INSTITUTO SUPERIOR TÉCNICO | UNIVERSIDADE DE LISBOA
COMPLEX NETWORKS 2017/18
LIST OF SUGGESTIONS FOR YOUR 2ND PROJECT
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FRANCISCO C. SANTOS & ALEXANDRE P. FRANCISCO
(franciscocsantos@tecnico.ulisboa.pt, aplf@tecnico.ulisboa.pt)

Below please find some suggestions for your 2nd homework/project. Please let us know which topic you prefer, such that we can point you to the relevant literature.

Mining large graphs

1. Explore how to compute vertex betweenness for large networks efficiently through sampling.
2. Explore how to answer as quick as possible queries for shortest paths on evolving graphs.
3. Explore how Average Path Length (APL) can be computed approximately through the use of approximated counters. Take a look on Webgraph HyperBall implementation, namely the use of HyperLogLog algorithm.
4. Explore the gtrie approach to enumerate motifs on graphs.
5. Implement the algorithm for finding k-cores in linear time on top of Webgraph.

Community finding, graph clustering and ranking

6. Consider the use of network clustering/partitioning methods for vertex reordering and its use for compressing graphs representations, as it is the case in Webgraph. Try different partitioning methods and compare compression results with LLP (Layered Label Propagation), which is presently used in Webgraph.
7. Consider the Affinity-Graph-Model for discovering overlapping communities. Try to find alternative methods for optimizing likelihood.
8. Implement the local optimization method based on Heat-Kernel and sweeping on top of Webgraph. Conduct experiments for finding local communities based on conductance optimization.
9. Explore the FastStep approach to find communities on networks.
10. Consider the alternative method InfoMap for finding communities and compare it with studied methods. We recommend that you rely on known benchmarks and on NMI for comparing partitions.
11. Conduct an experiment in order to compare Page-Rank and Heat-Kernel, comparing obtained rankings and convergence ratios. We suggest that you consider a Web graph in this study and the application on personalized ranking.
12. Explore the clique percolation method for uncovering the overlapping community structure of complex network.
13. Explore the resolution limit of modularity in community detection.

14. Explore how to use ranking for the local partitioning of graphs (directed and undirected).

Sampling of networks

15. Is a sample of a scale-free network, also a scale-free network? In this project you are expected to simulate computationally 2 sampling methods (random & percolated) on a given network (regular, random and scale-free) of large dimensions. To get into the general problem, you may start by reading Ref. [1].
16. Accuracy and scaling phenomena in Internet mapping. Analyze through computer simulations the advantages and problems of *traceroute* [2] as a sampling method through computer simulations. *Traceroute* has been used to extract the Internet graph Simulate *traceroute* on a given network (regular, random and scale-free) of large dimensions. Test the quality of the sampling resorting from a single source and multiple TTLs and multiple source with a given TTL. As an alternative, follow the analysis taken by Clauset et al [3-5] on this very same problem.

Cascading effects and load models

17. In many realistic situations the flow of physical quantities in the network, as characterized by the loads on nodes, is important. Here you're invited to implement the Motter-Lai model (2002) [6] which shows that intentional attacks can lead to a cascade of overload failures. The projects involves the creation of a computer model that shows that the heterogeneity of real-world networks makes them particularly vulnerable to attacks in that a large-scale cascade may be triggered by disabling a single key node.

Disease spreading

For a general overview of the topic, please see Ref. [7].

18. Implement a computer simulation of the SIS model in a given network. Compute the epidemic threshold for lattices, random graphs, BA model networks and minimal model. Discuss the results. Suggested reading: [7, 8] (see also [9, 10]).
19. Simulate and analyze (numerically or analytically) the impact of degree-degree correlations on the epidemic threshold of random and scale-free networks (see [11] to create artificial networks with a given assortativity).
20. Epidemics. Discuss analytically or by means of computer simulations the expected epidemic threshold in Random graphs and SF networks. In the later, analyze the epidemic threshold as a function of the exponent of the degree distribution. Assume that networks are infinitely large. Suggested reading: [7, 8] (see also [9, 10]).
21. Epidemics. Discuss and test (numerically or analytically) other forms of targeted immunization which do not assume global knowledge (e.g., please follow Refs. [12-14]).
22. Analytical methods for stochastic epidemic dynamics. Describe the stochastic SIS model through a Fokker-Plank Equation or, alternatively, as a Master equation (also known as Kolmogorov-forward equations). These equations provide an exact solution equivalent to multiple simulations of a model with stochasticity. See, for instance, Ref. [15], Section 6.6.
23. Epidemic spreading and temporal networks. Measurements indicate pairs of individuals will have periods of frequent interactions, when multiple contacts follow each other within a relatively short

