Tor Vergata University of Rome - Economics School
Bachelor of Arts in Global Governance

Microeconomics
Practice Sessions

Luisa Lorè

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Microeconomics
Practice Session 1 - Introduction & Consumption Theory (1)

Luisa Lorè*

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How Practice Sessions are going to be organized

All the practice sessions that are going to do together, will be followed by an handout like this. In which, there will be reported all the exercises done in class, and even some extra exercises to make some practice by yourself, or with your mates.
Before to start this excercise session, here you are some information about how to read the practice session handouts. For every excercise that we are going to do in class, you are going to see the following structure:

Exercise 1 - Cost functions

TOPICS TO REVIEW

- **Total Cost Function:** ...
- **Average Cost Function:** ...
- **Marginal Cost Function:** ...

Solution

So you are going to have the number of the exercice done in class, the topic on which that exercise is about, and the topics to review in order to do the exercice without any knowledge gap. All the solutions of the excercises done in class, will be presented with a step-by-step approach.

*luisa.lore@alumni.uniroma2.eu

Exercise 1 - Cost functions

TOPICS TO REVIEW

- **Total Cost Function:** denoted by $C(Q)$, shows how total cost varies with output, holding input prices fixed and selecting inputs to minimize cost.
- **Average Cost Function:** denoted by $AvC(Q)$, indicates firm's total cost per unit of output. It equals total cost divided by total quantity.
- **Marginal Cost Function:** denoted by $MrglC(Q)$, is the rate at which total cost changes with respect to change in output.

Given the following table:

Output	**Labour(L)**	**Capital(K)**
0	0	0
1	2	3
2	4	6
3	6	9
4	8	12
5	10	15

A fixed cost equal to 20, a unitary labour cost equal (C_L) to 3 and a unitary capital cost equal (C_K) to 4. Compute for every level of output:

1. the total cost beared by the producer
2. the average cost function value
3. the marginal cost function value

Solution

1. In order to compute the total cost beared by the producer, we can build the total cost function in this way:

   $$C(Q) = \text{Fixed Cost} + \text{Variable Cost} \cdot Q$$

   $$\text{Var Cost} = [(C_L)(\text{\# of L unit for each } Q) + (C_K)(\text{\# of K unit for each } Q)]$$
   $$C(Q) = 20 + [(3)(2) + (4)(3)](Q)$$
   $$\mathbf{C(Q) = 20 + 18(Q)}$$

   The solution for each level of output is in the next table first column.

2. For the average cost function value, we can build the average cost function in this way:

   $$\mathbf{AvC(Q) = \frac{C(Q)}{Q} = \frac{20 + 18(Q)}{Q}}$$

   The solution for each level of output is in the next table second column.

3. For the marginal cost function value, we can build the marginal cost function in this way:

$$\mathbf{MrglC(Q)} = \frac{\partial \mathbf{CT(Q)}}{\partial \mathbf{Q}} = \mathbf{18}$$

The solution for each level of output is in the next table third column.

Output	**TC**	**AvC**	**MrglC**
0	20	-	-
1	38	38	18
2	56	28	18
3	74	24,6	18
4	92	23	18
5	110	22	18

Exercise 2 - Price Elasticity of Demand

TOPICS TO REVIEW

- **Demand function:** the demand function is a relationship which associates different levels of price with different levels of demanded quantity. Namely, the direct demand function shows the quantity as a function of the price:

$$Q = f(P)$$

- **Price elasticity of demand:** the price elasticity of demand shows the percentage variation of demand with reference to a 1% variation of price. It is calculated in accordance with the following formula:

$$\varepsilon = \frac{\partial Q}{\partial P}\frac{P}{Q}$$

Where $\frac{\partial Q}{\partial P}$ is the derivative of the direct demand function.

With reference to these combinations of Price and Quantity, $A : (2, 3)$ and $B : (4, 2)$, determine:

1. The direct demand function

2. The inverse demand function

3. The price elasticity of demand with reference to both points

Solution

1. Given that the demand function is a straight line, the direct demand function is assumed to have such an equation:

$$Q = a + bP$$

Where a is the intercept and b is the slope. Since the demand curve usually slopes down, one should expect b to be a negative value. Having said that, we have to determine the slope and the intercept.
In order to determine the slope of our line we can use the general formula related to the determination of the slope of a straight line which joins two points:

$$b = \frac{\Delta Q}{\Delta P} = \frac{Q_2 - Q_1}{P_2 - P_1} = \frac{2-3}{4-2} = -\frac{1}{2}$$

In order to determine the intercept of the direct demand, we need to consider the general equation of a linear direct demand, the previously calculated value of the slope and the coordinates of one of the two points (in our case A):

$$Q = -\frac{1}{2}P + a \rightarrow 3 = -\frac{1}{2}2 + a \rightarrow a = 4$$

Consequently, the direct demand function is:

$$Q = -\frac{1}{2}P + 4$$

2. In order determine the inverse demand function, we simply can invert the direct demand function, by identifying an equation which shows P as a function of Q. Namely:

$$Q = -\frac{1}{2}P + 4 \rightarrow P = -2Q + 8$$

3. In order to determine ε_A and ε_B, it is needed to apply the formula of the elasticity:

$$\varepsilon_A = \frac{\partial Q}{\partial P}\frac{P}{Q} = -\frac{1}{2}\frac{2}{3} = -\frac{1}{3}$$

$$\varepsilon_B = \frac{\partial Q}{\partial P}\frac{P}{Q} = -\frac{1}{2}\frac{4}{2} = -1$$

Exercise 3 - Indifference Curve

TOPICS TO REVIEW

- **Indifference Curve:** an indifference curve is a collection of points on the (x_1, x_2) diagram which represent bundles of good 1 and good 2 that are associated with the same level of utility by the utility function. Mathematically, it is a level curve of the utility function.

Given the following utility function, $U = x_1^{\frac{1}{4}} x_2^{\frac{1}{2}}$, determine the equation of the indifference curve related to a utility level $\bar{U} = 3$.

Solution

Considering the equation of the utility function and the level of utility, it can be found that:

$$x_1^{\frac{1}{4}} x_2^{\frac{1}{2}} = 3 \rightarrow x_2^{\frac{1}{2}} = \frac{3}{x_1^{\frac{1}{4}}} \rightarrow x_2 = \frac{9}{x_1^{\frac{1}{2}}}$$

Extra Exercises

EXERCISE 2
With reference to these combinations of P and Q, determine: The direct demand function, the inverse demand function and the price elasticity of demand.

1. $A : (1, 7)$ and $B : (3, 1)$
2. $A : (10, 15)$ and $B : (4, 18)$
3. $A : (1, 5)$ and $B : (3, 3)$

EXERCISE 3
With reference to these utility functions and these levels of utility, determine the equations of the indifference curves.

