③ Seja:

$$f(x_1, x_2) = -\pi(0.072) x_1 x_2 + (x_1 - 0.5)^2 + (x_2 - 0.3)^2$$

$$0 \leq x_1 \leq 1, \qquad 0 \leq x_2 \leq 1$$

$$\min(x_1, x_2) = (0.54085096, \ 0.36117899)$$

Resolvido numericamente.

$$H(f(x)) = \begin{bmatrix} \dfrac{\partial^2 f}{\partial^2 x_1} & \dfrac{\partial f}{\partial x_1 x_2} \\[20pt] \dfrac{\partial f}{\partial x_2 x_1} & \dfrac{\partial^2 f}{\partial^2 x_2} \end{bmatrix} =$$

$$\frac{\partial f}{\partial x_1} = -0.22619 \, x_2 + 2 \cdot (x_1 - 0.5)$$

$$\frac{\partial f}{\partial x_2} = -0.22619 \, x_1 + 2 \cdot (x_2 - 0.3)$$

$$\frac{\partial^2 f}{\partial^2 x_1} = 2$$

$$\frac{\partial_2 f}{\partial_2 x_2} = 2$$

$$\frac{\partial^2 f}{\partial x_1 x_2} = \frac{\partial f}{\partial x_2 x_1} = -0.22619 \qquad \therefore \quad H(f(x)) = \begin{bmatrix} 2 & -0.22619 \\[12pt] -0.22619 & 2 \end{bmatrix}$$

$$\det(H - \lambda I) = 0$$

$$\det\left(\begin{bmatrix} 2 & -0.22619 \\ -0.22619 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right) =$$

$$(2 - \lambda)^2 - (-0.22619)^2$$

$$\lambda^2 - 4\lambda + 3.94884$$

$$\lambda_1 = 1.77381$$

$$\lambda_2 = 2.22619$$

Visto que $\forall \lambda_i > 0$, temos que a matriz é positiva definida.