Tor Vergata University of Rome - Economics School
Bachelor of Arts in Global Governance

Microeconomics
Practice Sessions

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Microeconomics
Practice Session 1
Consumption Theory 1

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Utility Maximization Problem

The Utility Maximization Problem, will be the fundamental problem of the Consumption Theory. In order to solve it we have to set some preliminary concepts.

Indifference Curves

An indifference curve is a collection of points on the (x_1, x_2) diagram which represent bundles of good 1 and good 2 that are associated with the same level of utility by the utility function. Mathematically, indifference curves are the level curves of the utility function. In order to be easily graph on a cartesian plan, it is important to know how to explicit the utility function in terms of x_2 as a function of x_1 (in the form $x_2 = f(x_1)$), after having fixed a utility level $\bar{U}$. Let us now move to discuss the slope of the indifference curves, all the curves have the same expression for the slope, which is represented by the opposite of the module of the ratio between the marginal utilities. The ratio between the marginal utilities, is the marginal rate of substitution:

$$MRS_{1,2} = \frac{MU_1}{MU_2} = \frac{\frac{\partial U(x_1,x_2)}{\partial x_1}}{\frac{\partial U(x_1,x_2)}{\partial x_2}}$$

So it follows that the slope is:

$$-|MRS_{1,2}|$$

Exercise 1 - Indifference Curve
Given the following utility function:

$$U(x_1, x_2) = x_1^{\frac{1}{4}} x_2^{\frac{1}{2}},$$

determine

a) the equation of any indifference curves,

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b) the equation of the indifference curve related to a utility level $\bar{U} = 3$,

c) the slope of the indifferent curves.

Solution

a) The equation of any indifference curves can be found simply fixing a utility level $\bar{U}$ and expliciting the equation in terms of x_2:

$$x_1^{\frac{1}{4}} x_2^{\frac{1}{2}} = \bar{U} \rightarrow x_2^{\frac{1}{2}} = \frac{\bar{U}}{x_1^{\frac{1}{4}}} \rightarrow x_2 = \frac{\bar{U}^2}{x_1^{\frac{1}{2}}}$$

b) Considering the equation of the utility function and the level of utility, it follows that:

$$x_1^{\frac{1}{4}} x_2^{\frac{1}{2}} = 3 \rightarrow x_2^{\frac{1}{2}} = \frac{3}{x_1^{\frac{1}{4}}} \rightarrow x_2 = \frac{9}{x_1^{\frac{1}{2}}}$$

c) The slope of the indifferent curves is given by the opposite of the module of the $MRS_{1,2}$. As a first thing we compute the marginal utilities:

$$MU_1 = \frac{\partial U(x_1, x_2)}{\partial x_1} = \frac{1}{4} x_1^{\frac{1}{4}-1} x_2^{\frac{1}{2}} = \frac{1}{4} x_1^{-\frac{3}{4}} x_2^{\frac{1}{2}}$$

$$MU_2 = \frac{\partial U(x_1, x_2)}{\partial x_2} = \frac{1}{2} x_1^{\frac{1}{4}} x_2^{\frac{1}{2}-1} = \frac{1}{2} x_1^{\frac{1}{4}} x_2^{-\frac{1}{2}}$$

And now let us compute the $MRS_{1,2}$ as the ratio of the the marginal ulilities:

$$MRS_{1,2} = \frac{MU_1}{MU_2} = \frac{\frac{\partial U(x_1,x_2)}{\partial x_1}}{\frac{\partial U(x_1,x_2)}{\partial x_2}} = \frac{\frac{1}{4} x_1^{-\frac{3}{4}} x_2^{\frac{1}{2}}}{\frac{1}{2} x_1^{\frac{1}{4}} x_2^{-\frac{1}{2}}} = \frac{1}{4} x_1^{-\frac{3}{4}} x_2^{\frac{1}{2}} \cdot 2 x_1^{-\frac{1}{4}} x_2^{\frac{1}{2}} = \frac{1}{2} x_1^{-\frac{3}{4}-\frac{1}{4}} x_2^{\frac{1}{2}+\frac{1}{2}}$$

$$MRS_{1,2} = \frac{1}{2} \frac{x_2}{x_1}$$

The slope of the indifference curves is then:

$$-|MRS_{1,2}| = -\frac{1}{2} \frac{x_2}{x_1}$$

Budget Constraint

The budget constraint expresses the consumption possibilities of the individual we are studing, it shows all the possible bundles (combination of goods) which are affordable for the consumer. In order to be easily graph on a cartesian plan, it always as to be expressed in the form $x_2 = f(x_1)$; but at the same time it is very important to know, and start our analysis from the form:

$$I = p_1 x_1 + p_2 x_2$$

And then re-write it as:

$$I = p_1 x_1 + p_2 x_2 \rightarrow p_2 x_2 = I - p_1 x_1$$

$$x_2 = \frac{I}{p_2} - \frac{p_1}{p_2}x_1$$

This is because the first formula helps us to understand better the concept of budget constraint and its functioning. Let us now move to understand which are the slope of the line and the intercepts with the axes.

- **Slope:** from the form $x_2 = f(x_1)$, it is to understand that the slope of the budget line is the coefficient for which x_1 is multiplied (remember $y = mx + q$), then $-\frac{p_1}{p_2}$. Then the slope of the budget line is given by the relative prices, the ratio of the prices.

- **Intercepts:** we know, from the form $x_2 = f(x_1)$ the intercept with x_2 axis, is the value that the line assumes when $x_1 = 0$ is $x_2 = \frac{I}{p_2}$. Which economic intuition can this suggests us? The intercepts with the axes represent how many units of a good the consumer is able to buy, if she does not buy any unit of the other good, then if she decides to allocate all her resources (her entire income) on one single good. It seems clear that the intecepts will be always: $x_2 = \frac{I}{p_2}$ and $x_1 = \frac{I}{p_1}$.

Exercise 2 - Budget Constraint

Given the price of good 1, $p_1 = 5$, the price of good 2, $p_2 = 10$, and the income of the consumer, $I = 100$, determine:

a) the equation of the BC as from the definition,

b) the equation of the BC in order to graph it on a diagram,

c) the slope and the intercepts with the axes.

