Problem 1. Carlos has a utility function that depends on the number of musicals and the number of operas seen each month. His utility function is given by $U = xy^2$, where x is the number of movies seen per month and y is the number of operas seen per month.

(a) Does Carlos believe that more is better for each good?

(b) Does Carlos have a diminishing marginal utility for each good?

Solution: The marginal utilities are given by: $MU_x = y^2$ and $MU_y = 2xy$

(a) Yes. When we say that *a consumer believes that more is better for a good*, we are referring to a utility function which is increasing in the variable relative to that good, indeed it has a positive marginal utility. In this case, Carlos' utility function is increasing in both x and y, both MU_x and MU_y are positive (for every x and y larger than 0).

(b) No. For what it concerns MU_x, it doesn't increase, neither decreases as x increases, it remains constant. While for MU_y, it increases as y increases. So, Carlos has a constant marginal utility for movies and an increasing marginal utility for operas.

Problem 2. Suppose a consumer's preferences for two goods can be represented by the Cobb-Douglas utility function $U = Ax^{\alpha}y^{\beta}$, where A, α, and β are positive constants.

(a) Is the assumption that more is better satisfied for both goods?

(b) Does the marginal utility of x diminish, remain constant, or increase as the consumer buys more x?

(c) What is $MRS_{x,y}$?

(d) Is $MRS_{x,y}$ diminishing, constant, or increasing as the consumer substitutes x for y along an indifference curve?

(e) On a graph with x on the horizontal axis and y on the vertical axis, draw a typical indifference curve (it need not be exactly to scale, but it needs to reflect accurately whether there is a diminishing $MRS_{x,y}$). Also indicate on your graph whether the indifference curve will intersect either or both axes. Label the curve U_1.

(f) On the same graph draw a second indifference curve U_2, with $U_2 > U_1$.

Solution:

(a) Yes, because the utility function is increasing in both variables, indeed the marginal utility for x and y are always positive.

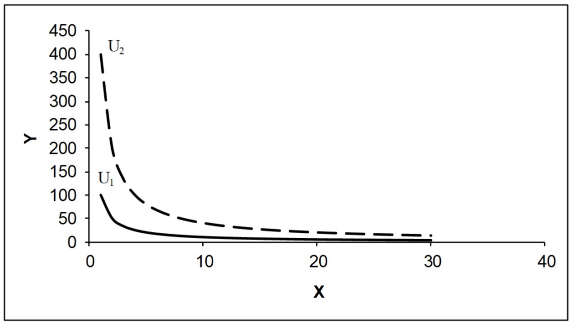
(b) The marginal utilities are $MU_x = \alpha Ax^{\alpha-1}y^{\beta}$ and $MU_y = \beta Ax^{\alpha}y^{\beta-1}$. In this case, we do not know the value of α, we only know that it is positive. So, we need to study the marginal utility and to specify three possible cases:

- When $\alpha < 1$, the marginal utility of x diminishes as x increases.
- When $\alpha = 1$, the marginal utility of x remains constant as x increases.
- When $\alpha > 1$, the marginal utility of x increases as x increases.

(c) $MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{\alpha A x^{\alpha-1} y^{\beta}}{\beta A x^{\alpha} y^{\beta-1}} = \frac{\alpha}{\beta}\frac{y}{x}$

(d) It diminishes constant, as the consumer substitutes x for y along an indifference curve.

(e) The graph below depicts indifference curves for the case where $A = 1$ and $\alpha = \beta = 0.5$.



(f) See graph above.

Problem 3. Pedro is a college student who receives a monthly salary from his parents of $\$1000$. He uses this salary to pay rent for housing and to go to the movies. In Pedro's town, each square foot of rental housing costs $\$2$ per month. Each movie he attends costs $\$10$. Let x denote the square feet of housing, and y denote the number of movies.

(a) What is the expression for Pedro's budget constraint?

(b) What is the maximum number of square feet of housing he can purchase given his income?

(c) What is the maximum number of movie tickets he could attend given his income?

