$$

$$\Downarrow$$

Ponto 1: $\begin{cases} x_1' = 0.7370 \\ x_1'' = 0.034006 \end{cases}$ Ponto 2: $\begin{cases} x_1 = -0.085231 \\ x_2 = 0.129944 \end{cases}$

Ambos os pontos violam restrições laterais

1ª condição: viabilidade de x_1 e x_2:

Encontrando os multiplicadores de Lagrange:

$$\begin{bmatrix} \pi x_2 \\ -\pi x_1 \end{bmatrix} + \lambda_1 \begin{bmatrix} \frac{0.07}{0.6} \\ 1 \end{bmatrix} + \lambda_2 \cdot \begin{bmatrix} \pi \cdot \left[3 - 2 \cdot \frac{1}{2} \frac{1}{\sqrt{x_1 + x_2}}\right] \\ \pi \cdot \left[3 - 2 \cdot \frac{1}{2} \frac{1}{\sqrt{x_1 + x_2}}\right] \end{bmatrix}$$

Aplicando no ponto 1:

$$\begin{cases} -0.106833 + 0.11667\lambda_1 + \lambda_2 \left[-3\pi + \frac{\pi}{0.878119}\right] = 0 \\ -2.315624 + \lambda_1 + \lambda_2 \left[-3\pi + \frac{\pi}{0.878119}\right] = 0 \end{cases}$$

$\Rightarrow$

$\Rightarrow \lambda_1 = 2.500522$, e (... resolvendo sistema anterior).
$\lambda_2 = 0.031621$

$\lambda_1 = 0.05384$
$x_2 = 0.062178$

Aplicando em P_2, temos: (~~aplicando~~)

$$\begin{cases} -0.408223 + 0.116667\lambda_1 + \lambda_2 \left[-3\pi \, e \dfrac{\pi}{0.211451} \right] = 0 \\ 0.267761 + \lambda_1 + \lambda_2 \cdot \left[-3\pi \cdot \dfrac{\pi}{0.211451} \right] = 0 \end{cases}$$

Ponto 2: $\left\{ \begin{array}{l} x_1'' = 2.9118 \\ x_2'' = 3.7128 \end{array} \right. \Rightarrow$

$h_1 = 1.236$ ñ ok
$h_2 = 0$ ok
$g_1 = -0.6765$ (ok) mas ñ ativa
$g_2 = 0$ (ok)

↳ viola restrição

~~Logo, o ponto viável é $x_1 = 1$ e $x_2 = 6$~~

$\nabla h_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $\quad \nabla h_2 = \begin{bmatrix} 2x_1 - 2 \\ 2x_2 - 8 \end{bmatrix}$ $\quad \nabla g_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\quad \nabla g_2 = \begin{bmatrix} 1 \\ -0.5x_2 \end{bmatrix}$

$\nabla F + \lambda_1 \nabla h_1 + \lambda_2 \nabla h_2 + \lambda_3 \nabla g_1 + \lambda_4 \nabla g_2 \Rightarrow$ desativado.

$\lambda_1 = 0$
$\lambda_2 = -3$
$\lambda_3 = 4$

(7) min $x_1 + x_2$, restrito a:

$$\begin{cases} 2x_1^2 - x_2 \leq 0 \\ 4 - x_1 - 3x_2 \leq 0 \\ 30 - x_1 - x_2^4 \leq 0 \end{cases} \Rightarrow \text{projeto inicial } (x_1, x_2) = (1, 1)$$

Condições de KKT:

Condição (I)

$x^* = (1, 1)$ deve ser viável

$g_1(1,1) = -2 - (1)^2 - 1 = 0$ ✓

$g_2(1,1) = 4 - 1 - 3 = 0$ ✓ $\therefore$ x^* satisfaz condição (I)

$g_3(1,1) = -30$ ✓

Condição (II)

(2) $\lambda_i g_i(x^*) = 0 \mid \lambda_i \geq 0$

$g_1(1,1) = g_2(1,1) = 0$ e $g_3(1,1) = -30$ $\therefore$ $\lambda_3 = 0$ (ok)

