Primitivas

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$

$\int a^x\,dx = \frac{a^x}{\ln a} + C$

$\int \ln x\,dx = x\ln x - x + C$

$\int \operatorname{sen} x\,dx = -\cos x + C$

$\int \cos x\,dx = \operatorname{sen} x + C$

- $\int f'(x)g(x)\,dx = f(x)g(x) - \int f(x)g'(x)\,dx$

- $\int g'(f(x))\,f'(x)\,dx = g(f(x)) + C$

	0	$\pi/6$	$\pi/2$	$\pi/4$	$\pi/3$
sen	0	$1/2$	1	$\sqrt{2}/2$	$\sqrt{3}/2$
cos	1	$\sqrt{3}/2$	0	$\sqrt{2}/2$	$1/2$

Folha 4

4. b) $\frac{\partial w}{\partial p}$ e $\frac{\partial w}{\partial q}$, $w = r^2 + s^2$, $r = pq^2$, $s = p^2 \operatorname{sen} q$

$2p \operatorname{sen} q$

$$\frac{\partial w}{\partial p}(p,q) = \frac{\partial w}{\partial r}\Big|_{r(p,q),s(p,q)} \cdot \frac{\partial r}{\partial p}\Big|_{(p,q)} + \frac{\partial w}{\partial s}\Big|_{r(p,q),s(p,q)} \cdot \boxed{\frac{\partial s}{\partial p}}\Big|_{(p,q)}$$

$$= 2r\Big|_{(pq^2,\,p^2\operatorname{sen} q)} \cdot q^2\Big|_{(p,q)} + 2s\Big|_{(pq^2,\,p^2\operatorname{sen} q)} \cdot 2p\operatorname{sen} q\Big|_{(p,q)}$$

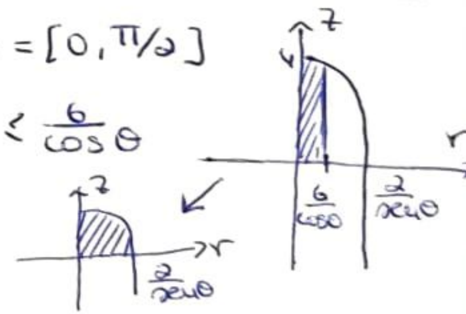
$$= 2pq^2q^2 + 2p^2\operatorname{sen} q \cdot 2p\operatorname{sen} q = \ldots$$

$D = \{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 \leq 4,\ 2 \leq z \leq 3\}$

- fixando $z_0 \in [2,3]$, desenha-se $D_{z_0} = D \cap \{(x,y,z_0) : x, y \in \mathbb{R}\}$
- Resolver o exercício.

Folha 6

6. cilindricas
$$\begin{cases} x \geq 0 \\ y \geq 0 \\ x \leq 6 \\ z \geq 0 \\ z \leq 4 - y^2 \end{cases} \qquad \begin{cases} r\cos\theta \geq 0 \\ r\operatorname{sen}\theta \geq 0 \end{cases} > \theta = [0, \pi/2] \\ r\cos\theta \leq 6 \to r \leq \frac{6}{\cos\theta} \\ z \geq 0 \\ z \leq 4 - r^2\operatorname{sen}^2\theta$$



$z = 0 \Rightarrow \frac{-4}{\operatorname{sen}^2\theta} \leq -r^2 \Leftrightarrow r \geq \frac{2}{\operatorname{sen}\theta}$ $\qquad r = r \Leftrightarrow \frac{6}{\cos\theta} = \frac{2}{\operatorname{sen}\theta} \Leftrightarrow \operatorname{tg}\theta \leq \frac{1}{3}$

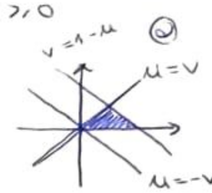
$$\int_0^{\operatorname{arctg}\frac{1}{3}} \int_0^{\frac{6}{\cos\theta}} \int_0^{4 - r^2\operatorname{sen}^2\theta} r\,dz\,dr\,d\theta + \int_{\operatorname{arctg} 1/3}^{\pi/2} \int_0^{\frac{2}{\operatorname{sen}\theta}} \int_0^{4 - r^2\operatorname{sen}^2\theta} r\,dz\,dr\,d\theta$$