(d) Draw a graph of Pedro's budget line.

(e) Suppose Pedro's parents increase his stipend by 10%. At the same time, suppose that in the college town he lives in, all prices, including housing rental rates and movie ticket prices, increase by 10%. What happens to the graph of Pedro's budget line?

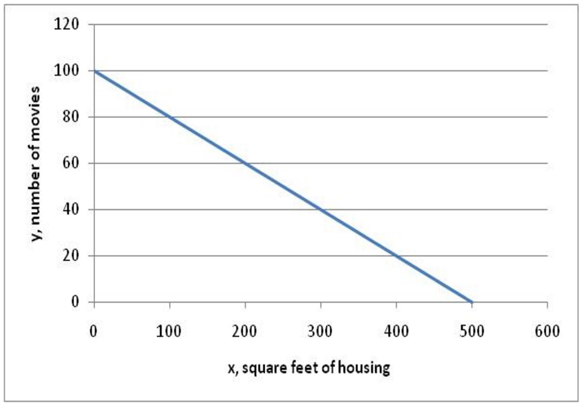
Solution:

(a) In order to build Pedro's budget constraint, we have to set the sum of goods weight by their relative prices smaller or equal to the maximum that Pedro can spend, namely his entire finances. Pedro's budget constraint is: $2x + 10y \leq 1000$

(b) $\$1000/\$2$ per square foot $= 500$ square feet

(c) $\$1000/\$10$ per ticket $= 100$ tickets

(d) **Graph**



(e) If everything increases by 10%, nothing changes in Pedro's budget line graph.

Problem 4. The utility that Ann receives by consuming food F and clothing C is given by $U(F,C) = FC + F$. Food costs \$1 a unit, and clothing costs \$2 a unit. Ann's income is \$22.

(a) Ann is currently spending all of her income. She is buying 8 units of food. How many units of clothing is she consuming?

(b) Graph her budget line. Place the number of units of clothing on the vertical axis and the number of units of food on the horizontal axis. Plot her current consumption basket.

(c) Find the utility-maximizing choice of food and clothing.

Solution:

(a) If Ann is currently spending all of her income, this is the equation of her budget line:

$$F + 2C = 22$$

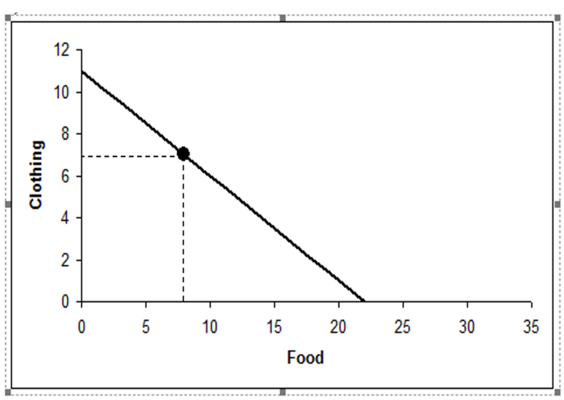
Since she is buying 8 units of food, we have to substitute $F = 8$, then:

$$8 + 2C = 22$$

$$2C = 14$$

$$C = 7$$

(b) **Graph**



(c) The marginal utilities of food and clothing are $MU_F = C + 1$ and $MU_C = F$.

$$\begin{cases} MRS_{F,C} = \frac{MU_F}{MU_C} = \frac{P_F}{P_C} \rightarrow \text{tangency condition (interior solution)} \\ P_F F + P_C C = I \rightarrow \text{budget constraint} \end{cases}$$

$$\begin{cases} \frac{C+1}{F} = \frac{1}{2} \Longrightarrow 2C + 2 = F \\ F + 2C = 22 \end{cases}$$

$$2C + 2 + 2C = 22 \rightarrow 4C = 20 \rightarrow C^* = 5$$

$$F = 2C + 2 \rightarrow F^* = 12$$

$$\text{Optimal Consumption Bundle } (F^*, C^*) = (12, 5)$$

Problem 5. Eric purchases food (measured by x) and clothing (measured by y) and has the utility function $U(x, y) = xy$. He has a monthly income of $\$800$. The price of food is $P_x = \$20$, and the price of clothing is $P_y = \$40$. Find Eric's optimal consumption bundle.

