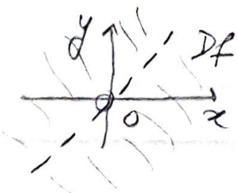
Folha 2 - Correcção

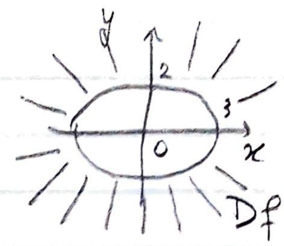
1) a) $Df=\{(x,y)\in\mathbb{R}^2: x-y\neq 0\}$
$=\{(x,y)\in\mathbb{R}^2: x\neq y\}$



b) $Df=\{(x,y)\in\mathbb{R}^2: 4x^2+9y^2-36\geq 0\}$
$=\{(x,y)\in\mathbb{R}^2: 4x^2+9y^2\geq 36\}$

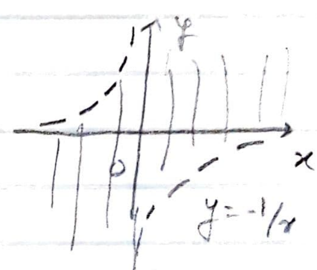
$4x^2+9y^2=36 \Leftrightarrow \dfrac{x^2}{3^2}+\dfrac{y^2}{2^2}=1$

Equação duma elipse

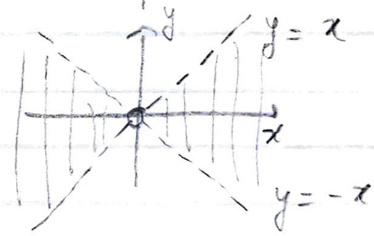


c) $Df=\{(x,y)\in\mathbb{R}^2: 1+xy>0\}$

$1+xy>0 \Leftrightarrow xy>-1 \Leftrightarrow \begin{cases} y>-\frac{1}{x} \wedge x>0 \\ \vee \\ y<-\frac{1}{x} \wedge x<0 \end{cases}$



d) $Df=\{(x,y)\in\mathbb{R}^2: x^2-y^2>0\}$
$=\{(x,y)\in\mathbb{R}^2: |x|>|y|\}$



2) a) $\lim\limits_{(x,y)\to(1,1)}(3x-2y)=3-2=1$

b) $\lim\limits_{(x,y)\to(1,2)}(3x^2-y)=3\cdot 1^2-2=1$

3) a) $\lim\limits_{(x,y)\to(1,1)}\dfrac{2(x-1)y^2}{x^2+y^2}=\dfrac{2\cdot 0\cdot 1^2}{1^2+1^2}=0$

$\lim\limits_{x\to 0}f(x,0)=\lim\limits_{x\to 0}\dfrac{0}{x^2}=0$

$\lim\limits_{y\to 0}f(0,y)=\lim\limits_{y\to 0}\dfrac{-2y^2}{y^2}=-2$

Como $\lim\limits_{x\to 0}f(x,0)\neq\lim\limits_{y\to 0}f(0,y)$, o limite $\lim\limits_{(x,y)\to(0,0)}f(x,y)$ não existe

(2) $\frac{\partial f}{\partial x}(0,0,0) = \lim_{h\to 0} \frac{f(h,0,0)-f(0,0,0)}{h} = \lim_{h\to 0} \frac{e^h - e^0}{h} = \lim_{h\to 0} \frac{e^h - 1}{h} = \frac{0}{0}$

$= \lim_{h\to 0} \frac{e^h}{1} = e^0 = 1$

Usando a Regra de l'Hôpital

$\frac{\partial f}{\partial y}(0,0,0) = \lim_{h\to 0} \frac{f(0,h,0)-f(0,0,0)}{h} = \lim_{h\to 0} \frac{e^h - 1}{h} = 1$

$\frac{\partial f}{\partial z}(0,0,0) = \lim_{h\to 0} \frac{f(0,0,h)-f(0,0,0)}{h} = \lim_{h\to 0} \frac{e^h - 1}{h} = 1$