1. $U = x_1^2 x_2^4$ with a $\bar{U} = 5$
2. $U = x_1^{\frac{1}{4}} x_2$ with a $\bar{U} = 3$
3. $U = x_1^{\frac{1}{2}} x_2^{\frac{1}{4}}$ with a $\bar{U} = 2$

Solutions

EXERCISE 2

1. $Q = 10 - 3p$
   $P = \frac{10}{3} - \frac{Q}{3}$
   $\varepsilon_A = \frac{3}{7}$
   $\varepsilon_B = -9$
2. $Q = 20 - \frac{1}{2}P$
   $P = 40 - 2Q$
   $\varepsilon_A = -\frac{1}{3}$
   $\varepsilon_B = -\frac{1}{9}$
3. $Q = 6 - P$
   $P = 6 - Q$
   $\varepsilon_A = -\frac{1}{5}$
   $\varepsilon_B = -1$

EXERCISE 3

1. $x_2 = \frac{5^{\frac{1}{4}}}{x_1^{\frac{1}{2}}}$
2. $x_2 = \frac{3}{x_1^{\frac{1}{4}}}$
3. $x_2 = \frac{16}{x_1^2}$

Microeconomics
Practice Session 2 - Consumption Theory (2)

Luisa Lorè*

24/03/2020

Exercise 1 - Utility function analysis

TOPICS TO REVIEW

- **Preferences**
- **Utility**
- **Marginal Utility**
- **Marginal Rate of Substitution**

An individual's preferences are represented by the following utility function:

$$U(x_1, x_2) = 3x_1^{\frac{1}{2}}x_2^{\frac{1}{2}} = 3\sqrt{x_1}\sqrt{x_2}$$

1. Given this function, verify the utility of the bundles corresponding to points $A = (9, 4)$ and $B = (4, 1)$. What can you say about the value of use related to the two bundles?
2. Which of the following functions can represent the same preferences expressed by our utility function:
   (a) $U(x_1, x_2) = 2x_1^{\frac{1}{2}}x_2^{\frac{1}{2}}$
   (b) $U(x_1, x_2) = 6x_1^{\frac{2}{3}}x_2^{\frac{5}{2}}$
   (c) $U(x_1, x_2) = 18x_1^{\frac{1}{2}}x_2^{\frac{1}{2}}$
   (d) $U(x_1, x_2) = 3x_1^{\frac{3}{2}}x_2^{\frac{1}{2}}$
3. Verify if the indifference curves associated to the utility function are increasing or decreasing.
4. Compute the indifference curves slope, which are the relevant considerations that we can make about the absolute value and the sign of the slope?
5. Compute the marginal utility function for the two goods.
6. Compute the MRS.

*luisa.lore@alumni.uniroma2.eu

Solution

1. In order to verify the utility of the bundles corresponding to points $A = (9, 4)$ and $B = (4, 1)$ we have to plug in the values of the two points in the utility function:

$$U_A = 3 \cdot \sqrt{9} \cdot \sqrt{4} = 3 \cdot 3 \cdot 2 = 18$$

$$U_B = 3 \cdot \sqrt{4} \cdot \sqrt{1} = 3 \cdot 2 \cdot 1 = 6$$

   What we can say it that the satisfaction the consumer derives from bundle A is higher than the one derived by bundle B. On the other hand, we cannot assess the entity of the difference between the satisfaction derived from bundle A and B, for example we cannot say that the utility of the first bundle is three times the utility of the second, this because utility is not measure in ordinal terms.

2. Correct answers are (a) and (c), because they are the only ones which are linear transformations of the utility function we are dealing with.

3. In order to derive the indifference curves it is necessary to fix a level of utility $\bar{U}$, and explicit the function in terms of x_2:

$$\bar{U} = 3x_1^{\frac{1}{2}}x_2^{\frac{1}{2}} \longrightarrow x_2^{\frac{1}{2}} = \frac{\bar{U}}{3x_1^{\frac{1}{2}}} \longrightarrow x_2 = \frac{\bar{U}^2}{9x_1}$$

   We can now observe that as good x_2 increases, then good x_1 decreases. We can then conclude that both goods presented can defined as "goods" (and not "bads"), and that consequently indifference curves have a negative slope.

4. In order to derive the slope of the indifference curves, we have to derive x_2 for x_1 in this way:

$$\frac{\partial x_2}{\partial x_1} = -\frac{\bar{U}^2}{9x_1^2} \longrightarrow \left|\frac{\partial x_2}{\partial x_1}\right| = \frac{\bar{U}^2}{9x_1^2}$$

   There are two things which we can notice: the slope of the indifference curve is negative, and this confirm our reasoning in the previous part of the exercise, and that as good x_1 increases, the absolute value of the slope tends to decrease and that the indifference curves tend to be flatter.

5. In order to compute the marginal utility function for a good i, MU_i it is necessary to derive the utility function for that good:

$$MU_1 = \frac{\partial U(x_1,\ x_2)}{\partial x_1} = 3\frac{1}{2}x_1^{-\frac{1}{2}}x_2^{\frac{1}{2}} = \frac{3}{2}\left(\frac{x_2}{x_1}\right)^{\frac{1}{2}}$$

$$MU_2 = \frac{\partial U(x_1,\ x_2)}{\partial x_2} = 3\frac{1}{2}x_1^{\frac{1}{2}}x_2^{-\frac{1}{2}} = \frac{3}{2}\left(\frac{x_1}{x_2}\right)^{\frac{1}{2}}$$

6. In order to compute the MRS, we compute the ratio between the margial utilities of the two goods:

$$MRS = \frac{MU_1}{MU_2} = \frac{3}{2}\left(\frac{x_2}{x_1}\right)^{\frac{1}{2}}\frac{2}{3}\left(\frac{x_2}{x_1}\right)^{\frac{1}{2}} = \frac{x_2}{x_1}$$

The MRS is the slope of the indifference curve of our consumer, indeed if we substitute the utility function inside the formula of the slope found in point 4, we obtaint the MRS:

$$\left|\frac{\partial x_2}{\partial x_1}\right| = \frac{\bar{U}^2}{9x_1^2} = \frac{(3x_1^{\frac{1}{2}}x_2^{\frac{1}{2}})^2}{9x_1^2} = \frac{9x_1x_2}{9x_1^2} = \frac{x_2}{x_1} = MRS$$

Utility Maximization Problem

The Utility Maximization Problem, will be the fundamental problem of the Consumption Theory. In order to solve it we have to set some preliminary concepts.

Budget Constraint

The budget constraint expresses the consumption possibilities of the individual we are studing, it shows all the possible bundles (combination of goods) which are affordable for the consumer. In order to be easily graph on a cartesian plan, it always as to be expressed in the form $x_2 = f(x_1)$; but at the same time it is very important to know, and start our analysis from the form:

$$I = p_1x_1 + p_2x_2$$

And then re-write it as:

$$I = p_1x_1 + p_2x_2 \to p_2x_2 = I - p_1x_1$$

$$x_2 = \frac{I}{p_2} - \frac{p_1}{p_2}x_1$$

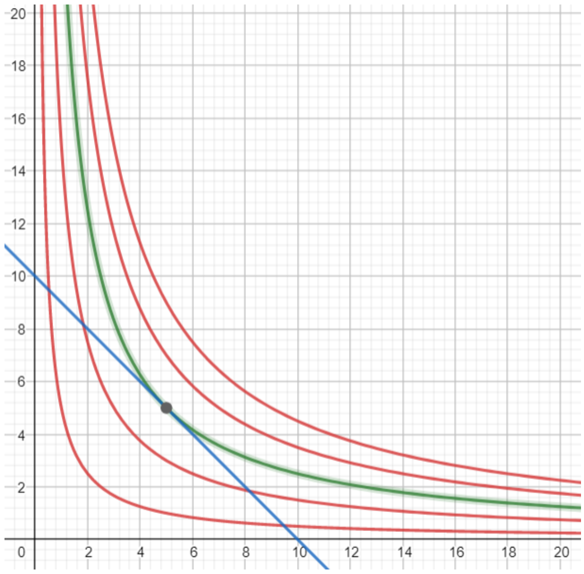
This is because the first formula helps us to understand better the concept of budget constraint and its functioning. Let us now move to understand which are the slope of the line and the intercepts with the axes.

