Cálculo EC - aula 8

34. Calcule os seguintes integrais:

1) $\int_0^{\sqrt{\pi/2}} x\,\mathrm{sen}\,(x^2)\,dx$

2) $\int_0^{\pi} (x+2)\cos x\,dx$

3) $\int_1^{2} x2^x\,dx$

4) $\int_0^{1} \frac{e^x}{\sqrt{e^x+1}}\,dx$

2) $\int_0^{\pi} \underbrace{(x+2)}_{v}\underbrace{\cos x}_{u'}\,dx =$

C.A.: $v = x+2 \qquad v' = 1$

$u' = \cos x \qquad u = \mathrm{sen}\,x$

$= (x+2)\,\mathrm{sen}\,x \Big|_0^{\pi} - \int_0^{\pi} \mathrm{sen}\,x\,dx = (\pi+2)\,\mathrm{sen}(\pi) - 2\,\mathrm{sen}\,0 + \cos x\Big|_0^{\pi} =$

$= \cos(\pi) - \cos(0) = -2$

4) $\int_0^{1} \frac{e^x}{\sqrt{e^x+1}}\,dx = \int_0^{1} e^x(e^x+1)^{-1/2}\,dx$

$u(x) = e^x+1$

$u'(x) = e^x$

$= \frac{(e^x+1)^{1/2}}{1/2}\Big|_0^{1} = 2\sqrt{e^x+1}\Big|_0^{1} = 2\sqrt{e+1} - 2\sqrt{2}.$

3) $\int_1^2 \underbrace{x}_{v}\, \underbrace{2^x}_{u'}\, dx$

CA: $\int 2^x dx = \int e^{(\ln 2)x} dx = \frac{1}{\ln 2} \int \underbrace{\ln 2}_{u'(x)}\, \underbrace{e^{(\ln 2)x}}_{e^{u(x)}} = \frac{1}{\ln 2} e^{(\ln 2)x} + \mathcal{C}$

$= \frac{2^x}{\ln(2)} + \mathcal{C}, \quad \mathcal{C} \in \mathbb{R}$

$$\int u'(x)\, a^{u(x)} dx = \frac{a^{u(x)}}{\ln(a)} + \mathcal{C}, \quad \mathcal{C} \in \mathbb{R} \qquad a>0 \text{ e } a \neq 1$$

$\int_1^2 \underbrace{x}_{v}\, \underbrace{2^x}_{u'}\, dx =$

CA: $v = x \qquad v' = 1$

$u' = 2^x \qquad u = \frac{2^x}{\ln 2}$

$= x \frac{2^x}{\ln 2} \Big|_1^2 - \int_1^2 \frac{2^x}{\ln 2} dx =$

$= 2 \frac{2^2}{\ln 2} - \frac{2}{\ln 2} - \left(\frac{2^x}{(\ln 2)^2} \Big|_1^2 \right) = \frac{6}{\ln 2} - \left(\frac{4}{(\ln 2)^2} - \frac{2}{(\ln 2)^2} \right) =$

$= \frac{6}{\ln 2} - \frac{2}{(\ln 2)^2}$

35. a) Calcule $\displaystyle\int_0^{\frac{\pi}{2}} e^x \operatorname{sen} x \, dx$.

b) Determine todas as primitivas de $f(x) = e^x \cos x$.

a) $I = \displaystyle\int_0^{\pi/2} \underbrace{e^x}_{u'} \underbrace{\operatorname{sen} x}_{v} \, dx$

CA: $u' = e^x \quad u = e^x$

$v = \operatorname{sen} x \quad v' = \cos x$

$= e^x \operatorname{sen} x \Big|_0^{\pi/2} - \displaystyle\int_0^{\pi/2} \underbrace{e^x}_{u'} \underbrace{\cos x}_{v} \, dx$

CA: $u' = e^x \quad u = e^x$

$v = \cos x \quad v' = -\operatorname{sen} x$

$= e^{\pi/2} - \left(e^x \cos x \Big|_0^{\pi/2} - \displaystyle\int_0^{\pi/2} e^x (-\operatorname{sen} x) \, dx \right) =$

$= e^{\pi/2} - \left(-1 + \displaystyle\int_0^{\pi/2} e^x \operatorname{sen} x \right) = e^{\pi/2} + 1 - I$

