Pablo Salazar 202124801
Luisa Silva 202124806

7. c. Muestre que las derivadas parciales de la métrica están dadas por

$$\frac{\partial \chi^2(\theta)}{\partial \theta_i} = -2 \sum_{i=1}^{N} (y_i - M(x_i, \vec{\theta})) \frac{\partial M(x_i, \vec{\theta})}{\partial \theta_i}$$

$$\chi^2(\vec{\theta}) = \sum_{i=1}^{n} \left(\frac{y_i - M(x_i, \vec{\theta})}{\partial \theta_i} \right)^2$$

$$\frac{\partial \chi^2(\vec{\theta})}{\partial \theta} \; 2 \left(y_i - M(x_i, \vec{\theta})\right)\left(- \frac{\partial M(x_i, \vec{\theta})}{\partial \theta_i}\right)$$

$$\frac{\partial \chi^2(\vec{\theta})}{\partial \theta_i} = -2 \left(y - M(x_i, \vec{\theta})\right) \left(\frac{\partial M(x_i, \vec{\theta})}{\partial \theta_i} \right)$$

d. Muestre que vectorialmente, el descenso del gradiente queda definido por:

$$\theta_{j+1} = \vec{\theta}_j - \gamma \left(-2 \sum_{i=1}^{N} (y_i - M(x_i - \vec{\theta}_j)) \nabla_\theta M(x_i, \vec{\theta}_j) \right)$$

donde el gradiente es respecto a los parámetros:

$$\nabla_\theta M(x_i, \vec{\theta}) = \left[\frac{\partial M(x_i, \vec{\theta})}{\partial \theta_0}, \frac{\partial M(x_i, \vec{\theta})}{\partial \theta_1}, \frac{\partial M(x_i, \vec{\theta})}{\partial \theta_2} \right]$$

$$\vec{x}^{\,1} = \vec{x}^{\,0} - \gamma \nabla \vec{F}(\vec{x}_0)$$

definición descenso de gradiente

$$\vec{x}_{j+1} = x_j - \gamma \underbrace{\nabla \vec{F}(\vec{x}_j)}_{\frac{\partial \chi^2(\vec{\theta})}{\partial \theta_i}}$$

$$\vec{\theta}_{j+1} = \theta_j - \gamma \nabla \frac{\partial \chi^2(\vec{\theta})}{\partial \theta_i} \rightarrow \vec{\theta}_{j+1} = \vec{\theta}_j - \gamma \left[-2 \left(y_i - M(x_i, \vec{\theta})\right) \left(\frac{\partial M(x_i, \vec{\theta})}{\partial \theta_i} \right) \right]$$

dado que se tiene $\theta_0, \theta_1, \theta_2 \rightarrow$ 3 derivadas parciales

por lo que :

$$\theta_{j+1} = \vec{\theta}_j - \gamma \left(-2 \sum_{i=1}^{N} (y_i - M(x_i - \vec{\theta}_j)) \nabla_\theta M(x_i, \vec{\theta}_j) \right)$$