Condição (III)

$$\nabla f(x) + \sum_{i=1}^{n} \lambda_i g_i(x^*) = 0$$

$$\nabla f = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \nabla g_1 = \begin{bmatrix} -2x_1 \\ -1 \end{bmatrix}, \quad \nabla g_1(1,1) = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$$

$$, \quad \nabla g_2 = \begin{bmatrix} -1 \\ -3 \end{bmatrix}, \quad \nabla g_2(1,1) = \begin{bmatrix} -1 \\ -4 \end{bmatrix}$$

$$\begin{cases} 1 + \lambda_1(-2) + \lambda_2(-1) + 0 \cdot (-1) = 0 \\ 1 + \lambda_1(-1) + \lambda_2(-3) + 0 \cdot (-4) = 0 \end{cases} \Rightarrow \begin{cases} 1 - 2\lambda_1 - \lambda_2 = 0 \\ 1 - \lambda_1 - 3\lambda_2 = 0 \end{cases} \begin{matrix} \\ \rightarrow \times(-2) \end{matrix}$$

$$-1 + 5\lambda_2 = 0 \Rightarrow \lambda_2 = \frac{1}{5} \Rightarrow$$

$\Rightarrow$ em I ... $\Rightarrow 1 - 2\lambda_1 - \frac{1}{5}$

$\lambda_1 = \frac{2}{5}$ //, visto que λ_1, λ_2 e λ_3 existem, então a 3ª condição está Ok!

O problema de minimização satisfaz as condições de KKT.

⑥ Dado a elipse:

$$\left(\frac{x_1}{2}\right)^2 + x_2^2 - 4$$

Temos que o maior retângulo inscrito nesta elipse é dado por:

$$\frac{x_1^2}{4} + x_2^2 = 4 \Rightarrow \frac{x_1^2}{16} + \frac{x_2^2}{4} = 1$$

$$\max A = 4x_1x_2 \text{ restrito a:}$$

$$g(x_1, x_2) = \frac{x_1^2}{16} + \frac{x_2^2}{4} = 1 \quad \text{I}$$

$$\frac{dA}{dx_1} = 4x_1 \frac{dx_2}{dx_1} + 4x_2$$

$$\text{I} \quad \frac{df}{dx_1} = \frac{2x_1}{16} + \frac{2x_2}{4} \frac{dx_2}{dx_1} = 0$$

$$\frac{dx_2}{dx_1} = \frac{-4x_1}{16x_2}$$

$$\frac{dA}{dx_1} = 4x_2 - \frac{16x_1^2}{16x_2}$$

$$4x_2 - \frac{16x_1^2}{16x_2} = 0$$

$x_2^2 = \frac{4x_1^2}{16}$, é sabido que $x_2^2 = 4 - \frac{4x_1^2}{16}$, logo $x_2^2 = 4 - x_2^2$, $2x_2^2 = 4$ e $\frac{x_2^2}{4} = \frac{1}{2}$.

Logo, $\frac{x^2}{16} = \frac{1}{2}$. Podemos concluir que a área máxima do retângulo é:

$$x_1 = \frac{4}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2} \quad \text{e} \quad x_2 = \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2} \Rightarrow \boxed{\max(x_1, x_2) = (2\sqrt{2}, \sqrt{2})}.$$

(II) $f_0 = 4xy$

$g = -4 + \frac{x^2}{4} + y^2$

$f_p = (-1) \cdot f_0(x,y) + r_p \cdot (\max(0, g(x,y))^2))$

(5) Dado o problema:

$$\min \; x_1 + \frac{1}{x_1} + x_2 + \frac{1}{x_2}$$

temos que $\vec{\nabla}_f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}\right)$

$$\nabla_f = \left(1 + x_1^{-2},\ 1 + x_2^{-2}\right),$$

logo $H(f(x)) = \begin{bmatrix} \frac{\partial^2 f}{\partial^2 x_1} & \frac{\partial f}{\partial x_1 x_2} \\ \frac{\partial f}{\partial x_2 x_1} & \frac{\partial^2 f}{\partial^2 x_2} \end{bmatrix} = \begin{bmatrix} 2x_1^{-3} & 0 \\ 0 & 2x_2^{-3} \end{bmatrix}$

