Cálculo EC - aula 7

30. Determine as seguintes primitivas:

$$1)\int (x^2-4x+\frac{5}{x})\,dx \quad 2)\int \frac{2x+1}{x^2+x+3}\,dx \quad 3)\int \frac{3}{2x-1}\,dx$$

$$4)\int \frac{1}{x}\cos(\ln x)\,dx \quad 5)\int \frac{\sqrt{1+2\ln x}}{x}\,dx \quad 6)\int \operatorname{sen} x \cos^4 x\,dx$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + \mathcal{C}, \quad n \neq -1, \quad \mathcal{C}\in\mathbb{R}$$

$$\int \frac{1}{x}\,dx = \ln|x| + \mathcal{C}, \quad \mathcal{C}\in\mathbb{R}$$

1) $$\int x^2 - 4x + \frac{5}{x}\,dx = \frac{x^3}{3} - 4\frac{x^2}{2} + 5\ln|x| + \mathcal{C}$$
$$= \frac{x^3}{3} - 2x^2 + 5\ln|x| + \mathcal{C}, \quad \mathcal{C}\in\mathbb{R}$$

$$\int \frac{u'(x)}{u(x)}\,dx = \ln|u(x)| + \mathcal{C}, \quad \mathcal{C}\in\mathbb{R}$$

2) $$\int \frac{2x+1}{x^2+x+3}\,dx = \ln|x^2+x+3| + \mathcal{C}, \quad \mathcal{C}\in\mathbb{R}$$

$u(x) = x^2+x+3$

$u'(x) = 2x+1$

3) $$\int \frac{3}{2x-1}\,dx = \frac{3}{2}\int \frac{2}{2x-1}\,dx = \frac{3}{2}\ln|2x-1| + \mathcal{C}, \quad \mathcal{C}\in\mathbb{R}$$

$u(x) = 2x-1$

$u'(x) = 2$

4) $\displaystyle\int \frac{1}{x}\cos(\ln x)\,dx = \operatorname{sen}(\ln x) + C$
$C \in \mathbb{R}$

$u(x) = \ln x$
$u'(x) = \frac{1}{x}$

$\displaystyle\int u'(x)\cos(u(x))\,dx =$
$= \operatorname{sen}(u(x)) + C$
$C \in \mathbb{R}$

$\displaystyle\int u'(x)\operatorname{sen}(u(x))\,dx =$
$= -\cos(u(x)) + C$
$C \in \mathbb{R}$

5) $\displaystyle\int \frac{\sqrt{1+2\ln x}}{x}\,dx =$

$\displaystyle\int u'(x)\,u^n(x)\,dx = \frac{u^{n+1}(x)}{n+1} + C$
$C \in \mathbb{R}$

$u(x) = 1+2\ln x \qquad n = 1/2$
$u'(x) = \frac{2}{x}$

$$= \frac{1}{2}\int \underbrace{\frac{2}{x}}_{u'(x)}\underbrace{(1+2\ln x)^{1/2}}_{u^{1/2}(x)}\,dx = \frac{1}{2}\,\frac{(1+2\ln x)^{3/2}}{3/2} + C, \quad C \in \mathbb{R} =$$

$$= \frac{1}{3}(1+2\ln x)^{3/2} + C, \quad C \in \mathbb{R}$$

6) $$\int \operatorname{sen}(x)\underbrace{\cos^4(x)}_{u^4(x)}\,dx = -\int \underbrace{-\operatorname{sen}(x)}_{u'(x)}\,\underbrace{\cos^4(x)}_{u^4(x)}\,dx = -\frac{\cos^5(x)}{5} + C$$
$C \in \mathbb{R}$

