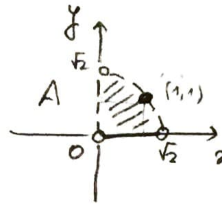
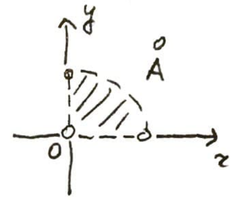
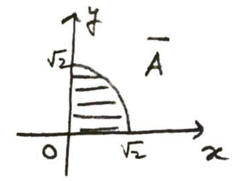
Correção do Exame de Recurso

Cálculo Vetorial 16/06/2021

1 a)







b) $\displaystyle\lim_{(x,y)\to(0,0)} x^2 \frac{\operatorname{sen}^2 y}{y} = \lim_{(x,y)\to(0,0)} x^2 \operatorname{sen} y \, \frac{\operatorname{sen} y}{y} = 0 \cdot 1 = 0,$

uma vez $\displaystyle\lim_{y\to 0} \frac{\operatorname{sen} y}{y} = 1$ e $\displaystyle\lim_{(x,y)\to(0,0)} x^2 \operatorname{sen} y = 0$

2 a) $\displaystyle 0 \leq \left| \frac{2x^2-4y^2}{\sqrt{x^2+y^2}} \right| \leq \frac{4(x^2+y^2)}{\sqrt{x^2+y^2}} = 4\sqrt{x^2+y^2} \xrightarrow[(x,y)\to(0,0)]{} 0$

Como $\displaystyle\lim_{(x,y)\to(0,0)} |g(x,y)| = 0$, também $\displaystyle\lim_{(x,y)\to(0,0)} g(x,y) = 0$ e, como este limite é igual a $g(0,0)$, g é contínua em $(0,0)$.

b) $\displaystyle\lim_{h\to 0} \frac{g(0,h)-g(0,0)}{h} = \lim_{h\to 0} \frac{\frac{-4h^2}{\sqrt{h^2}} - 0}{h} = \lim_{h\to 0} \frac{-4h^2}{|h|h}$

$\displaystyle = -4 \lim_{h\to 0} \frac{h}{|h|}$ não existe porque $\displaystyle\lim_{h\to 0^+} \frac{h}{|h|} = 1$ e $\displaystyle\lim_{h\to 0^-} \frac{h}{|h|} = -1$

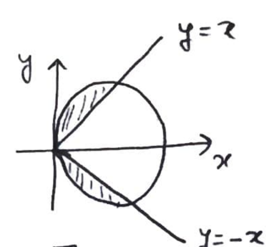
Então $\dfrac{\partial g}{\partial y}(0,0)$ não existe

c) $h(x,y) = \sqrt{x^2+y^2}\, g(x,y) = \begin{cases} 2x^2-4y^2 & \text{se } (x,y)\neq(0,0) \\ 0 & \text{se } (x,y)=(0,0) \end{cases}$ h é de classe C^1

$\dfrac{\partial h}{\partial x} = 4x, \quad \dfrac{\partial h}{\partial y} = -8y$

$h'((1,1);(1,2)) = \nabla h(1,1)\cdot(1,2) = (4,-8)\cdot(1,2) = 4-16 = -12$

3 a) $(x-1)^2+y^2 \leq 1 \iff x^2-2x+1+y^2 \leq 1 \iff x^2+y^2 \leq 2x$



$\begin{cases} x^2+y^2 \leq 2x \\ |y| \geq x \end{cases}$ $\begin{cases} r^2 \leq 2r\cos\theta \\ r\operatorname{sen}\theta \geq r\cos\theta \end{cases}$ $\begin{cases} r \leq 2\cos\theta \\ \operatorname{sen}\theta \geq \cos\theta \end{cases} \iff \begin{cases} \pi/4 \leq \theta \leq \pi/2 \\ -\pi/2 \leq \theta \leq -\pi/4 \end{cases}$

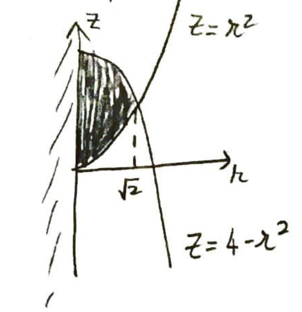
$$\iint_R d(x,y) = 2\int_{\pi/4}^{\pi/2}\int_0^{2\cos\theta} r\,dr\,d\theta = 2\int_{\pi/4}^{\pi/2}\left[\frac{r^2}{2}\right]_0^{2\cos\theta} d\theta = \int_{\pi/4}^{\pi/2} 4\cos^2\theta\,d\theta$$

$$= 4\int_{\pi/4}^{\pi/2} \frac{1+\cos(2\theta)}{2} = 2\left[\theta + \frac{\operatorname{sen}(2\theta)}{2}\right]_{\pi/4}^{\pi/2} = 2\left[\frac{\pi}{2} + 0 - \left(\frac{\pi}{4}+\frac{1}{2}\right)\right]$$

$$= \frac{\pi}{2} - 1.$$

b) $\begin{cases} z \le 4-(x^2+y^2) \\ z \ge x^2+y^2 \end{cases}$ $\begin{cases} z \le 4-r^2 \\ z \ge r^2 \end{cases}$ Então $0 \le \theta \le 2\pi$