Os valores de x_1 e x_2 que satisfazem $\vec{\nabla}_f(x) = 0$ são obtidos pelo sistema abaixo:

$\begin{cases} 1 - x_1^{-2} = 0 \\ 1 - x_2^{-2} = 0 \end{cases}$, temos que $x_1 = x_2 = \pm 1 \leftrightarrow x_1 = x$, senão $x_1 = -x_2 = \pm 1$, logo temos 4 soluções possíveis:

$p_1 = (1, 1)$ $\quad p_3 = (1, -1)$

$p_2 = (-1, -1)$ $\quad p_4 = (-1, 1)$

p_1 é mínimo global e p_2 é máximo global, visto que a Hessiana é positiva definida para p_1 e negativa definida para p_2.

$$\det(H - \lambda I) = 0$$

$$\lambda^2 - \frac{2\lambda}{x_1^3} + \frac{4}{x_1^3 x_2^3} - \frac{2\lambda}{x_2^3} = 0 \Rightarrow \text{multiplica por mmc}(a^3, a^3b^3, b^3)$$

$$\lambda^2 x_1^3 x_2^3 - \frac{2\lambda}{x_1^3} x_1^3 x_2^3 + \frac{4}{x_1^3 x_2^3} \cdot x_1^3 x_2^3 - \frac{2\lambda}{x_2^3} x_1^3 x_2^3 = 0 \Rightarrow$$

$$x_1^3\, x_2^3\, \lambda^2 - (2x_2^3 + 2x_1^3)\cdot \lambda + 4 = 0$$

$$\lambda_1, \lambda_2 = \frac{-(-2x_2^3 - 2x_1^3) \pm \sqrt{(-2x_2^3 - 2x_1^3)^2 - 4\, x_1^3\, x_2^3 \cdot 4}}{2x_1^3\, x_2^3}$$

$$\lambda_1 = \frac{2}{x_2^3} \qquad \lambda_2 = \frac{2}{x_1^3} \Big\} \text{ Autovalores.}$$

$\lambda_1(p_1) = 2$ | $\lambda_1(p_2) = -2$

$\lambda_1(p_1) = 2$ | $\lambda_2(p_2) = -2$

4) I) min $f(x_1, x_2) = 0.2262\, x_1 x_2$ restrito a:

$$0 \leq x_1 \leq 1 \,, \quad 0 \leq x_2 \leq 1$$

$$g(x_1, x_2) = 0.12x_2 - 0.07x_1 - 0.12 \leq 0$$

$$g_2(x_1, x_2) = -5.6549x_1 - 1.131x_2 + \pi\sqrt{(0.18x_1 + 0.12x_2)\cdot(0.6x_1 + 0.36x_2)} + 1.75 \leq 0$$

I) $\vec{\nabla} f + \sum_{i=1}^{m} \vec{\nabla} g_i\, \lambda_i + \sum_{j=m+1}^{m} \vec{\nabla} h_j\, \lambda_j = 0$

Calculando os gradients p/ montar função Lagrangiana:

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{bmatrix} = \begin{bmatrix} -0.2262x_2 \\ -0.2262x_1 \end{bmatrix}$$

$$\nabla g_1 \begin{bmatrix} \frac{\partial g_1}{\partial x_1} \\ \frac{\partial g_1}{\partial x_2} \end{bmatrix} = \begin{bmatrix} -0.07 \\ 0.12 \end{bmatrix}$$

$$\nabla g_2 = \begin{bmatrix} \frac{\partial g_2}{\partial x_1} \\ \frac{\partial g_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} -5.6549 + \frac{\pi\cdot(0.216x_1 + 0.1368x_2)}{2\sqrt{(0.18x_1 + 0.12x_2)\cdot(0.6x_1 + 0.36x_2)}} \\ -1.131 + \frac{\pi(0.1368x_1 + 0.0864x_2)}{2\sqrt{(0.18x_1 + 0.12x_2)\cdot(0.6x_1 + 0.36x_2)}} \end{bmatrix}$$