Solution: Eric's marginal utilities are $MU_x = y$ and $MU_y = x$.

$$\begin{cases} MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{P_x}{P_y} \rightarrow \text{tangency condition (interior solution)} \\ P_x x + P_y y = I \rightarrow \text{budget constraint} \end{cases}$$

$$\begin{cases} \frac{y}{x} = \frac{20}{40} = \frac{1}{2} \rightarrow 2y = x \\ 20x + 40y = 800 \end{cases}$$

$$40y + 40y = 800 \rightarrow 80y = 800 \rightarrow y^* = 10$$

$$2y = x \rightarrow x^* = 20$$

$$\text{Optimal Consumption Bundle } (x^*, y^*) = (20, 10)$$

Problem 6. Carina buys two goods, food F and clothing C, with the utility function $U = FC + F$. She has an income of 20. The price of clothing is 4

(a) Derive the equation representing Carina's demand for food, and draw this demand curve for prices of food ranging between 1 and 6.

(b) Calculate the income and substitution effects on Carina's consumption of food when the price of food rises from 1 to 4, and draw a graph illustrating these effects. Your graph need not be exactly to scale, but it should be consistent with the data.

Solution: Carina's marginal utility of food is $MU_F = C + 1$ and her marginal utility of clothing is $MU_C = F$.

(a) In order to find the demand for food, we solve the usual system:

$$\begin{cases} MRS_{C,F} = \frac{MU_F}{MU_C} = \frac{P_F}{P_C} \rightarrow \text{tangency condition (interior solution)} \\ P_F F + P_C C = I \rightarrow \text{budget constraint} \end{cases}$$

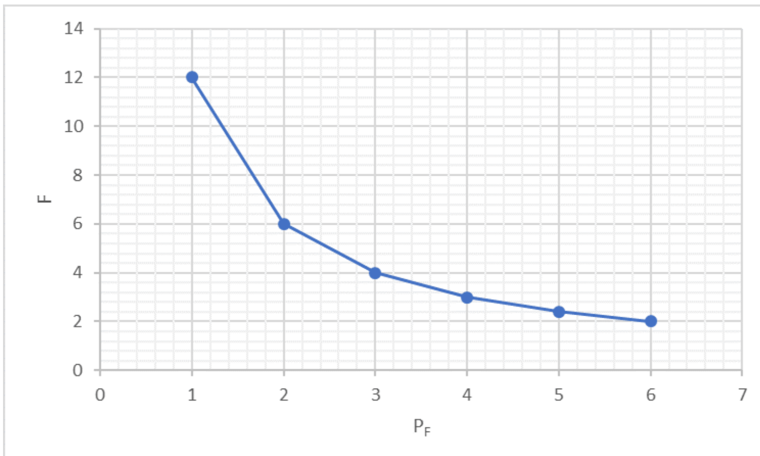
$$\begin{cases} \frac{C+1}{F} = \frac{P_F}{4} \to 4C + 4 = P_F F \to 4C = P_F F - 4 \\ P_F F + 4C = 20 \to P_F F + (P_F F - 4) = 20 \to 2P_F F = 24 \end{cases}$$

$$F = \frac{12}{P_F}$$

If we want to know also the demand for clothing, it is enough to continue solving the system:

$$4C = P_F \frac{12}{P_F} - 4 \to C = \frac{12-4}{4}$$

$$C = 2 \text{ (independently of the level of } P_F)$$



(b) **Initial Basket** From the demand for food in (a) $F = \frac{12}{1} = 12$, and $C = 2$. Also the initial level of utility is $U = FC + F = 12(2) + 12 = 36$.
Final Basket From the demand for food in (a), $F = \frac{12}{4} = 3$, and $C = 2$. Also, $U = 3(2)+3 = 9$.
Decomposition Basket Must be on initial indifference curve, with:

$$U = FC + F = 36$$

Tangency condition satisfied with final price:

$$\frac{MU_F}{MU_C} = \frac{P_F}{P_C} \to \frac{C+1}{F} = \frac{4}{4} \to C + 1 = F$$