- **Slope:** from the form $x_2 = f(x_1)$, it is to understand that the slope of the budget line is the coefficient for which x_1 is multiplied (remember $y = mx + q$), then $-\frac{p_1}{p_2}$. Then the slope of the budget line is given by the relative prices, the ratio of the prices.

- **Intercepts:** we know, from the form $x_2 = f(x_1)$ the intercept with x_2 axis, is the value that the line assumes when $x_1 = 0$ is $x_2 = \frac{I}{p_2}$. Which economic intuition can this suggests us? The intercepts with the axes represent how many units of a good the consumer is able to buy, if she does not buy any unit of the other good, then if she decides to allocate all her resources (her entire income) on one single good. It seems clear that the intecepts will be always: $x_2 = \frac{I}{p_2}$ and $x_1 = \frac{I}{p_1}$.

Optimal Bundle

The point of optimal consumption, the optimal bundle, is the point in which the consumer reaches the highest possible utility given her budget constraint, she is as happy as she could be with respect to her economic resources. In order to find the optimal bundle, it is important to get a look to the graphical representation of the problem:



The point of optimal consumption is the tangency point between the budget constraint and the indifference curve corresponding to the highest level of utility which still passes through affordable bundles, only one bundle is still affordable and it is one of the bundles for which the consumer spend her entire income. In order to find this bundle mathematically, we have to solve the following problem:

$$\begin{cases} MRS = \frac{p_1}{p_2} \rightarrow \text{Tangency condition (interior solution)} \\ I = p_1 x_1 + p_2 x_2 \rightarrow \text{Budget Constraint} \end{cases}$$

The first equation expresses the tangency condition between all the indifference curves and all the budget constraints with that ratio of prices, while the second equation anchor the tangency condition to the specific situation of the consumer we are analysing with the budget constraint.

Extra Exercises

EXERCISE 1

Repeat all points of Exercise 1, with the following data (data for points from 3 to 6 are not present, because you don't need any additional information to repeat them):

1. $U(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$
   1. $A : (4, 1)$ e $B : (1, 1)$
   2. (a) $U(x_1, x_2) = 3x_1^{\frac{4}{2}} x_2^{\frac{6}{2}}$
      (b) $U(x_1, x_2) = x_1 x_2$
      (c) $U(x_1, x_2) = 6x_1^{\frac{1}{4}} x_2^{\frac{3}{4}}$
      (d) $U(x_1, x_2) = \frac{1}{2}[ln(x_1) + ln(x_2)]$
2. $U(x_1, x_2) = ln(x_1) + ln(x_2)$
   1. $A : (3, 4)$ e $B : (8, 6)$
   2. (a) $U(x_1, x_2) = x_1 + x_2$
      (b) $U(x_1, x_2) = x_1 x_2$
      (c) $U(x_1, x_2) = ln(x_1 x_2)$
      (d) $U(x_1, x_2) = 6x_1^{\frac{1}{4}} x_2^{\frac{3}{4}}$

Solutions

EXERCISE 1

1. $U(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$
   1. $U_A = 2$ and $U_B = 1$
   2. (b) and (d)
   3. $x_2 = \frac{\bar{U}^2}{x_1}$
   4. $\frac{\partial x_2}{\partial x_1} = -\frac{\bar{U}^2}{x_1^2}$
   5. $MU_1 = \frac{1}{2}\left(\frac{x_2}{x_1}\right)^{\frac{1}{2}}$ and $MU_2 = \frac{1}{2}\left(\frac{x_1}{x_2}\right)^{\frac{1}{2}}$
   6. $MRS = \frac{x_2}{x_1}$
2. $U(x_1, x_2) = ln(x_1) + ln(x_2)$
   1. $U_A = ln(3) + ln(4) = 2.48$ and $U_B = ln(8) + ln(6) = 3.87$
   2. (b) and (c)
   3. $x_2 = \frac{e^{\bar{U}}}{x_1}$
   4. $\frac{\partial x_2}{\partial x_1} = -\frac{e^{\bar{U}}}{x_1^2}$
   5. $MU_1 = \frac{1}{x_1}$ and $MU_2 = \frac{1}{x_2}$
   6. $MRS = \frac{x_2}{x_1}$

Microeconomics
Practice Session 3 - Consumption Theory (3)

Luisa Lorè*

27/03/2020

Utility Maximization Problem (interior solution)

TOPICS TO REVIEW

- **Utility Maximization Problem**

Exercise 1

A consumer has a utility function $U(x_1, x_2) = x_1 x_2$ for good x_1 and x_2 and an income $I = 800$. The two goods have prices $p_1 = 20$ and $p_2 = 40$. Compute the optimal consumption bundle.

Solution

In order to solve a Utility Maximization Problem, we can rely on the system, we have seen during the previous Practice Session:

$$\begin{cases} MRS = \frac{MU_1}{MU_2} = \frac{p_1}{p_2} \rightarrow \text{tangency condition (interior solution)} \\ I = p_1 x_1 + p_2 x_2 \rightarrow \text{budget constraint} \end{cases}$$

We now can solve the exercise numerically:

$$MU_1 = x_2$$

$$MU_2 = x_1$$

$$MRS = \frac{MU_1}{MU_2} = \frac{x_2}{x_1}$$

$$\begin{cases} \frac{x_2}{x_1} = \frac{20}{40} = \frac{1}{2} \rightarrow 2x_2 = x_1 \\ 20x_1 + 40x_2 = 800 \end{cases}$$

$$40x_2 + 40x_2 = 800 \rightarrow 80x_2 = 800 \rightarrow x_2^* = 10$$

$$2x_2 = x_1 \rightarrow x_1^* = 20$$

$$\textbf{Optimal Consumption Bundle } (\mathbf{x_1^*, x_2^*}) = (\mathbf{20, 10})$$

*luisa.lore@alumni.uniroma2.eu

Exercise 2

A consumer has a utility function $U(x_1, x_2) = x_1x_2 + x_1$ for good x_1 and x_2 and an income $I = 22$. The two goods have prices $p_1 = 1$ and $p_2 = 2$. Compute the optimal consumption bundle.

Solution

As in the previous exercises we rely on the system that we learned last time.

$$MU_1 = x_2 + 1$$

$$MU_2 = x_1$$

$$MRS = \frac{MU_1}{MU_2} = \frac{x_2 + 1}{x_1}$$

$$\begin{cases} \frac{x_2+1}{x_1} = \frac{1}{2} \Longrightarrow 2x_2 + 2 = x_1 \\ x_1 + 2x_2 = 22 \end{cases}$$

$$2x_2 + 2 + 2x_2 = 22 \rightarrow 4x_2 = 20 \rightarrow x_2^* = 5$$

$$x_1 = 2x_2 + 2 \rightarrow x_1^* = 12$$

$$\textbf{Optimal Consumption Bundle } (\mathbf{x_1^*}, \mathbf{x_2^*}) = (\mathbf{12}, \mathbf{5})$$

Exercise 3

A consumer has a utility function $U(x_1, x_2) = ln(x_1x_2)$ for good x_1 and x_2 and an income $I = 120$. The two goods have prices $p_1 = 2$ and $p_2 = 3$. Compute the optimal consumption bundle.