Substituindo na Lagrangiana:

$$\vec{\nabla} f + \lambda_1 \vec{\nabla} g_1 + \lambda_2 \nabla g_2 + 0 = 0$$

$$\begin{cases} \text{I} \quad -0.2262\,x_2 - 0.07\lambda_1 + \left[-5.6549 + \dfrac{\pi \cdot (0.216x_1 + 0.1368x_2)}{2\sqrt{0.108x_1^2 + 0.1368x_1x_2 + 0.0432x_2^2}} \right] \cdot \lambda_2 = 0 \\ \text{II} \quad -0.2262x_1 + 0.12\lambda_1 + \left[-1.131 + \dfrac{\pi \cdot (0.1368x_1 + 0.0864x_2)}{2\sqrt{0.108x_1^2 + 0.1368x_1x_2 + 0.0432x_2^2}} \right] \cdot \lambda_2 = 0 \\ \text{III} \quad 0.012x_2 - 0.07x_1 - 0.012 = 0 \\ \text{IV} \quad -5.6549\lambda_1 - 1.131x_2 + \pi\sqrt{(0.18x_1 + 0.12x_2)\cdot(0.6x_1 + 0.36x_2)} + 1.75 = 0 \end{cases}$$

Em III $\Rightarrow$ $x_2 = \dfrac{0.07x_1 + 0.012}{0.012}$, substituindo em IV

IV $$1.75 - 5.6549\lambda_1 - 1.131 \cdot \left(\frac{0.07x_1 + 0.012}{0.012} \right) + \pi \cdot \sqrt{\left[0.18x_1 + 0.12 \cdot \left(\frac{0.07x_1 + 0.012}{0.012} \right) \right] \cdot \Rightarrow}$$

$$\Rightarrow \sqrt{\cdot \Rightarrow \left[0.6x_1 + 0.36 \left(\frac{0.07x_1 + 0.012}{0.012} \right) \right]} = 0$$

IV $$1.75 - 5.6549x_1 - \frac{0.07917}{0.012}x_1 - 1.131 + \pi \cdot \Rightarrow$$

$$\cdot \Rightarrow \sqrt{\left(0.18x_1 + \frac{0.0084x_1}{0.012} + 0.12 \right) \cdot \left(0.6x_1 + \frac{0.0252}{0.012}x_1 + 0.36 \right)} = 0$$

$$-3.3079x_1^2 - 0.9515x_1 + 0.043 = 0$$

$$\left. \begin{aligned} x_1' &= -0.045 \\ x_1'' &= 0.25 \end{aligned} \right\}$$ Apenas x_1'' satisfaz as restrições laterais $0 \leq x_1 \leq 1$

$\Rightarrow$

bstituindo estes valores em III

$$x_2'' = \frac{0.07 \cdot (0.25) + 0.012}{0.012} = 1.150$$

$$x_2' = \frac{0.07 \cdot (-0.0045) + 0.012}{0.012} = 0.973$$

Apenas x_2 satisfaz a restrição lateral $0 \leq x_2 \leq 1$

Os valores ótimos das variáveis de projeto são:

$(0.25,\ 1.15)$ e $(-0.0045,\ 0.997)$

(3) Seja:

$$f(x_1, x_2) = -\pi(0.072)x_1 x_2 + (x_1 - 0.5)^2 + (x_2 - 0.3)^2$$

$$0 \leq x_1 \leq 1 \quad , \quad 0 \leq x_2 \leq 1$$

$$\min(x_1, x_2) = (0.54085096, \ 0.36117899)$$

Resolvido numericamente.

