Folha 3 - correcção

1 a) $\frac{\partial f}{\partial x}(0,0) = \lim_{h\to 0} \frac{f((0,0)+h(1,0)) - f(0,0)}{h}$

$= \lim_{h\to 0} \frac{f(h,0)}{h} = \lim_{h\to 0} \frac{h^2 \cdot 0}{h} = \lim_{h\to 0} \frac{0}{h} = 0$

b) $\frac{\partial f}{\partial x}(x_0,2) = \lim_{h\to 0} \frac{f((x_0,2)+h(1,0)) - f(x_0,2)}{h}$

$= \lim_{h\to 0} \frac{f(x_0+h,2) - f(x_0,2)}{h}$

$= \lim_{h\to 0} \frac{(x_0+h)^2 \cdot 2 - x_0^2 \cdot 2}{h}$

$= \lim_{h\to 0} \frac{\cancel{2x_0^2} + 4x_0h + 2h^2 - \cancel{2x_0^2}}{h} = \lim_{h\to 0}(4x_0 + 2h) = 4x_0$

c) $\frac{\partial f}{\partial x}(x_0,y_0) = \lim_{h\to 0} \frac{f(x_0+h,y_0) - f(x_0,y_0)}{h}$

$= \lim_{h\to 0} \frac{(x_0+h)^2 y_0 - x_0^2 y_0}{h}$

$= \lim_{h\to 0} \frac{\cancel{x_0^2 y_0} + 2x_0y_0h + y_0h^2 - \cancel{x_0^2 y_0}}{h}$

$= \lim_{h\to 0}(2x_0y_0 + y_0h) = 2x_0y_0$

d) $\frac{\partial f}{\partial y}(0,0) = \lim_{h\to 0} \frac{f((0,0)+h(0,1)) - f(0,0)}{h} = \lim_{h\to 0} \frac{f(0,h) - f(0,0)}{h}$

$= \lim_{h\to 0} \frac{0^2 \cdot h}{h} = 0$

e) $\frac{\partial f}{\partial y}(x_0,2) = \lim_{h\to 0} \frac{f((x_0,2)+h(0,1)) - f(x_0,2)}{h} = \lim_{h\to 0} \frac{f(x_0,2+h) - 2x_0^2}{h}$

$= \lim_{h\to 0} \frac{x_0^2(2+h) - 2x_0^2}{h} = \lim_{h\to 0} x_0^2 = x_0^2$

f) $\frac{\partial f}{\partial y}(x_0,y_0) = \lim_{h\to 0} \frac{f(x_0,y_0+h) - f(x_0,y_0)}{h} = \lim_{h\to 0} \frac{x_0^2(y_0+h) - x_0^2 y_0}{h}$

$= \lim_{h\to 0} x_0^2 = x_0^2$