Conclusão: $I = e^{\pi/2} + 1 - I$

$\Rightarrow 2I = e^{\pi/2} + 1 \Rightarrow I = \dfrac{e^{\pi/2} + 1}{2}$

b) $P = \displaystyle\int \underbrace{e^x}_{u'} \underbrace{\cos x}_{v} \, dx$

CA: $u' = e^x \quad u = e^x$

$v = \cos x \quad v' = -\operatorname{sen} x$

$$= e^x \cos x - \int e^x(-\operatorname{sen} x)\,dx = e^x \cos x + \int \underbrace{e^x}_{u'} \underbrace{\operatorname{sen} x}_{v}\,dx =$$

$$= e^x \cos x + e^x \operatorname{sen} x - \int e^x \cos x\,dx$$

Conclusão: $\underline{P} = e^x(\cos x + \operatorname{sen} x) - \underline{P}$

$$\Rightarrow 2\underline{P} = e^x(\cos x + \operatorname{sen} x)$$

$$\Rightarrow \underline{P} = \frac{e^x}{2}(\cos x + \operatorname{sen} x) + \mathcal{C}, \quad \mathcal{C} \in \mathbb{R}$$

TPC: fazer a verificação.

36. Usando uma substituição, calcule os seguintes integrais

$$1)\int_{-1}^{1} e^{\operatorname{arcsen} x}\,dx \qquad 2)\int_{0}^{1} \frac{x^2}{\sqrt{x+1}}\,dx$$

$$3)\int_{0}^{3/2} 2^{\sqrt{2x+1}}\,dx \qquad 4)\int_{0}^{\sqrt{2}/2} \frac{x^2}{\sqrt{1-x^2}}\,dx$$

1) $\displaystyle\int_{-1}^{1} e^{\operatorname{arcsen} x}\,dx =$

$y = \operatorname{arcsen} x$

$x = \operatorname{sen} y$

$x' = \cos y$

$\dfrac{dx}{dy} = \cos y$

$dx = \cos y\, dy$

$x = -1$ — $y = -\pi/2$

$x = 1$ — $y = \pi/2$

$\displaystyle\int_{-\pi/2}^{\pi/2} e^{y} \cos y\, dy =$

$= \dfrac{e^{y}}{2}(\cos y + \operatorname{sen} y)\Big|_{-\pi/2}^{\pi/2}$, pelo exercício 35. b)

$= \dfrac{e^{\pi/2}}{2} - \dfrac{e^{-\pi/2}}{2}(-1) = \dfrac{e^{\pi/2} + e^{-\pi/2}}{2} = \operatorname{ch}(\pi/2).$

2) $\displaystyle\int_{0}^{1} \frac{x^2}{\sqrt{x+1}}\,dx$

$y = \sqrt{x+1}$

$x = 0$ — $y = 1$

$x = 1$ — $y = \sqrt{2}$

$x + 1 = y^2 \Rightarrow x = y^2 - 1$

$dx = 2y\, dy$

$$= \int_1^{\sqrt{2}} \frac{(y^2-1)^2}{\cancel{y}} \, 2\cancel{y} \, dy = \int_1^{\sqrt{2}} 2(y^4 - 2y^2 + 1) = \cdots = \frac{2}{15}(7\sqrt{2}-8)$$

3) $\displaystyle\int_0^{3/2} 2^{\sqrt{2x+1}} \, dx =$

$y = \sqrt{2x+1}$ $\qquad$ $2x+1 = y^2$

$x = 0 \text{ — } y = 1$ $\qquad$ $x = \dfrac{y^2-1}{2}$

$x = \frac{3}{2} \text{ — } y = 2$

$dx = y\,dy$

$$= \int_1^2 2^y \, y \, dy =$$

$$= \frac{6}{\ln 2} - \frac{2}{(\ln 2)^2} \text{ ,}$$ pelo exercício 34.3)