So in this case, we need to solve this system:

$$\begin{cases} C + 1 = F \\ FC + F = 36 \to F(C+1) = 36 \to F^2 = 36 \end{cases} \qquad \begin{cases} F^* = 6 \\ C^* = F - 1 = 5 \end{cases}$$

Income Effect on F: $F_{final\,basket} - F_{decomposition\,basket} = 3 - 6 = -3$
Substitution Effect on F: $F_{decomposition\,basket} - F_{initial\,basket} = 6 - 12 = -6$

Problem 1. Does a Cobb-Douglas production function, $Q = AL^\alpha K^\beta$ exhibit increasing, decreasing, or constant returns to scale?

Solution: Let us remeber that the returns to scale is the concept that tells us the percentage by which output will increase when all inputs are increased by a given percentage. In order to understand which kind of returns to scale a given function exhibit, let us increase all input quantities by the same proportional amount λ:

$$f(L,K) = AL^\alpha K^\beta \to f(\lambda L, \lambda K) = A(\lambda L)^\alpha(\lambda K)^\beta$$

We can now distinguish three cases:

(a) $A(\lambda L)^\alpha(\lambda K)^\beta > \lambda(AL^\alpha K^\beta) \to \lambda^{\alpha+\beta} > \lambda \to \alpha+\beta > 1$, the function exhibits an **increasing** return to scale,

(b) $A(\lambda L)^\alpha(\lambda K)^\beta = \lambda(AL^\alpha K^\beta) \to \lambda^{\alpha+\beta} = \lambda \to \alpha+\beta = 1$, the function exhibits an **constant** return to scale,

(c) $A(\lambda L)^\alpha(\lambda K)^\beta < \lambda(AL^\alpha K^\beta) \to \lambda^{\alpha+\beta} < \lambda \to \alpha+\beta < 1$, the function exhibits an **decreasing** return to scale.

This shows that the sum of the exponents $\alpha+\beta$ in the Cobb-Douglas production function determines whether returns to scale are increasing, constant, or decreasing. For this reason, economists have paid considerable attention to estimating this sum when studying production functions in specific industries.

Problem 2. Suppose the production function for automobiles is $Q = LK$ where Q is the quantity of automobiles produced per year, L is the quantity of labor (man-hours) and K is the quantity of capital (machine-hours).

(a) Sketch the isoquant corresponding to a quantity of $Q = 100$?

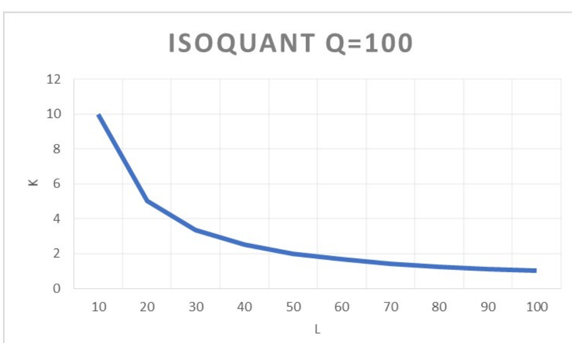
(b) What is the general equation for the isoquant corresponding to any level of output Q?

(c) Does the isoquant exhibit diminishing marginal rate of technical substitution?

(d) Does this production function has a diminishing return to scale?

Solution:

(a) **Graph**



(b) Let us fix a certain level $\bar{Q}$, the equation for the isoquant corresponding to this level is:

$$\bar{Q} = KL \to K = \frac{\bar{Q}}{L}$$

(c) Yes, the isoquant exhibits a diminishing marginal rate of technical substitution:

$$MRTS_{L,K} = \frac{MP_L}{MP_K} = \frac{K}{L}$$

We can notice that it diminishes as L increases and K falls as we move along an isoquant.