time frame, and long periods without any further contact. In this project the challenge will be to analyze the impact of bursts of interactions in disease spreading when compared to interaction patterns that are uniformly distributed in time through computer simulations. For more information please check [16-18]. As an alternative you may also discuss the origins of such bursts of interaction in networks [16, 19]. See also Ref. [20] and the dataset available. You may also explore the contradictory conclusions obtained in Refs. [20, 21] and [22].

24. Try to implement a simple computer model of co-evolution of disease states (e.g., SIS) and network structure. Assume that, at each time-step, individuals connected to infected nodes will try to rewire their links. In other words, network evolution is a natural outcome of information disease states at each neighborhood. Try out an agent-based version of the minimal model discussed in Ref. [23].

25. The recent Zika epidemic poses a major global public health emergency. While Zika is transmitted from human to human by bites of mosquitoes, recent evidence indicates that it can also be transmitted via sexual contact. In this project you're invited to explore the analytical model proposed in ref. [24] to investigate the impact of mosquito-borne and sexual transmission on the spread and control of Zika.

26. Illustrate the use of the GLEAMviz (www.gleamviz.org) large-scale simulation tool [25], creating and analyzing the results of a new simulation and providing a step-by-step tutorial on the use of this new platform.

Racial segregation models

27. Racial segregation has always been a pernicious social problem. Why is segregation such a difficult problem to eradicate? In 1971, the American economist Thomas Schelling created an agent-based model that might help explain why segregation is so difficult to combat. His model of segregation showed that even when individuals (or "agents") didn't mind being surrounded or living by agents of a different race, they would still choose to segregate themselves from other agents over time! Although the model is quite simple, it gives a fascinating look at how individuals might self-segregate, even when they have no explicit desire to do so. In this project it is expected that you create a computer simulation and analyze Schelling's model. For details please see here [26-29].

Frequency independent selection in networked populations

28. Population genetics and evolution on networks. Compute numerically (i.e., by means of agent-based computer simulations) the average fixation probability [30] of a mutant as a function of the degree of a network. Assume that the mutant has a fitness r>1, where resident traits share a fitness=1. Study this dependence as a function of r. Start by considering regular, random and scale-free networks (BA model). Randomize a BA model and repeat the simulations, checking if the degree is the only important factor.

Multi-level selection & cooperation

29. Evolution of cooperation by multi-level selection. Competition between groups can lead to selection of cooperative behavior. This idea can be traced back to Charles Darwin. In this project, you should repeat the analytical approach performed in ref. [31].

Reputation dynamics & indirect reciprocity

30. Reproduce the classical computer simulations by Nowak & Sigmund [32] which showed, for the first time, the evolution of cooperation through reputation dynamics. This is the first model of indirect reciprocity and reputation dynamics. It is funny to implement and brings interesting (unexplored) questions in complex networks. For a recent review on the marvelous topic of indirect reciprocity, see [33].

31. In ref. [32] (see previous project), the authors propose a social norm which, in general, fails to promote cooperation [34]. Here you should implement a simple computer simulation which shows that result and helps to offer a suitable alternative (please follow Ref.[34]).

32. Indirect reciprocity and dynamics of cooperation from an analytical / dynamical systems perspective. In this project you are invited to discuss two analytical models of indirect reciprocity extracting the phase diagrams for 2 famous social norms (image-scoring and standing). See [33] and references within.

Biodiversity, altruism, fairness and prosocial behaviors in space

33. Rock-Paper-Scissors in a Petri dish. The struggle for survival between competing bacteria species in a petri dish can create beautiful spiral patterns, until one type conquers all the available space, destroying the entangled structure. Kerr et al. made these observations when they mixed three E. coli populations exhibiting cyclic dominance [35, 36]: A beats B beats C beats A. In this project you should follow [36, 37] showing that the mobility of the bacteria, that is, their spatial relocation rate, is crucial for the stability of coexistence. You may also resort to [38], the Wolfram Demonstration Project associated with the above mentioned papers (involves numerical integration of differential equations).