Solution

As in the previous exercises we rely on the system that we learned last time.

$$MU_1 = \frac{1}{x_1}$$

$$MU_2 = \frac{1}{x_2}$$

$$MRS = \frac{MU_1}{MU_2} = \frac{x_2}{x_1}$$

$$\begin{cases} \frac{x_2}{x_1} = \frac{2}{3} \rightarrow x_2 = \frac{2}{3}x_1 \\ 120 = 2x_1 + 3x_2 \end{cases}$$

$$120 = 2x_1 + 3\frac{2}{3}x_1 \rightarrow 4x_1 = 120 \rightarrow x_1 = 30$$

$$x_2 = \frac{2}{3}30 = 20$$

$$\textbf{Optimal Consumption Bundle } (\mathbf{x_1^*}, \mathbf{x_2^*}) = (\mathbf{30}, \mathbf{20})$$

Extra Exercises

UTILITY MAXIMIZATION PROBLEM

1. $U(x_1, x_2) = x_1^2 x_2$
   $I = 240$
   $p_1 = 8$
   $p_2 = 2$

2. $U(x_1, x_2) = x_1 x_2 + x_1$
   $I = 20$
   $p_1 = 1$
   $p_2 = 4$

3. $U(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$
   $I = 200$
   $p_1 = 10$
   $p_2 = 10$

4. $U(x_1, x_2) = x_1^{\frac{1}{3}} x_2^{\frac{2}{3}}$
   $I = 60$
   $p_1 = 6$
   $p_2 = 1$

5. $U(x_1, x_2) = ln(x_1) + 2ln(x_2)$
   $I = 60$
   $p_1 = 3$
   $p_2 = 1$

6. $U(x_1, x_2) = x_1 x_2^2$
   $I = 60$
   $p_1 = 3$
   $p_2 = 3$

7. $U(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{4}}$
   $I = 12$
   $p_1 = 4$
   $p_2 = 1$

8. $U(x_1, x_2) = x_1^{\frac{2}{5}} x_2^{\frac{3}{5}}$
   $I = 200$
   $p_1 = 20$
   $p_2 = 30$

9. $U(x_1, x_2) = 2ln(x_1) + 3ln(x_2)$
   $I = 300$
   $p_1 = 20$
   $p_2 = 30$

Solutions

UTILITY MAXIMIZATION PROBLEM

1. $(x_1^*, x_2^*) = (20, 40)$
2. $(x_1^*, x_2^*) = (12, 2)$
3. $(x_1^*, x_2^*) = (10, 10)$
4. $(x_1^*, x_2^*) = (\frac{10}{3}, 40)$
5. $(x_1^*, x_2^*) = (\frac{20}{3}, 40)$
6. $(x_1^*, x_2^*) = (\frac{20}{3}, \frac{40}{3})$
7. $(x_1^*, x_2^*) = (2, 4)$
8. $(x_1^*, x_2^*) = (4, 4)$
9. $(x_1^*, x_2^*) = (6, 6)$

Microeconomics
Practice Session 4 - Production Theory (1)

Luisa Lorè*

15/05/2020

Exercise 1 - Production General Concepts

TOPICS TO REVIEW

- **Production function**: the production function is a relationship which associates different combinations of inputs with different levels of output; this function shows how the firm converts inputs into outputs.
- **Isoquant**: an isoquant is a curve on the (L;K) diagram which indicates combinations of labor and capital which are associated with the same level of output by the production function.
- **Marginal product**: the marginal product of a factor of production shows the variation of the output related to an infinitesimal variation of the factor of production, with all other factors kept constant.
- **Marginal substitution rate in production**: the MSRP shows how the inputs can substitute for each other in the production function; from a mathematical perspective, it is calculated as the ratio of the marginal product of one input to the marginal product of the other input; from a geometrical perspective, it can be thought of as the absolute value of the slope of the isoquant.
- **Isocost** an isocost line shows all combinations of inputs which cost the same total amount. Although similar to the budget constraint in consumer theory, the use of the isocost line pertains to cost-minimization in production, as opposed to utility-maximization.

Given the production function $f(L,K) = L^{\frac{1}{4}}K^{\frac{3}{4}}$:

1. Find the general equation of an isoquant, and the isoquant relative to the level $\bar{q} = 100$;
2. Calculate the marginal productivity of labour and capital and compute the marginal rate of technical substitution;
3. Find the general equation of an isocost, and the isocost relative to the input prices $w = 1$ and $r = 2$ and the level of total cost $\bar{c} = 5$;

*luisa.lore@alumni.uniroma2.eu

Solution

This excercise aims to review some basic concepts, which will be very useful during the second part of the lecture and in general in all the exercise which will do for the Production Theory part. Let us now solve all the tasks keeping in mind the theory:

1. As first thing we are asked to find the general equation of an isoquant, which we know it's the level curve of the production function, so it will be necessary just to set a certain level of $\bar{q}$ to which the production function will be equal:

$$\bar{q} = L^{\frac{1}{4}} K^{\frac{3}{4}}$$
$$K^{\frac{3}{4}} = \frac{\bar{q}}{L^{\frac{1}{4}}} \rightarrow K = \frac{\bar{q}^{\frac{4}{3}}}{L^{\frac{1}{4}\frac{4}{3}}} \rightarrow K = \frac{\bar{q}^{\frac{4}{3}}}{L^{\frac{1}{3}}}$$

   Now we can move to compute one specific isoquant, the one relative to the level $\bar{q} = 100$:

$$100 = L^{\frac{1}{4}} K^{\frac{3}{4}}$$
$$K^{\frac{3}{4}} = \frac{100}{L^{\frac{1}{4}}} \rightarrow K = \frac{100^{\frac{4}{3}}}{L^{\frac{1}{4}\frac{4}{3}}} \rightarrow K = \frac{100^{\frac{4}{3}}}{L^{\frac{1}{3}}}$$

2. Another important topic are marginal productivities and their ration, the marginal rate of technical substitution. In order to compute the marginal productivity of labour and capital we will simply need to derive the production function with respect to L for MP_L and capital for MP_K:

$$MP_L = \frac{\partial f(L,K)}{\partial L} = \frac{1}{4} L^{-\frac{3}{4}} K^{\frac{3}{4}} = \frac{1}{4}\left(\frac{K}{L}\right)^{\frac{3}{4}}$$
$$MP_K = \frac{\partial f(L,K)}{\partial K} = \frac{3}{4} L^{\frac{1}{4}} K^{-\frac{1}{4}} = \frac{3}{4}\left(\frac{L}{K}\right)^{\frac{1}{4}}$$

   In order to compute the $MRTS$ is enough to make the ratio of the two marginal productivity:

$$MRTS = \frac{MP_L}{MP_K} = \frac{\frac{1}{4}\left(\frac{K}{L}\right)^{\frac{3}{4}}}{\frac{3}{4}\left(\frac{L}{K}\right)^{\frac{1}{4}}} = \frac{1}{4}\frac{K}{L}$$