Solution

a) BC shows all the possible bundles (combination of goods) which are affordable for the consumer:

$$I = p_1x_1 + p_2x_2$$
$$100 = 5x_1 + 10x_2$$

b) In order to graph BC on a diagram, we have to express x_2 in terms of x_1:

$$100 = 5x_1 + 10x_2 \to 10x_2 = 100 - 5x_1$$
$$x_2 = 10 - \frac{1}{2}x_1$$

c) The slope of a budget constraint is given by the negative relative prices:

$$-\frac{p_1}{p_2} = -\frac{5}{10} = -\frac{1}{2}$$

While the intercepts are given by the consumption of a good when we are consuming only that good:

$$x_1 = 0 \to x_2 = \frac{R}{p_2} \to x_2 = \frac{100}{10} = 10$$
$$x_2 = 0 \to x_1 = \frac{R}{p_1} \to x_1 = \frac{100}{5} = 20$$

Microeconomics
Practice Session 2
Consumption Theory 2

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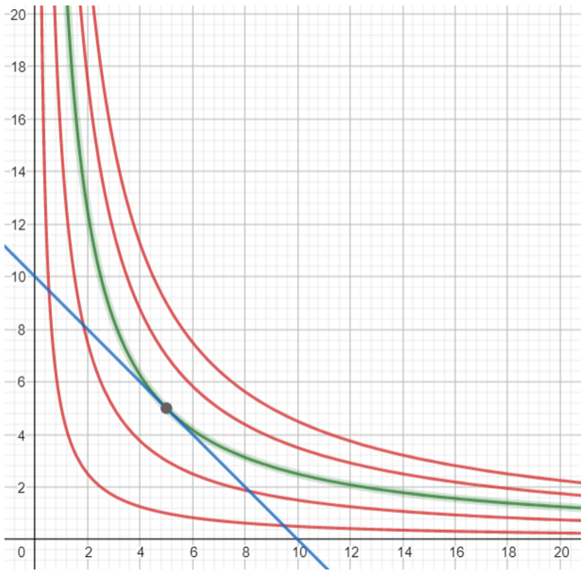
UMP (interior solution)

Optimal Bundle

The point of optimal consumption, the optimal bundle, is the point in which the consumer reaches the highest possible utility given her budget constraint, she is as happy as she could be with respect to her economic resources. In order to find the optimal bundle, it is important to get a look to the graphical representation of the problem:



The point of optimal consumption is the tangency point between the budget constraint and the indifference curve corresponding to the highest level of utility which still passes through affordable bundles, only one bundle is still affordable and it is one of the bundles for which the consumer spend her entire income. In order to find this bundle mathematically, we have to solve the following problem:

$$\begin{cases} MRS = \frac{p_1}{p_2} \rightarrow \text{Tangency condition (interior solution)} \\ I = p_1x_1 + p_2x_2 \rightarrow \text{Budget Constraint} \end{cases}$$

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The first equation expresses the tangency condition between all the indifference curves and all the budget constraints with that ratio of prices, while the second equation anchor the tangency condition to the specific situation of the consumer we are analysing with the budget constraint.

Exercise 1

A consumer has a utility function $U(x_1, x_2) = x_1 x_2$ for good x_1 and x_2 and an income $I = 800$. The two goods have prices $p_1 = 20$ and $p_2 = 40$. Compute the optimal consumption bundle.

Solution

In order to solve a Utility Maximization Problem, we can rely on the system, we have seen during the previous Practice Session:

$$\begin{cases} MRS = \frac{MU_1}{MU_2} = \frac{p_1}{p_2} \rightarrow \text{tangency condition (interior solution)} \\ I = p_1 x_1 + p_2 x_2 \rightarrow \text{budget constraint} \end{cases}$$

We now can solve the exercise numerically:

$$MU_1 = x_2$$

$$MU_2 = x_1$$

$$MRS = \frac{MU_1}{MU_2} = \frac{x_2}{x_1}$$

$$\begin{cases} \frac{x_2}{x_1} = \frac{20}{40} = \frac{1}{2} \rightarrow 2x_2 = x_1 \\ 20x_1 + 40x_2 = 800 \end{cases}$$

$$40x_2 + 40x_2 = 800 \rightarrow 80x_2 = 800 \rightarrow x_2^* = 10$$

$$2x_2 = x_1 \rightarrow x_1^* = 20$$

$$\textbf{Optimal Consumption Bundle } (\mathbf{x_1^*}, \mathbf{x_2^*}) = (\mathbf{20}, \mathbf{10})$$

Exercise 2

A consumer has a utility function $U(x_1, x_2) = x_1 x_2 + x_1$ for good x_1 and x_2 and an income $I = 22$. The two goods have prices $p_1 = 1$ and $p_2 = 2$. Compute the optimal consumption bundle.

Solution

As in the previous exercises we rely on the system that we learned last time.

$$MU_1 = x_2 + 1$$

$$MU_2 = x_1$$

$$MRS = \frac{MU_1}{MU_2} = \frac{x_2 + 1}{x_1}$$

$$\begin{cases} \frac{x_2+1}{x_1} = \frac{1}{2} = \rightarrow 2x_2 + 2 = x_1 \\ x_1 + 2x_2 = 22 \end{cases}$$

$$2x_2 + 2 + 2x_2 = 22 \rightarrow 4x_2 = 20 \rightarrow x_2^* = 5$$

$$x_1 = 2x_2 + 2 \rightarrow x_1^* = 12$$

$$\textbf{Optimal Consumption Bundle } (\mathbf{x_1^*}, \mathbf{x_2^*}) = (\mathbf{12}, \mathbf{5})$$

Exercise 3

A consumer has a utility function $U(x_1, x_2) = ln(x_1 x_2)$ for good x_1 and x_2 and an income $I = 120$. The two goods have prices $p_1 = 2$ and $p_2 = 3$. Compute the optimal consumption bundle.

Solution

As in the previous exercises we rely on the system that we learned last time.

$$MU_1 = \frac{1}{x_1}$$

$$MU_2 = \frac{1}{x_2}$$

$$MRS = \frac{MU_1}{MU_2} = \frac{x_2}{x_1}$$

$$\begin{cases} \frac{x_2}{x_1} = \frac{2}{3} \rightarrow x_2 = \frac{2}{3}x_1 \\ 120 = 2x_1 + 3x_2 \end{cases}$$

$$120 = 2x_1 + 3\frac{2}{3}x_1 \rightarrow 4x_1 = 120 \rightarrow x_1 = 30$$

$$x_2 = \frac{2}{3}30 = 20$$

$$\textbf{Optimal Consumption Bundle } (\mathbf{x_1^*}, \mathbf{x_2^*}) = (\mathbf{30}, \mathbf{20})$$

Extra Exercises

1. $U(x_1, x_2) = x_1^2 x_2$
   $I = 240$
   $p_1 = 8$
   $p_2 = 2$

2. $U(x_1, x_2) = x_1 x_2 + x_1$
   $I = 20$
   $p_1 = 1$
   $p_2 = 4$

3. $U(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$
   $I = 200$
   $p_1 = 10$
   $p_2 = 10$

4. $U(x_1, x_2) = x_1^{\frac{1}{3}} x_2^{\frac{2}{3}}$
   $I = 60$
   $p_1 = 6$
   $p_2 = 1$

5. $U(x_1, x_2) = ln(x_1) + 2ln(x_2)$
   $I = 60$
   $p_1 = 3$
   $p_2 = 1$

6. $U(x_1, x_2) = x_1 x_2^2$
   $I = 60$
   $p_1 = 3$
   $p_2 = 3$

7. $U(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{4}}$
   $I = 12$
   $p_1 = 4$
   $p_2 = 1$