34. Spatial dynamics of cooperation. Spatial constraints have been shown to deeply influence self-organized behavior. In this project, it is expected that you reproduce the classical results obtained by Nowak & May in Nature paper from 1992 (see Ref. [39]). If you like this type of challenge, this project begs for a nice 2D visualization of a computer simulation of evolution of traits on lattices.

35. Spatial dynamics in co-existence games. Project similar to the previous one, but with a different social dilemma (the Chicken or Snowdrift game [40]). In this case, you will be dealing with a co-existence dynamics and you're expected to replicate the results obtained in Ref. [41]. Again, this project begs for a nice 2D visualization of a computer simulation of the evolution of traits on lattices.

36. Even if social structure may sometimes promote cooperation, not all agents have access to the same level of resources. In this project, following Ref. [42], you shall investigate (through computer simulations) how inequality of resources among agents influence the emergence of cooperation.

37. The evolution of fairness and the ultimatum game (version 1) [43-45]: In the ultimatum game, two players are asked to split a certain sum of money. The proposer has to make an offer. If the responder accepts the offer, the money will be shared accordingly. If the responder rejects the offer, both players receive nothing. The rational solution is for the proposer to offer the smallest possible share, and for the responder to accept it. Human players, in contrast, usually prefer fair splits. In this project you shall investigate (through computer simulations) how a spatial setting drives (or not) evolution towards widespread fairness (for more information see [46]). If possible explore the possibility of having punishment of low offers and its impact in the evolution of fairness.

Cooperation in networked populations

38. Evolution of cooperation in networked populations through computer simulations. Consider a simple Prisoner's dilemma game [47](T, R=1, P=0, S), studied in the framework of evolutionary game theory [48]. Compute the stationary fraction of cooperators for lattices, random graphs and scale-free networks. Suggested reading list: Ref. [9] (section 10.5), and Ref. [49].

39. Evolution of cooperation in temporal networks. The structure of social networks is a key determinant in fostering cooperation and other altruistic behavior among naturally selfish individuals. However, most real social interactions are temporal, being both finite in duration and spread out over time. In this project you shall investigate the impact of temporal patterns on self-organized cooperation. Suggested reading: [50].

40. Collective action in heterogeneous political networks. Analyze the impact of structural diversity in the evolution of cooperative behavior in political networks. Check Fig. 3 in Ref. [51] and extend the methodology to other classes of N-person games (N-person Snowdrift-Game and N-person Stag-hunt game). [Involves computer simulations]

41. Develop a computer model to analyze the role of punishment and reputation in spatial public goods games (graph=lattice) (please follow [52]). Extend it to other networks if you have the time.

42. The ultimatum game (2) [43-45]: Following the ultimatum game (1) proposal suggested above, here you are expected to implement the ultimatum game on scale-free networks and analyze the levels of fairness obtained (you may follow the approach proposed in Ref. [53]) [Involves computer simulations].

43. The ultimatum game (3) [43-45]: Following the ultimatum game (1) proposal suggested above, here you are expected to analyze the emergence of fairness in populations of agents interacting following a N-person version of the ultimatum game. Instead of having 2 individuals, proposal are made to a collective of individuals who may collectively accept and reject an offer. This type of situations are similar to the ones observed in international negotiations [Involves computer simulations].

44. Information sharing, interdependent networks and prosocial behaviors. Implement a computer simulation with two interdependent lattices (you may extend it to a general network). The goal is to assess the impact of information sharing about strategy choice between players residing on two different networks on the evolution of cooperation. See [54] for further details.

45. Leadership and conformism in social dilemmas. Develop a computer simulation in order to assess the role of conformism in social dilemmas played in heterogeneous networks. You may follow the recent discussion presented in Ref. [55].

46. Try to implement a simple computer model of co-evolution of strategies and network structure. For an agent-based example see Ref. [56].

47. Try to implement a simple analytical model of co-evolution of strategies and network structure. Discuss the "Active Linking" model introduced in Ref. [57] and address the game transformations also discussed in the same paper.

