LUISS Guido Carli
Bachelor Degree in Business Administration

Microeconomics
Practice Sessions

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A.Y. 2022/2023

Math Review

Microeconomics
Bachelor in Business Administration
LUISS Guido Carli

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September 19, 2022

1 Laws of exponents

$$x^n x^m = x^{n+m}$$
$$\frac{x^n}{x^m} = x^{n-m}$$
$$(x^n)^m = x^{nm}$$
$$x^n y^n = (xy)^n$$
$$\frac{x^n}{y^n} = \left(\frac{x}{y}\right)^n$$

Exercise 1

Simplify the following expression using the laws of exponents:

(a) $5^3 5$

(b) $2^2 2^{0.25}$

(c) $\frac{6^5}{6}$

(d) $\frac{30^9}{30^{10}}$

(e) $\frac{6}{2^2 3^2}$

(f) $\frac{7^{\frac{1}{2}}}{7}$

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Solution

(a) $5^3 5 = 5^{3+1} = 5^4$

(b) $2^2 2^{0.25} = 2^{2+0.25} = 2^{2.25} = 2^{\frac{9}{4}}$

(c) $\frac{6^5}{6} = 6^{5-1} = 6^4$

(d) $\frac{30^9}{30^{10}} = \frac{30^9}{30^{9+1}} = \frac{\cancel{30^9}}{\cancel{30^9}\,30^1} = \frac{1}{30}$

(e) $\frac{6}{2^2 3^2} = \frac{6}{(2\cdot 3)^2} = \frac{\cancel{6}}{6^{\cancel{2}}} = \frac{1}{6}$

(f) $\frac{7^{\frac{1}{2}}}{7} = \frac{7^{\frac{1}{2}}}{7^{\frac{1}{2}+\frac{1}{2}}} = \frac{\cancel{7^{\frac{1}{2}}}}{\cancel{7^{\frac{1}{2}}}\,7^{\frac{1}{2}}} = \frac{1}{7^{\frac{1}{2}}}$

2 System of equations

Exercise 2

Solve the following system:

$$\begin{cases} y = 3x + 2 \\ 5 = 2y + 3x \end{cases}$$

Solution

$$\begin{cases} y = 3x + 2 \\ 5 = 2y + 3x \end{cases} \quad \begin{cases} y = 3x + 2 \\ 5 = 2(3x + 2) + 3x \end{cases} \quad \begin{cases} y = 3x + 2 \\ 5 = 6x + 4 + 3x \end{cases} \quad \begin{cases} y = 3x + 2 \\ 9x = 1 \end{cases}$$

$$\begin{cases} y = 3\frac{1}{9} + 2 \\ x = \frac{1}{9} \end{cases} \quad \begin{cases} y = \frac{3}{9} + \frac{18}{9} \\ x = \frac{1}{9} \end{cases} \quad \begin{cases} y = \frac{21}{9} = \frac{7}{3} \\ x = \frac{1}{9} \end{cases}$$

$$x^*, y^* = \frac{1}{9}, \frac{7}{3}$$

3 Derivatives

function	**first derivative**
$f(x) = k$	$\frac{\partial f(x)}{\partial x} = 0$
$f(x) = x$	$\frac{\partial f(x)}{\partial x} = 1$
$f(x) = kh(x)$	$\frac{\partial f(x)}{\partial x} = k\frac{\partial h(x)}{\partial x}$
$f(x) = kx$	$\frac{\partial f(x)}{\partial x} = k$
$f(x) = x^{(n)}$	$\frac{\partial f(x)}{\partial x} = nx^{(n-1)}$
$f(x) = g(h(x))$	$\frac{\partial f(x)}{\partial x} = \frac{\partial g(x)}{\partial x}\frac{\partial g(h(x))}{\partial x}$
$f(x) = \ln(x)$	$\frac{\partial f(x)}{\partial x} = \frac{1}{x}$

Exercise 3

Compute the first derivatives of the following functions:

(a) $f(x) = 10x + 100$

(b) $f(x) = x^2 + 4x + 10$

(c) $f(x) = 200x - x^2 - 20x - 200$

(d) $f(x) = x + 10 + \frac{100}{x} + \ln(x)$

(e) $f(x) = x^3 + x^{\frac{1}{2}} - \ln(x)$

Solution

(a) $f(x) = 10x + 100 \rightarrow \frac{\partial f(x)}{\partial x} = 10 + 0 = 10$

(b) $f(x) = x^2 + 4x + 10 \rightarrow \frac{\partial f(x)}{\partial x} = 2x + 4 + 0 = 2x + 4$

(c) $f(x) = 200x - x^2 - 20x - 200 \rightarrow \frac{\partial f(x)}{\partial x} = 200 - 2x - 20 - 0 = 180 - 2x$

(d) $f(x) = x + 10 + \frac{100}{x} + \ln(x) \rightarrow \frac{\partial f(x)}{\partial x} = 1 + 0 + \frac{100}{x^2} + \frac{1}{x} = 1 + \frac{100}{x^2} + \frac{1}{x}$

(e) $f(x) = x^3 + x^{\frac{1}{2}} - \ln(x) \rightarrow \frac{\partial f(x)}{\partial x} = 3x^2 + \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{x} = 3x^2 + \frac{1}{2}\frac{1}{x^{\frac{1}{2}}} - \frac{1}{x}$

4 Partial derivatives

Exercise 4

Compute the partial derivatives of the following functions (wtr to both x and y):

(a) $f(x,y) = x^{\frac{1}{2}}y^{\frac{1}{2}}$

(b) $f(x,y) = x^2 + y$

(c) $f(x,y) = \ln(x) + \ln(y)$

(d) $f(x,y) = x^{\frac{1}{3}}y^{\frac{1}{4}}$

(e) $f(x,y) = x^2y^3$

Solution

(a) · $\frac{\partial f(x,y)}{\partial x} = \frac{1}{2}x^{-\frac{1}{2}}y^{\frac{1}{2}}$
· $\frac{\partial f(x,y)}{\partial y} = \frac{1}{2}x^{\frac{1}{2}}y^{-\frac{1}{2}}$

