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1. Demuestre la fórmula alternativa para la estimación de la segunda derivada discreta

$$\frac{d^2 f(x_i)}{dx^2} = \frac{f(x_{i+2}) - 2f(x_i) + f(x_{i-2})}{4h^2}$$

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h}$$

$$f''(x) = \frac{f'(x+h) - f(x-h)}{2h}$$

$$f''(x) = \frac{\frac{f(x+2h) - f(x)}{2h} - \frac{f(x) - f(x-2h)}{2h}}{2h}$$

$$f''(x) = \frac{f(x+2h) - 2f(x) + f(x-2h)}{4h^2}$$

4a. Dar la expresión matemática:

$$D^2 f(x) = \frac{1}{h^2} \sum_{m=-1}^{1} M[m+1]\, f(x_{n-m}) \quad ; \quad M = [1, -2, 1]$$

5. $$D^4 f(x_j) \simeq \frac{f(x_{j+2}) - 4f(x_{j+1}) + 6f(x_j) - 4f(x_{j-1}) + f(x_{j-2})}{h^4}$$

$$f(x+2h) = f(x) + 2hf'(x) + \frac{4h^2}{2} f''(x) + \frac{8h^3}{3!} f^{(3)}(x) + \frac{16h^4}{4!} f^{(4)}(x) + O(h^5)$$

$$f(x-2h) = f(x) - 2hf'(x) + \frac{4h^2}{2} f^{(2)}(x) - \frac{8h^3}{3!} f^{(3)}(x) + \frac{16h^4}{4!} f^{(4)}(x) + O(h^5)$$

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2} f^{(2)}(x) + \frac{h^3}{3!} f^{(3)}(x) + \frac{h^4}{4!} f^{(4)}(x) + O(h^5)$$

$$f(x-h) = f(x) - hf'(x) + \frac{h^2}{2} f^{(2)}(x) - \frac{h^3}{3!} f^{(3)}(x) + \frac{h^4}{4!} f^{(4)}(x) + O(h^5)$$

$$f(x+2h) + f(x-2h) = 2f(x) + 4h^2 f^{(2)}(x) + \frac{4}{3} h^4 f^{(4)}(x) + \frac{8}{45} h^6 f^{(6)}(x) + O(h^8)$$

$$4f(x+h) + 4f(x-h) = 8f(x) + 4h^2 f^{(2)}(x) + \frac{1}{3} h^4 f^{(4)}(x) + \frac{1}{90} h^6 f^{(6)}(x) + O(h^8)$$

$$f(x+2h)+f(x-2h)-4f(x+h)-4f(x-h)=-6f(x)+h^4f^{(4)}(x)+\frac{17}{90}h^6f^{(6)}(x)+O(h^8)$$

$$f^{(4)}(x)=\frac{f(x+2h-4f(x+h)+6f(x)-4f(x-h)+f(x-2h)}{h^4}-O(h^2)$$

El orden de la aproximación es de $O(h^2)$