Cálculo EC: aula 1

1. Determine o maior domínio possível das seguintes funções:

$$(a)\ f(x) = \frac{1}{x^2-1};\quad (b)\ f(x) = \sqrt{2-3x} + \sqrt{x};\quad (c)\ f(x) = \sqrt{1-\cos(3x^3+x)}.$$

a) $D_f = \{x \in \mathbb{R} : x^2 - 1 \neq 0\} = \mathbb{R} \setminus \{-1, 1\}$

b) $D_f = \{x \in \mathbb{R} : 2 - 3x \geq 0 \wedge x \geq 0\} = [0, 2/3]$

CA: $2 - 3x \geq 0 \Leftrightarrow 2 \geq 3x \Leftrightarrow x \leq 2/3$

c) $D_f = \{x \in \mathbb{R} : 1 - \cos(3x^3 + x) \geq 0\} = \mathbb{R}$

CA: $1 - \cos(3x^3 + x) \geq 0 \Leftrightarrow \cos(3x^3 + x) \leq 1$ condição universal

2. (a) Sejam $f : [0, +\infty[\to \mathbb{R}$ e $g : \mathbb{R} \to \mathbb{R}$ as funções definidas por $f(x) = \sqrt{x} + 1$ e $g(x) = \cos x - 2x^2 + 5x$. Descreva a função $g \circ f$.

(b) Para a função h dada indique duas funções f e g tais que $h = g \circ f$:

$$(i)\ h(x) = \operatorname{sen}\left(\frac{x}{x^2 - 3}\right);$$
$$(ii)\ h(x) = \sqrt{x^2 + 1} + \frac{2}{x^2 + 1}.$$

a

$D_{g\circ f} = \{x \in \mathbb{R} : x \in D_f \text{ e } f(x) \in D_g\}$

$= \{x \in \mathbb{R} : x \in [0, +\infty[\text{ e } f(x) \in \mathbb{R}\}$

$= [0, +\infty[$

$D_f = [0, +\infty[$

$D_g = \mathbb{R}$

$g\circ f : [0, +\infty[\longrightarrow \mathbb{R}$

$x \longmapsto (g\circ f)(x) = g(\sqrt{x}+1) = \cos(\sqrt{x}+1) - 2(\sqrt{x}+1)^2 + 5(\sqrt{x}+1)$

b

i $h = id \circ h$

$h = h \circ id$

$id : \mathbb{R} \longrightarrow \mathbb{R}$

$x \longmapsto x$

$h = g \circ f$ onde $g(x) = \operatorname{sen}(x)$ e $f(x) = \dfrac{x}{x^2 - 3}$

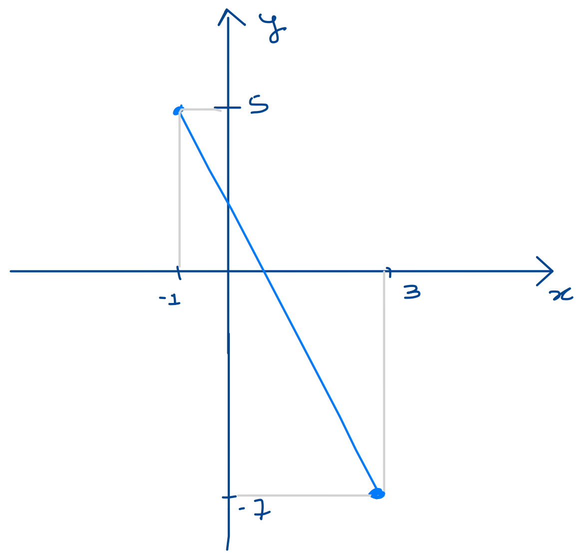
ii $h = g \circ f$ onde $g(x) = \sqrt{x} + \dfrac{2}{x}$ e $f(x) = x^2 + 1$

ou

$h = g \circ f$ onde $g(x) = \sqrt{x+1} + \dfrac{2}{x+1}$ e $f(x) = x^2$

3. Determine a imagem das seguintes funções:

(a) $f : [-1, 3] \to \mathbb{R}, \quad x \mapsto 2 - 3x;$
(b) $f :]-4, 2[\to \mathbb{R}, \quad x \mapsto |2x - 1|.$



• $f : D \to \mathbb{R}, \quad Im(f) = \{ y \in \mathbb{R} : y = f(x), \, x \in D \}$

a

$f(-1) = 5$

$f(3) = -7$

$Im(f) = [-7, 5]$

b

$$|2x-1| = \begin{cases} 2x-1, & \text{se } 2x-1 \geq 0 \\ -2x+1, & \text{se } 2x-1 < 0 \end{cases} = \begin{cases} 2x-1, & \text{se } x \geq 1/2 \\ -2x+1, & \text{se } x < 1/2 \end{cases}$$