$$H(f(x)) = \begin{bmatrix} \frac{\partial^2 f}{\partial^2 x_1} & \frac{\partial f}{\partial x_1 x_2} \\ \frac{\partial f}{\partial x_2 x_1} & \frac{\partial^2 f}{\partial^2 x_2} \end{bmatrix} =$$

$$\frac{\partial f}{\partial x_1} = -0.22619\, x_2 + 2 \cdot (x_1 - 0.5)$$

$$\frac{\partial f}{\partial x_2} = -0.22619\, x_1 + 2 \cdot (x_2 - 0.3)$$

$$\frac{\partial^2 f}{\partial^2 x_1} = 2$$

$$\frac{\partial_2 f}{\partial_2 x_2} = 2$$

$$\frac{\partial^2 f}{\partial x_1 x_2} = \frac{\partial f}{\partial x_2 x_1} = -0.22619 \quad \therefore \quad H(f(x)) = \begin{bmatrix} 2 & -0.22619 \\ -0.22619 & 2 \end{bmatrix}$$

$$\det(H - \lambda I) = 0$$

$$\det\left(\begin{bmatrix} 2 & -0.22619 \\ -0.22619 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right) =$$

$$(2-\lambda)^2 - (-0.22619)^2$$

$$\lambda^2 - 4\lambda + 3.94884$$

$$\lambda_1 = 1.77381$$

$$\lambda_2 = 2.22619$$

Visto que $\forall \lambda_i > 0$, temos que a matriz é positiva definida.

② Dada a função:

$$f(x_1, x_2) = \pi\left[3(x_1 - x_2) - \sqrt{(3x_1 + x_2)(x_1 + 3x_2)}\right]$$

· Temos que o gradiente é:

$$\vec{\nabla} f(x_1, x_2) = \left(\frac{df}{dx_1}, \frac{df}{dx_2}\right)$$

$$\frac{df}{dx_1} = \pi \frac{d}{dx_1}\left(\underbrace{3\cdot(x_1 - x_2)}_{\Downarrow\, \mathrm{I} \atop 3} - \underbrace{\sqrt{(3x_1 + x_2)\cdot(x_1 + 3x_2)}}_{\Downarrow \atop \mathrm{II}}\right)$$

II $\frac{df}{dx_1}\left(\sqrt{(3x_1 + x_2)\cdot(x_1 + 3x_2)}\right) =$

$$\frac{d}{du}\underbrace{\sqrt{u}}_{\Downarrow \atop \frac{1}{2\sqrt{u}}} \cdot \frac{d}{dx_1}\underbrace{((3x_1 + x_2)\cdot(x_1 + 3x_2))}_{\Downarrow \atop \mathrm{III}}$$

III $\frac{d}{dx_1}((3x_1 + x_2)\cdot(x_1 + 3x_2))$

III $\frac{d}{x_1}\underbrace{(3x_1 + x_2)}_{\Downarrow \atop 3}\cdot(x_1 + 3x_2) + \frac{d}{dx_1}\underbrace{(x_1 + 3x_2)}_{\Downarrow \atop 1}\cdot(3x_1 + x_2)$

III $3\cdot(x_1 + 3x_2) + 1\cdot(3x_1 + x_2)$

III $6x_1 + 10x_2$

... →

$$\frac{1}{2\sqrt{v}}(6x_1+10x_2)$$

$v=(3x_1+x_2)\cdot(x_1+3x_2)$ substituindo...

$$\frac{\partial f}{\partial x_1}\left(\frac{1}{2\cdot\sqrt{(3x_1+x_2)\cdot(x_1+3x_2)}}\cdot(6x_1+10x_2)\right)=$$

$$\frac{\partial f}{\partial x_1}=\pi\cdot\left(3-\frac{3x_1+5x_2}{(3x_1+x_2)\cdot(x_1+3x_2)}\right)$$

$$\frac{\partial f}{\partial x_2}=\pi\cdot\left(\frac{d}{dx_2}\underbrace{(3\cdot(x_1-x_2))}_{-3}-\frac{d}{dx_2}\underbrace{\left(\sqrt{(3x_1+x_2)\cdot(x_1+3x_2)}\right)}_{\mathrm{I}}\right)=$$

$$\frac{\partial \mathrm{I}}{\partial x_2} \Rightarrow f=\sqrt{v}$$

$$\Downarrow$$

$$\frac{d}{dv}=\frac{1}{2\sqrt{v}} \qquad \frac{d}{dx_2}=6x_2+10x_1 \Rightarrow \frac{1}{2\sqrt{v}}\cdot(6x_2+10x_1)$$

