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conjunto soporte

$$\Omega = \{(x_0, f(x_0)), (x_1, f(x_1)), (x_2, f(x_2))\}$$

$$\{(x, f(x)), (x+h, f(x+h)), (x+2h, f(x+2h))\}$$

a) Calcular analíticamente el polinomio que interpola el conjunto soporte.

$(x_0, y_0), \dots, (x_k, y_k)$

$$L(x) = \sum_{j=0}^{k} y_j \, \ell_j(x)$$

Polinomio interpolador combinación lineal de los L_i

$$L_0(x) = \frac{x - x_1}{x_0 - x_1} \cdot \frac{x - x_2}{x_0 - x_2}$$

$$L_1(x) = \frac{x - x_0}{x_1 - x_0} \cdot \frac{x - x_2}{x_1 - x_2}$$

$$C_k = \frac{y_k - P_{k-1}(x_k)}{(x_k - x_0)(x_k - x_1) \cdots (x_k - x_{k-1})}$$

$$L_2(x) = \frac{x - x_0}{x_2 - x_0} \cdot \frac{x - x_1}{x_2 - x_1}$$

$$p(x) = f(x_0)\left(\frac{x - x_1}{x_0 - x_1} \cdot \frac{x - x_2}{x_0 - x_2}\right) + f(x_1)\left(\frac{x - x_0}{x_1 - x_0} \cdot \frac{x - x_2}{x_1 - x_2}\right) + f(x_2)\left(\frac{x - x_0}{x_2 - x_0} \cdot \frac{x - x_1}{x_2 - x_1}\right)$$

$$p_0(x) = C_0$$
$$p_1(x) = C_0 + C_1(x - x_0)$$
$$p_2(x) = C_0 + C_1(x - x_0) + C_2(x - x_0)(x - x_1)$$

$$C_0 = f(x_0)$$
$$C_1 = \frac{f(x_1) - f(x_0)}{(x_1 - x_0)}$$
$$C_2 = \frac{f(x_2) - P_1(x_2)}{(x_2 - x_0)(x_2 - x_1)}$$

$$p_2(x) = f(x_0) + \left(\frac{f(x_1) - f(x_0)}{(x_1 - x_0)}\right)(x - x_0) + \left(\frac{f(x_2) - p_1(x_2)}{(x_2 - x_0)(x_2 - x_1)}\right)(x - x_0)(x - x_1)$$

$$p(x) = \frac{f(x_0)}{(x_0 - x_1)(x_0 - x_2)}(x - x_1)(x - x_2) + \frac{f(x_1)}{(x_1 - x_0)(x_1 - x_2)}(x - x_0)(x - x_2) + \frac{f(x_2)}{(x_2 - x_0)(x_2 - x_1)}(x - x_0)(x - x_1)$$

6. Derivar el polinomio interpolador para encontrar la derivada en el punto x_0:

$$f'(x_0) \approx p'(x_0) = \frac{1}{2h}\left(-3f(x_0) + 4f(x_1) - f(x_2)\right)$$

si la discretización es equidistante, tenemos:

$$f'(x) \cong \frac{1}{2h}\left(-3f(x) + 4f(x+h) - f(x+2h)\right)$$

$$\frac{f(x_0)}{\quad}$$

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