32. Recorde que $\cos^2 x = \dfrac{\cos 2x + 1}{2}$ e determine $\displaystyle\int \cos^2 x \, dx$.

$$\int \cos^2 x \, dx = \int \frac{\cos(2x) + 1}{2} \, dx = \frac{1}{2} \int \cos(2x) + 1 \, dx =$$

$$= \frac{1}{2} \left[\frac{1}{2} \int 2\cos(2x) + 1 \, dx \right] = \frac{1}{2} \left[\frac{1}{2} \operatorname{sen}(2x) + x + c \right] =$$

$$= \frac{1}{4} \operatorname{sen}(2x) + \frac{x}{2} + k \; , \; k \in \mathbb{R}.$$

31. Recorrendo à primitivação por partes, determine as seguintes primitivas:

$$1)\int x\,\mathrm{sen}\,2x\,dx \quad 2)\int (2x^2-1)e^x\,dx \quad 3)\int \mathrm{arctg}\,x\,dx$$

Primitivação por partes: $\displaystyle\int u'(x)\,v(x)\,dx = u(x)\,v(x) - \int u(x)\,v'(x)\,dx$

1) $\displaystyle\int \underbrace{x}_{u'}\,\underbrace{\mathrm{sen}(2x)}_{v}\,dx$

$u'(x) = x \Rightarrow u(x) = \dfrac{x^2}{2}$

$v(x) = \mathrm{sen}(2x) \Rightarrow v'(x) = 2\cos(2x)$

$\displaystyle\int x\,\mathrm{sen}(2x)\,dx = \frac{x^2}{2}\,\mathrm{sen}(2x) - \int \frac{x^2}{2}\,2\cos(2x)\,dx$

escolha errada!

$\displaystyle\int \underbrace{x}_{u}\,\underbrace{\mathrm{sen}(2x)}_{v'}\,dx$

$u(x) = x \Rightarrow u'(x) = 1$

$v'(x) = \mathrm{sen}(2x) \Rightarrow v(x) = -\dfrac{\cos(2x)}{2}$

$$= \underbrace{x}_{u}\underbrace{\left(-\frac{\cos(2x)}{2}\right)}_{v} - \int \underbrace{1}_{u'} \cdot \underbrace{\left(-\frac{\cos(2x)}{2}\right)}_{v} dx =$$

$$= -\frac{x}{2}\cos(2x) + \frac{1}{2}\int \cos(2x)\,dx = -\frac{x}{2}\cos(2x) + \frac{1}{2}\,\frac{\operatorname{sen}(2x)}{2} + C$$

$$= -\frac{x}{2}\cos(2x) + \frac{1}{4}\operatorname{sen}(2x) + C$$

Verificação: $\left(-\frac{x}{2}\cos(2x) + \frac{1}{4}\operatorname{sen}(2x) + C\right)' =$

$$-\frac{1}{2}\cos(2x) - \frac{x}{2}\,2\left(-\operatorname{sen}(2x)\right) + \frac{1}{4}\,2\cos(2x) =$$

$$= -\frac{1}{2}\cancel{\cos(2x)} + x\operatorname{sen}(2x) + \frac{1}{2}\cancel{\cos(2x)} = x\operatorname{sen}(2x) \quad \checkmark$$

2) $\int \underbrace{(2x^2-1)}_{u}\,\underbrace{e^x}_{v'}\,dx$

$u = 2x^2 - 1 \qquad u' = 4x$

$v' = e^x \qquad v = e^x$

$$= \underbrace{(2x^2-1)}_{u}\underbrace{e^x}_{v} - \int \underbrace{4x}_{u'}\underbrace{e^x}_{v}\,dx = (2x^2-1)e^x - 4\int \underbrace{x}_{\alpha}\underbrace{e^x}_{\beta'}\,dx$$

