③ $f(x_1, x_2) = -\pi(0.072)x_1 x_2 + (x_1 - 0.5)^2 + (x_2 - 0.3)^2$

$$0 \leq x_1 \leq 1$$
$$0 \leq x_2 \leq 1$$

$$\vec{\nabla}_f = \begin{bmatrix} 2x_1 - 0.072\pi x_2 - 1 \\ -0.072\pi x_1 + 2x_2 - 0.6 \end{bmatrix}$$

$$\vec{\nabla}_f = 0 \Rightarrow \begin{cases} 2x_1 - 0.072\pi x_2 - 1 = 0 \\ -0.072\pi x_1 + 2x_2 - 0.6 = 0 \end{cases}$$

$x_1 = 0.54085096$

$x_2 = 0.36117899$

$$H(f(x)) = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial f}{\partial x_1 x_2} \\ \frac{\partial f}{\partial x_2 x_1} & \frac{\partial^2 f}{\partial^2 x_2} \end{bmatrix} =$$

$$\frac{\partial f}{\partial x_1} = -0.22619\, x_2 + 2\cdot(x_1 - 0.5)$$

$$\frac{\partial f}{\partial x_2} = -0.22619\, x_1 + 2\cdot(x_2 - 0.3)$$

$$\frac{\partial^2 f}{\partial^2 x_1} = 2$$

$$\frac{\partial_2 f}{\partial_2 x_2} = 2$$

$$\frac{\partial^2 f}{\partial x_1 x_2} = \frac{\partial f}{\partial x_2 x_1} = -0.22619 \quad \therefore \quad H(f(x)) = \begin{bmatrix} 2 & -0.22619 \\ -0.22619 & 2 \end{bmatrix}$$

$\det(H - \lambda I) = 0$

$$\det\left(\begin{bmatrix} 2 & -0{,}22619 \\ -0{,}22619 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right) =$$

$$(2-\lambda)^2 - (-0.22619)^2$$

$$\lambda^2 - 4\lambda + 3.94884$$

$\lambda_1 = 1.77381$

$\lambda_2 = 2.22619$

Visto que $\forall \lambda_i > 0$, temos que a matriz é positiva definida.