(b) · $\frac{\partial f(x,y)}{\partial x} = 2x$
· $\frac{\partial f(x,y)}{\partial y} = 1$

(c) · $\frac{\partial f(x,y)}{\partial x} = \frac{1}{x}$
· $\frac{\partial f(x,y)}{\partial y} = \frac{1}{y}$

(d) · $\frac{\partial f(x,y)}{\partial x} = \frac{1}{3}x^{-\frac{2}{3}}y^{\frac{1}{4}}$
· $\frac{\partial f(x,y)}{\partial y} = \frac{1}{4}x^{\frac{1}{3}}y^{-\frac{3}{4}}$

(e) · $\frac{\partial f(x,y)}{\partial x} = 2xy^3$
· $\frac{\partial f(x,y)}{\partial y} = 3x^2y^2$

Practice Session 1

Microeconomics

Bachelor in Business Administration
LUISS Guido Carli

Luisa Lorè*

September 26, 2022

Exercise 1

Suppose demand for seats at volleyball games is $P = 1,900 - \frac{1}{50}Q$ and supply is fixed at $Q = 90,000$ seats.

a) Find the equilibrium price and quantity of seats for a volleyball game.

b) Suppose the government prohibits tickets scalping (selling tickets above their face value), and the face value of tickets is \$50 (this policy places a price ceiling at \$50). How many consumers will be dissatisfied (how large is excess demand)?

c) Suppose the next game is a major rivalry, and so demand jumps to $P = 2,100 - \frac{1}{50}Q$. How many consumers will be dissatisfied for the big game?

Solution

a) The equilibrium quantity is $Q = 90,000$ seats and the equilibrium price is $P = \$100$.

$$\begin{cases} P = 1,900 - \frac{1}{50}Q \\ Q = 90,000 \end{cases} \quad \begin{cases} P = 1,900 - \frac{1}{50}(90,000) \\ Q = 90,000 \end{cases}$$

$$\begin{cases} P = 1,900 - 1,800 \\ Q = 90,000 \end{cases} \quad \begin{cases} P^* = 100 \\ Q^* = 90,000 \end{cases}$$

$$(Q^*, P^*) = (90,000; \$100)$$

b) With a price ceiling of $P = \$50$, quantity demanded is $Q = 92,500$ seats. Since there are only $Q = 90,000$ seats in the stadium, there will be 2,500 dissatisfied fans who want to buy a ticket at $P = \$50$ but cannot find one available.

$$\begin{cases} P = 1,900 - \frac{1}{50}Q \\ P = 50 \end{cases} \quad \begin{cases} 50 = 1,900 - \frac{1}{50}Q \\ P = 50 \end{cases} \quad \begin{cases} \frac{1}{50}Q = 1,900 - 50 \\ P = 50 \end{cases}$$

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$$\begin{cases} Q = (1,850) \cdot 50 \\ P = 50 \end{cases} \quad \begin{cases} \bar{Q} = 92,500 \\ \bar{P} = 50 \end{cases}$$

$$\text{excess demand}^* = \bar{Q} - Q^* = 92,500 - 90,000 = 2,500$$

c) Quantity demanded for the higher demand is $Q = 102,500$ seats. Now there will be $12,500$ dissatisfied fans who want to buy a ticket at $P = \$50$ but cannot find one available.

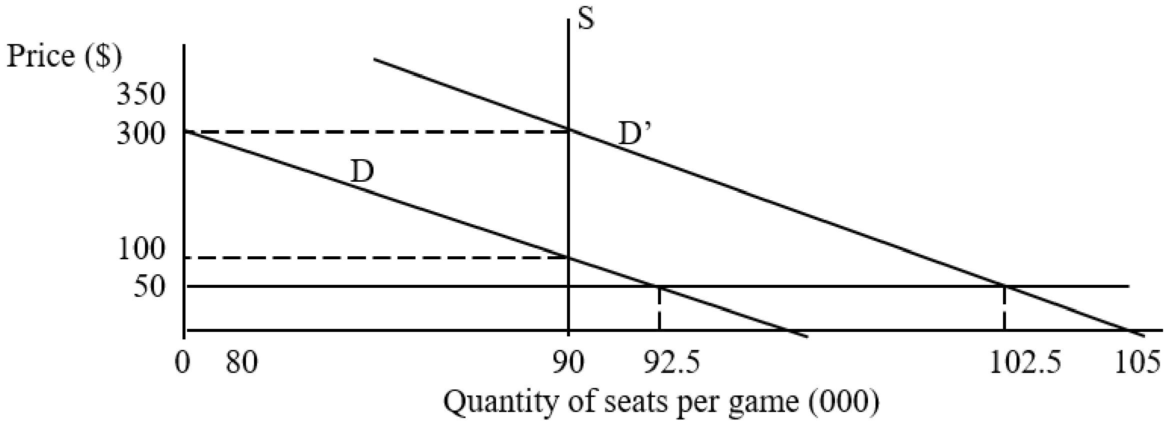
$$\begin{cases} P = 2,100 - \frac{1}{50}Q \\ P = 50 \end{cases} \quad \begin{cases} 50 = 2,100 - \frac{1}{50}Q \\ P = 50 \end{cases} \quad \begin{cases} \frac{1}{50}Q = 2,100 - 50 \\ P = 50 \end{cases}$$

$$\begin{cases} Q = (2050) \cdot 50 \\ P = 50 \end{cases} \quad \begin{cases} \bar{Q} = 102,500 \\ \bar{P} = 50 \end{cases}$$

$$\text{excess demand}^* = \bar{Q} - Q^* = 102,500 - 90,000 = 12,500$$

* Reference point for the excess demand is always the equilibrium quantity in point a).

