Funções trigonométricas, hiperbólicas e suas inversas

Funções trigonométricas

$\operatorname{sen}: \mathbb{R} \to \mathbb{R}$, $\quad \cos: \mathbb{R} \to \mathbb{R}$, $\quad \operatorname{tg}: \mathbb{R} \setminus \{\frac{\pi}{2} + k\pi : k \in \mathbb{Z}\} \to \mathbb{R}$,

$\operatorname{cotg}: \mathbb{R} \setminus \{k\pi : k \in \mathbb{Z}\} \to \mathbb{R}$, $\quad \sec: \mathbb{R} \setminus \{\frac{\pi}{2} + k\pi : k \in \mathbb{Z}\} \to \mathbb{R}$, $\quad \operatorname{cosec}: \mathbb{R} \setminus \{k\pi : k \in \mathbb{Z}\} \to \mathbb{R}$.

$$\operatorname{tg}(x) = \frac{\operatorname{sen} x}{\cos x}, \qquad \operatorname{cotg}(x) = \frac{\cos x}{\operatorname{sen} x}, \qquad \sec(x) = \frac{1}{\cos x}, \qquad \operatorname{cosec}(x) = \frac{1}{\operatorname{sen} x}.$$

Fórmulas importantes:

$\forall x \in \mathbb{R} \quad \operatorname{sen}^2 x + \cos^2 x = 1$, $\qquad \forall x \in \mathbb{R} \setminus \{\frac{\pi}{2} + k\pi : k \in \mathbb{Z}\} \quad 1 + \operatorname{tg}^2 x = \sec^2 x$,

$\forall x \in \mathbb{R} \quad \cos^2 x = \frac{1+\cos(2x)}{2}$, $\qquad \forall x \in \mathbb{R} \setminus \{k\pi : k \in \mathbb{Z}\} \quad 1 + \operatorname{cotg}^2 x = \operatorname{cosec}^2 x$,

$\forall x \in \mathbb{R} \quad \operatorname{sen}^2 x = \frac{1-\cos(2x)}{2}$.

Funções trigonométricas inversas

$\operatorname{arcsen}(x) = \left(\operatorname{sen}_{|[-\frac{\pi}{2}, \frac{\pi}{2}]} \right)^{-1}(x)$, $\qquad \arccos(x) = \left(\cos_{|[0,\pi]} \right)^{-1}(x)$

$\operatorname{arctg}(x) = \left(\operatorname{tg}_{|]-\frac{\pi}{2}, \frac{\pi}{2}[} \right)^{-1}(x)$, $\qquad \operatorname{arccotg}(x) = \left(\operatorname{cotg}_{|]0,\pi[} \right)^{-1}(x)$,

$\operatorname{arcsec}(x) = \left(\sec_{|[0, \frac{\pi}{2}[} \right)^{-1}(x)$, $\qquad \operatorname{arccosec}(x) = \left(\operatorname{cosec}_{|]0, \frac{\pi}{2}]} \right)^{-1}(x)$

Funções hiperbólicas

$$\begin{array}{rccc} \operatorname{sh}: & \mathbb{R} & \to & \mathbb{R} \\ & x & \mapsto & \frac{e^x - e^{-x}}{2} \end{array} \qquad \begin{array}{rccc} \operatorname{ch}: & \mathbb{R} & \to & \mathbb{R} \\ & x & \mapsto & \frac{e^x + e^{-x}}{2} \end{array} \qquad \begin{array}{rccc} \operatorname{th}: & \mathbb{R} & \to & \mathbb{R} \\ & x & \mapsto & \frac{e^x - e^{-x}}{e^x + e^{-x}} \end{array}$$

$$\begin{array}{rccc} \operatorname{coth}: & \mathbb{R} \setminus \{0\} & \to & \mathbb{R} \\ & x & \mapsto & \frac{e^x + e^{-x}}{e^x - e^{-x}} \end{array} \qquad \begin{array}{rccc} \operatorname{sech}: & \mathbb{R} & \to & \mathbb{R} \\ & x & \mapsto & \frac{2}{e^x + e^{-x}} \end{array} \qquad \begin{array}{rccc} \operatorname{cosech}: & \mathbb{R} & \to & \mathbb{R} \\ & x & \mapsto & \frac{2}{e^x - e^{-x}} \end{array}$$

$$\operatorname{th}(x) = \frac{\operatorname{sh} x}{\operatorname{ch} x}, \qquad \operatorname{coth}(x) = \frac{\operatorname{ch} x}{\operatorname{sh} x}, \qquad \operatorname{sech}(x) = \frac{1}{\operatorname{ch} x}, \qquad \operatorname{cosech}(x) = \frac{1}{\operatorname{sh} x}.$$

Fórmulas importantes:

$\forall x \in \mathbb{R} \quad \operatorname{ch}^2 x - \operatorname{sh}^2 x = 1$, $\qquad \forall x \in \mathbb{R} \quad \operatorname{th}^2 x + \operatorname{sech}^2 x = 1$,

$\forall x \in \mathbb{R} \quad \operatorname{ch}^2 x = \frac{\operatorname{ch}(2x)+1}{2}$, $\qquad \forall x \in \mathbb{R} \setminus \{0\} \quad \operatorname{coth}^2 x - \operatorname{cosech}^2 x = 1$,

$\forall x \in \mathbb{R} \quad \operatorname{sh}^2 x = \frac{\operatorname{ch}(2x)-1}{2}$.

Funções hiperbólicas inversas

$\operatorname{argsh}(x) = (\operatorname{sh})^{-1}(x)$, $\qquad \operatorname{argch}(x) = \left(\operatorname{ch}_{|\mathbb{R}_0^+} \right)^{-1}(x)$,

$\operatorname{argth}(x) = (\operatorname{th})^{-1}(x)$, $\qquad \operatorname{argcoth}(x) = \left(\operatorname{coth}_{|\mathbb{R}\setminus\{0\}} \right)^{-1}(x)$,

$\operatorname{argsech}(x) = \left(\operatorname{sech}_{|\mathbb{R}_0^+} \right)^{-1}(x)$, $\qquad \operatorname{arccosec}(x) = \left(\operatorname{cosech}|_{\mathbb{R}\setminus\{0\}} \right)^{-1}(x)$