$\alpha = x \qquad \beta' = e^x$

$\alpha' = 1 \qquad \beta = e^x$

$$= (2x^2-1)e^x - 4\left[\underbrace{x}_{\alpha}\underbrace{e^x}_{\beta} - \int \underbrace{1}_{\alpha'}\cdot\underbrace{e^x}_{\beta}\,dx\right] =$$

$$= (2x^2-1)e^x - 4x\,e^x + 4e^x + \mathcal{C} = (2x^2-1-4x+4)e^x =$$

$$= (2x^2-4x+3)e^x + \mathcal{C}, \quad \mathcal{C}\in\mathbb{R}$$

TPC: Verificação

3) $\displaystyle\int \operatorname{arctg} x\,dx = \int \underbrace{1}_{u'}\cdot\underbrace{\operatorname{arctg}(x)}_{v}\,dx$

$u' = 1 \qquad u = x$

$v = \operatorname{arctg} x \qquad v' = \dfrac{1}{1+x^2}$

$$= x\operatorname{arctg}(x) - \int x\,\frac{1}{1+x^2}\,dx$$

$$= x\operatorname{arctg}(x) - \frac{1}{2}\int \frac{2x}{1+x^2}\,dx$$

$$= x\operatorname{arctg}(x) - \frac{1}{2}\ln(1+x^2) + \mathcal{C}, \ \mathcal{C}\in\mathbb{R}$$

33. Determine as primitivas seguintes :

1) $\displaystyle\int \ln x\,dx$ 2) $\displaystyle\int \frac{e^{\operatorname{arctg} x}}{1+x^2}\,dx$ 3) $\displaystyle\int \frac{-3}{x(\ln x)^3}\,dx$

4) $\displaystyle\int -3x^2\cos x\,dx$ 5) $\displaystyle\int \frac{\operatorname{sen} x}{\sqrt{1+\cos x}}\,dx$ 6) $\displaystyle\int \operatorname{arcsen} x\,dx$

1) $$\int \ln(x)\,dx = \int \underbrace{1}_{u'} \cdot \underbrace{\ln(x)}_{v}\,dx = x\ln(x) - \int x\,\frac{1}{x}\,dx =$$

$$= x\ln(x) - \int 1\,dx =$$

$u' = 1 \qquad v = \ln x$

$u = x \qquad v' = \frac{1}{x}$

$$= x\ln(x) - x + C, \quad C\in\mathbb{R}.$$

2) $$\int \frac{e^{\operatorname{arctg} x}}{1+x^2}\,dx =$$

$$\int u'(x)\,e^{u(x)}\,dx = e^{u(x)} + C$$
$C\in\mathbb{R}$

$$= \int \underbrace{\frac{1}{1+x^2}}_{u'(x)}\,\underbrace{e^{\operatorname{arctg} x}}_{e^{u(x)}}\,dx = e^{\operatorname{arctg} x} + C, \quad C\in\mathbb{R}$$

3)
$$\int \frac{-3}{x(\ln x)^3}\,dx = \int -\frac{3}{x}(\ln x)^{-3}\,dx =$$

$u(x) = \ln x$

$u'(x) = \frac{1}{x}$

$n = -3$

$$= -3\int \frac{1}{x}\ln(x)^{-3}\,dx = -3\,\frac{(\ln(x))^{-2}}{-2} + C,\quad C\in\mathbb{R} =$$

$$= \frac{3}{2(\ln(x))^2} + C,\quad C\in\mathbb{R}$$

4)
$$\int \underbrace{-3x^2}_{u}\,\underbrace{\cos x}_{v'}\,dx = -3x^2\,\operatorname{sen} x - \int -6x\,\operatorname{sen} x\,dx =$$

$u = -3x^2 \qquad u' = -6x$

$v' = \cos x \qquad v = \operatorname{sen} x$

$$= -3x^2\operatorname{sen} x + \int \underbrace{6x}_{u}\,\underbrace{\operatorname{sen} x}_{v'}\,dx$$