For a graphical intuition:



Exercise 2

Suppose demand is $P = 600 - Q$ and supply is $P = Q$ in the soybean market, where Q is tons of soybeans per year. The government sets a price support at $P = \$500$/ton and purchases any excess supply at this price. In response, as a long-run adjustment, farmers switch their crops from corn to soybeans, expanding supply to $P = \frac{1}{2}Q$.

a) How does excess supply with the larger supply compare to excess supply prior to the farmers switching crops?

b) How much more does the government have to spend to buy up the excess supply?

Solution

a) The equilibrium quantity, without the price support, is $Q = 300$ tons and the equilibrium price is $P = \$300$.

$$\begin{cases} P = 600 - Q \\ P = Q \end{cases} \quad \begin{cases} P = 600 - P \\ P = Q \end{cases} \quad \begin{cases} 2P = 600 \\ P = Q \end{cases}$$

$$(Q^*, P^*) = (300; \$300)$$

With a price support of $P = \$500/\text{ton}$ and the original supply of $P = Q$, quantity supplied must be $Q = 500$ tons.

$$\begin{cases} P = 500 \\ P = Q \end{cases} \quad \begin{cases} P = 500 \\ Q^{Supplied\ 1} = 500 \end{cases}$$

Meanwhile, quantity demanded is $Q = 100$ tons, so excess supply is 400 tons.

$$\begin{cases} P = 600 - Q \\ P = 500 \end{cases} \quad \begin{cases} 500 = 600 - Q \\ P = 500 \end{cases} \quad \begin{cases} Q^{Demanded} = 100 \\ P = 500 \end{cases}$$

$$\text{excess supply} = ES_1 = Q^{Demanded} - Q^{Supplied\ 1} = 500 - 100 = 400$$

With the expanded supply of $P = \frac{1}{2}Q$, quantity supplied grows to $Q = 1,000$ tons. Quantity demanded is still Q = 100 tons, so excess supply grows to 1,000 – 100 = 900 tons.

$$\begin{cases} P = 500 \\ P = \frac{1}{2}Q \end{cases} \quad \begin{cases} P = 500 \\ Q = 2P \end{cases} \quad \begin{cases} P = 500 \\ Q^{Supplied\ 2} = 1,000 \end{cases}$$

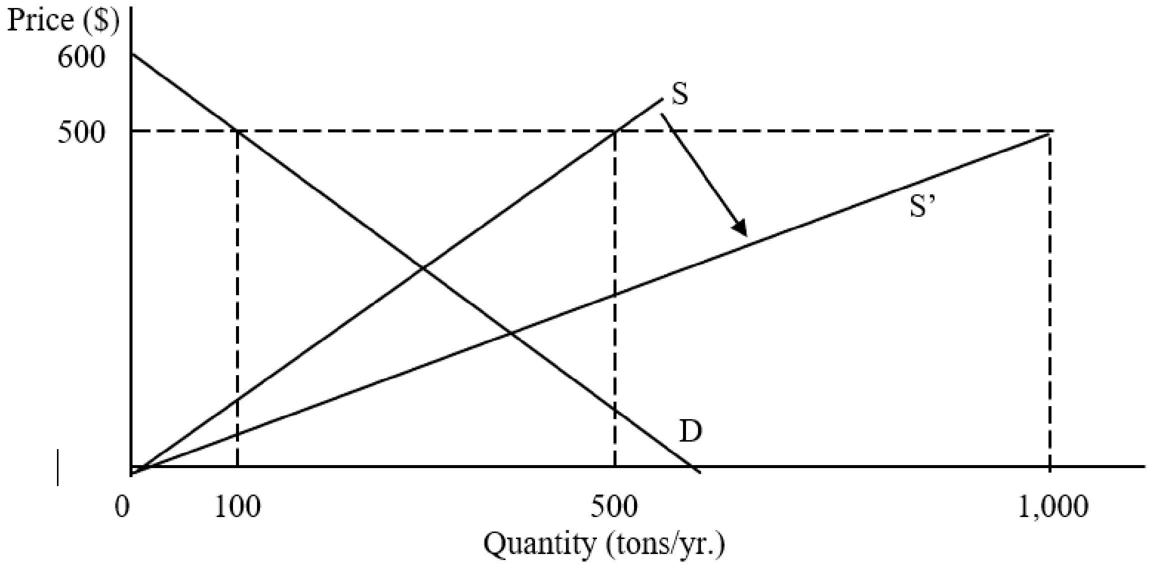
$$\text{excess supply} = ES_2 = Q^{Demanded} - Q^{Supplied\ 2} = 1,000 - 100 = 900$$

b) The extra 500 tons the government has to buy of excess supply costs the government $\$500/\text{ton}$, so the added expenditure is $\$250,000$.

$$\text{extra tons} = ES_2 - ES_1 = 900–400 = 500$$

$$\text{extra expenditure} = 500 \cdot 500 = 250,000$$

For a graphical intuition:



Practice Session 2

Microeconomics

Bachelor in Business Administration
LUISS Guido Carli

Luisa Lorè*

October 3, 2022

Exercise 1

Eric purchases food (measured by x) and clothing (measured by y) and has the utility function $U(x,y) = xy$. His marginal utilities are $MU_x = y$ and $MU_y = x$. He has a monthly income of \$800. The price of food is $P_x = \$20$, and the price of clothing is $P_y = \$40$.
Find Eric's optimal consumption bundle.