8. $U(x_1, x_2) = x_1^{\frac{2}{5}} x_2^{\frac{3}{5}}$
   $I = 200$
   $p_1 = 20$
   $p_2 = 30$

9. $U(x_1, x_2) = 2ln(x_1) + 3ln(x_2)$
   $I = 300$
   $p_1 = 20$
   $p_2 = 30$

Solutions

1. $(x_1^*, x_2^*) = (20, 40)$
2. $(x_1^*, x_2^*) = (12, 2)$
3. $(x_1^*, x_2^*) = (10, 10)$
4. $(x_1^*, x_2^*) = (\frac{10}{3}, 40)$
5. $(x_1^*, x_2^*) = (\frac{20}{3}, 40)$
6. $(x_1^*, x_2^*) = (\frac{20}{3}, \frac{40}{3})$
7. $(x_1^*, x_2^*) = (2, 4)$
8. $(x_1^*, x_2^*) = (4, 4)$
9. $(x_1^*, x_2^*) = (6, 6)$

Microeconomics
Practice Session 3
Consumption Theory 3

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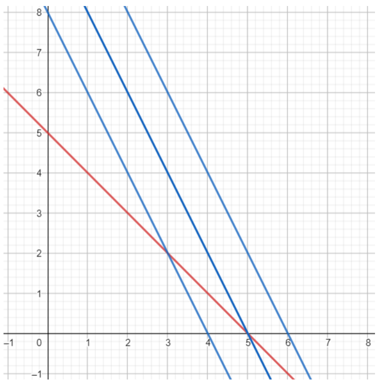
23/04/2021

Perfect Substitutes

The case of perfect substitute goods is a particular case, in which the consumer considers the consumption of the two goods to be equivalent (the classic example is that of tea and coffee), the choice between the two goods depends, as we shall see, solely by their price. To understand how to compute the optimal basket at the indicated prices and income, we use a graphical representation. We can observe three cases (in blue there are the indifference curves, while in red the budget constraint):

1. If the budget constraint and the indifference curves have different slopes, and the slope of the former is greater than that of the latter, then $-\frac{p_1}{p_2} > -MRS \rightarrow MRS > \frac{p_1}{p_2}$, the highest utility point that the consumer can reach economically is the intercept with the x_1 axis, as can be seen from the following graph:



So the demand for the good x_1 is given by the whole income divided by the price of the good 1, as we have seen in the previous practice session, while the demand for the good x_2 is equal to zero.

$$x_1 = \frac{I}{p_1} \text{ if } MRS > \frac{p_1}{p_2}$$

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$$x_2 = 0 \text{ if } MRS > \frac{p_1}{p_2}$$

From a mathematical point of view, we know that if the condition holds:

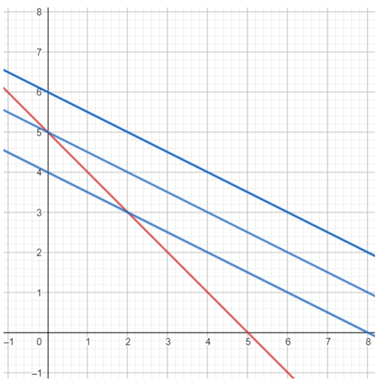
$$MRS > \frac{p_1}{p_2}$$

$$\frac{Umg_1}{Umg_2} > \frac{p_1}{p_2}$$

$$\frac{Umg_1}{p_1} > \frac{Umg_2}{p_2}$$

And that therefore, the marginal utility of good 1 relative to its price is higher than that of good 2, always relative to its price. In other words, we know that the value associated with the consumption of good 1 considering its cost is greater than the value associated with the consumption of good 2 considering its cost. It is therefore easy to understand why, if the consumer is indifferent between the two goods, he chooses to allocate his entire income in the purchase of good 1.

2. If the budget constraint and the indifference curves have different slopes, and the slope of the former is smaller than that of the latter, then $-\frac{p_1}{p_2} < -MRS \rightarrow MRS < \frac{p_1}{p_2}$, the highest utility point that the consumer can reach economically is the intercept with the x_2 axis, as can be seen from the following graph:



So the demand for the good x_2 is given by the whole income divided by the price of the good 2, as we have seen in the previous practice session, while the demand for the good x_1 is equal to zero.

$$x_1 = 0 \text{ if } MRS < \frac{p_1}{p_2}$$

$$x_2 = \frac{I}{p_2} \text{ if } MRS < \frac{p_1}{p_2}$$

From a mathematical point of view, we know that if the condition holds:

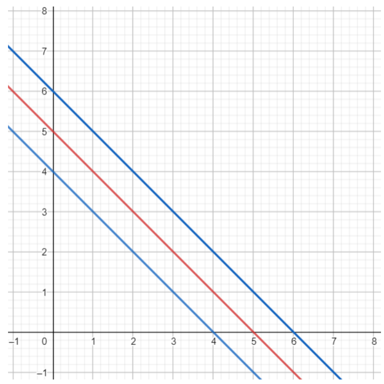
$$MRS < \frac{p_1}{p_2}$$

$$\frac{MU_1}{MU_2} < \frac{p_1}{p_2}$$

$$\frac{MU_1}{p_1} < \frac{MU_2}{p_2}$$

And that therefore, the marginal utility of good 1 relative to its price is lower than that of good 2, always relative to its price. In other words, we know that the value associated with the consumption of good 1 considering its cost is smaller than the value associated with the consumption of good 2 considering its cost. It is therefore easy to understand why, if the consumer is indifferent between the two goods, he chooses to allocate his entire income in the purchase of good 2.

3. If the budget constraint and the indifference curves have the same slope, then $-\frac{p_1}{p_2} = -MRS \rightarrow MRS = \frac{p_1}{p_2}$, the highest utility point that the consumer can reach economically is the any point of the budget constraint, as can be seen from the following graph:



Thus the demand for the goods x_1 and x_2 is given by any linear combination of the two points that lies on the budget constraint.

$$x_1 = [0; \frac{I}{p_1}] \text{ if } MRS = \frac{p_1}{p_2}$$

$$x_2 = [0; \frac{I}{p_2}] \text{ if } MRS = \frac{p_1}{p_2}$$

From a mathematical point of view, we know that if the condition holds:

$$MRS = \frac{p_1}{p_2}$$

$$\frac{MU_1}{MU_2} = \frac{p_1}{p_2}$$

$$\frac{MU_1}{p_1} = \frac{MU_2}{p_2}$$

And that therefore, the marginal utility of good 1 relative to its price is equal than that of good 2, always relative to its price. In other words, we know that the value associated with the consumption of good 1 considering its cost is equal than the value associated with the consumption of good 2 considering its cost. It is therefore easy to understand why, if the consumer is indifferent between the two goods, he chooses to allocate his entire income to any combination of the two goods.

Exercise 1

Given the following utility function

$$U(x_1, x_2) = 2x_1 + x_2$$

Compute the optimal bundle for an income of the consumer $I = 30$ and market prices $p_1 = 3$ and $p_2 = 1$.