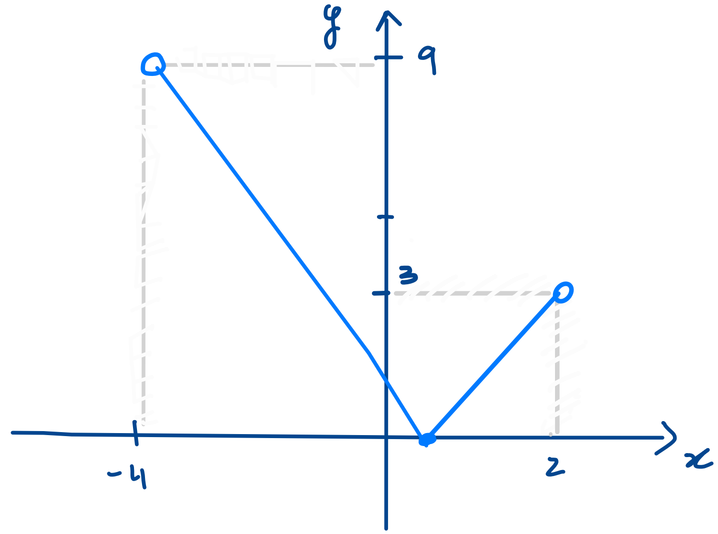
$$f(x) = \begin{cases} -2x+1, & \text{se } -4 < x \leq 1/2 \\ 2x-1, & \text{se } 1/2 \leq x < 2 \end{cases}$$

$\lim\limits_{x \to -4} f(x) = 9$

$\lim\limits_{x \to 2} f(x) = 3$

$f(1/2) = 0$

$Im(f) = [0, 9[$



4. Estude a paridade das seguintes funções definidas em $\mathbb{R}$:

$$(a)\ f(x) = 3x - x^3; \quad (b)\ g(x) = |x+1| + |x-1|; \quad (c)\ h(x) = x^3 - x^2.$$

a $f(-x) = 3(-x) - (-x)^3 = -3x + x^3 = -(3x - x^3) = -f(x)$

logo f é ímpar

b $g(-x) = |-x+1| + |-x-1| = |-(x-1)| + |-(x+1)| = |x-1| + |x+1| = g(x)$

logo g é par

c $h(1) = 0$ Como $h(-1) \neq h(1)$ então h não é par

$h(-1) = -2$ Como $h(-1) \neq -h(1)$ então h não é ímpar

logo h não é par nem ímpar

Nota: polinómios do tipo $p(x) = x^{2k}$, $k \in \mathbb{N}_0$, são funções pares

polinómios do tipo $p(x) = x^{2k+1}$, $k \in \mathbb{N}_0$ são funções ímpares

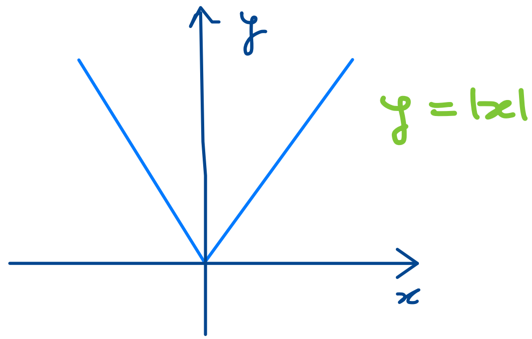
5. Seja $f(x) = |x|$. Esboce o gráfico de $g(x)$:

   (a) $g(x) = f(x) + 2$;

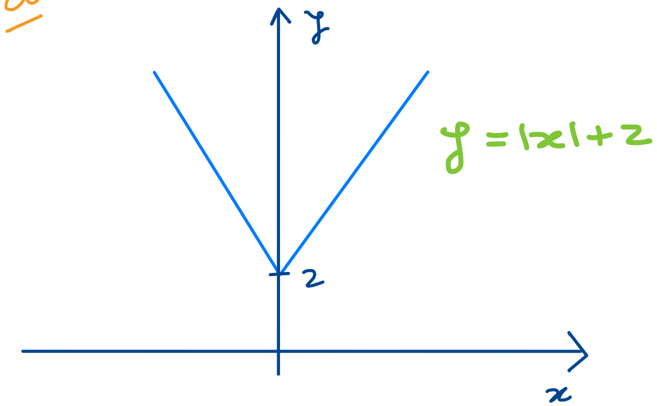
   (b) $g(x) = f(x - 1)$;

   (c) $g(x) = 2f(x)$;

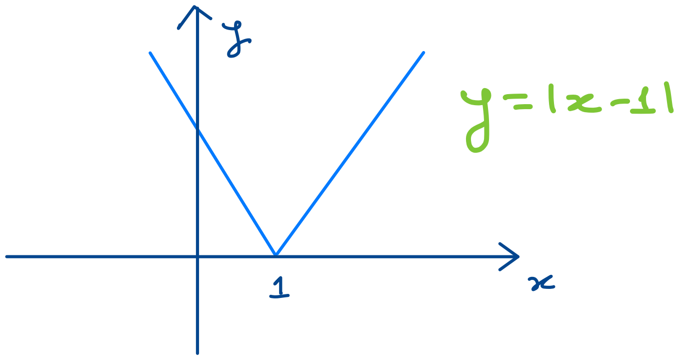
   (d) $g(x) = f(-x)$.