Solution

$$\begin{cases} MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{P_x}{P_y} \rightarrow \text{tangency condition (interior solution)} \\ P_x x + P_y y = I \rightarrow \text{budget constraint} \end{cases}$$

$$\begin{cases} \frac{y}{x} = \frac{20}{40} = \frac{1}{2} \rightarrow 2y = x \\ 20x + 40y = 800 \end{cases}$$

$$40y + 40y = 800 \rightarrow 80y = 800 \rightarrow y^* = 10$$

$$2y = x \rightarrow x^* = 20$$

$$\text{Optimal Consumption Bundle } (x^*, y^*) = (20, 10)$$

Exercise 2

David is considering his purchases of food x and clothing y. He has the utility function $U(x,y) = xy + 10x$, with marginal utilities $MU_x = y + 10$ and $MU_y = x$. His income is $I = 10$. He faces a price of food $P_x = \$1$ and a price of clothing $P_y = \$2$.
What is David's optimal basket?

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Solution

$$\begin{cases} MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{P_x}{P_y} \to \text{tangency condition (interior solution)} \\ P_x x + P_y y = I \to \text{budget constraint} \end{cases}$$

$$\begin{cases} \frac{y+10}{x} = \frac{1}{2} \to 2y + 20 = x \to 2y = x - 20 \\ x + 2y = 10 \end{cases}$$

$$x + x - 20 = 10 \to 2x = 30 \to x^* = 15$$

$$2y = x - 20 \to 2y = 15 - 20 \to y^* = -\frac{5}{2}$$

This result cannot be accepted (remeber $x \geq 0$, $y \geq 0$), it means it is not possible to have an interior solution, then it will be a corner solution (either $x^* = 0$ and $y^* > 0$, or $x^* > 0$ and $y^* = 0$).
Let us suppose $x^* = 0$, it follows then:

$$x + 2y = 10 \to y^* = 5 \to U(x^*, y^*) = (0)(5) + 10(0) = 0$$

Let us now suppose $y^* = 0$, it follows then:

$$x + 2y = 10 \to x^* = 10 \to U(x^*, y^*) = (10)(0) + 10(10) = 100$$

We understand that the Optimal Consumption Bundle $(x^*, y^*) = (10, 0)$

Exercise 3

Sara views chocolate and vanilla ice cream as perfect substitutes. She likes both and is always willing to trade one scoop of chocolate for two scoops of vanilla ice cream. In other words, her marginal utility for chocolate is twice as large as her marginal utility for vanilla. Thus, $MRS_{C,V} = \frac{MU_C}{MU_V} = 2$. If the price of a scoop of chocolate ice cream (P_C) is three times the price of vanilla (P_V), will Sara buy both types of ice cream? If not, which will she buy?

Solution

Sara will buy both types of ice cream in presence of an interior solution, meaning if the tangency condition is confirmed:

$$\frac{MU_C}{MU_V} = \frac{P_C}{P_V} \to 2 \neq 3 \to \text{Corner Solution}$$

Given the MUs and the ice creams prices, it is not possible for Sara to buy both types of ice cream. We need now to understand which of the two types of ice cream she is going to buy.

$$\frac{MU_C}{MU_V} = \frac{P_C}{P_V} \to \frac{MU_C}{P_C} = \frac{MU_V}{P_V}$$

$$\frac{MU_C}{MU_V} > \frac{P_C}{P_V} \to \frac{MU_C}{P_C} > \frac{MU_V}{P_V} \to \text{Optimal Consumption Bundle } (C^*, V^*) = (C, 0)$$

$$\frac{MU_C}{MU_V} < \frac{P_C}{P_V} \to \frac{MU_C}{P_C} < \frac{MU_V}{P_V} \to \text{Optimal Consumption Bundle } (C^*, V^*) = (0, V)$$

In our case Sara is going to buy only Vanilla ice cream.

Exercise 4

Consider a consumer with utily function $U(x, y) = min(3x, 5y)$ that is, the two goods are perfect complements in the ratio 3 : 5. The prices of the two goods are $P_x = \$5$ and $P_y = \$10$, and the consumer's income is \$220. Determine the optimum consumption basket.

Solution

As for the previous exercises, we have to set the maximization problem as a system of equation in which we have the tangency condition and the budget constraint:

$$\begin{cases} \text{tangency condition} \\ \text{budget constraint} \end{cases}$$

Differently from before, the tangency condition has to be formulated differently. Perfect complements imply that $3x = 5y \rightarrow y = \frac{3}{5}x$, and that is the tangency condition. While the budget line is $5x + 10y = 220$. Having these two elements we are ready to solve the otpimization problem.

$$\begin{cases} 3x = 5y \rightarrow y = \frac{3}{5}x \\ 5x + 10y = 220 \end{cases}$$

$$5x + \frac{30}{5}x = 220 \rightarrow 11x = 220 \rightarrow x^* = 20$$

$$y = \frac{3}{5}x \rightarrow y = \frac{3}{5}20 \rightarrow y^* = 12$$

$$\text{Optimal Consumption Bundle } (x^*, y^*) = (20, 12)$$

Practice Session 3

Microeconomics

Bachelor in Business Administration

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October 10, 2022

Exercise

A consumer has an income of $I = 24$ euro and her consumption is made of two good, x and y, which prices are respectively $p_x = 2$ e $p_y = 1$.
Let consider the following utility functions: $U = 2x^{\frac{1}{2}}y^{\frac{1}{2}}$

1. Find the optimal bundle (x^A, y^A).
2. Assuming and increase in the good x such that $p'_x = 4$, compute the new optimal bundle (x^B, y^B).
3. Decompose **price effect** into **income effect** and **substitution effect** with the Hicks method. And then, represent the effects graphically.