Solution

We just have to comupte the MRS and compare it with the relative prices:

$$MU_1 = \frac{\partial U(x_1, x_2)}{\partial x_1} = 2$$

$$MU_2 = \frac{\partial U(x_1, x_2)}{\partial x_2} = 1$$

$$MRS = \frac{MU_1}{MU_2} = 2$$

$$\frac{p_1}{p_2} = \frac{3}{1}$$

$$2 < 3 \rightarrow MRS < \frac{p_1}{p_2}$$

$$x_1^* = 0$$

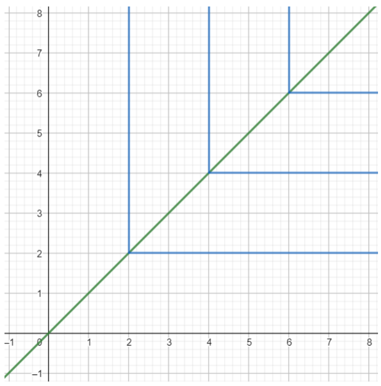
$$x_2^* = \frac{I}{p_2} = \frac{30}{1} = 30$$

Perfect Complements

The case of perfect complement goods is a particular case, in which the consumer wishes to consume the two goods jointly and in a specific proportion expressed by the utility function (the classic example is that of coffee and sugar), which comes in the following formula:

$$U(x_1, x_2) = min\{\alpha x_1; \beta x_2\}$$

From a graphical point of view, the indifference curves look like this (in blue):



The geometrical place in which the consumer's consumption is optimal is given by the corner points, those in which the optimal proportion between the quantities is respected. Which lies on the line (green in the graph):

$$\alpha x_1 = \beta x_2 \rightarrow x_2 = \frac{\alpha}{\beta} x_1$$

Therefore the optimal basket is given by the intersection between the budget constraint and the line $x_2 = \ frac\ alpha\ beta x_1$. We can therefore build the system in the following way:

$$\begin{cases} x_2 = \frac{\alpha}{\beta} x_1 \\ I = p_1 x_1 + p_2 x_2 \end{cases}$$

Exercise 1

Given the following utility function

$$U(x_1, x_2) = min\{3x_1; 5x_2\}$$

Compute the optimal bundle for an income of the consumer $I = 220$ and market prices $p_1 = 5$ and $p_2 = 10$.

Solution

Let's set the system we have just learned, with the information provided by the utility function:

$$\begin{cases} x_2 = \frac{\alpha}{\beta} x_1 \\ I = p_1 x_1 + p_2 x_2 \end{cases}$$

$$\begin{cases} x_2 = \frac{3}{5} x_1 \\ 220 = 5x_1 + 10x_2 \end{cases}$$

$$220 = 5x_1 + 10\frac{3}{5} x_1 \rightarrow x_1 = \frac{220}{11} = 20$$

$$x_2 = \frac{3}{5} 20 = 12$$

Microeconomics
Practice Session 4
Production Theory 1

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06/05/2021

Production General Concepts

- **Production function**: the production function is a relationship which associates different combinations of inputs with different levels of output; this function shows how the firm converts inputs into outputs.
- **Isoquant**: an isoquant is a curve on the (L;K) diagram which indicates combinations of labor and capital which are associated with the same level of output by the production function.
- **Marginal product**: the marginal product of a factor of production shows the variation of the output related to an infinitesimal variation of the factor of production, with all other factors kept constant.
- **Marginal substitution rate in production**: the MSRP shows how the inputs can substitute for each other in the production function; from a mathematical perspective, it is calculated as the ratio of the marginal product of one input to the marginal product of the other input; from a geometrical perspective, it can be thought of as the absolute value of the slope of the isoquant.
- **Isocost** an isocost line shows all combinations of inputs which cost the same total amount. Although similar to the budget constraint in consumer theory, the use of the isocost line pertains to cost-minimization in production, as opposed to utility-maximization.

Exercise 1

Given the production function $f(L,K) = L^{\frac{1}{4}}K^{\frac{3}{4}}$:

1. Find the general equation of an isoquant, and the isoquant relative to the level $\bar{q} = 100$;

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2. Calculate the marginal productivity of labour and capital and compute the marginal rate of technical substitution;

3. Find the general equation of an isocost, and the isocost relative to the input prices $w = 1$ and $r = 2$ and the level of total cost $\bar{c} = 5$;

Solution

This excercise aims to review some basic concepts, which will be very useful during the second part of the lecture and in general in all the exercise which will do for the Production Theory part. Let us now solve all the tasks keeping in mind the theory:

1. As first thing we are asked to find the general equation of an isoquant, which we know it's the level curve of the production function, so it will be necessary just to set a certain level of $\bar{q}$ to which the production function will be equal:

$$\bar{q} = L^{\frac{1}{4}} K^{\frac{3}{4}}$$
$$K^{\frac{3}{4}} = \frac{\bar{q}}{L^{\frac{1}{4}}} \to K = \frac{\bar{q}^{\frac{4}{3}}}{L^{\frac{1}{4}\frac{4}{3}}} \to K = \frac{\bar{q}^{\frac{4}{3}}}{L^{\frac{1}{3}}}$$

   Now we can move to compute one specific isoquant, the one relative to the level $\bar{q} = 100$:

$$100 = L^{\frac{1}{4}} K^{\frac{3}{4}}$$
$$K^{\frac{3}{4}} = \frac{100}{L^{\frac{1}{4}}} \to K = \frac{100^{\frac{4}{3}}}{L^{\frac{1}{4}\frac{4}{3}}} \to K = \frac{100^{\frac{4}{3}}}{L^{\frac{1}{3}}}$$

2. Another important topic are marginal productivities and their ration, the marginal rate of technical substitution. In order to compute the marginal productivity of labour and capital we will simply need to derive the production function with respect to L for MP_L and capital for MP_K:

$$MP_L = \frac{\partial f(L,K)}{\partial L} = \frac{1}{4} L^{-\frac{3}{4}} K^{\frac{3}{4}} = \frac{1}{4}\left(\frac{K}{L}\right)^{\frac{3}{4}}$$
$$MP_K = \frac{\partial f(L,K)}{\partial K} = \frac{3}{4} L^{\frac{1}{4}} K^{-\frac{1}{4}} = \frac{3}{4}\left(\frac{L}{K}\right)^{\frac{1}{4}}$$

   In order to compute the $MRTS$ is enough to make the ratio of the two marginal productivity:

$$MRTS = \frac{MP_L}{MP_K} = \frac{\frac{1}{4}\left(\frac{K}{L}\right)^{\frac{3}{4}}}{\frac{3}{4}\left(\frac{L}{K}\right)^{\frac{1}{4}}} = \frac{1}{4}\frac{K}{L}$$