Collective action, public goods, and sanctioning

48. Collective action in well-mixed populations (analytical project). Compute analytically (through, for instance, a *Mathematica* notebook) the gradient of selection for N-person games with thresholds. Check the following two refs: [58, 59].

49. A project in ecological economics: Tourists and traditional divers in a common fishing ground (try to reproduce analytically or by means of computer simulations some of the results in Ref. [60]). Discuss the general approach to the problem.

50. Cooperation and Ostracism. Analyze a game-theoretical model of social exclusion in which a punishing cooperator can exclude free-riders from benefit sharing. You may opt to repeat the calculations described in Ref. [61] or propose a computer model replicating the same idea.

51. Social diversity and Public goods games in complex networks. Repeat the computer simulations proposed in Ref. [62] and discuss the results obtained.

52. Game theory has been used to investigate possible climate negotiation solutions and strategies for accomplishing them. Here you shall analyze the evolutionary dynamics of cooperation in climate dilemmas through a stochastic process (see ref. [51]). You shall identify the role or risk, group size and diversity in the chances of reaching to cooperation. This project can be dealt both through computer simulations and analytically, computing a stationary distribution of a Markov chain.

53. Game theory has been used to investigate possible climate negotiation solutions and strategies for accomplishing them. Here we you shall implement an N-player bargaining game in an agent-based model (see here [63]) to examine the past failures of and future prospects for a robust international climate agreement. This project involves computer simulations.

54. Graduated punishment (analytical project). Following [64], you shall discuss/present a recent analytical model addressing Elinor Ostrom's [65] idea that successful cooperative communities apply graduated punishment, i.e., that the punishment level gradually increases with the amount of harm of the selfish action.

55. Spatial dynamics of ecological public goods. The production, consumption, and exploitation of common resources ranging from extracellular products in microorganisms to global issues of climate change refer to public goods interactions. In this project you shall study the diffusion of cooperation taking into account that benefits of the common resource enable cooperators to maintain higher population densities. This leads to a natural feedback between population dynamics and interaction group sizes as captured by ”ecological public goods”. You are invited to follow the formulation described in ref. [66] in terms of continuous space based on partial differential equations (PDE), allowing for a deep understanding of the relevant mechanisms that drive the spontaneous generation of spatial heterogeneity and temporal fluctuations, which may be responsible for supporting cooperation in spatial settings.

Rumor spreading & peer-influence

56. Explore how (dynamic) structure of social networks affect peer influence and sentiment about topics. Data from Twitter is available for this suggestion.

57. Rumor and information spreading in complex networks (analytical or numerical simulations). Implement one the models discussed in [9](see section 10.2) on a network. Discuss the efficiency of rumor spreading in 3 network classes (e.g., WS strogatz, BA model and Minimal model). As an alternative, discuss the impact of community structures in the overall efficiency of information spreading.

58. Rumor and information spreading in complex networks (analytical or numerical simulations). Implement one the variants of the Voter model discussed in [9] (see section 10.3).

59. The origin of 3-degrees of influence. In 2007 it was found that social influence does not end with the people to whom a person is directly tied [67, 68]. We influence our friends, who in their turn influence their friends, and so our actions can influence people we have never met, to whom we are only indirectly tied. As Fowler and Christakis, the authors of this idea posit, "ripple through our network, having an impact on our friends (one degree), our friends' friends (two degrees), and even our friends' friends' friends (three degrees). Our influence gradually dissipates and ceases to have a noticeable effect on people beyond the social frontier that lies at three degrees of separation". Implement the Voter Model in a random graph. Analyze the emergent correlations among nodes at different distances and compare it with the correlations obtained in the random case. Try to understand the emergence of the phenomena through this very simple model. For details please follow [69].

Spatial and ad-hoc networks

60. Discuss the possibility of having scale-free spatial networks (see [70, 71]) and implement one these models through computer simulations or discuss the models proposed in refs. [70, 72]. For a review on spatial network please check ref. [73].

61. Networks and population of robots. Consider a spatial network of moving agents. Implement a computer simulation where a scale-free ad-hoc network emerges from such interacting system.

Other topics

62. Follow Evelina Gabasova's footsteps [74] and analyze the Star Wars social network.

63. Propose your own idea and let us know about it. If you are already involved in a particular project (say, MSc or PhD thesis) related with complex systems or network science, please let us know. You may try to use part of your work for this 2nd project.