4) $\displaystyle\int_0^{\frac{\sqrt{2}}{2}} \frac{x^2}{\sqrt{1-x^2}} \, dx$

$x = \operatorname{sen} y$ $\qquad$ $dx = \cos y \, dy$

$y = \operatorname{arcsen} x$ $\qquad$ $x = 0 \text{ — } y = 0$

$x = \frac{\sqrt{2}}{2} \text{ — } y = \frac{\pi}{4}$

$$= \int_0^{\frac{\pi}{4}} \frac{\operatorname{sen}^2 y}{\sqrt{1-\operatorname{sen}^2 y}} \cos y \, dy =$$

↳ usando a FFT

$$= \int_0^{\pi/4} \frac{\operatorname{sen}^2 y}{\sqrt{\cos^2 y}} \cos y \, dy = \left[\text{Como } y \in [0, \pi/4], \ \cos(y) > 0\right]$$

$$= \int_0^{\pi/4} \frac{\operatorname{sen}^2 y}{\cancel{\cos y}} \cancel{\cos} y \, dy = \int_0^{\frac{\pi}{4}} \operatorname{sen}^2 y \, dy = (*)$$

$$= \int_0^{\pi/4} \frac{1 - \cos(2y)}{2}\, dy = \frac{1}{2}\int_0^{\pi/4} 1 - \cos(2y)\, dy =$$

$$= \frac{1}{2}\left(y - \frac{\operatorname{sen}(2y)}{2} \right) \Big|_0^{\pi/4} = \frac{1}{2}\left(\frac{\pi}{4} - \frac{\operatorname{sen}(\pi/2)}{2} \right) = \frac{1}{8}(\pi - 2)$$

(*) Recordar: $\operatorname{sen}^2 y = \dfrac{1 - \cos(2y)}{2}$

37. Represente graficamente o conjunto A dado e calcule a sua área.

a) A é o conjunto do plano limitado pelas rectas $x = 1$, $x = 4$, $y = 0$ e pela curva de $f(x) = \sqrt{x}$.

b) $A = \{(x, y) \in \mathbb{R}^2 | \, 0 \leq x \leq 1 \quad \text{e} \quad \sqrt{x} \leq y \leq -x + 2\}$.

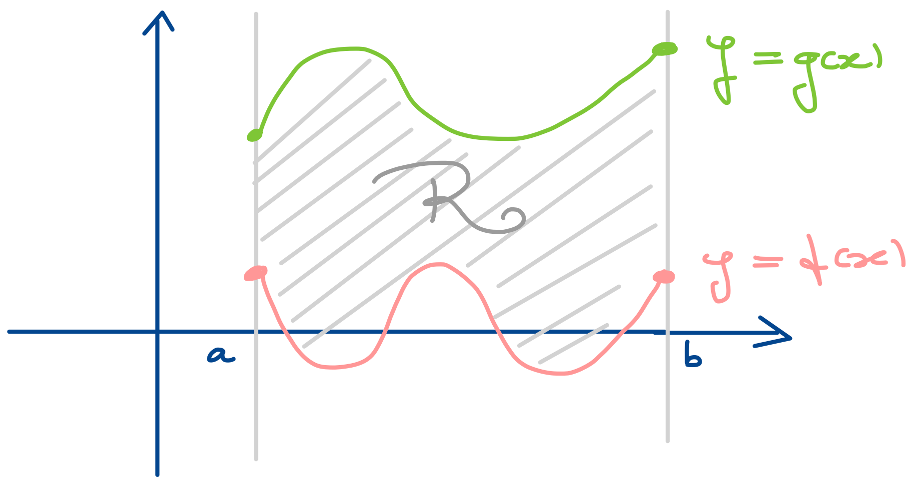
c) A é o conjunto do plano limitado superiormente pela parábola de equação $y = -x^2 + \dfrac{7}{2}$ e inferiormente pela parábola de equação $y = x^2 - 1$.

Seja R uma região do plano que se pode escrever na forma:

$$R = \{(x,y) \in \mathbb{R}^2 : \ a \leq x \leq b \ \text{ e } \ f(x) \leq y \leq g(x)\}$$

Então área$(R) = \displaystyle\int_a^b g(x) - f(x) \, dx$.