Folha 4

$$J(x,y)f = \begin{pmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \\ \frac{\partial f_3}{\partial x} & \frac{\partial f_3}{\partial y} \end{pmatrix}$$

mudança de variável

$\begin{cases} x = u+v \\ y = u-v \end{cases}$ $\begin{cases} 0 \leq y \leq x \\ 0 \leq x \leq 1 \end{cases}$ $\begin{cases} 0 \leq u-v \leq u+v \\ 0 \leq u+v \leq 1 \end{cases}$ ① $\begin{cases} u-v \geq 0 \\ u-v \leq u+v \\ u+v \geq 0 \\ u+v \leq 1 \end{cases}$ $\begin{cases} u \geq v \\ -v \leq v \Rightarrow v \geq 0 \\ v \geq -u \\ v \leq 1-v \end{cases}$ ②



$J(u,v)\varphi = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ $|\det J(u,v)\varphi| = |-1-1| = 2$ ③

$f(\varphi(u,v)) = f(u+v, u-v) = u+v+u-v = 2u$ ④

$\iint_D (x+y)\, d(x,y) = \int_0^{1/2} \int_v^{1-v} 2 \cdot 2u \, du\, dv = \dots$

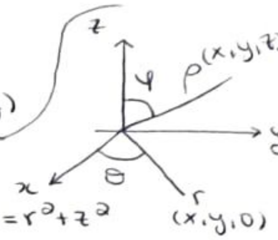
$= \iint_{\varphi^{-1}(D)} |\det J(u,v)\varphi| \, f(\varphi(u,v))\, d(x,y)$

Coordenadas polares

$r = \sqrt{x^2+y^2}$ $\varnothing : \begin{cases} x = r\cos\theta \\ y = r \operatorname{sen}\theta \end{cases}$ $\theta \in [0, 2\pi]$

$\iint_D f(x,y)\, d(x,y) = \iint_{\varnothing^{-1}(D)} r f(r\cos\theta, r\operatorname{sen}\theta)\, d(r,\theta)$

coordenadas esféricas (ρ, θ, φ)



- $z = \rho\cos\varphi$ · $x = \rho \operatorname{sen}\varphi \cos\theta$ · $\rho^2 = r^2 + z^2$
- $r = \rho\operatorname{sen}\varphi$ · $y = \rho\operatorname{sen}\varphi \,\cancel{\cos}\, \operatorname{sen}\theta$ · $|J(\rho,\theta,\varphi)| = \rho^2 \operatorname{sen}\varphi$

Integrais em linha

$\int_c f\, ds = \int_a^b f(c(t))\, \|c'(t)\|\, dt$

$\int_c F \cdot ds = \int_a^b F(c(t))\, c'(t)\, dt$

- $\int_c F\cdot ds = f(c(b)) - f(c(a))$ → não depende do caminho

para mostrar que é conservativo

$\frac{\partial F_1}{\partial y} = \frac{\partial F_2}{\partial x}$ em $\mathbb{R}^3 = \frac{\partial F_1}{\partial y} = \frac{\partial F_2}{\partial x}$; $\frac{\partial F_1}{\partial z} = \frac{\partial F_3}{\partial x}$; $\frac{\partial F_2}{\partial z} = \frac{\partial F_3}{\partial y}$

Comprimento do gráfico

$L(c) = \int_a^b \|c'(t)\|\, dt = \dots$

Teorema de Green $F : D \to \mathbb{R}^2$ $\boxed{F \text{ de classe } C^1}$

$(x,y) \mapsto (P(x,y), Q(x,y))$

- $\int_c P\,dx + Q\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) d(x,y)$

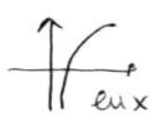
obs • se F for um campo de gradientes então:

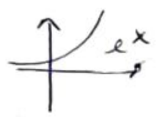
$\int_{c+} P\,dx + Q\,dy = 0$ (porque c+ é um caminho fechado)

$\iint_D (\dots) = 0$ (porque $\frac{\partial Q}{\partial \cancel{y} x} = \frac{\partial P}{\partial y}$)

reta tangente à curva.

$x = c(t) + k c'(t), \ k \in \mathbb{R}$





$\cos^2\theta = \frac{1+\cos(2\theta)}{2}$

$\operatorname{sen}^2\theta = \frac{1-\cos(2\theta)}{2}$

$\nabla f(x_0) = \left(\frac{\partial f}{\partial x_1}(x_0), \dots, \frac{\partial f}{\partial x_n}(x_0)\right)$

gradiente de f em x_0

$\nabla f(1,2)\cdot(3,0) = \frac{\partial f}{\partial x}(1,2)\cdot 3 + \frac{\partial f}{\partial y}(1,2)\cdot 0$

Derivadas:

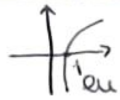
$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$