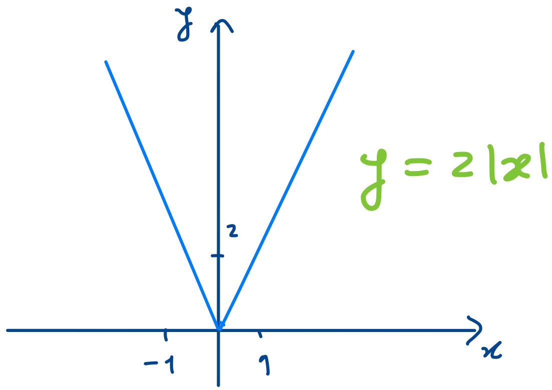
a



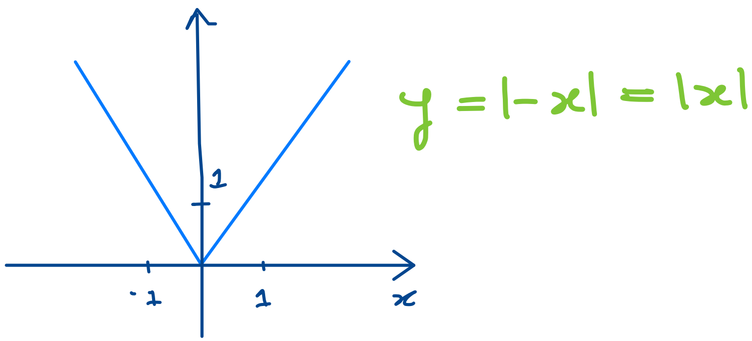
b



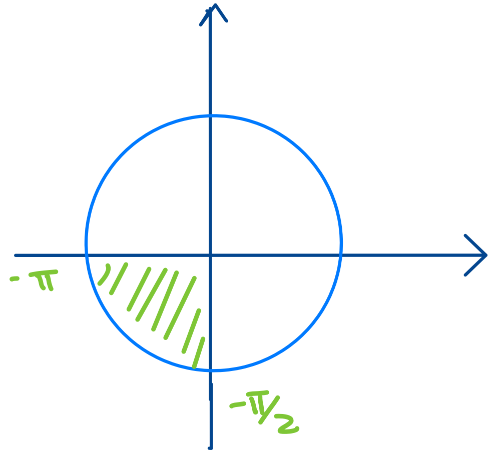
c



d



6. Calcule os números $\operatorname{sen}\alpha$ e $\operatorname{tg}\alpha$ sabendo que $\cos\alpha = -3/5$ e $-\pi < \alpha < -\pi/2$.



Como $\alpha \in]-\pi, -\pi/2[$ então $\alpha \in 3^{\circ}$ Q

Usando a fórmula fundamental da trigonometria (FFT)

$$\cos^2\alpha + \operatorname{sen}^2\alpha = 1$$

logo $\left(-\frac{3}{5}\right)^2 + \operatorname{sen}^2\alpha = 1 \Rightarrow \operatorname{sen}^2\alpha = 1 - \frac{9}{25}$

$$\Rightarrow \operatorname{sen}^2\alpha = \frac{16}{25} \Rightarrow \operatorname{sen}\alpha = \pm\frac{4}{5}$$

Como $\alpha \in 3^{\circ}$ Q então $\operatorname{sen}\alpha = -\frac{4}{5}$.

Temos $\operatorname{tg}\alpha = \frac{\operatorname{sen}\alpha}{\cos\alpha} = \frac{-4/5}{-3/5} = \frac{4}{3}$.

7. Resolva a equação $\operatorname{sen}(2x) = 1/2$.

$\operatorname{sen}(2x) = \operatorname{sen}\left(\frac{\pi}{6}\right) \Leftrightarrow 2x = \frac{\pi}{6} + 2k\pi \vee 2x = \pi - \frac{\pi}{6} + 2k\pi, k \in \mathbb{Z}$

$\Leftrightarrow x = \frac{\pi}{12} + k\pi \vee x = \frac{5\pi}{12} + k\pi, k \in \mathbb{Z}.$

8. Mostre que $\cos^2 x = \dfrac{\cos 2x + 1}{2}$.

- $\cos(2x) = \cos(x+x) = \cos(x)\cos(x) - \operatorname{sen}(x)\operatorname{sen}(x) = \cos^2 x - \operatorname{sen}^2 x$

- $\dfrac{\cos(2x)+1}{2} = \dfrac{\cos^2 x - \operatorname{sen}^2 x + 1}{2} = \dfrac{\cos^2 x + \cos^2 x}{2}$, pela FFT

$= \cos^2 x$ c.q.d.

TPC: $\operatorname{sen}^2 x = \dfrac{1 - \cos(2x)}{2}$