64. Continue to explore interesting challenges identified in your 1st project.

References

1. Stumpf, M.P., C. Wiuf, and R.M. May, *Subnets of scale-free networks are not scale-free: sampling properties of networks.* Proc Natl Acad Sci USA, 2005. **102**(12): p. 4221-4224 Available from: http://www.pnas.org/content/102/12/4221.full.pdf.
2. Wikipedia. *Traceroute*. Available from: https://en.wikipedia.org/wiki/Traceroute.
3. Achlioptas, D., A. Clauset, D. Kempe, and C. Moore, *On the bias of traceroute sampling: or, power-law degree distributions in regular graphs.* Journal of the ACM (JACM), 2009. **56**(4): p. 21 Available from.
4. Clauset, A. and C. Moore, *Traceroute sampling makes random graphs appear to have power law degree distributions.* arXiv preprint cond-mat/0312674, 2003 Available from.
5. Clauset, A. and C. Moore, *Accuracy and scaling phenomena in Internet mapping.* Physical Review Letters, 2005. **94**(1): p. 018701 Available from.
6. Motter, A.E. and Y.-C. Lai, *Cascade-based attacks on complex networks.* Physical Review E, 2002. **66**(6): p. 065102 Available from.
7. Pastor-Satorras, R., C. Castellano, P. Van Mieghem, and A. Vespignani, *Epidemic processes in complex networks.* Reviews of modern physics, 2015. **87**(3): p. 925 Available from: https://www.nas.ewi.tudelft.nl/people/Piet/papers/RMP2015_EpidemicsReview.pdf.
8. Pastor-Satorras, R. and A. Vespignani, *Epidemic spreading in scale-free networks.* Physical Review Letters, 2001. **86**(14): p. 3200 Available from: http://www-fen.upc.es/~romu/Papers/virus.pdf.
9. Barrat, A., M. Barthelemy, and A. Vespignani, *Dynamical processes on complex networks*. 2008: Cambridge University Press.
10. Barabási, A.-L., *Network science*. 2016: Cambridge University Press Cambridge.
11. Wikipedia. *Xulvi-Brunet - Sokolov algorithm*. Available from: https://en.wikipedia.org/wiki/Xulvi-Brunet_-_Sokolov_algorithm.
12. Cohen, R., S. Havlin, and D. Ben-Avraham, *Efficient immunization strategies for computer networks and populations.* Physical Review Letters, 2003. **91**(24): p. 247901 Available from: http://people.clarkson.edu/~dbenavra/paper/103.pdf.