3. As last task, we have to find the general equation of an isocost. As for the budget constraint, the total cost is given by a sum of input times their prices:

$$R = p_1 x_1 + p_2 x_2 \rightarrow TC = wL + rK$$

   If we want the generic formula an isocost having $w = 1$ and $r = 2$, we will simply set a certain level of cost $\bar{c}$:

$$\bar{c} = wL + rK \rightarrow K = -\frac{w}{r}L + \frac{\bar{c}}{r}$$

Or fix $\bar{c} = 5$, if we want to have an isocost relative to a specific level of cost:

$$5 = L + 2K \rightarrow K = -\frac{1}{2}L + \frac{5}{2}$$

Cost Minimization Problem

The objective of the firm is to maximize its profit; for the sake of this goal, it has to produce any possible quantity at lowest possible cost. As a consequence, when we talk about how the firm produces, we are talking about a cost minimization problem. Namely, this problem can be thought of as a constrained minimization problem, in which the objective function (i.e. the function to be minimized) is the total cost function and the constraint is the quantity which the firm aims to produce. Mathematically speaking:

$$min\, TC = wL + rK\ s.t.\, \bar{q} = f(L, K)$$

Geometrically speaking, it is important to keep in mind that the constraint represents the equation of the isoquant associated with the desired quantity; as a consequence, solving this problems means finding the lowest possible isocost which touches the given isoquant. The solution of this problem represents the input bundle which allows the firm to produce the desired quantity at the lowest cost: this combination of labor and capital satisfies the property of economic efficiency.

Cost Minimization Problem in the Short Run

In the short run, at least an input is fixed (i.e. it cannot be used beyond a maximum level). In this course of Microeconomics, capital is the fixed factor in the short run. With the aim of minimizing its costs, the firm will use its capital at the maximum level with reference to any possible quantity and it will try to minimize the use of labor. Given that $\bar{K}$ is the highest possible use of capital, we obtain a modified version of the constrained minimization problem:

$$min\, wL + rK\ s.t.\, \bar{q} = f(L, K),\ s.t.\, K = \bar{K}$$

$$min\, wL + r\bar{K}\ s.t.\, \bar{q} = f(L, \bar{K})$$

The second constraint $(K = \bar{K})$ is typical of the short run time horizon.

Exercise 2 - Cost Minimization in the Short Run

TOPICS TO REVIEW

- **Cost Minimization Problem in the Short Run**

Given that $f(L, K) = L^{\frac{1}{2}} K^{\frac{1}{2}}$, $w = 16$, $r = 4$, $\bar{K} = 100$ and $\bar{q} = 200$:

1. Carry out the cost minimization problem, assuming a short run time horizon,
2. Find the optimal input bundle,
3. Compute the total cost beared by the producer.

Solution

1. In order to minimize producer's cost, we simply can use the setting which we have discussed in the previous section using the data we have:

$$min\, 16L + 4K\ s.t.\, 200 = L^{\frac{1}{2}} K^{\frac{1}{2}},\ s.t.\, K = 100$$

$$min\, 16L + 4 \cdot 100\ s.t.\, 200 = L^{\frac{1}{2}} 100^{\frac{1}{2}}$$

As you can observe, in order to solve the entire problem just using the constraints:

$$200 = 10L^{\frac{1}{2}}$$

$$L^{\frac{1}{2}} = 20$$

$$(L^{\frac{1}{2}})^2 = 20^2$$

$$L^* = 400$$

2. The optimal input bundle is simply given by the fixed level of capital and the result of the minimization problem:

$$\text{Optimal input bundle: } (L^*, \bar{K}) = (400, 100)$$

3. Again, in order to compute the total cost, we simply have to plug in all the data we have:

$$TC = wL^* + r\bar{K} = 16 \cdot 400 + 4 \cdot 100 = 6400 + 400 = 6800$$

Extra Exercises

COST MINIMIZATION IN THE SHORT RUN

1. $f(L,K) = L^{\frac{1}{4}}K^{\frac{1}{2}}$; $\bar{q} = 2$
   $w = 20$; $r = 30$
   $\bar{K} = 9$

2. $f(L,K) = LK^{\frac{1}{4}}$; $\bar{q} = 5$
   $w = 2$; $r = 3$
   $\bar{K} = 16$

3. $f(L,K) = L^{\frac{1}{5}}K$; $\bar{q} = 1$
   $w = 4$; $r = 3$
   $\bar{K} = 2$

4. $f(L,K) = L^{\frac{1}{2}}K^{\frac{1}{2}}$; $\bar{q} = 32$
   $w = 2$; $r = 2$
   $\bar{K} = 16$

5. $f(L,K) = L^{\frac{1}{2}}K^{\frac{1}{2}}$; $\bar{q} = 10$
   $w = 3$; $r = 5$
   $\bar{K} = 16$

Solutions

COST MINIMIZATION IN THE SHORT RUN

1. $L^* = 0.1975$
2. $L^* = 2.5$
3. $L^* = 0.03$
4. $L^* = 2$
5. $L^* = 6.25$

Microeconomics
Practice Session 5 - Production Theory (2)

Luisa Lorè*

15/05/2020

Cost Minimization Problem

Cost Minimization Problem in the Long Run

Let us start reviewing what we have learned last time. In order to minimize costs, we have to set the following problem:

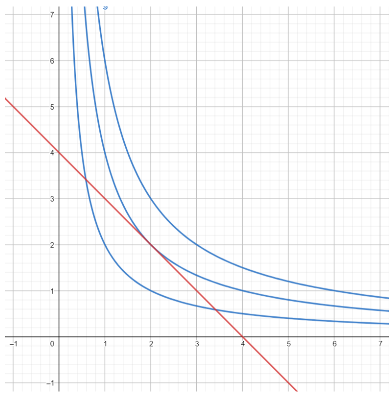
$$min\, wL + rK s.t. \bar{q} = f(L, K)$$

In the short run horizon we were setting a certain level of capital $K = \bar{K}$, while in the long run horizon, we can optimize the level of both productive inputs.
In order to solve this minimization problem we have to follow a very similar procedure to the one used in order to maximize utility, for this reason doing a comparison between this two problems will allow us to better undersand to procedure:

Utility maximization problem

$$max\, U(x_1, x_2) s.t. I = p_1x_1 + p_2x_2$$

In this problem we have a fixed level of income that consumer can spend and many possible indifference curves on which she can position herself. We are looking for the further from the origin curves (because we want the highest possible utility) but still on the budget line (because of non-satiation assumption), that's the reason why we are looking for the indifference curve which is tangent to the budget constraint.



*luisa.lore@alumni.uniroma2.eu

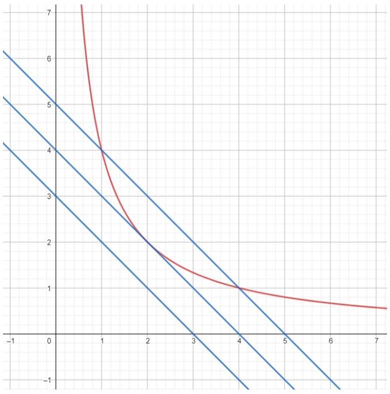
For this reasong we set the following system:

$$\begin{cases} MRS = \frac{MU_1}{MU_2} = \frac{p_1}{p_2} \rightarrow \text{tangency condition} \\ I = p_1x_1 + p_2x_2 \rightarrow \text{budget constraint} \end{cases}$$

Cost minimization problem

$$min\, wL + rK s.t.\bar{q} = f(L, K)$$

In this problem we have a fixed level of output that producer want to produce and many possible isocost lines on which she can position herself. We are looking for the closest to the origin line (because we want the lowest possible cost) but still on the isoquant (because of efficiency), that's the reason why we are looking for the isocost which is tangent to the isoquant.



