Singular Value Decomposition

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Gliederung

1. Grundbegriffe
2. SVD Definition
3. Dimensionsreduktion
4. Rechenbeispiel mit Numpy
5. Anwendungsmöglichkeiten
6. Literaturverzeichnis

Grundbegriffe

Diagonalmatrix $\forall i \neq j: D_{i,j} = 0$

$$D = \begin{pmatrix} d_{11} & 0 & \cdots & 0 \\ 0 & d_{22} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & d_{nn} \end{pmatrix}$$

Grundbegriffe

Diagonalmatrix $\forall i \neq j\colon D_{i,j} = 0$

$$D = \begin{pmatrix} d_{11} & 0 & \cdots & 0 \\ 0 & d_{22} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & d_{nn} \end{pmatrix}$$

Transponierte Matrix $\forall i, j\colon\ A_{i,j} = A^T_{j,i}$

A

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

Grundbegriffe

Inverse Matrix $A \cdot A^{-1} = A^{-1} \cdot A = E$

$$A \cdot A^{-1} = E$$

$$\begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & -1 \\ 2 & 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

https://www.youtube.com/watch?v=LmiF_iCV-Fk

Grundbegriffe

Orthogonalmatrix

$$Q^TQ = \begin{vmatrix} q_1^T \\ q_2^T \\ q_3^T \\ \vdots \\ q_n^T \end{vmatrix} [q_1\ q_2\ q_3\ \dots\ q_n] = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix} = I$$

Beispiel:

$$Q^TQ = \begin{pmatrix} 1/3 & 2/3 & -2/3 \\ -2/3 & 2/3 & 1/3 \\ 2/3 & 1/3 & 2/3 \end{pmatrix} \begin{pmatrix} 1/3 & -2/3 & 2/3 \\ 2/3 & 2/3 & 1/3 \\ -2/3 & 1/3 & 2/3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

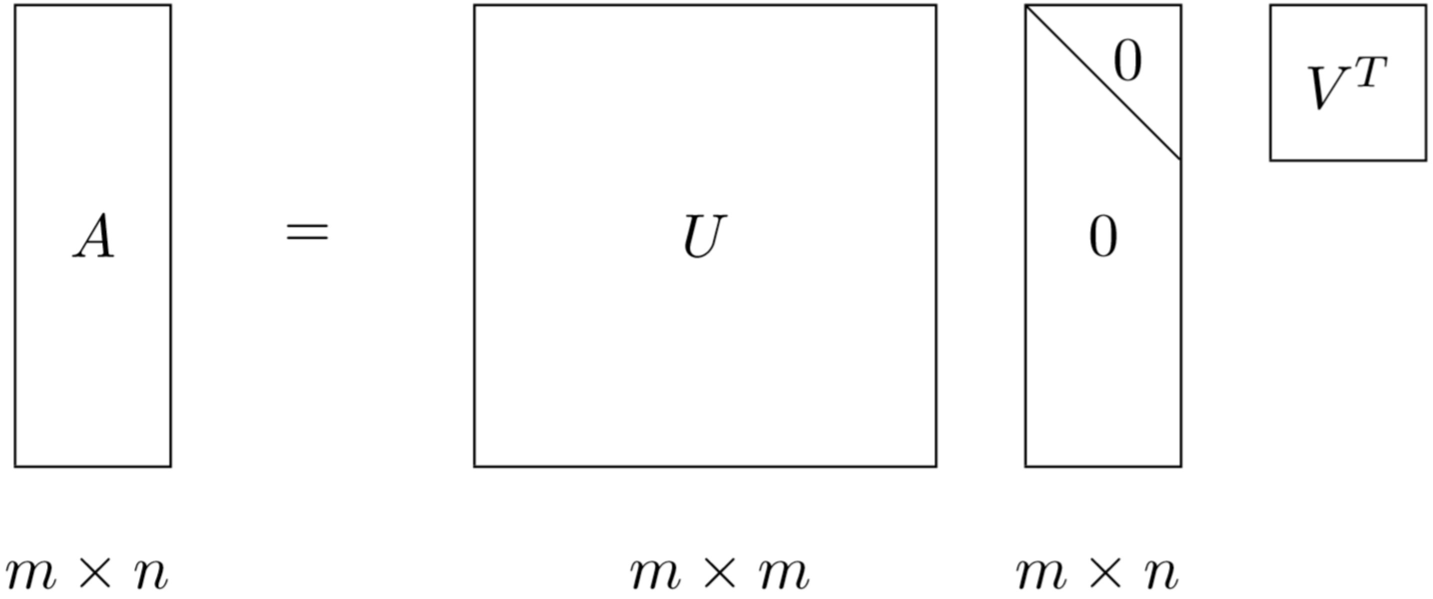
SVD Definition

Jede Matrix A$\in \mathbb{R}^{m\times n}$, mit $m \geq n$, kann zerlegt werden als:

$$A = U \begin{pmatrix} \Sigma \\ 0 \end{pmatrix} V^T$$

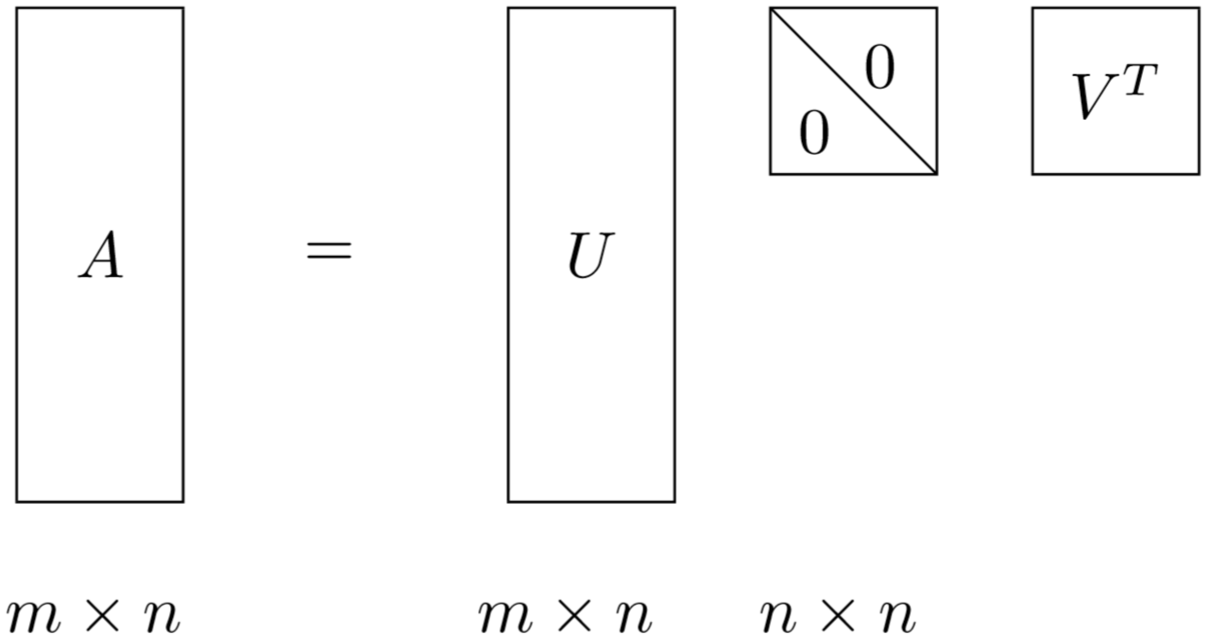
Wobei $U \in \mathbb{R}^{m\times m}, V \in \mathbb{R}^{n\times n}$ orthogonal, und $\Sigma \in \mathbb{R}^{n\times n} = \mathrm{diag}(\sigma_1, \sigma_2, \cdots, \sigma_n)$ so dass $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_n \geq 0$

SVD Definition



$$U = (U_1\, U_2)$$

Dimensionsreduktion



$$A = U_1 \Sigma V^T$$

Rechenbeispiel mit Numpy

https://github.com/arkocal/SVD-Notebook

Anwendungen

- Wortähnlichkeit berechnen
- Sentimentanalyse
- Latent Semantic Analysis, LSA
- Recommender system
- How does Netflix recommend movies? Matrix Factorization
  https://www.youtube.com/watch?v=ZspR5PZemcs

Literaturverzeichnis

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- Roth, Benjamin. Schütze Hinrich. Embeddings Learned By Matrix Factorization.Center for Information and Language Processing, LMU Munich. 2018
- How does Netflix recommend movies? Matrix Factorization https://www.youtube.com/watch?v=ZspR5PZemcs