④ (I) $\min \quad f(x_1, x_2) = 0.2262\, x_1 x_2$ restrito a:

$$0 \leq x_1 \leq 1, \quad 0 \leq x_2 \leq 1$$

$$g_1(x_1, x_2) = 0.12 x_2 - 0.07 x_1 - 0.12 \leq 0$$

$$g_2(x_1, x_2) = -5.6549 x_1 - 1.131 x_2 + \pi \sqrt{(0.18 x_1 + 0.12 x_2) \cdot (0.6 x_1 + 0.36 x_2)} + 1.75 \leq 0$$

(I) $$\vec{\nabla} f + \sum_{i=1}^{m} \vec{\nabla} g_i \cdot \lambda_i + \sum_{j=m+1}^{m+l} \vec{\nabla} h_j \cdot \lambda_j = 0$$

Calculando os gradientes p/ montar função Lagrangiana:

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{bmatrix} = \begin{bmatrix} -0.2262 x_2 \\ -0.2262 x_1 \end{bmatrix}$$

$$\nabla g_1 \cdot \begin{bmatrix} \frac{\partial g_1}{\partial x_1} \\ \frac{\partial g_1}{\partial x_2} \end{bmatrix} = \begin{bmatrix} -0.07 \\ 0.12 \end{bmatrix}$$

$$\nabla g_2 = \begin{bmatrix} \frac{\partial g_2}{\partial x_1} \\ \frac{\partial g_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} -5.6549 + \frac{\pi \cdot (0.216 x_1 + 0.1368 x_2)}{2\sqrt{(0.18 x_1^2 + 0.12 x_2) \cdot (0.8 x_1 + 0.36 x_2^2)}} \\ -1.131 + \frac{\pi (0.1368 x_1 + 0.0864 x_2)}{2\sqrt{(0.18 x_1 + 0.12 x_2) \cdot (0.6 x_1 + 0.36 x_2)}} \end{bmatrix}$$

Substituindo na Lagrangiana:

$$\nabla f + \lambda_1 \nabla g_1 + \lambda_2 \nabla g_{02} + 0 = 0$$

(I) $$-0.2262 x_2 - 0.07 x_1 + \left[-5.6549 + \pi \cdot (0.216 x_1 + 0.1368 x_2) \right] \cdot \lambda_2 = 0$$

$$2\sqrt{0.108 x_1^2 + 0.1368 x_1 x_2 + 0.0432 x_2^2}$$

(II) $$-0.2262 x_1 + 0.12 \lambda_1 + \left[-1.131 + \pi \cdot (0.1368 x_1 + 0.0864 x_2) \right] \cdot \lambda_2 = 0$$

$$2\sqrt{0.108 x_1^2 + 0.1368 x_1 x_2 + 0.0432 x_2^2}$$

(III) $$0.012 x_2 - 0.07 x_1 - 0.012 = 0$$

(IV) $$-5.6549 x_1 - 1.131 x_2 + \pi \sqrt{(0.18 x_1 + 0.12 x_2) \cdot (0.8 x_1 + 0.36 x_2)} + 1.75 = 0$$

Em (III) $\Rightarrow x_2 = \dfrac{0.07 x_1 + 0.012}{0.012}$, substituindo em (IV)

(IV) $$1.75 - 5.6549 \lambda_1 - 1.131 \cdot \left(\frac{0.07 x_1 + 0.012}{0.012} \right) + \pi \cdot \sqrt{\left[0.18 x_1 + 0.12 \cdot \left(\frac{0.07 x_1 + 0.012}{0.012} \right) \right] \cdot} \Rightarrow$$

$$\Rightarrow \sqrt{\cdot} \Rightarrow \left[0.6 x_1 + 0.36 \left(\frac{0.07 x_1 + 0.012}{0.012} \right) \right] = 0$$

(IV) $$1.75 - 5.6549 x_1 - \frac{0.07917}{0.012} x_1 - 1.131 + \pi \cdot \Rightarrow$$

$$\cdot \Rightarrow \sqrt{\left(0.18 x_1 + \frac{0.0084 x_1}{0.012} + 0.12 \right) \cdot \left(0.6 x_1 + \frac{0.0252}{0.012} x_1 + 0.36 \right)} = 0$$

$$-3.3079 x_1^2 - 0.9515 x_1 + 0.043 = 0$$

$x_1' = -0.045$ } Apenas x_1'' satisfaz as restrições laterais $0 \leq x_1 \leq 1$

$x_1'' = 0.25$

$\Rightarrow$

Substituindo estes valores em (III)

$$x_2'' = \frac{0.07 \cdot (0.25) + 0.012}{0.012} = 1.150$$

$$x_2' = \frac{0.07 \cdot (-0.0045) + 0.012}{0.012} = 0.973$$

Apenas x_2 satisfaz a restrição lateral $0 \leq x_2 \leq 1$

Os valores ótimos das variáveis de projeto são:

$(0.25, 1.15)$ e $(-0.0045, 0.997)$