13. Schneider, C.M., T. Mihaljev, S. Havlin, and H.J. Herrmann, *Suppressing epidemics with a limited amount of immunization units.* Physical Review E, 2011. **84**(6): p. 061911 Available from: http://polymer.bu.edu/~hes/networks/smhh11.pdf.
14. Holme, P. and N. Litvak, *Cost-efficient vaccination protocols for network epidemiology.* PLoS Computational Biology, 2017. **13**(9): p. e1005696 Available from.
15. Keeling, M.J. and P. Rohani, *Modeling infectious diseases in humans and animals*. 2008: Princeton University Press.
16. Barabasi, A.-L., *The origin of bursts and heavy tails in human dynamics.* Nature, 2005. **435**(7039): p. 207-211 Available from.
17. Rocha, L.E. and V.D. Blondel, *Bursts of vertex activation and epidemics in evolving networks.* PLoS Comput Biol, 2013. **9**(3): p. e1002974 Available from.
18. Masuda, N. and P. Holme, *Predicting and controlling infectious disease epidemics using temporal networks.* F1000 prime reports, 2013. **5**: p. 6 Available from.
19. Gonzalez, M.C., C.A. Hidalgo, and A.-L. Barabasi, *Understanding individual human mobility patterns.* Nature, 2008. **453**(7196): p. 779-782 Available from.
20. Rocha, L.E., F. Liljeros, and P. Holme, *Simulated epidemics in an empirical spatiotemporal network of 50,185 sexual contacts.* PLoS computational biology, 2011. **7**(3): p. e1001109 Available from.
21. Takaguchi, T., N. Masuda, and P. Holme, *Bursty communication patterns facilitate spreading in a threshold-based epidemic dynamics.* PloS one, 2013. **8**(7): p. e68629 Available from.
22. Karsai, M., M. Kivelä, R.K. Pan, K. Kaski, J. Kertész, A.-L. Barabási, and J. Saramäki, *Small but slow world: How network topology and burstiness slow down spreading.* Physical Review E, 2011. **83**(2): p. 025102 Available from.
23. Van Segbroeck, S., F.C. Santos, and J.M. Pacheco, *Adaptive contact networks change effective disease infectiousness and dynamics.* PLoS Comput Biol, 2010. **6**(8): p. e1000895 Available from: http://journals.plos.org/ploscompbiol/article?id=10.1371/journal.pcbi.1000895.
24. Gao, D., Y. Lou, D. He, T.C. Porco, Y. Kuang, G. Chowell, and S. Ruan, *Prevention and control of Zika as a mosquito-borne and sexually transmitted disease: a mathematical modeling analysis.* Scientific reports, 2016. **6**: p. 28070 Available from.
25. Van den Broeck, W., C. Gioannini, B. Gonçalves, M. Quaggiotto, V. Colizza, and A. Vespignani, *The GLEaMviz computational tool, a publicly available software to explore realistic epidemic spreading scenarios at the global scale.* BMC infectious diseases, 2011. **11**(1): p. 1 Available from.
26. McCown, F. *Schelling's Model of Segregation*. Available from: http://nifty.stanford.edu/2014/mccown-schelling-model-segregation/.
27. Easley, D. and J. Kleinberg, *Networks, crowds, and markets: Reasoning about a highly connected world*. 2010: Cambridge University Press.
28. Schelling, T.C., *Dynamic models of segregation.* Journal of Mathematical Sociology, 1971. **1**(2): p. 143-186 Available from: http://norsemathology.org/longa/classes/stuff/DynamicModelsOfSegregation.pdf.