For this reasong we set the following system:

$$\begin{cases} MRTS = \frac{MP_L}{MP_K} = \frac{w}{r} \rightarrow \text{tangency condition} \\ \bar{q} = f(L, K) \rightarrow \text{isoquant} \end{cases}$$

Exercise 1 - Cost Minimization in the Long Run

TOPICS TO REVIEW

- **Cost Minimization Problem in the Long Run**

Given that $f(L,K) = L^{\frac{1}{4}}K^{\frac{1}{4}}$, $w = 30$, $r = 30$ and $\bar{q} = 5$:

1. Compute the value of L and K that minimize cost
2. Calculate the corrisponding total cost

Solution

1. We have to set a cost minimization problem in the following way:

$$min\, 30L + 30K s.t. 5 = L^{\frac{1}{4}}K^{\frac{1}{4}}$$

Let us compute the $MRTS$, and then directly set the system:

$$MRTS = \frac{MP_L}{MP_K}$$

$$MP_L = \frac{\partial f(L,K)}{\partial L} = \frac{1}{4}L^{\frac{1}{4}-1}K^{\frac{1}{4}} = \frac{1}{4}L^{-\frac{3}{4}}K^{\frac{1}{4}}$$

$$MP_K = \frac{\partial f(L,K)}{\partial K} = \frac{1}{4}L^{\frac{1}{4}}K^{\frac{1}{4}-1} = \frac{1}{4}L^{\frac{1}{4}}K^{-\frac{3}{4}}$$

$$MRTS = \frac{\frac{1}{4}L^{-\frac{3}{4}}K^{\frac{1}{4}}}{\frac{1}{4}L^{\frac{1}{4}}K^{-\frac{3}{4}}} = L^{-\frac{4}{4}}K^{\frac{4}{4}} = \frac{K}{L}$$

$$\begin{cases} \frac{K}{L} = \frac{30}{30} \rightarrow K = L \\ 5 = L^{\frac{1}{4}}K^{\frac{1}{4}} \end{cases}$$

$$5 = L^{\frac{1}{4}}L^{\frac{1}{4}} \rightarrow 5 = L^{\frac{1}{2}} \rightarrow 5^2 = L^{\frac{1}{2}\cdot 2}$$

$$L^* = 25$$

$$K^* = 25$$

2. Let us simply use the total cost formula:

$$CT = wL^* + rK^* = 30 \cdot 25 + 30 \cdot 25 = 750 + 750 = 1500$$

Exercise 2 - Return to Scale

TOPICS TO REVIEW

- **Returns to scale**: returns to scale regard the relationship between an increase in the use of inputs and the consequent variation in the output.

Given the following production functions establish the return to scale that each of the function exhibits, giving a clear definition of the return to scale type:

1. $f(K,\, L) = 2(L + K)$
2. $f(K,\, L) = L^{\frac{1}{2}} K^{\frac{2}{6}}$
3. $f(K,\, L) = 2(LK)^{\frac{1}{2}}$
4. $f(K,\, L) = L + K^2$
5. $f(K,\, L) = L^3 K^5$

Solution

First of all let us clearly state the different types of return to scale:
Increasing returns to scale (IRS): as the use of inputs increases, the output increases in a more than linear way.
Constant returns to scale (CRS): as the use of inputs increases, the output increases in a linear way.
Decreasing returns to scale (DRS): as the use of inputs increases, the output increases in a less than linear way.
In general, in order to verify return to scale we can simply multiply the entire function for a constant λ (where $\lambda > 0$), and the function will exhibit:
IRS if $f(\lambda K, \lambda L) > \lambda f(K, L)$, because increasing inputs by a constant λ, we have a more than proportional increase in output.
CRS if $f(\lambda K, \lambda L) = \lambda f(K, L)$ because increasing inputs by a constant λ, we have a proportional increase in output.
DRS if $f(\lambda K, \lambda L) < \lambda f(K, L)$, because increasing inputs by a constant λ, we have a less than proportional increase in output.
If we are dealing with a Cobb-Douglas production function, we can rely on analysing inputs' exponents. Let us recall a Cobb-Douglas production function:

$$f(K,\, L) = K^{\alpha} L^{\beta}$$

If we apply the same reasoning done until now, we obtained that:

$$f(\lambda K,\, \lambda L) \lesseqgtr \lambda \left[f(K,\, L) \right]$$
$$\lambda^{\alpha} K^{\alpha} \lambda^{\beta} L^{\beta} \lesseqgtr \lambda K^{\alpha} L^{\beta}$$
$$\lambda^{\alpha+\beta} K^{\alpha} L^{\beta} \lesseqgtr \lambda K^{\alpha} L^{\beta}$$
$$\lambda^{\alpha+\beta} \lesseqgtr \lambda$$

$$\alpha + \beta \lesseqgtr 1$$

Then, this function will exhibit:
IRS if $\alpha + \beta > 1$
CRS if $\alpha + \beta = 1$
DRS if $\alpha + \beta < 1$

Now let us move to the solution of the exercise implementing the two strategies we have learned:

1. $f\left(K,\, L\right) = 2(L + K)$
   $f\left(\lambda K,\, \lambda L\right) = 2(\lambda L + \lambda K) = 2\lambda(L + K)$
   $\lambda\left[f\left(K,\, L\right)\right] = 2\lambda(L + K)$
   $2\lambda(L + K) = 2\lambda(L + K) \rightarrow$ **CRS**

2. $f\left(K,\, L\right) = L^{\frac{1}{2}}K^{\frac{2}{6}}$
   $\frac{1}{2} + \frac{2}{6} = \frac{5}{6} < 1 \rightarrow$ **DRS**

3. $f\left(K,\, L\right) = 2(LK)^{\frac{1}{2}}$
   $\frac{1}{2} + \frac{1}{2} = 1 \rightarrow$ **CRS**

4. $f\left(K,\, L\right) = L + K^2$
   $f\left(\lambda K,\, \lambda L\right) = (\lambda L + (\lambda K)^2) = (\lambda L + \lambda^2 K^2) = \lambda(L + \lambda K^2)$
   $\lambda\left[f\left(K,\, L\right)\right] = \lambda(L + K^2)$
   $\lambda(L + \lambda K^2) > \lambda(L + K^2) \rightarrow$ **IRS**