(d) No, the function does not exhibit a diminishing return to scale, but an increasing one:

$$(\lambda K)(\lambda L) > \lambda(KL) \to \lambda^2 > \lambda$$

Problem 3. Consider the production function $Q = 50\sqrt{LK}$. What is the short-run total cost curve for this production function when capital is fixed at a level $\bar{K}$, and the input prices of labor and capital are $w = 25$ and $r = 100$, respectively?

Solution: First of all we have to compute the cost-minimizing quantity of labour in the short run, the optimal amount of labour, in the following way:

$$Q = 50\sqrt{L\bar{K}} \to L = \frac{Q^2}{2500\bar{K}}$$

We can obtain the short-run total cost curve directly from:

$$STC(Q) = wL + r\bar{K} = 25L + 100\bar{K} = 25\frac{Q^2}{2500\bar{K}} + 100\bar{K} = \frac{Q^2}{100\bar{K}} + 100\bar{K}$$

The total variable and total fixed cost curves follow: $TVC(Q) = \frac{Q^2}{100\bar{K}}$ and $TFC = 100\bar{K}$. Note that, holding Q constant, total variable cost is decreasing in the quantity of capital $\bar{K}$. The reason is that, for a given amount of output, a firm that uses more capital can reduce the amount of labor it employs. Since TVC is the firm's labor expense, it follows that TVC should decrease in $\bar{K}$.

Problem 4. A firm has the production function $Q = LK$. For this production function, $MP_L = K$ and $MP_K = L$. The firm initially faces input prices $w = \$1$ and $r = \$1$ and is required to produce $Q = 100$ units. Later the price of labor w goes up to $\$4$. Find the optimal input combinations for each set of prices and use these to calculate the firm's price elasticity of demand for labor over this range of prices.

Solution: In order to find the optimal input combinations, we have to impose the tangency condition for the interior solution, and a specific isoquant. In our case, we know

$$\begin{cases} MRTS_{L,K} = \frac{MP_L}{MP_K} = \frac{w}{r} \to \text{Tangency Condition} \\ \bar{Q} = f(L,K) \to \text{Level of Production} \end{cases}$$

We can now solve the two systems with the information that we have:

1. $w_1 = 1$

$$\begin{cases} \frac{MP_L}{MP_K} = \frac{w_1}{r} \\ \bar{Q} = KL \end{cases} \quad \begin{cases} \frac{K}{L} = \frac{1}{1} \to K = L \\ 100 = KL \to K^2 = 100 \end{cases}$$

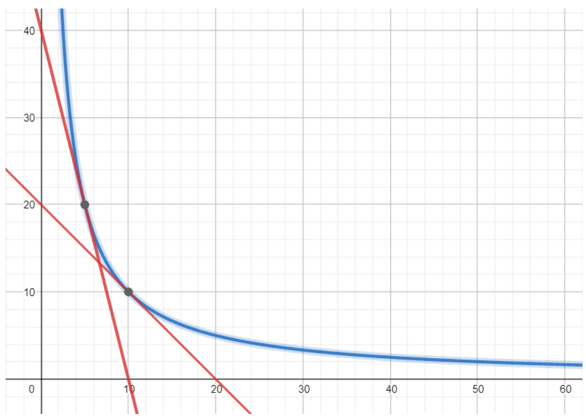
$$\begin{cases} L_1^* = 10 \\ K_1^* = 10 \end{cases}$$

2. $w_2 = 4$

$$\begin{cases} \frac{MP_L}{MP_K} = \frac{w_2}{r} \\ \bar{Q} = KL \end{cases} \quad \begin{cases} \frac{K}{L} = \frac{4}{1} \to K = 4L \\ 100 = KL \to (4L)(L) = 100 \to L^2 = \frac{100}{4} = 25 \end{cases}$$

$$\begin{cases} L_2^* = 5 \\ K_2^* = 20 \end{cases}$$

In the next graph it is possible to observe the optimal input combinations for each set of prices:



Let us now move to the calculation of the firm's price elasticity of demand for labor over this range of prices:

$$\varepsilon_{L,w} = \frac{\Delta L}{\Delta w}\frac{w}{L} = \frac{L_2 - L_1}{w_2 - w_1}\frac{w}{L}$$

In this specific case,

$$\varepsilon_{L_1,w_1} = \frac{5-10}{4-1}\frac{1}{10} = -\frac{5}{3}\frac{1}{10} = -\frac{1}{6}$$

$$\varepsilon_{L_2,w_2} = \frac{5-10}{4-1}\frac{4}{5} = -\frac{5}{3}\frac{4}{5} = -\frac{4}{3}$$

Problem 5. Suppose the input demand curves of a firm are:

$$L = \frac{Q}{30}\sqrt{\frac{r}{w}}$$

$$K = \frac{Q}{30}\sqrt{\frac{w}{r}}$$

Derive the production function of such firm.

Solution: In order to derive the production function, let us rewrite the labour demand curve in the following way:

$$L = \frac{Q}{30}\sqrt{\frac{r}{w}} \rightarrow Q = 30L\sqrt{\frac{w}{r}}$$

Now, let us rewrite the capital demand curve, expliciting the square root of the ratio between the input prices:

$$K = \frac{Q}{30}\sqrt{\frac{w}{r}} \rightarrow \sqrt{\frac{w}{r}} = 30\frac{K}{Q}$$

Doing so, we now can substitute the the square root of the ratio between the input prices into the first function, in order to have the production function:

$$Q = 30L30\frac{K}{Q}$$

$$Q^2 = 900LK$$

$$Q = 30\sqrt{L}\sqrt{K}$$

Problem 1. Suppose that a firm has a short-run total cost curve given by $STC = 100 + 20Q + Q^2$, where the total fixed cost is 100 and the total variable cost is $20Q+Q^2$. The corresponding short-run marginal cost curve is $SMC = 20 + 2Q$. All of the fixed cost is sunk.

(a) What is the equation for average variable cost (AVC)?

(b) What is the minimum level of average variable cost?

(c) What is the firm's short-run supply curve?

Solution:

(a) Average variable cost is total variable cost divided by output. Thus,

$$AVC(Q) = \frac{SVC(Q)}{Q} = \frac{20Q + Q^2}{Q} = 20 + Q$$

(b) We know that the minimum level of average variable cost occurs at the point at which AVC and SMC are equal in this case,

$$AVC(Q) = SMC(Q) \rightarrow 20 + Q = 20 + 2Q \rightarrow Q_{min} = 0$$

$$AVC(Q_{min}) = 20$$

(c) For prices below 20 (the minimum level of average variable cost), the firm will not produce. For prices above 20, we can find the supply curve by equating price to marginal cost and solving for Q:

$$P = SMC(Q) \rightarrow P = 20 + 2Q \rightarrow Q = \frac{1}{2}P - 10$$

The firm's short-run supply curve, which we denote by $s(P)$, is thus:

$$s(P) = \begin{cases} 0 & P < 20 \\ \frac{1}{2}P - 10 & P \geq 20 \end{cases}$$

Problem 2. Consider the previous exercise and

(a) Suppose that $SFC = 36$, while $NSFC = 64$. What is the firm's average nonsunk cost curve?

(b) What is the minimum level of average nonsunk cost?

(c) What is the firm's short-run supply curve?