29. Hart, V. and N. Case. *Parable of the polygons: A playable post on the shape of society*. Available from: http://ncase.me/polygons/.
30. Wikipedia. *Fixation Probability (population genetics)*. Available from: https://en.wikipedia.org/wiki/Fixation_(population_genetics).
31. Traulsen, A. and M.A. Nowak, *Evolution of cooperation by multilevel selection.* Proc Natl Acad Sci USA, 2006. **103**(29): p. 10952-10955 Available from: http://web.evolbio.mpg.de/~traulsen/paper/10.pdf.
32. Nowak, M.A. and K. Sigmund, *Evolution of indirect reciprocity by image scoring.* Nature, 1998. **393**(6685): p. 573-577 Available from: http://estamine.net/fc0809/Nature98.pdf.
33. Nowak, M.A. and K. Sigmund, *Evolution of indirect reciprocity.* Nature, 2005. **437**(7063): p. 1291-1298 Available from.
34. Leimar, O. and P. Hammerstein, *Evolution of cooperation through indirect reciprocity.* Proceedings of the Royal Society of London B: Biological Sciences, 2001. **268**(1468): p. 745-753 Available from.
35. Kerr, B., M.A. Riley, M.W. Feldman, and B.J. Bohannan, *Local dispersal promotes biodiversity in a real-life game of rock–paper–scissors.* Nature, 2002. **418**(6894): p. 171-174 Available from.
36. Reichenbach, T., M. Mobilia, and E. Frey, *Mobility promotes and jeopardizes biodiversity in rock–paper–scissors games.* Nature, 2007. **448**(7157): p. 1046-1049 Available from.
37. Reichenbach, T., M. Mobilia, and E. Frey, *Self-organization of mobile populations in cyclic competition.* Journal of Theoretical Biology, 2008. **254**(2): p. 368-383 Available from.
38. Weidl, M. *Biodiversity in Spatial Rock-Paper-Scissors Games*. Available from: Wolfram Demonstrations Project http://demonstrations.wolfram.com/BiodiversityInSpatialRockPaperScissorsGames/.
39. Nowak, M.A. and R.M. May, *Evolutionary games and spatial chaos.* Nature, 1992. **359**(6398): p. 826-829 Available from: http://ped.fas.harvard.edu/files/ped/files/nature92_0.pdf.
40. Wikipedia. *Chicken or Snowdrift Game*. Available from: https://en.wikipedia.org/wiki/Chicken_(game).
41. Hauert, C. and M. Doebeli, *Spatial structure often inhibits the evolution of cooperation in the snowdrift game.* Nature, 2004. **428**(6983): p. 643-646 Available from: http://www.math.ubc.ca/~hauert/publications/reprints/hauert_nature04.pdf.
42. Kun, Á. and U. Dieckmann, *Resource heterogeneity can facilitate cooperation.* Nature communications, 2013. **4** Available from.
43. Wikipedia. *Ultimatum game*. Available from: https://en.wikipedia.org/wiki/Ultimatum_game.
44. Fehr, E. and U. Fischbacher, *The nature of human altruism.* Nature, 2003. **425**(6960): p. 785-791 Available from.
45. Sigmund, K., E. Fehr, and M.A. Nowak, *The economics of fair play.* SCIENTIFIC AMERICAN-AMERICAN EDITION-, 2002. **286**(1): p. 80-85 Available from.
46. Page, K.M., M.A. Nowak, and K. Sigmund, *The spatial ultimatum game.* Proceedings of the Royal Society of London B: Biological Sciences, 2000. **267**(1458): p. 2177-2182 Available from.
47. Wikipedia. *Prisoner's Dilemma*. Available from: https://en.wikipedia.org/wiki/Prisoner%27s_dilemma.
48. Wikipedia. *Evolutionary Game Theory*. Available from: https://en.wikipedia.org/wiki/Evolutionary_game_theory.