3. As last task, we have to find the general equation of an isocost. As for the budget constraint, the total cost is given by a sum of input times their prices:

$$R = p_1x_1 + p_2x_2 \rightarrow TC = wL + rK$$

If we want the generic formula an isocost having $w = 1$ and $r = 2$, we will simply set a certain level of cost $\bar{c}$:

$$\bar{c} = wL + rK \rightarrow K = -\frac{w}{r}L + \frac{\bar{c}}{r}$$

Or fix $\bar{c} = 5$, if we want to have an isocost relative to a specific level of cost:

$$5 = L + 2K \rightarrow K = -\frac{1}{2}L + \frac{5}{2}$$

Cost Minimization Problem

The objective of the firm is to maximize its profit; for the sake of this goal, it has to produce any possible quantity at lowest possible cost. As a consequence, when we talk about how the firm produces, we are talking about a cost minimization problem. Namely, this problem can be thought of as a constrained minimization problem, in which the objective function (i.e. the function to be minimized) is the total cost function and the constraint is the quantity which the firm aims to produce. Mathematically speaking:

$$min\, TC = wL + rK\ s.t.\, \bar{q} = f(L, K)$$

Geometrically speaking, it is important to keep in mind that the constraint represents the equation of the isoquant associated with the desired quantity; as a consequence, solving this problems means finding the lowest possible isocost which touches the given isoquant. The solution of this problem represents the input bundle which allows the firm to produce the desired quantity at the lowest cost: this combination of labor and capital satisfies the property of economic efficiency.

Cost Minimization Problem in the Short Run

In the short run, at least an input is fixed (i.e. it cannot be used beyond a maximum level). In this course of Microeconomics, capital is the fixed factor in the short run. With the aim of minimizing its costs, the firm will use its capital at the maximum level with reference to any possible quantity and it will try to minimize the use of labor. Given that $\bar{K}$ is the highest possible use of capital, we obtain a modified version of the constrained minimization problem:

$$min\, wL + rK\ s.t.\, \bar{q} = f(L, K),\ s.t.\, K = \bar{K}$$

$$min\, wL + r\bar{K}\ s.t.\, \bar{q} = f(L, \bar{K})$$

The second constraint ($K = \bar{K}$) is typical of the short run time horizon.

Exercise 2

Given that $f(L, K) = L^{\frac{1}{2}}K^{\frac{1}{2}}$, $w = 16$, $r = 4$, $\bar{K} = 100$ and $\bar{q} = 200$:

1. Carry out the cost minimization problem, assuming a short run time horizon,
2. Find the optimal input bundle,
3. Compute the total cost beared by the producer.

Solution

1. In order to minimize producer's cost, we simply can use the setting which we have discussed in the previous section using the data we have:

$$min\, 16L + 4K \; s.t.\, 200 = L^{\frac{1}{2}}K^{\frac{1}{2}}, \; s.t.\, K = 100$$

$$min\, 16L + 4 \cdot 100 \; s.t.\, 200 = L^{\frac{1}{2}}100^{\frac{1}{2}}$$

   As you can observe, in order to solve the entire problem just using the constraints:

$$200 = 10L^{\frac{1}{2}}$$

$$L^{\frac{1}{2}} = 20$$

$$(L^{\frac{1}{2}})^2 = 20^2$$

$$L^* = 400$$

2. The optimal input bundle is simply given by the fixed level of capital and the result of the minimization problem:

$$\text{Optimal input bundle: } (L^*, \bar{K}) = (400, 100)$$

3. Again, in order to compute the total cost, we simply have to plug in all the data we have:

$$TC = wL^* + r\bar{K} = 16 \cdot 400 + 4 \cdot 100 = 6400 + 400 = 6800$$

Cost Minimization Problem in the Long Run

Let us start reviewing what we have learned last time. In order to minimize costs, we have to set the following problem:

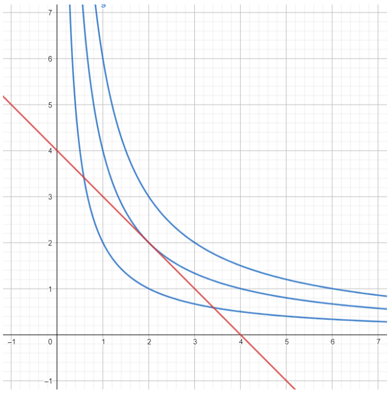
$$min\, wL + rK s.t.\bar{q} = f(L, K)$$

In the short run horizon we were setting a certain level of capital $K = \bar{K}$, while in the long run horizon, we can optimize the level of both productive inputs.
In order to solve this minimization problem we have to follow a very similar procedure to the one used in order to maximize utility, for this reason doing a comparison between this two problems will allow us to better undersand to procedure:

Utility maximization problem

$$max\, U(x_1, x_2) s.t. I = p_1 x_1 + p_2 x_2$$

In this problem we have a fixed level of income that consumer can spend and many possible indifference curves on which she can position herself. We are looking for the further from the origin curves (because we want the highest possible utility) but still on the budget line (because of non-satiation assumption), that's the reason why we are looking for the indifference curve which is tangent to the budget constraint.



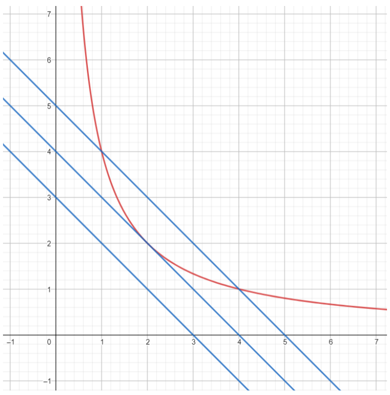
For this reasong we set the following system:

$$\begin{cases} MRS = \frac{MU_1}{MU_2} = \frac{p_1}{p_2} \rightarrow \text{tangency condition} \\ I = p_1 x_1 + p_2 x_2 \rightarrow \text{budget constraint} \end{cases}$$

Cost minimization problem

$$min\, wL + rK s.t. \bar{q} = f(L, K)$$

In this problem we have a fixed level of output that producer want to produce and many possible isocost lines on which she can position herself. We are looking for the closest to the origin line (because we want the lowest possible cost) but still on the isoquant (because of efficiency), that's the reason why we are looking for the isocost which is tangent to the isoquant.