Solution

1. Let us compute the optimal bundle with the usual system:

$$\begin{cases} MRS = \frac{p_x}{p_y} \\ I = p_x x + p_y y \end{cases} \quad \begin{cases} \frac{y}{x} = 2 \\ 24 = 2x + y \end{cases} \quad \begin{cases} y = 2x \\ 24 = 2y \end{cases} \quad \begin{cases} x^A = 6 \\ y^A = 12 \end{cases}$$

$$(x^A, y^A) = (6, 12)$$

2. Let us repeat the previous exercise, with the new price for x:

$$\begin{cases} MRS = \frac{p_x}{p_y} \\ I = p_x x + p_y y \end{cases} \quad \begin{cases} \frac{y}{x} = 4 \\ 24 = 4x + y \end{cases} \quad \begin{cases} y = 4x \\ 24 = 2y \end{cases} \quad \begin{cases} x^B = 3 \\ y^B = 12 \end{cases}$$

$$(x^B, y^B) = (3, 12)$$

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3. In order to understand price effect components, let us compute a decomposition bundle with the following characteristics: it lies on the initial indifference curve (it gives the same utility level as bundle A)

$$U(x^A, y^A) = 2 \cdot \sqrt{6} \cdot \sqrt{12} = 2\sqrt{2^3 \cdot 3^2} = 12\sqrt{2}$$

and the tangency condition is satisfied with final price ratio

$$MRS = \frac{p'_x}{p_y} \rightarrow \frac{y}{x} = 4$$

So,

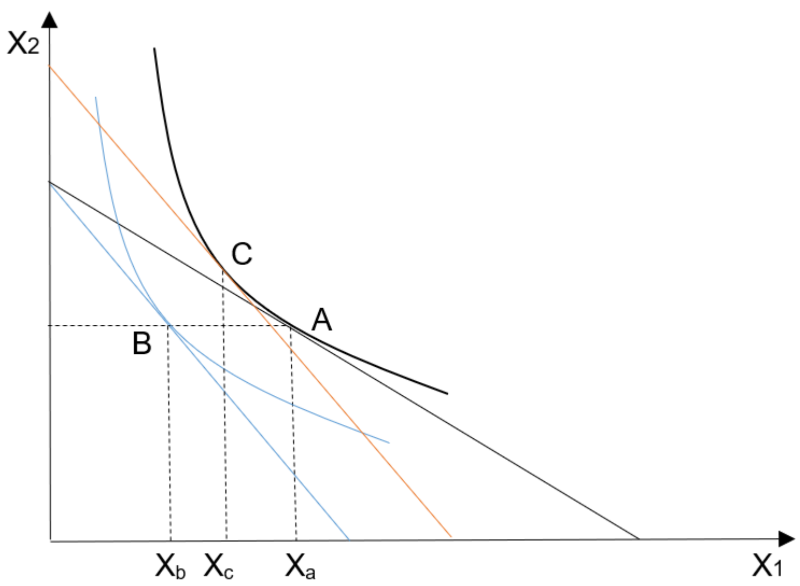
$$\begin{cases} MRS = \frac{p'_x}{p_y} \\ U(x,y) = U^A \end{cases} \quad \begin{cases} \frac{y}{x} = 4 \\ 2x^{\frac{1}{2}}y^{\frac{1}{2}} = 12\sqrt{2} \end{cases} \quad \begin{cases} y = 4x \\ 2x^{\frac{1}{2}}(4x)^{\frac{1}{2}} = 12\sqrt{2} \end{cases}$$

$$\begin{cases} y = 4x \\ 4x = 12\sqrt{2} \end{cases} \quad \begin{cases} x^C = 3\sqrt{2} \\ y^C = 12\sqrt{2} \end{cases}$$

$$(x^C, y^C) = (3\sqrt{2}, 12\sqrt{2})$$

Now we can compute the price effect relative to the change in p_x:
Price effect: $x^B - x^A = -3$
Substitution effect: $x^C - x^A = 3\sqrt{2} - 6$
Income effect: $x^B - x^C = 3 - 3\sqrt{2}$

The graphical representation is in the following figure:



Practice Session 4

Microeconomics

Bachelor in Business Administration

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Luisa Lorè*

October 24, 2022

Exercise 1

Let us consider the following production function: $q = L^{1/2}K^{1/2}$. Labour unitary cost is $w = 90$, while capital unitary cost is $r = 10$.

1. Find the demand functions of inputs L and K and the equation of the expansion path.
2. Find the capital-labor combination that minimizes the cost the firm faces to produce 9 units of output.
3. Assume that the firm wants to increase its production to 15 units of output. In the short run (capital is constant), how many units of labour will the firm hire?
4. Find the cost minimizing capital-labor combination to produce 15 units of output.
5. Write the long run total cost, average cost, and marginal cost function.
6. Represent on the same graph the equilibria identified in points (2), (3) and (4) and the path of expansion.
7. Indicate whether the given production function exhibits constant, increasing or decreasing returns to scale.

Solution

1. In order to find the demand functions for the inputs of production, we can solve the following system:

$$\begin{cases} MRTS_{L,K} = \frac{MP_L}{MP_K} = \frac{w}{r} \rightarrow \text{tangency condition} \\ q = f(L,K) \rightarrow \text{production function} \end{cases}$$

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$$\begin{cases} \frac{K}{L} = \frac{w}{r} \\ q = L^{1/2}K^{1/2} \end{cases} \quad \begin{cases} K = \frac{w}{r}L \\ q = L^{1/2}(\frac{w}{r}L)^{1/2} \end{cases} \quad \begin{cases} K = \frac{w}{r}L \\ q = L\sqrt{\frac{w}{r}} \end{cases} \quad \begin{cases} K = \frac{w}{r}(q\sqrt{\frac{r}{w}}) \\ L = q\sqrt{\frac{r}{w}} \end{cases}$$

$$\begin{cases} K = q\sqrt{\frac{w}{r}} \\ L = q\sqrt{\frac{r}{w}} \end{cases}$$

While for the equation of the expansion path, we can substitute into the demand for K, the output q, such that:

$$\begin{cases} K = q\sqrt{\frac{w}{r}} \\ L = q\sqrt{\frac{r}{w}} \rightarrow q = L\sqrt{\frac{w}{r}} \end{cases} \rightarrow K = L\sqrt{\frac{w}{r}}\sqrt{\frac{w}{r}}$$

$$K = \frac{w}{r}L$$

2. In order to solve this point, we can either use the system built in point (1):

$$\begin{cases} MRTS_{L,K} = \frac{MP_L}{MP_K} = \frac{w}{r} \rightarrow \text{tangency condition} \\ q = f(L,K) \rightarrow \text{production function} \rightarrow \bar{q} = f(L,K) \rightarrow \text{isoquant} \end{cases}$$

$$\begin{cases} \frac{K}{L} = \frac{90}{10} \\ 9 = L^{1/2}K^{1/2} \end{cases}$$

Or we can take a shortcut and use the input demand functions computed in point (1), in the following way:

$$\begin{cases} K = q\sqrt{\frac{w}{r}} \\ L = q\sqrt{\frac{r}{w}} \end{cases} \quad \begin{cases} K = 9\sqrt{\frac{90}{10}} = 9 \cdot 3 = 27 \\ L = 9\sqrt{\frac{10}{9}} = 9 \cdot \frac{1}{3} = 3 \end{cases} \quad \begin{cases} K' = 27 \\ L' = 3 \end{cases}$$

3. Let us keep constant the capital found in point (2), $\bar{K} = 27$, and let us solve the new isoquant equation:

$$\bar{q} = L^{1/2}\bar{K}^{1/2}$$

$$15 = L^{1/2}\sqrt{27} \rightarrow L^{1/2} = \frac{15}{\sqrt{27}} \rightarrow L = \frac{225}{27}$$

$$L'' = \frac{225}{27} = 8.\bar{3}$$

4. Let us now compute, as we did in point (2) the cost minimising capital-labour combination:

$$\begin{cases} K = q\sqrt{\frac{w}{r}} \\ L = q\sqrt{\frac{r}{w}} \end{cases} \quad \begin{cases} K = 15\sqrt{\frac{90}{10}} = 15 \cdot 3 = 45 \\ L = 15\sqrt{\frac{10}{9}} = 15 \cdot \frac{1}{3} = 5 \end{cases} \quad \begin{cases} K''' = 45 \\ L''' = 5 \end{cases}$$

5. In order to find the long run total cost function we have to use the values of w, r and the input demand functions found in point (1):

$$TC(q) = w \cdot L + r \cdot K = w \cdot q\sqrt{\frac{r}{w}} + r \cdot q\sqrt{\frac{w}{r}}$$

$$TC(q) = q\left(w \cdot \sqrt{\frac{r}{w}} + r \cdot \sqrt{\frac{w}{r}}\right) = q\left(90 \cdot \sqrt{\frac{10}{90}} + 10 \cdot \sqrt{\frac{90}{10}}\right) = q\left(\frac{90}{3} + 10 \cdot 3\right)$$

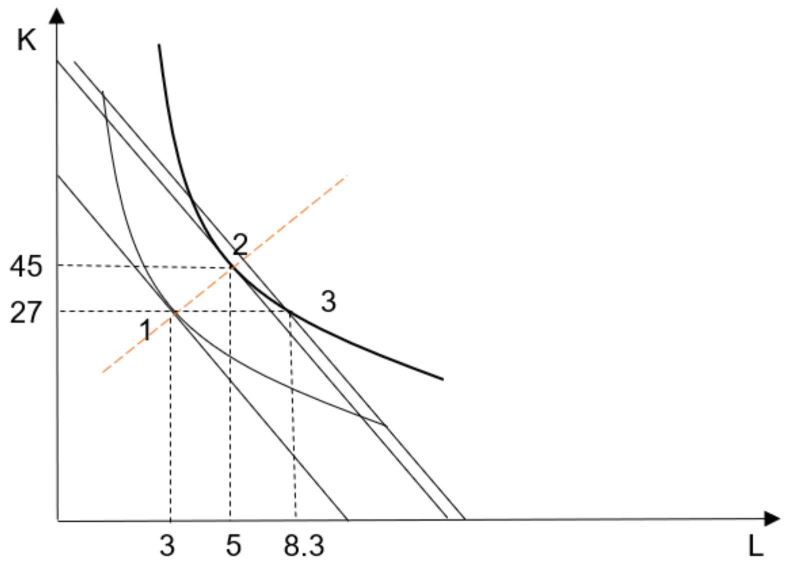
$$TC(q) = 60q$$

To get the average cost and the marginal cost functions, we can simply use their definition:

$$AC(q) = \frac{TC(q)}{q} = \frac{60q}{q} = 60$$

$$MC(q) = \frac{\partial TC(q)}{\partial q} = 60$$

6. We can observe on the graph, that the cost minimising capital-labour combinations are on the expansion path, while the capital-labour combination in the short run deviates from it.



7. We have to verify the relation between $q(\lambda L, \lambda K)$ and $\lambda q(L, K)$:

- $q(\lambda L, \lambda K) > \lambda q(L, K)$, the function exhibits an **increasing** return to scale,
- $q(\lambda L, \lambda K) = \lambda q(L, K)$, the function exhibits a **constant** return to scale,
- $q(\lambda L, \lambda K) < \lambda q(L, K)$, the function exhibits a **decreasing** return to scale.

In this case:

$$q(\lambda L, \lambda K) = (\lambda L)^{\frac{1}{2}} (\lambda K)^{\frac{1}{2}} = \lambda L^{\frac{1}{2}} K^{\frac{1}{2}}$$
$$\lambda q(L, K) = \lambda L^{\frac{1}{2}} K^{\frac{1}{2}}$$
$$q(\lambda L, \lambda K) = \lambda q(L, K)$$

then, the function exhibits a constant return to scale.

