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Taller 4.

1. Hacer pasos intermedios para regla de trapecio simple.

$$I = \int_a^b f(x)dx \cong \int_a^b p_1(x)\,dx = \frac{b-a}{2}\left(f(a)+f(b)\right)$$

$$p_1(x) = \frac{x-b}{a-b} f(a) + \frac{x-a}{b-a} f(b)$$

$$I = \int_a^b f(x)dx \cong \int_a^b \left[\frac{x-b}{a-b} f(a) + \frac{x-a}{b-a} f(b)\right] dx$$

$$= \int_a^b \frac{x-b}{a-b} f(a)\,dx + \int_a^b \frac{x-a}{b-a} f(b)\,dx$$

$$= \frac{f(a)}{a-b}\int_a^b (x-b)\,dx + \frac{f(b)}{b-a}\int_a^b (x-a)\,dx$$

$$= \frac{f(a)}{a-b}\left[\frac{x^2}{2} - bx\right]_a^b + \frac{f(b)}{b-a}\left[\frac{x^2}{2} - ax\right]_a^b$$

$$= \frac{f(a)}{a-b}\left[\left(\frac{b^2}{2} - b^2\right) - \left(\frac{a^2}{2} - ba\right)\right] + \frac{f(b)}{b-a}\left[\left(\frac{b^2}{2} - ab\right) - \left(\frac{a^2}{2} - a^2\right)\right]$$

$$= \frac{f(a)}{a-b}\left[\frac{b^2}{2} - b^2 - \frac{a^2}{2} + ba\right] + \frac{f(b)}{b-a}\left[\frac{b^2}{2} - ab - \frac{a^2}{2} + a^2\right]$$

$$= \frac{f(a)}{a-b}\left[-\frac{b^2}{2} + ba - \frac{a^2}{2}\right] + \frac{f(b)}{b-a}\left[\frac{b^2}{2} - ab + \frac{a^2}{2}\right]$$

$$= -\frac{f(a)}{a-b}\left[\frac{b^2}{2} - ba + \frac{a^2}{2}\right] + \frac{f(b)}{b-a}\left[\frac{b^2}{2} - ab + \frac{a^2}{2}\right]$$

$$= -\frac{f(a)}{a-b}\left[\frac{b^2 - 2ab + a^2}{2}\right] + \frac{f(b)}{b-a}\left[\frac{b^2 - 2ab + a^2}{2}\right]$$

$$= -\frac{f(a)}{(a-b)}\frac{(b-a)^2}{2} + \frac{f(b)}{(b-a)}\frac{(b-a)^2}{2}$$

$$= \frac{f(a)}{\cancel{(b-a)}} \frac{(b-a)^{\cancel{2}}}{2} + \frac{f(b)}{\cancel{(b-a)}} \frac{(b-a)^{\cancel{2}}}{2}$$

$$I \cong \frac{(b-a)}{2} [f(a) + f(b)]$$

2. Encontrar el error para regla de trapecio simple

$$E = \int_a^b \epsilon(x)\,dx = -\frac{h^3}{12} f''(\xi)$$

El error en la estimación está asociado al proceso de interpolación. Suponiendo que $f(x)$ es continua y derivable de clase C^2 en el intervalo $[a, b]$:

$f(x) = p_1(x) + \epsilon(x)$ donde $\epsilon(x) = \frac{f''(\xi)}{2}(x-a)(x-b)$, $a \leq \xi \leq b$

$$E = \int_a^b \epsilon(x)\,dx = \int_a^b \frac{f''(\xi)}{2}(x-a)(x-b)\,dx$$

$$= \frac{f''(\xi)}{2} \int_a^b (x-a)(x-b)\,dx$$

$$= \frac{f''(\xi)}{2} \int_a^b (x^2 - bx - ax + ab)\,dx$$

$$= \frac{f''(\xi)}{2} \left[\frac{x^3}{3} - \frac{bx^2}{2} - \frac{ax^2}{2} + abx \right]_a^b$$

$$= \frac{f''(\xi)}{2} \left[\left(\frac{b^3}{3} - \frac{b^3}{2} - \frac{ab^2}{2} + ab^2 \right) - \left(\frac{a^3}{3} - \frac{ba^2}{2} - \frac{a^3}{2} + a^2 b \right) \right]$$

$$= \frac{f''(\xi)}{2} \left[\frac{2b^3 - 3b^3 - 3ab^2 + 6ab^2 - 2a^3 + 3ba^2 + 3a^3 - 6a^2b}{6} \right]$$

$$= \frac{f''(\xi)}{2} \left[\frac{-b^3 + 3ab^2 + 3a^3 - 3a^2b}{6} \right]$$

$$= \frac{f''(\xi)}{2} \left[\frac{-(b^3 - 3ab^2 + 3a^2b - 3a^3)}{6} \right]$$

$h = b - a$

$$= \frac{f''(\xi)}{2} \left[\frac{-(b-a)^3}{6} \right]$$

$$= \frac{-h^3}{12} [f''(\xi)]$$