Universidade do Minho
Departamento de Matemática

LFIS e MIEFIS

Cálculo Vetorial

Folha 2 — fevereiro de 2020

Exercício 1. Indique o domínio das seguintes funções:

a) $f(x,y) = \dfrac{x+y}{x-y}$;

b) $f(x,y) = \sqrt{4x^2+9y^2-36}$;

c) $f(x,y) = \log(1+xy)$;

d) $f(x,y) = \dfrac{1}{\sqrt{x^2-y^2}}$.

Exercício 2. Calcule:

a) $\displaystyle\lim_{(x,y)\to(1,1)} 3x-2y = 1$;

b) $\displaystyle\lim_{(x,y)\to(1,2)} 3x^2-y = 1$.

Exercício 3. Considere as funções:

$$f : \mathbb{R}^2 \setminus \{(0,0)\} \longrightarrow \mathbb{R} \text{ tal que } f(x,y) = \frac{2(x-1)y^2}{x^2+y^2};$$

$$g : \mathbb{R}^2 \setminus \{(0,0)\} \longrightarrow \mathbb{R} \text{ tal que } g(x,y) = \frac{y^2-2y}{x^4+y^2};$$

$$h : \{(x,y) \in \mathbb{R}^2 : \ y \neq -1\} \longrightarrow \mathbb{R} \text{ tal que } h(x,y) = \frac{x^2}{y+1}.$$

Calcule, caso existam, os seguintes limites:

a) $\displaystyle\lim_{(x,y)\to(1,1)} f(x,y)$ e $\displaystyle\lim_{(x,y)\to(0,0)} f(x,y)$;

b) $\displaystyle\lim_{(x,y)\to(0,2)} g(x,y)$ e $\displaystyle\lim_{(x,y)\to(0,0)} g(x,y)$;

c) $\displaystyle\lim_{(x,y)\to(0,0)} h(x,y)$ e $\displaystyle\lim_{(x,y)\to(0,-1)} h(x,y)$.

Exercício 4. Determine, caso existam, os seguintes limites:

a) $\displaystyle\lim_{(x,y)\to(0,0)} (x^2+y^2) \operatorname{sen} \tfrac{1}{\sqrt{x^2+y^2}}$;

b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$;

c) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x+y}{x^2+y^2}$;

d) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^2+y^6}$;

e) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}$;

f) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^2}$;

g) $\displaystyle\lim_{(x,y,z)\to(0,0,0)} \frac{xyz}{x^2+y^2+z^2}$;

h) $\displaystyle\lim_{(x,y,z)\to(0,0,0)} \frac{x^3+yz^2}{x^4+y^2+z^4}$.

Exercício 5. Estude a continuidade das seguintes funções:

a) $f(x,y) = \begin{cases} \dfrac{x^2y^2}{x^2+y^4} & \text{se} \quad (x,y) \neq (0,0), \\ 0 & \text{se} \quad (x,y) = (0,0); \end{cases}$

b) $f(x,y) = \begin{cases} \dfrac{x}{y} & \text{se} \quad y \neq 0, \\ 0 & \text{se} \quad y = 0; \end{cases}$

c) $f(x,y) = \begin{cases} \dfrac{4xy(x^2-y^2)}{x^2+y^2} & \text{se} \quad (x,y) \neq (0,0), \\ 0 & \text{se} \quad (x,y) = (0,0); \end{cases}$

d) $f(x,y) = \begin{cases} \dfrac{2x^3y^3}{(x^2+y^2)^2} & \text{se} \quad (x,y) \neq (0,0), \\ 0 & \text{se} \quad (0,0); \end{cases}$

e) $f(x,y) = \begin{cases} \dfrac{x^3y}{x^4+y^2} & \text{se} \quad (x,y) \neq (0,0), \\ 0 & \text{se} \quad (0,0); \end{cases}$

f) $f(x,y) = \begin{cases} 0 & \text{se} \quad 0 < y < x^2, \\ 1 & \text{se} \quad y \leq 0 \vee x^2 \leq y; \end{cases}$

g) $f(x,y) = \begin{cases} \dfrac{x+y}{x-y} & \text{se} \quad x \neq y, \\ 2 & \text{se} \quad x = y; \end{cases}$

h) $f(x,y,z) = \begin{cases} \dfrac{xyz}{x^2+y^2+z^2} & \text{se} \quad (x,y,z) \neq (0,0,0), \\ 0 & \text{se} \quad (x,y,z) = (0,0,0). \end{cases}$

Exercício 6. Considere as funções

$$\begin{array}{rccc} f: & \mathbb{R}^2 \setminus \{(0,0)\} & \longrightarrow & \mathbb{R} \\ & (x,y) & \longmapsto & \frac{1-\cos(x^2+y^2)}{x^2+y^2} \end{array} \quad \text{e} \quad \begin{array}{rccc} g: & \mathbb{R}^2 \setminus \{(0,0)\} & \longrightarrow & \mathbb{R}. \\ & (x,y) & \longmapsto & \frac{xy}{x^2+y^2} \end{array}$$

Verifique se as função f e g admitem prolongamentos contínuos a $\mathbb{R}^2$.