(3) a) $\frac{\partial f}{\partial x} = 6x$, $\frac{\partial f}{\partial y} = 4y$

b) $\frac{\partial f}{\partial x} = \cos(x^2 - 3xy) \frac{\partial}{\partial x}(x^2 - 3xy) = \cos(x^2 - 3xy)(6x - 3y)$

$\frac{\partial f}{\partial y} = \cos(x^2 - 3xy) \frac{\partial}{\partial y}(x^2 - 3xy) = \cos(x^2 - 3xy)(-3x)$

c) $\frac{\partial f}{\partial x} = 2xye^{2xy} + x^2y \cdot 2ye^{2xy}$ $\quad \frac{\partial f}{\partial y} = x^2 \cdot 2ye^{2xy} + x^2y^2 \cdot 2xe^{2xy}$

d) $\frac{\partial f}{\partial x} = e^{\operatorname{sen}(xy^{1/2})} \cos(xy^{1/2})\, y^{1/2}$

$\frac{\partial f}{\partial y} = e^{\operatorname{sen}(xy^{1/2})} \cos(xy^{1/2})\, x \cdot \frac{1}{2} y^{-1/2}$

e) $\frac{\partial f}{\partial x} = \frac{\frac{\partial}{\partial x}(x^2y^3)}{1 + (x^2y^3)^2} = \frac{2x \cdot y^3}{1 + x^4y^6}$

$\frac{\partial f}{\partial y} = \frac{\frac{\partial}{\partial y}(x^2y^3)}{1 + (x^2y^3)^2} = \frac{x^2 \cdot 3y^2}{1 + x^4y^6}$

f) $\frac{\partial f}{\partial x} = 1 + y^2 + \frac{\frac{\partial}{\partial x}(x^2+y)}{\operatorname{sen}(x^2+y)} = 1 + y^2 + \frac{2x}{\operatorname{sen}(x^2+y)}$

$\frac{\partial f}{\partial y} = 2yx + \frac{\frac{\partial}{\partial y}(x^2+y)}{\operatorname{sen}(x^2+y)} = 2xy + \frac{1}{\operatorname{sen}(x^2+y)}$

g) $\frac{\partial f}{\partial x} = \frac{e^x}{e^x + z^y}$ $\quad \frac{\partial f}{\partial y} = \frac{\frac{\partial}{\partial y}(z^y)}{e^x + z^y} = \frac{z^y \ln z}{e^x + z^y}$

$z^y = e^{\ln z^y} = e^{y \ln z}$

$\frac{\partial}{\partial y}(z^y) = e^{y\ln z} \ln z$
$= z^y \ln z$

$\frac{\partial f}{\partial z} = \frac{yz^{y-1}}{e^x + z^y}$

h) $\frac{\partial f}{\partial x} = \frac{y^3(x^3y - e^z) - 3x^2y\,(xy^3+e^z)}{(x^3y - e^z)^2}$

$\frac{\partial f}{\partial y} = \frac{3xy^2(x^3y - e^z) - x^3(xy^3+e^z)}{(x^3y - e^z)^2}$

$\frac{\partial f}{\partial z} = \frac{e^z(x^3y - e^z) + e^z(xy^3+e^z)}{(x^3y - e^z)^2}$

4) $f_x(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h} = \lim_{h\to 0} \frac{\frac{h\cdot 0}{h^2+0^2} - 0}{h} = 0$

$f_y(0,0) = \lim_{k\to 0} \frac{f(0,k) - f(0,0)}{k} = \lim_{k\to 0} \frac{\frac{0k}{0^2+k^2} - 0}{k} = 0$

5) a) Saltei - dá muita conta

b) $\frac{\partial f}{\partial x} = -sen(xy^2)y^2$ $\quad\frac{\partial f}{\partial y} = -sen(xy^2)\,2xy$

$\frac{\partial^2 f}{\partial x^2} = -cos(xy^2)y^4$ $\quad\frac{\partial^2 f}{\partial y^2} = -cos(xy^2)(2xy)^2 - sen(xy^2)\,2x$