Practice Session 5

Microeconomics
Bachelor in Business Administration
LUISS Guido Carli

Luisa Lorè*

October 31, 2022

Exercise 1

Suppose that a firm has a short-run total cost curve given by $STC = 100 + 20Q + Q^2$, where the total fixed cost is 100 and the total variable cost is $20Q+Q^2$. The corresponding short-run marginal cost curve is $SMC = 20 + 2Q$. All of the fixed cost is sunk.

a) What is the equation for average variable cost (AVC)?

b) What is the minimum level of average variable cost?

c) What is the firm's short-run supply curve?

Solution

a) Average variable cost is total variable cost divided by output. Thus,

$$AVC(Q) = \frac{SVC(Q)}{Q} = \frac{20Q + Q^2}{Q} = 20 + Q$$

b) We know that the minimum level of average variable cost occurs at the point at which AVC and SMC are equal in this case,

$$AVC(Q) = SMC(Q) \rightarrow 20 + Q = 20 + 2Q \rightarrow Q_{min} = 0$$

$$AVC(Q_{min}) = 20$$

c) For prices below 20 (the minimum level of average variable cost), the firm will not produce. For prices above 20, we can find the supply curve by equating price to marginal cost and solving for Q:

$$P = SMC(Q) \rightarrow P = 20 + 2Q \rightarrow Q = \frac{1}{2}P - 10$$

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The firm's short-run supply curve, which we denote by $s(P)$, is thus:

$$s(P) = \begin{cases} 0 & P < 20 \\ \frac{1}{2}P - 10 & P \geq 20 \end{cases}$$

Exercise 2

A market consists of 300 identical firms, and the market demand curve is given by $D(P) = 60 - P$. Each firm has a short-run total cost curve $STC = 0.1 + 150Q^2$, and all fixed costs are sunk. The corresponding short-run marginal cost curve is $SMS = 300Q$, and the corresponding average variable cost curve is $AVC = 150Q$. The minimum level of AVC is 0; thus, a firm will continue to produce as long as price is positive. (You can verify this by sketching the SMC and AVC curves.)
What is the short-run equilibrium price in this market?

Solution

Each firm's profit-maximizing quantity is given by equating marginal cost and price:

$$P = SMC(Q) \rightarrow P = 300Q \rightarrow Q = \frac{P}{300}$$

Thus the supply curve $s(P)$ of an individual firm is

$$s(P) = \frac{P}{300}$$

Since the firms in this market are all identical, short-run market supply $S(P)$ equals

$$S(P) = ns(P) = 300\frac{P}{300} = P$$

The short-run equilibrium occurs where market supply equals market demand, thus

$$S(P) = D(P) \rightarrow P = 60 - P \rightarrow 2P = 60 \rightarrow P = 30$$

Exercise 3

In a market, all firms and potential entrants are identical. Each has a long-run average cost curve $AC(Q) = 40 - Q + 0.01Q^2$ and a corresponding long-run marginal cost curve $MC(Q) = 40 - 2Q + 0.03Q^2$, where Q is thousands of units per year. The market demand curve is $D(P) = 25,000 - 1,000P$ where $D(P)$ is also measured in thousands of units. Find the long-run equilibrium quantity per firm, price, and number of firms.

Solution

The long-run competitive equilibrium satisfies the following three equations:

$$\begin{cases} P = MC(Q) \rightarrow \text{ profit maximization} \\ P = AC(Q) \rightarrow \text{ zero profit} \\ S(P) = D(P) \rightarrow \text{ supply equals demand (market clearing)} \end{cases}$$

$$\begin{cases} P = 40 - 2Q + 0.03Q^2 \\ P = 40 - Q + 0.01Q^2 \\ nQ = 25,000 - 1,000P \to n = \frac{25,000-1,000P}{Q} \end{cases}$$

$$40 - 2Q + 0.03Q^2 = 40 - Q + 0.01Q^2$$

$$(0.03 - 0.01)Q^2 = (2 - 1)Q$$

$$Q^* = \frac{1}{0.02} = 50$$

$$P = 40 - 50 + 0.01(2500) = 40 - 50 + 25 \to P^* = 15$$

$$n = \frac{25,000 - 1,000(15)}{50} = \frac{25,000 - 15,000}{50} = \frac{10,000}{50} \to n^* = 200$$

Practice Session 6

Microeconomics
Bachelor in Business Administration
LUISS Guido Carli

October 31, 2022

Exercise

The market for a given good operates under perfect competition. The total cost function of a single firm is given by: $C(q) = 5q^3 - 10q^2 + 30q$ where q indicates the quantity produced.

1. Calculate the quantity produced by the firm in the short-run equilibrium if the price is $p = 50$ and determine the level of profits. Provide a graphical representation.

2. Calculate the quantity produced by the firm in the long-run equilibrium.

3. Assuming the market demand function is: $Q = 100 - p$ (where p is the price and Q the quantity), calculate the total quantity produced in the market and the equilibrium price in the long-run.

4. How many firms are operating in this market in the long-run?

Solution

1. Imposing $p = MC(q)$ we get $q_i = 2$.

2. Given that the $MC(q)$ curve intersects the $AVC(q)$ curve in its minumum, the solution is found imposing $MC(q) = AVC(q)$. Doing so we get $q = 1$.

3. We substitute $q = 1$ in $AVC(q)$, and we obtain $p = 25$. We plug this value in the demand function getting $Q = 100 - 25 = 75$.

4. $n = \frac{Q}{q} = 75$

Practice Session 7

Microeconomics
Bachelor in Business Administration
LUISS Guido Carli

Luisa Lorè*

November 7, 2022

Exercise 1

The market demand curve for a monopolist is given by $P = 40 - 2Q$.

a) What is the marginal revenue function for the firm?

b) What is the maximum possible revenue that the firm can earn?