NOTA: área$(R) > 0$

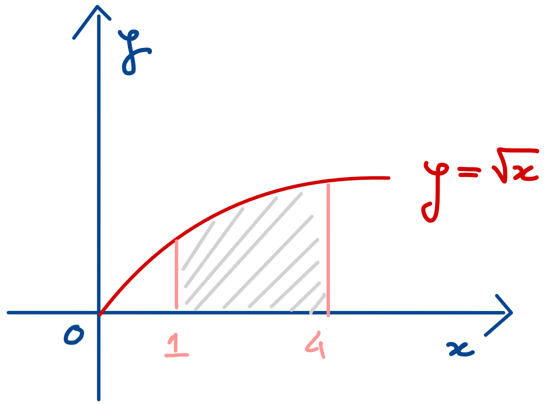


a)

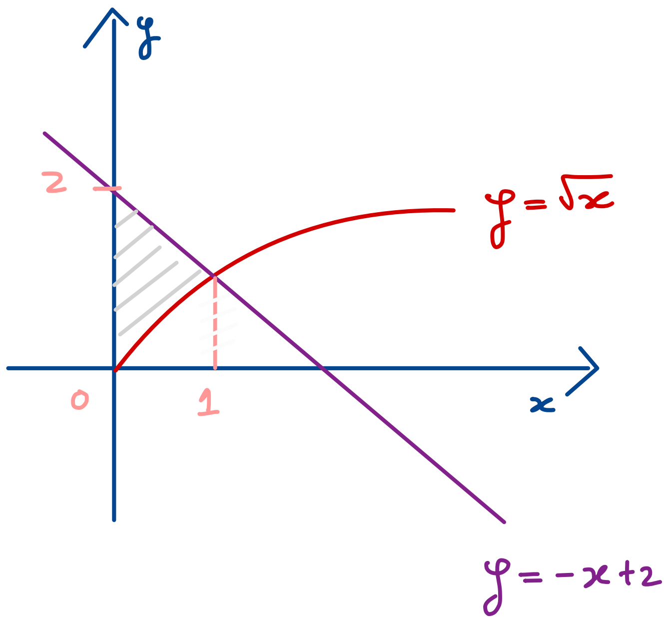
$$\text{área}(A) = \int_1^4 \sqrt{x} - 0 \, dx =$$

$$= \left. \frac{x^{3/2}}{3/2} \right|_1^4 = \left. \frac{2}{3}\sqrt{x^3} \right|_1^4 =$$

$$= \dots = \frac{14}{3}$$



b)

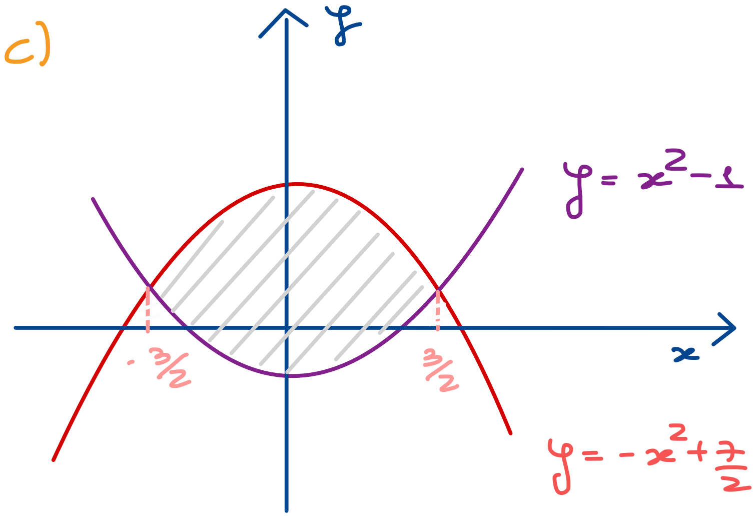


$$\text{área}(A) = \int_0^1 -x+2-\sqrt{x} \, dx =$$

$$= \left. -\frac{x^2}{2} + 2x - \frac{x^{3/2}}{3/2} \right|_0^1 =$$

$$= \dots = \frac{5}{6}$$

c)



CA: $-x^2 + \frac{7}{2} = x^2 - 1 \iff 2x^2 = \frac{9}{2}$

$\iff x^2 = \frac{9}{4} \iff x = \pm \frac{3}{2}$

$$\text{área}(A) = \int_{-3/2}^{3/2} -x^2 + \frac{7}{2} - (x^2 - 1)\, dx = \cdots = 9.$$