5. $f\left(K,\, L\right) = L^3K^5$
   $3 + 5 = 8 > 1 \rightarrow$ **IRS**

Extra Exercises

COST MINIMIZATION IN THE SHORT RUN

1. $f(L,K) = L^{\frac{1}{2}}K^{\frac{1}{2}}$; $\bar{q} = 20$
   $w = 2$; $r = 2$

2. $f(L,K) = L^{\frac{1}{5}}K^{\frac{4}{5}}$; $\bar{q} = 10$
   $w = 1$; $r = 4$

3. $f(L,K) = L^{\frac{3}{4}}K^{\frac{1}{4}}$; $\bar{q} = 10$
   $w = 3$; $r = 1$

4. $f(L,K) = L^2K$; $\bar{q} = 32$
   $w = 2$; $r = 2$

5. $f(L,K) = LK$; $\bar{q} = 15$
   $w = 5$; $r = 3$

RETURN TO SCALE

1. $f(L,K) = L^{\frac{1}{4}}K$

2. $f(L,K) = L + K$

3. $f(L,K) = L^{\frac{1}{5}}K^{\frac{4}{5}}$

4. $f(L,K) = L^3 + K^2$

5. $f(L,K) = L^{\frac{1}{6}}K^{\frac{1}{8}}$

Solutions

COST MINIMIZATION IN THE SHORT RUN

1. $L^* = 20$; $K^* = 20$
2. $L^* = 10$; $K^* = 10$
3. $L^* = 5$; $K^* = 5$
4. $L^* = 4$; $K^* = 2$
5. $L^* = 3$; $K^* = 5$

RETURN TO SCALE

1. IRS
2. CRS
3. CRS
4. IRS
5. DRS

Microeconomics
Practice Session 6 - Markets

Luisa Lorè*

28/05/2020

Perfect Competition

A market is defined as perfectly competitive providing that four requirements are fulfilled:

1. **Homogeneity of products**: all the products that are marketed by the various firms are exactly identical or perfect subsitutes; this means that the only lever that can be exploited by a single firm to compete is price.

2. **High number of economic agents**: there is a significant number of consumers and producers that are not able to individually influence the level of the market demand and the market supply.

3. **Perfect information**: any producer is perfectly informed about the preferences of all consumers and any consumer is perfectly informed about the characteristics of the products marketed by all firms.

4. **Absence of barriers to entry and exit**: firms can easily enter or exit the market.

The combination of these requirements causes each firm to consider the market price as a given.

Perfect Competition in the Short Run

A market supply curve is generally built as the horizontal summation of the individual supply curves. In a competitive market, as firms are identical, the market supply curve is obtained as the individual supply curve multiplied by the number of firms which are present in the marketplace. Each firm supplies a quantity which allows it to maximize its profit and therefore to fulfill this condition:

$$max\, \pi = TR(Q) - TC(Q)$$

Since in perfect competition the market price is a given, we can rewrite the condition in the following way:

$$max\, \pi = TR(Q) - TC(Q) = pQ - TC(Q)$$

*luisa.lore@alumni.uniroma2.eu

$$\frac{\partial \pi(Q)}{\partial Q} = 0$$

$$\frac{\partial pQ}{\partial Q} - \frac{\partial TC(Q)}{\partial Q} = 0$$

$$p - MC(Q) = 0$$

$$p = MC(Q)$$

Perfect Competition in the Long Run

In order to build the supply curve in the long run under competitive conditions one has to pay attention to the forth requirement, according to which there is no barrier to entry and exit. This means that in the long run there is a market equilibrium providing that no potential entrant is interested in entering the market and no firm is interested in exiting the market. Such a situation is achieved if the individual surplus of each firm equals zero. This implies that each firm, in equilibrium, supplies the quantity that minimizes the average cost and therefore fulfills the following condition:

$$p = AC(Q^{min})$$

As consequence, the supply curve in the long run is a horizontal straight line, at the level of the minimum average cost.

Market Equilibrium

The market equilibrium is a situation in which the market demand equals the market supply. Geometrically speaking, it is represented by the point on the (Q,P) cartesian plane in which the supply curve and the demand curve intersect.

Exercise 1 - PC in the Short Run

In a perfectly competitive market, there are 10 firms operating, each one has the following total cost function:

$$TC(Q_i) = Q_i^2$$

The market is also characterized by the following demand function:

$$Q^d = 100 - 20p$$

Compute:

1. The short run supply curve of the firm
2. The short run supply curve of the industry
3. Price and Quantity at the equilibrium
4. The level of production and the profit realized by one firm in the short run

Solution

1. In order to compute the short run supply curve of the firm, we need to start from the profit maximization condition that we learned in the previous paragraph and then express the quantity in function of the price. As a first thing let us compute the marginal cost and then derive the supply curve:

$$MC(Q_i) = \frac{\partial TC(Q_i)}{\partial Q} = 2Q_i$$

$$p = CMa(Q_i) \longrightarrow p = 2Q_i \longrightarrow Q_i = \frac{1}{2}p$$

2. In order to compute the short run supply curve of the entire industry, we simply sum linearly the supply curves of the each firm, in the following way:

$$Q^s = 10\left(\frac{1}{2}p\right) = 5p$$

3. In order to compute market equilibrium we have to solve a system in two unknown, this is because we have to find the intersection point between the demand and the supply curve:

$$\begin{cases} Q^d(p) = 100 - 20p \\ Q^s(p) = 5p \end{cases}$$

In order to solve this system, we simply have to set the two functions as equal, because both express the quantity as a function of price:

$$100 - 20p = 5p \longrightarrow 25p = 100 \longrightarrow p = \frac{100}{25} = 4$$

Let us now substitute price in to one of the two function, for example in the supply:

$$Q^s(p) = 5 \cdot 4 = 20$$

And the equilibrium condition is:

$$Eq. = \{Q^E = 20,\, p^E = 4\}$$

4. Since there are 10 firms in the industry, each one produces one tenth on the quantity demanded by the market:

$$Q_i^E = \frac{Q^E}{\#\, firms} = \frac{20}{10} = 2$$

Let us now substitute price and quantity into the profit formula:

$$\pi_i = p^E \cdot Q_i^E - CT(Q_i^E) = 4 \cdot 2 - 2^2 = 8 - 4 = 4$$

All firms have an individual profit of 4.

Exercise 2 - PC in the Long Run

In a perfectly competitive market, there are 100 firms operating, each one has the following total cost function:

$$TC(Q_i) = Q_i^2 + 10$$

The market is also characterized by the following demand function:

$$Q^d = 300 - 20p$$

Compute:

1. Equilibrium price and quantity of the firm in the long run
2. Equilibrium quantity in the market in the long run
3. The number of firms operating in the long run
4. Long run profit of a firm operating in the long run

Solution

1. In order to compute price and quantity, we need to start from the profit maximization condition that we learned in the previous paragraph and then express the quantity in function of the price:

   $$p = AC(Q^{min})$$

   As a first thing let us compute the minimum level of average cost:

   $$\frac{\partial AC(Q)}{\partial Q} = 0$$

   $$\frac{\partial\left(Q + \frac{10}{Q}\right)}{\partial Q} = 1 - \frac{10}{Q^2} = 0 \longrightarrow Q^2 = 10 \longrightarrow Q_i^{min} = \sqrt{10}$$

   And then compute the profit maximization condition:

   $$p = AC(\sqrt{10}) = \sqrt{10} + \frac{10}{\sqrt{10}} = \sqrt{10} + \sqrt{10} \longrightarrow p^{LR} = 2\sqrt{10}$$

2. In order to compute the supply of the entire industry, let us multiply the equilibrium quantity of each firm:

   $$nQ_i = 100\sqrt{10} \approx 316$$

3. The number of firms operating in the long run is given by the ratio between the quantity demanded and the price found in the first part of the exercise:

   $$Q^d(2\sqrt{10}) = 300 - 20p = 300 - 40\sqrt{10} \approx 300 - 126 = 174 \longrightarrow Q^E \approx 174$$

   and the maximum quantity that each firm wants to supply for p^{LP}:

   $$n^{LR} = \frac{Q^E}{Q_i} = \frac{174}{\sqrt{10}} \approx 55$$

   We can notice that in the long run, supply exceeds demand, and more or less of the half of firms will enter the market.