Solution

a) The MR curve is the first derivative of the total revenues TR:

$$TR(Q) = P(Q)Q = (40 - 2Q)Q = 40Q - 2Q^2$$

$$MR(Q) = \frac{\partial TR(Q)}{\partial Q} = 40 - 4Q$$

b) Total revenue will be maximized when $MR = 0$:

$$40 - 4Q = 0 \rightarrow Q = \frac{40}{4} \rightarrow Q = 0$$

At that quantity, the price will be:

$$P = 40 - 2(10) \rightarrow P = 20$$

While total revenue is

$$TR(Q) = PQ = 20 \cdot 10 = 200$$

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Exercise 2

A monopolist operates in an industry where the demand curve is given by $Q = 1000 - 20P$. The monopolist's constant marginal cost is \$8. What is the monopolist's profit-maximizing price?

Solution

Let rewrite the demand curve in the inverse form and let derive the TR from it:

$$Q = 1000 - 20P \rightarrow P = 50 - \frac{Q}{20}$$

$$TR = \left(50 - \frac{Q}{20}\right) Q$$

Make the first derivative of TR:

$$MR = \frac{\partial TR(Q)}{\partial Q} = 50 - \frac{Q}{10}$$

Let use the rule $MR = MC$

$$50 - \frac{Q}{10} = 8 \rightarrow Q = 420$$

$$P = 50 - \frac{420}{20} \rightarrow P = 29$$

Exercise 3

Let a monopolist face a linear demand, $P_0(Q) = 100 - 2Q$ and marginal cost $MC(Q) = 20$. Suppose that now demand shifts to $P_1(Q) = 120 - 2Q$. How does the equilibrium price change?

Solution

For inverse demand curve $P_0(Q)$, we have

$$TR_0(Q) = (100 - 2Q)Q = 100Q - 2Q^2$$

$$MR_0(Q) = 100 - 4Q$$

Setting this equal to marginal cost, we have

$$100 - 4Q = 20 \rightarrow Q_0 = 20$$

$$P = 100 - 2(20) \rightarrow P_0 = 60$$

For inverse demand curve $P_1(Q)$, we have

$$TR_1(Q) = (120 - 2Q)Q = 120Q - 2Q^2$$

$$MR_1(Q) = 120 - 4Q$$

Setting this equal to marginal cost, we have

$$120 - 4Q = 20 \rightarrow Q_1 = 25$$

$$P = 100 - 2(25) \rightarrow P_1 = 70$$

Therefore, $P_1 > P_0$ and so the equilibrium price has risen.

Practice Session 8

Microeconomics
Bachelor in Business Administration
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Luisa Lorè*

November 14, 2022

Exercise 1

A homogeneous products duopoly faces a market demand function given by $P = 300 - 3Q$, where $Q = Q_l + Q_2$. Both firms have a constant marginal cost $MC = 100$.

a) Derirve the equation of each firm's reaction curve and then graph these curves.

b) What is the Cournot equilibrum, quantity per firm and price in this market?

c) What would the equilibrium price in this market be if it were perfectly competitive?

d) What would the equilillrium price in this market be if the two firms colluded to set the monopoly price?

e) What is the Bertrand equilibrium price is this market?

Solution

a) For Firm 1, we know Demand is given by:

$$P = (300 - 3Q_2) - 3Q_1$$

Setting $MR = MC$ implies

$$(300 - 3Q_2) - 6Q_1 = 100 \rightarrow 6Q_1 = 200 - 3Q_2$$

$$Q_1^{BR} = 33.33 - \frac{1}{2}Q_2$$

Since the marginal costs are the same for both firms, symmetry implies:

$$Q_2^{BR} = 33.33 - \frac{1}{2}Q_1$$

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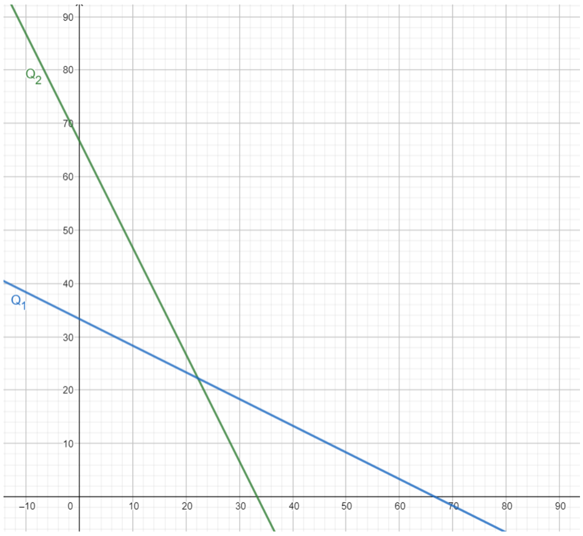

b) Because of symmetry, in equilibrium both firms will choose the same level of output. Thus we can set $Q_1 = Q_2$ and solve

$$Q_1 = 33.33 - \frac{1}{2}Q_1 \rightarrow \frac{3}{2}Q_1 = 33.33\frac{2}{3}$$

$$Q_1^C = 22.22$$
$$Q_2^C = 22.22$$

Since both firms will choose the same level of output, both firms will produce 22.22 units. Price can be found by substituting the quantity for each firm into market demand. This implies price will be

$$P = 300 - 3(44.44) \rightarrow P^C = 166.67$$

c) If this market were perfectly competitive, then equilibrium would occur at the point where

$$P = MC \rightarrow P^{PC} = 100$$

d) If the firms colluded to set the monopoly price, then

$$300 - 6Q = 100$$

$$Q = \frac{200}{6} = 33.33$$

$$P = 300 - 3(\frac{200}{6}) \rightarrow P^{Collusion} = 200$$

e) If the firms acted as Bertrand oligopolists, the equilibrium would coincide with the perfectly competitive equilibrium of

$$P^{PC} = P^B \rightarrow P^B = 100$$