49. Santos, F.C., J.M. Pacheco, and T. Lenaerts, *Evolutionary dynamics of social dilemmas in structured heterogeneous populations.* Proc Natl Acad Sci USA, 2006. **103**(9): p. 3490-3494 Available from: http://web.ist.utl.pt/franciscocsantos/MyArticles/SantosPachecoLenaerts.PNAS2006.pdf.
50. Li, A., L. Zhou, Q. Su, S.P. Cornelius, Y.-Y. Liu, and L. Wang, *Evolution of Cooperation on Temporal Networks.* arXiv preprint arXiv:1609.07569, 2016 Available from.
51. Santos, F.C. and J.M. Pacheco, *Risk of collective failure provides an escape from the tragedy of the commons.* Proc Natl Acad Sci USA, 2011. **108**(26): p. 10421-10425 Available from: http://web.ist.utl.pt/franciscocsantos/MyArticles/FCSantos-JMPacheco-PNAS2011.pdf.
52. Brandt, H., C. Hauert, and K. Sigmund, *Punishment and reputation in spatial public goods games.* Proceedings of the Royal Society of London B: Biological Sciences, 2003. **270**(1519): p. 1099-1104 Available from: http://homepage.univie.ac.at/Karl.Sigmund/punspatialprocb03.pdf.
53. Sinatra, R., J. Iranzo, J. Gomez-Gardenes, L.M. Floria, V. Latora, and Y. Moreno, *The ultimatum game in complex networks.* Journal of Statistical Mechanics: Theory and Experiment, 2009. **2009**(09): p. P09012 Available from: http://complex.unizar.es/~jesus/pub_files/Jstat09.pdf.
54. Szolnoki, A. and M. Perc, *Information sharing promotes prosocial behaviour.* New Journal of Physics, 2013. **15**(5): p. 053010 Available from: http://iopscience.iop.org/article/10.1088/1367-2630/15/5/053010/meta.
55. Szolnoki, A. and M. Perc, *Leaders should not be conformists in evolutionary social dilemmas.* Scientific reports, 2016. **6** Available from: http://www.ncbi.nlm.nih.gov/pmc/articles/PMC4804302/.
56. Santos, F.C., J.M. Pacheco, and T. Lenaerts, *Cooperation prevails when individuals adjust their social ties.* PLoS Comput Biol, 2006. **2**(10): p. e140 Available from: http://journals.plos.org/ploscompbiol/article?id=10.1371/journal.pcbi.0020140.
57. Pacheco, J.M., A. Traulsen, and M.A. Nowak, *Coevolution of strategy and structure in complex networks with dynamical linking.* Physical review letters, 2006. **97**(25): p. 258103 Available from: http://web.evolbio.mpg.de/~traulsen/paper/13.pdf.
58. Pacheco, J.M., F.C. Santos, M.O. Souza, and B. Skyrms, *Evolutionary dynamics of collective action in N-person stag hunt dilemmas.* Proceedings of the Royal Society of London B: Biological Sciences, 2009. **276**(1655): p. 315-321 Available from: http://web.ist.utl.pt/franciscocsantos/MyArticles/Pachecoetal.PRSB08FirstCite.pdf.
59. Souza, M.O., J.M. Pacheco, and F.C. Santos, *Evolution of cooperation under N-person snowdrift games.* Journal of Theoretical Biology, 2009. **260**(4): p. 581-588 Available from: http://web.ist.utl.pt/franciscocsantos/MyArticles/SPS.JTB2009.pdf.
60. Lee, J.-H. and Y. Iwasa, *Tourists and traditional divers in a common fishing ground.* Ecological Economics, 2011. **70**(12): p. 2350-2360 Available from.
61. Sasaki, T. and S. Uchida, *The evolution of cooperation by social exclusion.* Proceedings of the Royal Society of London B: Biological Sciences, 2013. **280**(1752): p. 20122498 Available from: http://izt.ciens.ucv.ve/ecologia/Archivos/ECO_POB 2013/ECOPO2_2013/Sasiki y Uchida 2013.pdf.
62. Santos, F.C., M.D. Santos, and J.M. Pacheco, *Social diversity promotes the emergence of cooperation in public goods games.* Nature, 2008. **454**(7201): p. 213-216 Available from.
63. Smead, R., R.L. Sandler, P. Forber, and J. Basl, *A bargaining game analysis of international climate negotiations.* Nature Climate Change, 2014. **4**(6): p. 442-445 Available from.
64. Iwasa, Y. and J.-H. Lee, *Graduated punishment is efficient in resource management if people are heterogeneous.* Journal of theoretical biology, 2013. **333**: p. 117-125 Available from.
65. Wikipedia. *Elinor Ostrom*. Available from: https://en.wikipedia.org/wiki/Elinor_Ostrom.
66. Wakano, J.Y., M.A. Nowak, and C. Hauert, *Spatial dynamics of ecological public goods.* Proc Natl Acad Sci USA, 2009. **106**(19): p. 7910-7914 Available from.
67. Christakis, N.A. and J.H. Fowler, *The spread of obesity in a large social network over 32 years.* New England journal of medicine, 2007. **357**(4): p. 370-379 Available from.
68. Christakis, N.A. and J.H. Fowler, *The collective dynamics of smoking in a large social network.* New England journal of medicine, 2008. **358**(21): p. 2249-2258 Available from.
69. Pinheiro, F.L., M.D. Santos, F.C. Santos, and J.M. Pacheco, *Origin of peer influence in social networks.* Physical Review Letters, 2014. **112**(9): p. 098702 Available from: http://web.ist.utl.pt/franciscocsantos/MyArticles/PinheiroSantosSantosPacheco.PRL2014.pdf.
70. Andrade Jr, J.S., H.J. Herrmann, R.F. Andrade, and L.R. Da Silva, *Apollonian networks: Simultaneously scale-free, small world, euclidean, space filling, and with matching graphs.* Physical Review Letters, 2005. **94**(1): p. 018702 Available from: http://www.repositorio.ufba.br:8080/ri/bitstream/ri/6609/1/(174).pdf.
71. Wikipedia-Appolonian-Network. *Appolonian Network*. Available from: https://en.wikipedia.org/wiki/Apollonian_network.
72. Rozenfeld, A.F., R. Cohen, D. Ben-Avraham, and S. Havlin, *Scale-free networks on lattices.* Physical Review Letters, 2002. **89**(21): p. 218701 Available from: http://polymer.bu.edu/~hes/networks/rcbh02.pdf.
73. Barthélemy, M., *Spatial networks.* Physics Reports, 2011. **499**(1): p. 1-101 Available from: http://scf.berkeley.edu/~aldous/206-SNET/Papers/barthelemy_survey.pdf.
74. Gabasova, E. *The Star Wars social network*. 2015; Available from: http://evelinag.com/blog/2016/01-25-social-network-force-awakens/index.html - .V_PmlrwrLVF.