Corte por θ = constante



$4-r^2 = r^2 \iff r = \sqrt{2}$

$$\iiint_V y\, d(x,y,z) = \int_0^{2\pi}\int_0^{\sqrt{2}}\int_{r^2}^{4-r^2} r \operatorname{sen}\theta \, r \, dz\, dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\sqrt{2}} \left[r^2 \operatorname{sen}\theta \, z \right]_{r^2}^{4-r^2} dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\sqrt{2}} \left(r^2(4-r^2) - r^3 \right) \operatorname{sen}\theta \, dr\, d\theta$$

$$= \int_0^{2\pi} \left[\frac{4r^3}{3} - \frac{r^5}{5} - \frac{r^4}{4} \right]_0^{\sqrt{2}} \operatorname{sen}\theta\, d\theta = \left(\frac{8\sqrt{2}}{3} - \frac{4\sqrt{2}}{5} - 1 \right) \left[-\cos\theta \right]_0^{2\pi} = 0$$

4) a) $\int_C x\,dx + yz\,dy - z\,dz =$

$x(t) = 1 \quad y(t) = \cos t \quad z(t) = \operatorname{sen} t$

$x'(t) = 0 \quad y'(t) = -\operatorname{sen} t \quad z'(t) = \cos t$

$$= \int_0^{2\pi} 1.0\,dt + \cos t \operatorname{sen} t\,(-\operatorname{sen} t)\,dt - \operatorname{sen} t \cos t\,dt$$

$$= \int_0^{2\pi} \left(-\cos t \operatorname{sen}^2 t - \operatorname{sen} t \cos t \right) dt = \left[-\frac{\operatorname{sen}^3 t}{3} - \frac{\operatorname{sen}^2 t}{2} \right]_0^{2\pi} = 0$$

b) O campo F não é conservativo porque

$\frac{\partial F_1}{\partial y} = 0 \quad \frac{\partial F_2}{\partial x} = 0 \quad \frac{\partial F_1}{\partial z} = 0 \quad \frac{\partial F_3}{\partial x} = 0$

$$\boxed{\frac{\partial F_2}{\partial z} = y \neq \frac{\partial F_3}{\partial y} = 0}$$

5) Como f é de classe C^1 e

$$(x) = \lim_{h \to 0^+} \frac{f(2h,2h^2) - 2f(h,h^2) + f(0,0)}{h^2} = \frac{0}{0}, \text{ posso aplicar a Regra de l'Hôpital e então}$$

$$(x) = \lim_{h\to 0^+} \frac{\frac{\partial f}{\partial x}(2h,2h^2)2 + \frac{\partial f}{\partial y}(2h,2h^2)4h - 2\frac{\partial f}{\partial x}(h,h^2) - 2\frac{\partial f}{\partial y}(h,h^2)2h}{2h} = \frac{0}{0}$$

Como f é de classe C^2, posso aplicar novamente a Regra de l'Hôpital, obtendo

$$(x) = \lim_{h\to 0^+} \frac{1}{2}\Big(\frac{\partial^2 f}{\partial x^2}(2h,2h^2)4 + \frac{\partial^2 f}{\partial y \partial x}(2h,2h^2)8h + \frac{\partial^2 f}{\partial x \partial y}(2h,2h^2)8h + \frac{\partial^2 f}{\partial y^2}(2h,2h^2)16h^2 + \frac{\partial f}{\partial y}(2h,2h^2)4 -$$

$$- 2\frac{\partial^2 f}{\partial x^2}(h,h^2) - 2\frac{\partial^2 f}{\partial y \partial x}(h,h^2)2h - 2\frac{\partial^2 f}{\partial x \partial y}(h,h^2)2h - 2\frac{\partial^2 f}{\partial y^2}(h,h^2)4h^2 - 2\frac{\partial f}{\partial y}(h,h^2)2 \Big) =$$

$$= \frac{1}{2}\left(4\frac{\partial^2 f}{\partial x^2}(0,0) + 0 + 0 + 0 + \cancel{4\frac{\partial f}{\partial y}(0,0)} - 2\frac{\partial^2 f}{\partial x^2}(0,0) - 0 - 0 - 0 - \cancel{4\frac{\partial f}{\partial y}(0,0)} \right) = \frac{\partial^2 f}{\partial x^2}(0,0)$$