For this reasong we set the following system:

$$\begin{cases} MRTS = \frac{MP_L}{MP_K} = \frac{w}{r} \rightarrow \text{tangency condition} \\ \bar{q} = f(L, K) \rightarrow \text{isoquant} \end{cases}$$

Exercise 3

Given that $f(L,K) = L^{\frac{1}{4}}K^{\frac{1}{4}}$, $w = 30$, $r = 30$ and $\bar{q} = 5$:

1. Compute the value of L and K that minimize cost
2. Calculate the corrisponding total cost

Solution

1. We have to set a cost minimization problem in the following way:

$$min\, 30L + 30K s.t. 5 = L^{\frac{1}{4}}K^{\frac{1}{4}}$$

Let us compute the $MRTS$, and then directly set the system:

$$MRTS = \frac{MP_L}{MP_K}$$

$$MP_L = \frac{\partial f(L,K)}{\partial L} = \frac{1}{4}L^{\frac{1}{4}-1}K^{\frac{1}{4}} = \frac{1}{4}L^{-\frac{3}{4}}K^{\frac{1}{4}}$$

$$MP_K = \frac{\partial f(L,K)}{\partial K} = \frac{1}{4}L^{\frac{1}{4}}K^{\frac{1}{4}-1} = \frac{1}{4}L^{\frac{1}{4}}K^{-\frac{3}{4}}$$

$$MRTS = \frac{\frac{1}{4}L^{-\frac{3}{4}}K^{\frac{1}{4}}}{\frac{1}{4}L^{\frac{1}{4}}K^{-\frac{3}{4}}} = L^{-\frac{4}{4}}K^{\frac{4}{4}} = \frac{K}{L}$$

$$\begin{cases} \frac{K}{L} = \frac{30}{30} \rightarrow K = L \\ 5 = L^{\frac{1}{4}}K^{\frac{1}{4}} \end{cases}$$

$$5 = L^{\frac{1}{4}}L^{\frac{1}{4}} \rightarrow 5 = L^{\frac{1}{2}} \rightarrow 5^2 = L^{\frac{1}{2}\cdot 2}$$

$$L^* = 25$$

$$K^* = 25$$

2. Let us simply use the total cost formula:

$$CT = wL^* + rK^* = 30 \cdot 25 + 30 \cdot 25 = 750 + 750 = 1500$$

Extra Exercises

COST MINIMIZATION IN THE SHORT RUN

1. $f(L,K) = L^{\frac{1}{4}}K^{\frac{1}{2}}$; $\bar{q} = 2$
   $w = 20$; $r = 30$
   $\bar{K} = 9$

2. $f(L,K) = LK^{\frac{1}{4}}$; $\bar{q} = 5$
   $w = 2$; $r = 3$
   $\bar{K} = 16$

3. $f(L,K) = L^{\frac{1}{5}}K$; $\bar{q} = 1$
   $w = 4$; $r = 3$
   $\bar{K} = 2$

4. $f(L,K) = L^{\frac{1}{2}}K^{\frac{1}{2}}$; $\bar{q} = 32$
   $w = 2$; $r = 2$
   $\bar{K} = 16$

5. $f(L,K) = L^{\frac{1}{2}}K^{\frac{1}{2}}$; $\bar{q} = 10$
   $w = 3$; $r = 5$
   $\bar{K} = 16$

COST MINIMIZATION IN THE LONG RUN

1. $f(L,K) = L^{\frac{1}{2}}K^{\frac{1}{2}}$; $\bar{q} = 20$
   $w = 2$; $r = 2$

2. $f(L,K) = L^{\frac{1}{5}}K^{\frac{4}{5}}$; $\bar{q} = 10$
   $w = 1$; $r = 4$

3. $f(L,K) = L^{\frac{3}{4}}K^{\frac{1}{4}}$; $\bar{q} = 10$
   $w = 3$; $r = 1$

4. $f(L,K) = L^2K$; $\bar{q} = 32$
   $w = 2$; $r = 2$

5. $f(L,K) = LK$; $\bar{q} = 15$
   $w = 5$; $r = 3$

Solutions

COST MINIMIZATION IN THE SHORT RUN

1. $L^* = 0.1975$
2. $L^* = 2.5$
3. $L^* = 0.03$
4. $L^* = 2$
5. $L^* = 6.25$

COST MINIMIZATION IN THE LONG RUN

1. $L^* = 20$; $K^* = 20$
2. $L^* = 10$; $K^* = 10$
3. $L^* = 5$; $K^* = 5$
4. $L^* = 4$; $K^* = 2$
5. $L^* = 3$; $K^* = 5$

Microeconomics
Practice Session 5
Production Theory 2

Luisa Lorè*

15/05/2020

Return to Scale

Eeturns to scale regard the relationship between an increase in the use of inputs and the consequent variation in the output. There are different types of return to scale:
Increasing returns to scale (IRS): as the use of inputs increases, the output increases in a more than linear way.
Constant returns to scale (CRS): as the use of inputs increases, the output increases in a linear way.
Decreasing returns to scale (DRS): as the use of inputs increases, the output increases in a less than linear way.
In general, in order to verify return to scale we can simply multiply the entire function for a constant λ (where $\lambda > 0$), and the function will exhibit:
IRS if $f(\lambda K, \lambda L) > \lambda f(K, L)$, because increasing inputs by a constant λ, we have a more than proportional increase in output.
CRS if $f(\lambda K, \lambda L) = \lambda f(K, L)$ because increasing inputs by a constant λ, we have a proportional increase in output.
DRS if $f(\lambda K, \lambda L) < \lambda f(K, L)$, because increasing inputs by a constant λ, we have a less than proportional increase in output.
If we are dealing with a Cobb-Douglas production function, we can rely on analysing inputs' exponents. Let us recall a Cobb-Douglas production function:

$$f\left(K,\, L\right) = K^{\alpha} L^{\beta}$$

If we apply the same reasoning done until now, we obtained that:

$$f\left(\lambda K,\, \lambda L\right) \lesseqgtr \lambda \left[f\left(K,\, L\right)\right]$$
$$\lambda^{\alpha} K^{\alpha} \lambda^{\beta} L^{\beta} \lesseqgtr \lambda K^{\alpha} L^{\beta}$$
$$\lambda^{\alpha+\beta} K^{\alpha} L^{\beta} \lesseqgtr \lambda K^{\alpha} L^{\beta}$$
$$\lambda^{\alpha+\beta} \lesseqgtr \lambda$$

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$$\alpha + \beta \lesseqgtr 1$$

Then, this function will exhibit:
IRS if $\alpha + \beta > 1$
CRS if $\alpha + \beta = 1$
DRS if $\alpha + \beta < 1$

Exercise 1

Given the following production functions establish the return to scale that each of the function exhibits, giving a clear definition of the return to scale type:

1. $f(K, L) = 2(L + K)$
2. $f(K, L) = L^{\frac{1}{2}} K^{\frac{2}{6}}$
3. $f(K, L) = 2(LK)^{\frac{1}{2}}$
4. $f(K, L) = L + K^2$
5. $f(K, L) = L^3 K^5$

Solution

Now let us move to the solution of the exercise implementing the two strategies we have learned:

1. $f(K, L) = 2(L + K)$
   $f(\lambda K, \lambda L) = 2(\lambda L + \lambda K) = 2\lambda(L + K)$
   $\lambda [f(K, L)] = 2\lambda(L + K)$
   $2\lambda(L + K) = 2\lambda(L + K) \rightarrow$ **CRS**
2. $f(K, L) = L^{\frac{1}{2}} K^{\frac{2}{6}}$
   $\frac{1}{2} + \frac{2}{6} = \frac{5}{6} < 1 \rightarrow$ **DRS**
3. $f(K, L) = 2(LK)^{\frac{1}{2}}$
   $\frac{1}{2} + \frac{1}{2} = 1 \rightarrow$ **CRS**
4. $f(K, L) = L + K^2$
   $f(\lambda K, \lambda L) = (\lambda L + (\lambda K)^2) = (\lambda L + \lambda^2 K^2) = \lambda(L + \lambda K^2)$
   $\lambda [f(K, L)] = \lambda(L + K^2)$
   $\lambda(L + \lambda K^2) > \lambda(L + K^2) \rightarrow$ **IRS**
5. $f(K, L) = L^3 K^5$
   $3 + 5 = 8 > 1 \rightarrow$ **IRS**

Profit Maximisation

We define profit as the difference between total revenues (so everything producer can produce for the price to which she can sell) and total costs (all the factors of production we use times their price), as follows:

$$\pi = pY - (wL + rK) = pY - wL - rK$$

What is the constraint that a producer could have in such a problem? Surely it concerns its possibility and production capacity of Y, hence its production function $f(L, K)$. So our problem will be to maximize profit, under the constraint of the production function. This time, however, we will not build a system to solve the problem, but we will directly replace the constraint within the function to be maximized as follows:

Maximization of Profit $\underset{K,L}{max}\, \pi = pY - wL - rK \; s.t.c. \; f(L, K)$

Maximization of Profit $\underset{K,L}{max}\, \pi = p\,[f(L, K)] - wL - rK$

We also need to characterize this problem to achieve short-term maximization. Conventionally we consider L, labor, a factor of production that can be easily modified in a firm, while K, capital, a factor of production that can hardly be modified in a firm. So in the short term we "fix", keep constant, the capital and we optimize only in function of the work, in the following way:

Maximization of Profit (in short period) $\underset{L}{max}\, \pi = p\,[f(L, K)] - wL - rK \, s.t.c. K = \overline{K}$

Maximization of Profit (in short period) $\underset{L}{max}\, \pi = p\left[f(L, \overline{K})\right] - wL - r\overline{K}$

So doing what we get is a function in a single variable, L, and to optimize this function we just need to compute its first derivative, equal it to zero, and solve for L^*.

Exercise 2

Given the following production function and the following data, calculate the labor demand function that solves the short-run profit maximization problem.

$$f\,(K,\, L) = L^{\frac{1}{2}} K^{\frac{1}{2}}$$

$$p = 10$$

$$\overline{K} = 100$$

$$w = 5$$

$$r = 5$$

Solution

Let us compute the optimal amout of labour in order to maximise the profit:

$$\underset{L}{max}\,\pi = pY - wL - r\overline{K}\; s.v.\; f\left(L,\,\overline{K}\right) \longrightarrow \underset{L}{max}\,\pi = p\left[f\left(L,\,\overline{K}\right)\right] - wL - r\overline{K}$$

$$\underset{L}{max}\,\pi = 10Y - 5L - 500\; s.v.\; L^{\frac{1}{2}}10 \longrightarrow \underset{L}{max}\,\pi = 100L^{\frac{1}{2}} - 5L - 500$$

$$\frac{\partial\pi}{\partial L} = 100L^{-\frac{1}{2}} - 5 = 0$$

$$\frac{10}{\sqrt{L}} = 1 \longrightarrow \left(\sqrt{L}\right)^2 = 10^2 \longrightarrow L = 100$$

Let us now calculate the optimal level of output to be produced and the relative profit:

$$q^* = f\left(K,\,L\right) = 100^{\frac{1}{2}}100^{\frac{1}{2}} = 100$$

$$\pi = pQ - CT(Q) = 1000 - 1000 = 0$$

Extra Exercises

RETURN TO SCALE

1. $f(L, K) = L^{\frac{1}{4}} K$
2. $f(L, K) = L + K$
3. $f(L, K) = L^{\frac{1}{5}} K^{\frac{4}{5}}$
4. $f(L, K) = L^3 + K^2$
5. $f(L, K) = L^{\frac{1}{6}} K^{\frac{1}{8}}$

Solutions

RETURN TO SCALE

1. IRS
2. CRS
3. CRS
4. IRS
5. DRS

Microeconomics
Practice Session 6
Markets

Luisa Lorè*

28/05/2020

Perfect Competition

A market is defined as perfectly competitive providing that four requirements are fulfilled:

1. **Homogeneity of products**: all the products that are marketed by the various firms are exactly identical or perfect subsitutes; this means that the only lever that can be exploited by a single firm to compete is price.

2. **High number of economic agents**: there is a significant number of consumers and producers that are not able to individually influence the level of the market demand and the market supply.

3. **Perfect information**: any producer is perfectly informed about the preferences of all consumers and any consumer is perfectly informed about the characteristics of the products marketed by all firms.

4. **Absence of barriers to entry and exit**: firms can easily enter or exit the market.

The combination of these requirements causes each firm to consider the market price as a given.

Perfect Competition in the Short Run

A market supply curve is generally built as the horizontal summation of the individual supply curves. In a competitive market, as firms are identical, the market supply curve is obtained as the individual supply curve multiplied by the number of firms which are present in the marketplace. Each firm supplies a quantity which allows it to maximize its profit and therefore to fulfill this condition:

$$max\,\pi = TR(Q) - TC(Q)$$

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Since in perfect competition the market price is a given, we can rewrite the condition in the following way:

$$max\, \pi = TR(Q) - TC(Q) = pQ - TC(Q)$$

$$\frac{\partial \pi(Q)}{\partial Q} = 0$$

$$\frac{\partial pQ}{\partial Q} - \frac{\partial TC(Q)}{\partial Q} = 0$$

$$p - MC(Q) = 0$$

$$p = MC(Q)$$

Market Equilibrium

The market equilibrium is a situation in which the market demand equals the market supply. Geometrically speaking, it is represented by the point on the (Q,P) cartesian plane in which the supply curve and the demand curve intersect.

Exercise 1

In a perfectly competitive market, there are 10 firms operating, each one has the following total cost function:

$$TC(Q_i) = Q_i^2$$

The market is also characterized by the following demand function:

$$Q^d = 100 - 20p$$

Compute:

1. The short run supply curve of the firm
2. The short run supply curve of the industry
3. Price and Quantity at the equilibrium
4. The level of production and the profit realized by one firm in the short run

Solution

1. In order to compute the short run supply curve of the firm, we need to start from the profit maximization condition that we learned in the previous paragraph and then express the quantity in function of the price. As a first thing let us compute the marginal cost and then derive the supply curve:

$$MC(Q_i) = \frac{\partial TC(Q_i)}{\partial Q} = 2Q_i$$

$$p = CMa(Q_i) \longrightarrow p = 2Q_i \longrightarrow Q_i = \frac{1}{2}p$$

2. In order to compute the short run supply curve of the entire industry, we simply sum linearly the supply curves of the each firm, in the following way:

$$Q^s = 10\left(\frac{1}{2}p\right) = 5p$$

3. In order to compute market equilibrium we have to solve a system in two unknown, this is because we have to find the intersection point between the demand and the supply curve:

$$\begin{cases} Q^d(p) = 100 - 20p \\ Q^s(p) = 5p \end{cases}$$

   In order to solve this system, we simply have to set the two functions as equal, because both express the quantity as a function of price:

$$100 - 20p = 5p \longrightarrow 25p = 100 \longrightarrow p = \frac{100}{25} = 4$$