$$\frac{1}{2\sqrt{(3x_1+x_2)(x_1+3x_2)}}\cdot 6x_2+10x_1$$ | Substituir em $v=(3x_1+x_2)\cdot(x_1+3x_2)$

$$\frac{3x_2+5x_1}{\sqrt{(x_2+3x_1)\cdot(3x_2+x_1)}}=$$

$$\frac{\partial f}{\partial x_2}=\pi\cdot\left(-3-\frac{3x_2+5x_1}{\sqrt{(x_2+3x_1)\cdot(3x_2+x_1)}}\right)$$

$$\nabla f(x_1, x_2) = \left(\pi \cdot \left(3 - \frac{3x_1 + 5x_2}{(3x_1 + x_2)\cdot(x_1 + 3x_2)}\right), \ \pi\left(-3 - \frac{3x_2 + 5x_1}{\sqrt{(x_2 + 3x_1)\cdot(3x_2 + x_1)}}\right)\right)$$

$$\nabla f(0.3, 0.4) = (2.90053, -15.4991)$$

Direção de descida máxima $S = -\nabla f(x)$.

$$S = \begin{bmatrix} -2.90053 \\ 15.4991 \end{bmatrix}$$

(II) $x_1 = x_0 + \alpha S$

$$x_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \alpha = 0.05$$

$$\cancel{x_1 = \begin{bmatrix} 1.1450265 \\ 1.774955 \end{bmatrix} \Rightarrow \begin{matrix} x_1 = 1.1450265 \\ x_2 = 1.774955 \end{matrix}}$$

$$x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 0.05 \cdot \begin{bmatrix} -2.90053 \\ 15.4991 \end{bmatrix} = \begin{bmatrix} 0.8549735 \\ 1.774955 \end{bmatrix} \Rightarrow$$

$$F = \pi \cdot \left(3 \cdot (0.8549735\alpha - 1.774955\alpha) - \sqrt{(3 \cdot 0.8549735\alpha + 1.774955\alpha)\cdot(0.8549735\alpha + 3 \cdot 1.774955\alpha)}\right)$$

• A Jacobiana J de $f(x)$ é:

$$J(f(x)) = \left[\pi\cdot\left(3 - \frac{3x_1+5x_2}{(3x_1+x_2)\cdot(x_1+3x_2)}\right), \pi\cdot\left(-3 - \frac{3x_2+5x_1}{\sqrt{(x_2+3x_1)\cdot(3x_2+x_1)}}\right)\right]$$

A Hessiana H de $f(x)$ é:

$$H(f(x)) = \begin{bmatrix} \frac{\partial^2 f}{\partial^2 x_1} & \frac{\partial f}{\partial x_1 x_2} \\ \frac{\partial^2 f}{\partial x_2 x_1} & \frac{\partial^2 f}{\partial^2 x_2} \end{bmatrix}$$

Sabendo que $\frac{\partial f}{\partial x_1} = \pi\cdot\left(3 - \frac{3x_1+5x_2}{\boxed{\sqrt{(3x_1+x_2)\cdot(x_1+3x_2)}}}\right)$

Termos que $\frac{\partial^2 f}{\partial^2 x_1} \Rightarrow$ $3 \xrightarrow{\frac{d}{dx_1}} 0$, $\frac{3x_1+5x_2}{\sqrt{(3x_1+x_2)\cdot(x_1+3x_2)}} \xrightarrow{\frac{d}{dx_1}}$ II

II $\frac{\partial \text{II}}{\partial x_1} = \frac{\underbrace{\frac{\partial}{\partial x}\left((3x_1+5x_2)\right)}_{3}\cdot\sqrt{(3x_1+x_2)\cdot(x_1+3x_2)} - \overbrace{\frac{\partial}{\partial x}\left(\sqrt{(3x_1+x_2)\cdot(x_1+3x_2)}\right)}^{\text{III}}\cdot(3x_1+5x_2)}{\left(\sqrt{(3x_1+x_2)\cdot(x_1+3x_2)}\right)^2}$

