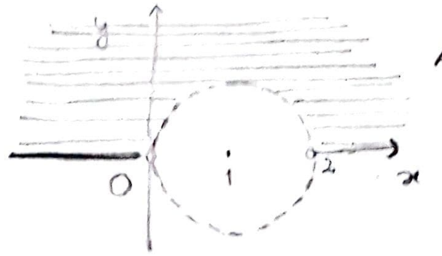
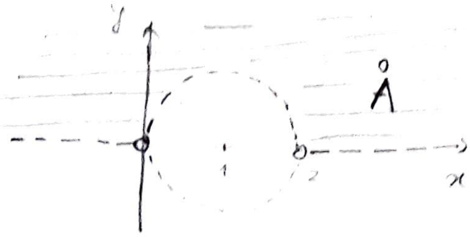
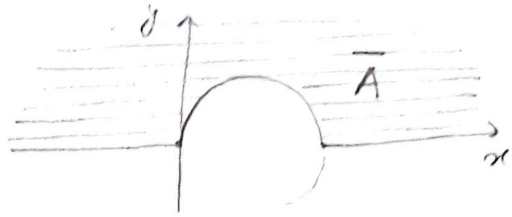
Correcção do Teste TPC

(1) a)



b)





c) O conjunto A não é limitado porque os pontos da forma $(n,0)$, com $n \in \mathbb{N}\setminus\{1,2\}$ pertencem a A e $\lim\limits_{n\to\infty} (n,0) = (\infty, 0)$

(2) a) $\gamma'(t) = (2t\cos t - t^2 \operatorname{sen} t,\ 2t \operatorname{sen} t + t^2 \cos t)$

$$\gamma(t)\cdot\gamma'(t) = (t^2\cos t,\ t^2 \operatorname{sen} t)\cdot(2t\cos t - t^2\operatorname{sen} t,\ 2t\operatorname{sen} t + t^2\cos t)$$
$$= 2t^3\cos^2 t - \cancel{t^4 \operatorname{sen} t \cos t} + 2t^3 \operatorname{sen}^2 t + \cancel{t^4 \operatorname{sen} t \cos t}$$
$$= 2t^3(\cos^2 t + \operatorname{sen}^2 t) = 2t^3 \neq 0 \quad , \quad \forall t \in \mathbb{R}\setminus\{0\}$$

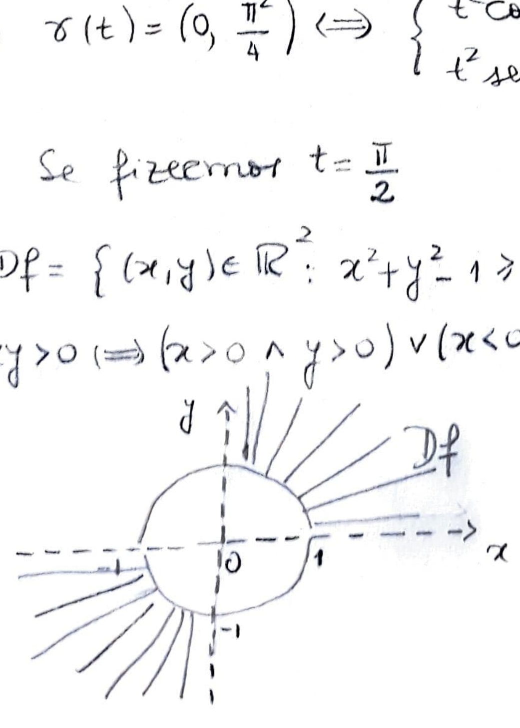
Então não existe $t_0 \neq 0$ tal que $\gamma(t_0)$ e $\gamma'(t_0)$ sejam ortogonais

b) $\gamma(t) = \left(0, \frac{\pi^2}{4}\right) \iff \begin{cases} t^2\cos t = 0 \\ t^2 \operatorname{sen} t = \frac{\pi^2}{4} \end{cases}$

Se fizermos $t = \frac{\pi}{2}$

(3) $D_f = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 - 1 \geq 0 \ \wedge\ xy > 0\}$

$xy > 0 \iff (x > 0 \wedge y > 0) \vee (x < 0 \text{ e } y < 0)$



(4) a) $\lim_{x\to 0} f(x,x^3) = \lim_{x\to 0} \frac{3x^3 . x^3}{x^6+x^6} = \lim_{x\to 0} 3 = 3$

b) A função f é descontínua, uma vez que, quando $x \to 0$ então $(x,x^3) \to (0,0)$ e

$$\lim_{x\to 0} f(x,x^3) = 3 \neq f(0,0)=0$$

c) $$\frac{\partial f}{\partial y}(0,0) = \lim_{h\to 0} \frac{f((0,0)+h(0,1)) - f(0,0)}{h} = \lim_{h\to 0} \frac{f(0,h)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{3.0^3.h}{0^6+h^2}}{h} = \lim_{h\to 0} 0 = 0$$

$(x,y) \neq (0,0)$

$$\frac{\partial f}{\partial y} = \frac{3x^2(x^6+y^2) - 3x^3y \cdot 2y}{(x^6+y^2)^2} = \frac{3x^8+3x^2y^2-6x^3y^2}{(x^6+y^2)^2}$$

Então $\frac{\partial f}{\partial y} : \mathbb{R}^2 \to \mathbb{R}$ é a função definida do seguinte modo:

$$\frac{\partial f}{\partial y}(x,y) = \begin{cases} \frac{3x^8+3x^2y^2-6x^3y^2}{(x^6+y^2)^2} & \text{se } (x,y) \neq (0,0) \\ 0 & \text{se } (x,y) = (0,0) \end{cases}$$

d) $f'((0,0);(0,0)) = 0$

Se $(a,b) \neq (0,0)$ (e $b \neq 0$

$$f'((0,0);(a,b)) = \lim_{h\to 0} \frac{f((0,0)+h(a,b)) - f(0,0)}{h}$$

$$= \lim_{h\to 0} \frac{f(ha,hb)}{h} = \lim_{h\to 0} \frac{\frac{3a^3h^3.bh}{h^6a^6+h^2b^2}}{h}$$

$$= \lim_{h\to 0} \frac{3a^3bh^4}{(h^6a^6+h^2b^2)h}$$

$$= \lim_{h\to 0} \frac{3a^3bh^{\cancel{4}}}{\cancel{h^3}(a^6h^4+b^2)} = \frac{0}{b^2} = 0 \text{ porque } b \neq 0$$

Se $a \neq 0$ e $b = 0$

$$f'((0,0);(a,0)) = \lim_{h\to 0} \frac{f(ha,0)}{h} = \lim_{h\to 0} \frac{\frac{0}{h^6a^6}}{h} = \lim_{h\to 0} 0 = 0$$

Então,

$\forall (a,b) \in \mathbb{R}^2 \quad f'((0,0);(a,b)) = 0$

(5)

$$J_{(x,y,z)}\Psi = \begin{pmatrix} yz & xz & xy \\ y^2 & 2xy - z & -y \end{pmatrix}$$