   Let us now substitute price in to one of the two function, for example in the supply:

$$Q^s(p) = 5 \cdot 4 = 20$$

   And the equilibrium condition is:

$$Eq. = \{Q^E = 20,\ p^E = 4\}$$

4. Since there are 10 firms in the industry, each one produces one tenth on the quantity demanded by the market:

$$Q_i^E = \frac{Q^E}{\#\,firms} = \frac{20}{10} = 2$$

   Let us now substitute price and quantity into the profit formula:

$$\pi_i = p^E \cdot Q_i^E - CT(Q_i^E) = 4 \cdot 2 - 2^2 = 8 - 4 = 4$$

   All firms have an individual profit of 4.

Monopoly

If in a market the demand is satisfied entirely by only one firm, this market is a monopoly, and the firm operating in it is a monopolist. A monopolist has not a supply curve, it simply provides a price to the market, knowing that at that price the consumers will be willing to buy the quantity that maximize his profit. In order to find the quantity that maximize its output, he has indeed to take in consideration market demand, and it does so maximizing the following formula for its profit:

$$\pi = TR(Q) - TC(Q) = p(Q)Q - MC(Q)$$

In which $p(Q)$ is the inverse market demand function. From the profit maximization, we arrive to this profit-maximization condition:

$$max\,\pi = TR(Q) - TC(Q)$$

$$\frac{\partial \pi(Q)}{\partial Q} = 0$$
$$\frac{\partial TR(Q)}{\partial Q} - \frac{\partial TC(Q)}{\partial Q} = 0$$
$$MR(Q) - MC(Q) = 0$$
$$MR(Q) = MC(Q)$$

Monopoly mark-up

As we have seen a monopolist can set a price above the perfect competition price, but it always take in consideration the market demand. In order to have a measure about how much above the marginal cost he can price, we can compute its mark-up in the following way:

$$\mu = \frac{p^m - CMa(Q^m)}{CMa(Q^m)} = \frac{p^m}{CMa(Q^m)} - 1$$

Exercise 2

In a market there is only one firm operating with the following total cost function:

$$CT(Q) = 100Q$$

the market demand is given by

$$Q^d = 400 - 2p$$

Compute:

1. Market equilibrium when firm is price-setter (behave as a monopoly)
2. Monopoly mark-up and profit
3. Market equilibrium when firm is price-taker (behave as in perfect competition)
4. Monopoly net gain

Solution

1. When the firm is price-setter, it maximize the following profit function:

$$max\, \pi^m = RT(Q) - CT(Q) = p(Q)Q - CT(Q) \longrightarrow RMa(Q) - CMa(Q) = 0$$
$$RMa(Q) = CMa(Q)$$

We have to derive $p(Q)$, the market demand inverse function, in the following way:

$$Q^d = 400 - 2p \longrightarrow 2p = 400 - Q \longrightarrow p = 200 - \frac{1}{2}Q$$

We derive revenues and costs and we set them equal:

$$\frac{\partial RT(Q)}{\partial Q} = \frac{\partial \left(200Q - \frac{1}{2}Q^2\right)}{\partial Q} = 200 - Q$$

$$\frac{\partial CT(Q)}{\partial Q} = \frac{\partial\,(100Q)}{\partial Q} = 100$$

$$RMa(Q) = CMa(Q) \longrightarrow 200 - Q = 100 \longrightarrow Q^m = 100$$

In order to find the price, let us plug the quantity into the demand function:

$$p = 200 - \frac{1}{2}Q = 200 - \frac{1}{2}100 \longrightarrow p^m = 150$$

$$Eq^{mon} = \{Q^m = 100,\, p^m = 150\}$$

2. In order to compute the monopolist's mark-up we know that:

$$\mu = \frac{p^m - CMa(Q^m)}{CMa(Q^m)} = \frac{150 - 100}{100} = \frac{50}{100} = \frac{1}{2} \longrightarrow 50\%$$

While profit is given by the formula:

$$\pi^m = RT(Q^m) - CT(Q^m) = p^m Q^m - CT(Q^m)$$

$$\pi^m = 150 \cdot 100 - 100 \cdot 100 = 15000 - 10000 = 5000$$

3. Now, suppose that for any reason monopolist cannot (or decide not to) influence the market, then it decide to be a price-taker and to behave as a firm in perfect competition:

$$p = CMa(Q) \longrightarrow 200 - \frac{1}{2}Q = 100 \longrightarrow \frac{1}{2}Q = 100 \longrightarrow Q^{cp} = 200$$

$$p = 200 - \frac{1}{2}200 = 200 - 100 \longrightarrow p^{cp} = 100$$

$$Eq^{cp} = \{Q^{cp} = 200,\, p^{cp} = 100\}$$

As we should have expected, monopoly equilibrium present an higher price and a lower quantity with respect to the equilibrium generated by the price-taker firm, indeed a monopoly produce less and sell at an higher price compare to a firm operating in a perfectly competitive regime.

$$\pi^{cp} = RT(Q^{cp}) - CT(Q^{cp}) = p^{cp} Q^{cp} - CT(Q^{cp})$$

$$\pi^{cp} = 100 \cdot 200 - 100 \cdot 200 = 20000 - 20000 = 0$$

4. Let us now compute the difference between the monopoly and the perfect competition profits:

$$\pi^m - \pi^{cp} = 5000 - 0 = +5000$$

Extra Exercises

PERFECT COMPETITION IN THE SHORT RUN

1. $Q^d = 500 - 25p$; $CT(Q_i) = Q_i^2 + 25$; $n = 50$
2. $Q^d = 40 - 4p$; $CT(Q_i) = Q_i^2$; $n = 40$
3. $Q^d = 10 - \frac{1}{5}p$; $CT(Q_i) = Q_i^2$; $n = 10$
4. $Q^d = 420 - 20p$; $CT(Q_i) = Q_i^2 + 1$; $n = 100$
5. $Q^d = 100 - 2p$; $CT(Q_i) = Q_i^2 + 9$; $n = 4$

MONOPOLY

1. $Q^d = 200 - p$; $CT(Q_i) = 40Q_i + 10$;
2. $Q^d = 100 - \frac{1}{2}p$; $CT(Q_i) = 40Q_i + 10$;
3. $Q^d = 400 - 2p$; $CT(Q_i) = 100Q_i$;
4. $Q^d = 100 - p$; $CT(Q_i) = 20Q_i$;
5. $Q^d = 120 - p$; $CT(Q_i) = Q_i^2 + 100$;