$\frac{\partial \text{III}}{\partial x_1} \Rightarrow f = \sqrt{u} \Rightarrow u = (3x_1+x_2)\cdot(x_1+3x_2)$

$\frac{\partial f}{\partial u} = \frac{1}{2\sqrt{u}}$, $u \xrightarrow{\frac{d}{dx_1}} 6x_1+10x_2 \Rightarrow \frac{1}{2\sqrt{u}}\cdot 6x_1+10x_2$

Substituindo em u, temos:

$$\frac{1}{2\sqrt{(3x_1+x_2)\cdot(x_1+3x_2)}}\cdot(6x_1+10x_2) = \frac{3x_1+5x_2}{\sqrt{(3x_1+x_2)\cdot(x_1+3x_2)}} \Rightarrow \ldots$$

$$\Rightarrow \ldots \frac{\cancel{3}\; 3x_1+5x_2}{\sqrt{(3x_1+x_2)\cdot(x_1+3x_2)}} = \frac{3\sqrt{(3x_1+x_2)\cdot(x_1+3x_2)} - \frac{3x_1+5x_2}{\sqrt{(3x_1+x_2)\cdot(x_1+3x_2)}}\cdot(3x_1+5x_2)}{\left(\sqrt{(3x_1+x_2)\cdot(x_1+3x_2)}\right)^2} =$$

$$= \ldots \frac{16x_2^2}{(3x_1+x_2)\cdot(x_1+3x_2)\cdot\sqrt{(3x_1+x_2)\cdot(x_1+3x_2)}}$$

$$\frac{\partial^2 f}{\partial^2 x_1} = \frac{16\pi x_2^2}{(3x_1+x_2)\cdot(x_1+3x_2)\cdot\sqrt{(3x_1+x_2)\cdot(x_1+3x_2)}} \quad //$$

Sabendo que:

$$\frac{\partial f}{\partial x_2} = \pi\cdot\left(\underbrace{-3}_{\frac{d}{dx_2}=0} - \underbrace{\frac{3x_2+5x_1}{\sqrt{(x_2+3x_1)\cdot(3x_2+x_1)}}}_{\frac{d}{dx_2}=\mathrm{II}}\right)$$

Temos que:

$$\frac{\partial^2 \mathrm{II}}{\partial^2 x_2} = \frac{\underbrace{\frac{\partial}{\partial x_2}(3x_2+5x_1)}_{3}\cdot\sqrt{(x_2+3x_1)\cdot(3x_2+x_1)} - \overbrace{\frac{\partial}{\partial x_2}\left(\sqrt{(x_2+3x_1)\cdot(3x_2+x_1)}\right)}^{\mathrm{III}}\cdot(3x_2+5x_1)}{\left(\sqrt{(x_2+3x_1)\cdot(3x_2+x_1)}\right)^2}$$

$$\frac{\partial \mathrm{III}}{\partial x_2} \Rightarrow f=\sqrt{u}, \quad u=(x_2+3x_1)\cdot(3x_2+x_1)$$

$$\frac{\partial f}{\partial u} = \frac{1}{2\sqrt{u}}, \quad \frac{d}{du} = 6x_2+10x_1 \;\therefore$$

$$\frac{1}{2\sqrt{u}}\cdot(6x_2+10x_1) \Rightarrow \text{Substituindo em o...}$$

=> Substituindo em u:

$$\frac{1}{2\sqrt{(x_2+3x_1)\cdot(3x_2+x_1)}}\cdot(6x_2+10x_1)=\frac{3x_2+5x_1}{\sqrt{(x_2+3x_1)\cdot(3x_2+x_1)}}$$

$$\frac{3\sqrt{(x_2+3x_1)\cdot(3x_2+x_1)}-\frac{3x_2+5x_1}{\sqrt{(x_2+3x_1)\cdot(3x_2+x_1)}}\cdot(3x_2+5x_1)}{\left(\sqrt{(x_2+3x_1)\cdot(3x_2+x_1)}\right)^2}=$$

$$\frac{16x_1^2}{(x_2+3x_1)\cdot(3x_2+x_1)\cdot\sqrt{(x_2+3x_1)\cdot(3x_2+x_1)}}\therefore$$

$$\frac{\partial^2 f}{\partial x_2^2}=\frac{16\pi x_1^2}{(x_2+3x_1)\cdot(3x_2+x_1)\cdot\sqrt{(x_2+3x_1)\cdot(3x_2+x_1)}}$$