Solution:

(a) The average nonsunk cost curve is

$$ANSC = AVC + \frac{NSFC}{Q} = 20 + Q + \frac{64}{Q}$$

(b) The average nonsunk cost curve $ANSC$ reaches its minimum when average nonsunk cost equals short-run marginal cost:

$$ANSC = SMC \rightarrow 20 + Q + \frac{64}{Q} = 20 + 2Q \rightarrow \frac{64}{Q} = 2Q - Q \rightarrow Q^2 = 64 \rightarrow Q_{min} = 8$$

$$ANSC(Q_{min}) = 20 + 8 + \frac{64}{8} = 36$$

(c) For prices below the minimum level of ANSC (i.e., for $P = 36$), the firm does not produce. For prices above this level, the firm's profit-maximizing quantity is given by equating price to marginal cost and solving for Q:

$$P = SMC(Q) \rightarrow P = 20 + 2Q \rightarrow Q = \frac{1}{2}P - 10$$

The firm's short-run supply curve, which we denote by $s(P)$, is thus:

$$s(P) = \begin{cases} 0 & P < 36 \\ \frac{1}{2}P - 10 & P \geq 36 \end{cases}$$

When the market price is between 36 and 40, the firm will continue to produce in the short run, even though its economic profit is negative. Its losses from operating will be less than its losses if it shuts down.

Problem 3. A market consists of 300 identical firms, and the market demand curve is given by $D(P) = 60 - P$. Each firm has a short-run total cost curve $STC = 0.1 + 150Q^2$, and all fixed costs are sunk. The corresponding short-run marginal cost curve is $SMS = 300Q$, and the corresponding average variable cost curve is $AVC = 150Q$. The minimum level of AVC is 0; thus, a firm will continue to produce as long as price is positive. (You can verify this by sketching the SMC and AVC curves.)
What is the short-run equilibrium price in this market?

Solution: Each firm's profit-maximizing quantity is given by equating marginal cost and price:

$$P = SMC(Q) \rightarrow P = 300Q \rightarrow Q = \frac{P}{300}$$

Thus the supply curve $s(P)$ of an individual firm is

$$s(P) = \frac{P}{300}$$

Since the firms in this market are all identical, short-run market supply $S(P)$ equals

$$S(P) = ns(P) = 300\frac{P}{300} = P$$

The short-run equilibrium occurs where market supply equals market demand, thus

$$S(P) = D(P) \rightarrow P = 60 - P \rightarrow 2P = 60 \rightarrow P = 30$$

Problem 4. In a market, all firms and potential entrants are identical. Each has a long-run average cost curve $AC(Q) = 40 - Q + 0.01Q^2$ and a corresponding long-run marginal cost curve $MC(Q) =$

$40 - 2Q + 0.03Q^2$, where Q is thousands of units per year. The market demand curve is $D(P) = 25,000 - 1,000P$ where $D(P)$ is also measured in thousands of units. Find the long-run equilibrium quantity per firm, price, and number of firms.

Solution: The long-run competitive equilibrium satisfies the following three equations:

$$\begin{cases} P = MC(Q) \rightarrow \text{ profit maximization} \\ P = AC(Q) \rightarrow \text{ zero profit} \\ S(P) = D(P) \rightarrow \text{ supply equals demand (market clearing)} \end{cases}$$

$$\begin{cases} P = 40 - 2Q + 0.03Q^2 \\ P = 40 - Q + 0.01Q^2 \\ nQ = 25,000 - 1,000P \rightarrow n = \frac{25,000-1,000P}{Q} \end{cases}$$

$$40 - 2Q + 0.03Q^2 = 40 - Q + 0.01Q^2$$

$$(0.03 - 0.01)Q^2 = (2 - 1)Q$$

$$Q^* = \frac{1}{0.02} = 50$$

$$P = 40 - 50 + 0.01(2500) = 40 - 50 + 25 \rightarrow P^* = 15$$

$$n = \frac{25,000 - 1,000(15)}{50} = \frac{25,000 - 15,000}{50} = \frac{10,000}{50} \rightarrow n^* = 200$$

Problem 1. A monopolist operates in an industry where the demand curve is given by $Q = 1000 - 20P$. The monopolist's constant marginal cost is \$8. What is the monopolist's profit-maximizing price?