4. Then, let us compute the profit of a firm operating in the long run:

$$\pi_i = p^{LR} \cdot Q_i^{min} - CT(Q_i^{min}) = 2\sqrt{10} \cdot \sqrt{10} - \left(\sqrt{10}\right)^2 - 10 = 20 - 10 - 10 = 0$$

All firms taking part to the market have an individual profit of 0.

Monopoly

If in a market the demand is satisfied entirely by only one firm, this market is a monopoly, and the firm operating in it is a monopolist. A monopolist has not a supply curve, it simply provides a price to the market, knowing that at that price the consumers will be willing to buy the quantity that maximize his profit. In order to find the quantity that maximize its output, he has indeed to take in consideration market demand, and it does so maximizing the following formula for its profit:

$$\pi = TR(Q) - TC(Q) = p(Q)Q - MC(Q)$$

In which $p(Q)$ is the inverse market demand function. From the profit maximization, we arrive to this profit-maximization condition:

$$max\, \pi = TR(Q) - TC(Q)$$

$$\frac{\partial \pi(Q)}{\partial Q} = 0$$

$$\frac{\partial TR(Q)}{\partial Q} - \frac{\partial TC(Q)}{\partial Q} = 0$$

$$MR(Q) - MC(Q) = 0$$

$$MR(Q) = MC(Q)$$

Monopoly mark-up

As we have seen a monopolist can set a price above the perfect competition price, but it always take in consideration the market demand. In order to have a measure about how much above the marginal cost he can price, we can compute its mark-up in the following way:

$$\mu = \frac{p^m - CMa(Q^m)}{CMa(Q^m)} = \frac{p^m}{CMa(Q^m)} - 1$$

Exercise 3 - Monopoly

In a market there is only one firm operating with the following total cost function:

$$CT(Q) = 100Q$$

the market demand is given by

$$Q^d = 400 - 2p$$

Compute:

1. Market equilibrium when firm is price-setter (behave as a monopoly)
2. Monopoly mark-up and profit
3. Market equilibrium when firm is price-taker (behave as in perfect competition)
4. Monopoly net gain

Solution

1. When the firm is price-setter, it maximize the following profit function:

$$max\, \pi^m = RT(Q) - CT(Q) = p(Q)Q - CT(Q) \longrightarrow RMa(Q) - CMa(Q) = 0$$

$$RMa(Q) = CMa(Q)$$

We have to derive $p(Q)$, the market demand inverse function, in the following way:

$$Q^d = 400 - 2p \longrightarrow 2p = 400 - Q \longrightarrow p = 200 - \frac{1}{2}Q$$

We derive revenues and costs and we set them equal:

$$\frac{\partial RT(Q)}{\partial Q} = \frac{\partial \left(200Q - \frac{1}{2}Q^2\right)}{\partial Q} = 200 - Q$$

$$\frac{\partial CT(Q)}{\partial Q} = \frac{\partial \left(100Q\right)}{\partial Q} = 100$$

$$RMa(Q) = CMa(Q) \longrightarrow 200 - Q = 100 \longrightarrow Q^m = 100$$

In order to find the price, let us plug the quantity into the demand function:

$$p = 200 - \frac{1}{2}Q = 200 - \frac{1}{2}100 \longrightarrow p^m = 150$$

$$Eq^{mon} = \{Q^m = 100,\, p^m = 150\}$$

2. In order to compute the monopolist's mark-up we know that:

$$\mu = \frac{p^m - CMa(Q^m)}{CMa(Q^m)} = \frac{150 - 100}{100} = \frac{50}{100} = \frac{1}{2} \longrightarrow 50\%$$

While profit is given by the formula:

$$\pi^m = RT(Q^m) - CT(Q^m) = p^m Q^m - CT(Q^m)$$

$$\pi^m = 150 \cdot 100 - 100 \cdot 100 = 15000 - 10000 = 5000$$

3. Now, suppose that for any reason monopolist cannot (or decide not to) influence the market, then it decide to be a price-taker and to behave as a firm in perfect competition:

$$p = CMa(Q) \longrightarrow 200 - \frac{1}{2}Q = 100 \longrightarrow \frac{1}{2}Q = 100 \longrightarrow Q^{cp} = 200$$

$$p = 200 - \frac{1}{2}200 = 200 - 100 \longrightarrow p^{cp} = 100$$
$$Eq^{cp} = \{Q^{cp} = 200,\, p^{cp} = 100\}$$

As we should have expected, monopoly equilibrium present an higher price and a lower quantity with respect to the equilibrium generated by the price-taker firm, indeed a monopoly produce less and sell at an higher price compare to a firm operating in a perfectly competitive regime.

$$\pi^{cp} = RT(Q^{cp}) - CT(Q^{cp}) = p^{cp}Q^{cp} - CT(Q^{cp})$$

$$\pi^{cp} = 100 \cdot 200 - 100 \cdot 200 = 20000 - 20000 = 0$$

4. Let us now compute the difference between the monopoly and the perfect competition profits:
$$\pi^m - \pi^{cp} = 5000 - 0 = +5000$$

Extra Exercises

PERFECT COMPETITION IN THE SHORT RUN

1. $Q^d = 500 - 25p$; $CT(Q_i) = Q_i^2 + 25$; $n = 50$
2. $Q^d = 40 - 4p$; $CT(Q_i) = Q_i^2$; $n = 40$
3. $Q^d = 10 - \frac{1}{5}p$; $CT(Q_i) = Q_i^2$; $n = 10$
4. $Q^d = 420 - 20p$; $CT(Q_i) = Q_i^2 + 1$; $n = 100$
5. $Q^d = 100 - 2p$; $CT(Q_i) = Q_i^2 + 9$; $n = 4$

PERFECT COMPETITION IN THE LONG RUN

1. $Q^d = 500 - 25p$; $CT(Q_i) = Q_i^2 + 25$; $n = 50$
2. $Q^d = 40 - 4p$; $CT(Q_i) = Q_i^2$; $n = 40$
3. $Q^d = 10 - \frac{1}{5}p$; $CT(Q_i) = Q_i^2$; $n = 10$
4. $Q^d = 420 - 20p$; $CT(Q_i) = Q_i^2 + 1$; $n = 100$
5. $Q^d = 100 - 2p$; $CT(Q_i) = Q_i^2 + 9$; $n = 4$

MONOPOLY

1. $Q^d = 200 - p$; $CT(Q_i) = 40Q_i + 10$;
2. $Q^d = 100 - \frac{1}{2}p$; $CT(Q_i) = 40Q_i + 10$;
3. $Q^d = 400 - 2p$; $CT(Q_i) = 100Q_i$;
4. $Q^d = 100 - p$; $CT(Q_i) = 20Q_i$;
5. $Q^d = 120 - p$; $CT(Q_i) = Q_i^2 + 100$;