Sabendo que:

$$\frac{\partial f}{\partial x_1 x_2}=\frac{\partial f}{\partial x_2\,\partial x_1} \quad \text{e} \quad \frac{\partial f}{\partial x_1}=\left(\pi\cdot\left(3-\underbrace{\frac{3x_1+5x_2}{\sqrt{(3x_1+x_2)\cdot(x_1+3x_2)}}}_{\mathrm{I}}\right)\right)$$, temos que:

$$\mathrm{I}\quad \frac{\partial \mathrm{I}}{\partial x_2}=\frac{\underbrace{\frac{\partial}{\partial x_2}(3x_1+5x_2)}_{5}\cdot\sqrt{(3x_1+x_2)\cdot(x_1+3x_2)}-\frac{\partial}{\partial x_2}\overbrace{\left(\sqrt{(3x_1+x_2)\cdot(x_1+3x_2)}\right)}^{\mathrm{II}}\cdot(3x_1+5x_2)}{\left(\sqrt{(3x_1+x_2)\cdot(x_1+3x_2)}\right)^2}$$

$$\frac{\partial\,\mathrm{II}}{\partial x_2}\Rightarrow f=\sqrt{u},\ u=(3x_1+x_2)\cdot(x_1+3x_2)$$

$$\frac{df}{du}=\frac{1}{2\sqrt{u}},\quad \frac{d}{du}=6x_2+10x_1\Rightarrow\frac{1}{2\sqrt{u}}\cdot 6x_2+10x_1\Rightarrow$$ Substituindo em u... =>

$\Rightarrow$ Substituindo em a...

$$\frac{1}{2\sqrt{(3x_1+x_2)\cdot(x_1+3x_2)}}\cdot(6x_2+10x_1) =$$

$$\frac{3x_2+5x_1}{\sqrt{(x_2+3x_1)\cdot(3x_2+x_1)}} =$$

$$\frac{5\sqrt{(3x_1+x_2)\cdot(x_1+3x_2)} - \frac{3x_2+5x_1}{\sqrt{(x_2+3x_1)\cdot(3x_2+x_1)}}\cdot(3x_1+5x_2)}{\left(\sqrt{(3x_1+x_2)\cdot(x_1+3x_2)}\right)^2} =$$

$$\frac{16x_1x_2}{(3x_1+x_2)\cdot(x_1+3x_2)\cdot\sqrt{(x_2+3x_1)\cdot(3x_2+x_1)}} \Rightarrow \cdot\pi$$

$$\frac{\partial f}{\partial x\partial y} = \frac{\partial f}{\partial y\partial x} = -\frac{16\pi x_1x_2}{(3x_1+x_2)\cdot(x_1+3x_2)\cdot\sqrt{(3x_1+x_2)\cdot(x_1+3x_2)}}$$

Logo, a Hessiana de f é:

$$H(f(x)) = \begin{bmatrix} \pi\left(-3 - \frac{3x_2+5x_1}{\sqrt{(x_2+3x_1)\cdot(3x_2+x_1)}}\right) & -\frac{16\pi x_1x_2}{(3x_1+x_2)\cdot(x_1+3x_2)\cdot\sqrt{(3x_1+x_2)\cdot(x_1+3x_2)}} \\ -\frac{16\pi x_2x_1}{(x_2+3x_1)\cdot(3x_2+x_1)\cdot\sqrt{(x_2+3x_1)\cdot(3x_2+x_1)}} & \frac{16\pi x_1^2}{(x_2+3x_1)\cdot(3x_2+x_1)\sqrt{(x_2+3x_1)\cdot(3x_2+x_1)}} \end{bmatrix}$$