Solution: Let rewrite the demand curve in the inverse form and let derive the MR from it:

$$Q = 1000 - 20P \rightarrow P = 50 - \frac{Q}{20}$$

$$MR = 50 - \frac{Q}{10}$$

Let use the rule $MR = MC$

$$50 - \frac{Q}{10} = 8 \rightarrow Q = 420$$

$$P = 50 - \frac{420}{20} \rightarrow P = 29$$

Problem 2. A monopolist faces a demand curve $P = 210 - 4Q$ and initially faces a constant marginal cost $MC = 10$.

1. Calculate the profit-maximizing monopoly quantity and compute the monopolist's total revenue at the optimal price.
2. Suppose that the monopolist's marginal cost increases to $MC = 20$. Verify that the monopolist's total revenue goes down.
3. Suppose that all firms in a perfectly competitive equilibrium had a constant marginal cost $MC = 10$. Find the long-run perfectly competitive industry price and quantity.
4. Suppose that al firms' marginal costs increased to $MC = 20$. Verify that the increase in marginal cost causes total industry revenue to go up.

Solution:

1. With demand $P = 210 - 4Q$, $MR = 210 - 8Q$. Setting $MR = MC$ implies

$$210 - 8Q = 10 \rightarrow Q = 25$$

$$P = 210 - 4(25) \rightarrow P = 110$$

$$TR = 110(25) = 2750$$

2. If $MC = 20$, than setting $MR = MC$ implies

$$210 - 8Q = 20 \rightarrow Q = 23.75$$

$$P = 210 - 4(20) \rightarrow P = 115$$

$$TR = 115(23.75) = 2731,25$$

Therefore, the increase in marginal cost will result in lower total revenue for the firm.

3. Competitive firms produce until $P = MC$, so in this case we know the market price would be

$$P_1 = 10$$

$$210 - 4Q = 10 \rightarrow Q_1 = 50$$

4. In this case, the market price will be

$$P_2 = 20$$

$$210 - 4Q = 20 \rightarrow Q_2 = 47,50$$

Let us now compare total revenue first and after the increase in marginal cost:

$$TR_1 = 10(50) = 500$$

$$TR_2 = 20(47,50) = 950$$

$$500 < 950$$

$$TR_1 < TR_2$$

Total industry revenue increases in the perfectly competitive market after the increases in the perfectly competitive market after the increase in marginal cost.

Problem 3. Let a monopolist face a linear demand, $P_0(Q) = 100 - 2Q$ and marginal cost $MC(Q) = 20$. Suppose that now demand shifts to $P_1(Q) = 120 - 2Q$. How does the equilibrium price change?

Solution: For inverse demand curve $P_0(Q)$, we have

$$MR_0(Q) = 100 - 4Q$$

Setting this equal to marginal cost, we have

$$100 - 4Q = 20 \rightarrow Q_0 = 20$$

$$P = 100 - 2(20) \rightarrow P_0 = 60$$

For inverse demand curve $P_1(Q)$, we have

$$MR_1(Q) = 120 - 4Q$$

Setting this equal to marginal cost, we have

$$120 - 4Q = 20 \rightarrow Q_1 = 25$$

$$P = 100 - 2(25) \rightarrow P_1 = 70$$

Therefore, $P_1 > P_0$ and so the equilibrium price has risen.

Problem 4. Suppose a monopolist has an inverse demand function given by $P = 100Q^{-\frac{1}{2}}$. What is the monopolist's optimal mark-up of price above marginal cost?

Solution: Remeber that the demand elasticity in a constant elasticity demand function is the exponent on P when the demand function is written in the regular form, i.e. $Q = f(P)$. Let us now rewrite the inverse demand function as a direct demand function:

$$P = 100Q^{-\frac{1}{2}} \to Q = 10000P^{-2}$$

We know now that the demand elasticity $\varepsilon_{Q,P} = -2$, we can use the IEPR:

$$\frac{P - MC}{P} = -\frac{1}{\varepsilon_{Q,P}} = -\frac{1}{-2} = \frac{1}{2}$$

The optimal percentage mark-up is 50%.