$u = 6x \qquad u' = 6$

$v' = \operatorname{sen} x \qquad v = -\cos x$

$$= -3x^2\operatorname{sen} x + 6x(-\cos x) - \int 6(-\cos x)\,dx$$

$$= -3x^2\operatorname{sen} x - 6x\cos x + 6\int \cos x\,dx$$

$$= -3x^2\operatorname{sen} x - 6x\cos x + 6\operatorname{sen} x + C$$

$C\in\mathbb{R}$

5) $$\int \frac{\operatorname{sen} x}{\sqrt{1+\cos x}}\,dx = \int \operatorname{sen} x\,(1+\cos x)^{-1/2}\,dx =$$

$$u(x) = 1+\cos x$$
$$u'(x) = -\operatorname{sen} x$$

$$= -\int -\operatorname{sen} x\,(1+\cos x)^{-1/2}\,dx = -\frac{(1+\cos x)^{1/2}}{1/2} + \mathcal{C} =$$

$$= -2\sqrt{1+\cos x} + \mathcal{C}, \quad \mathcal{C} \in \mathbb{R}$$

6) $$\int \operatorname{arcsen} x\,dx = \int \underbrace{1}_{u'} \cdot \underbrace{\operatorname{arcsen} x}_{v}\,dx =$$

$$u' = 1 \qquad u = x$$
$$v = \operatorname{arcsen} x \qquad v' = \frac{1}{\sqrt{1-x^2}}$$

$$= x \operatorname{arcsen} x - \int x \frac{1}{\sqrt{1-x^2}}\,dx =$$

$$= x \operatorname{arcsen} x + \frac{1}{2}\int (-2x)(1-x^2)^{-1/2}\,dx$$

$$= x \operatorname{arcsen} x + \frac{1}{2}\,\frac{(1-x^2)^{1/2}}{1/2} + \mathcal{C}$$

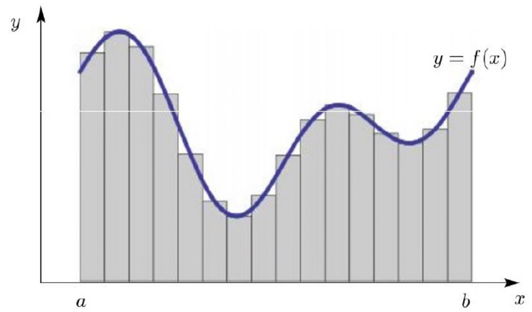
$$= x \operatorname{arcsen} x + \sqrt{1-x^2} + \mathcal{C}, \quad \mathcal{C} \in \mathbb{R}$$

34. Calcule os seguintes integrais:

1) $\displaystyle\int_0^{\sqrt{\pi/2}} x\mathrm{sen}\,(x^2)\,dx$ 2) $\displaystyle\int_0^{\pi} (x+2)\cos x\,dx$

3) $\displaystyle\int_1^{2} x2^x\,dx$ 4) $\displaystyle\int_0^{1} \frac{e^x}{\sqrt{e^x+1}}\,dx$

Integral de Riemann



$$\sum_{k=1}^{n-1} f(\tilde{x}_i)(x_{i+1}-x_i) \quad , \quad \tilde{x}_i \in [x_i, x_{i+1}]$$

$$\xrightarrow[n \to +\infty]{} \int_a^b f(x)\,dx$$

Teorema fundamental do Cálculo (fórmula de Barrow)

Seja $f : [a,b] \longrightarrow \mathbb{R}$ uma função contínua. Então, para qualquer primitiva $F : [a,b] \longrightarrow \mathbb{R}$ de f:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Notação : $\int_a^b f(x)\,dx = F(x)\Big|_a^b$

1) $\int_0^{\sqrt{\pi/2}} x \operatorname{sen}(x^2)\,dx = \frac{1}{2}\int_0^{\sqrt{\frac{\pi}{2}}} 2x \operatorname{sen}(x^2)\,dx = -\frac{1}{2}\cos(x^2)\Big|_0^{\sqrt{\pi/2}} =$

$= -\frac{1}{2}\cos\left(\frac{\pi}{2}\right) + \frac{1}{2}\cos(0) = \frac{1}{2}$