$\frac{\partial^2 f}{\partial x\partial y} = \frac{\partial^2 f}{\partial y\partial x} = -cos(xy^2)\,2xy.\,y^2 - sen(xy^2)\,2y$

c) $\frac{\partial f}{\partial x} = e^{-xy^2}(-y^2) + y^3.\,4x^3$ $\quad\frac{\partial f}{\partial y} = e^{-xy^2}(-2xy) + 3y^2x^4$

$\frac{\partial^2 f}{\partial x^2} = e^{-xy^2}(-y^2)^2 + y^3.\,12x^2$ $\quad\frac{\partial^2 f}{\partial y^2} = e^{-xy^2}(-2xy)^2 + 6y\,x^4$

$\frac{\partial^2 f}{\partial x\partial y} = \frac{\partial^2 f}{\partial y\partial x} = e^{-xy^2}(-2xy)(-y^2) + e^{-xy^2}(-2y) + 3y^2.\,4x^3$

d) $f(x,y) = (cos^2x + e^{-y})^{-1}$

$\frac{\partial f}{\partial x} = -(cos^2x + e^{-y})^{-2}\,2cos\,x.\,(-sen\,x)$

$\frac{\partial f}{\partial y} = -(cos^2x + e^{-y})^{-2}(-e^{-y})$

$\frac{\partial^2 f}{\partial x^2} = 2(cos^2x + e^{-y})^{-3}(-2cos\,x\,sen\,x)^2 - (cos^2x + e^{-y})^{-2}(2sen^2x - 2cos^2x)$

$\frac{\partial^2 f}{\partial y^2} = 2(cos^2x + e^{-y})^{-3}(-e^{-y})^2 + (cos^2x + e^{-y})\,e^{-y}$

$\frac{\partial^2 f}{\partial x\partial y} = \frac{\partial^2 f}{\partial y\partial x} = 2(cos^2x + e^{-y})^{-3}(-e^{-y})(-2sen\,x\,cos\,x$

6-b) $f_x = 1 + \dfrac{1\cdot(y-z) - (x-y)\cdot 0}{(y-z)^2} = 1 + \dfrac{y-z}{(y-z)^2}$

$f_y = \dfrac{-(y-z) - (x-y)\cdot 1}{(y-z)^2} = \dfrac{z-x}{(y-z)^2}$

$f_z = \dfrac{0\,(y-z) - (x-y)(-1)}{(y-z)^2} = \dfrac{x-y}{(y-z)^2}$

$f_x + f_y + f_z = 1 + \dfrac{\cancel{y} - \cancel{z} + \cancel{z} - \cancel{x} + \cancel{x} - \cancel{y}}{(y-z)^2} = 1$

8) a) $\dfrac{\partial f}{\partial x} = 2x \qquad \dfrac{\partial^2 f}{\partial x^2} = 2 \qquad \dfrac{\partial f}{\partial y} = 2y \qquad \dfrac{\partial^2 f}{\partial x^2} = 2$

$\dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} = 2+2 = 4 \neq 0$ f nañ resolve a eq. de Laplace

b) $\dfrac{\partial f}{\partial x} = 3x^2 - 3y^2 \qquad \dfrac{\partial^2 f}{\partial x^2} = 6x \qquad \dfrac{\partial f}{\partial y} = -6xy \qquad \dfrac{\partial^2 f}{\partial y^2} = -6x$

$\dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} = 6x - 6x = 0$ f resolve a eq. de Laplace

d) $\dfrac{\partial f}{\partial x} = -e^{-x}\cos y + e^{-y}\operatorname{sen} x \qquad \dfrac{\partial^2 f}{\partial x^2} = e^{-x}\cos y + e^{-y}\cos x$

$\dfrac{\partial f}{\partial y} = -e^{-x}\operatorname{sen} y + e^{-y}\cos x \qquad \dfrac{\partial^2 f}{\partial y^2} = -e^{-x}\cos y - e^{-y}\cos x$

$\dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} = e^{-x}\cos y + e^{-y}\cos x - e^{-x}\cos y - e^{-y}\cos x = 0$

f resolve a equação de Laplace