$(f(x)^n)' = n f(x)^{n-1} f'(x)$

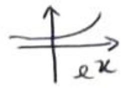
$(a^x)' = x' a^x \ln a$

Limites:

$\lim_{x \to 0} \frac{\operatorname{sen} x}{x} = 1$



$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$



$\lim_{x \to +\infty} \frac{\ln x}{x} = 0$

Graus:

$f(x,y) = \frac{x^2 y}{(x^2+y^2)^{3/2}}$

grau numerador $2+1=3$

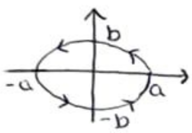
grau denominador $2 \times \frac{3}{2} = 3$

Cálculo de limites

- calcular $\lim_{x \to x_0} f(x, y_0 + m(x - x_0))$ e $\lim_{y \to y_0} f(x_0, y)$
- fazer o enquadramento
- procurar alguma curva que se aproxime do ponto e segundo o qual o limite ou não existe ou não dá 0.

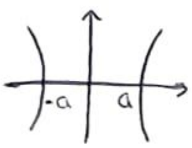
- $\operatorname{sen} 2x = 2 \operatorname{sen} x \cos x$
- $\cos 2x = \cos^2 x - \operatorname{sen}^2 x$
- $\operatorname{sen}(x+y) = \operatorname{sen} x \cos y + \cos x \operatorname{sen} y$ // • $\cos(x+y) = \cos x \cos y - \operatorname{sen} x \operatorname{sen} y$

Elipse de equação:



$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Hipérbole de equação:



$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Derivadas parciais de 1ª ordem

Uma função é de classe 1, C^1 se f for contínua e se existirem e forem contínuas as derivadas de 1ª ordem de f.

- 1º calcular a equação da 1ª derivada
- calcular a derivada para o ponto (pela definição do "h")
- calcular $\lim_{x \to x_0} \frac{df}{dx}(x,x)$ e ver se dá igual.

Derivadas direcionais

$f'(x_0, \vec{v}) = \lim_{h \to 0} \frac{f(x_0 + h\vec{v}) - f(x_0)}{h}$

se a função estiver em ramos temos de ver se é de classe C^1

Gradiente

→ tem de ser classe C^1

Gradiente → $f'((1,2),(3,0))$ → $\nabla f(x_0) = (1,2) \cdot (3,0)$

- calcular $\underbrace{\frac{df}{dx}(1,2)}_{a}$ e $\underbrace{\frac{df}{dy}(1,2)}_{b}$
- $(a,b) \cdot (3,0) = \ldots$

Derivadas parciais de 2ª ordem

- f é uma função de classe C^2 se existirem e forem contínuas todas as derivadas parciais de 2ª ordem. Se f for de classe C^2 então f e as derivadas de C^1 também são contínuas.

$\bar{A}$ → linha e se tiver interior → aderência

$fr(A)$ → fronteira, sem interior

$\mathring{A}$ → pontos interiores (sem a linha)

- $\mathring{A} = A$ → conjunto aberto
- $\bar{A} = A$ → conjunto fechado

- para um conjunto ser limitado,

$\exists R > 0 : A \subseteq \underbrace{B(0,R)}$

circunferência de centro $(0,0)$ e raio R.

Enquadramentos

→ não podes sacar raízes no denominador.

$$\left.\begin{array}{l} |x| = \sqrt{x^2} \leq \sqrt{x^2+y^2} \\ |y| = \sqrt{y^2} \leq \sqrt{x^2+y^2} \end{array}\right\}$$

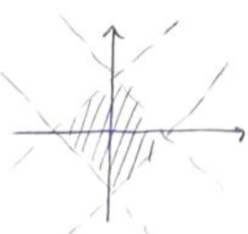
Teorema do Confronto

Uma parte tem de ser limitada e outra tender para 0.

$\{(x,y) \in \mathbb{R}^2 : |x| + |y| < 2\}$

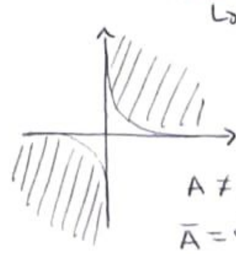
$\hookrightarrow |y| < 2 - |x|$

$$\begin{cases} y < 2 - x \\ y < 2 + x \\ -y < 2 - x \\ -y < 2 + x \end{cases} \Rightarrow \begin{cases} y < 2 - x \\ y < 2 + x \\ y > -2 + x \\ y > -2 - x \end{cases}$$



$\{(x,y) \in \mathbb{R}^2 : xy > 1\}$

$\hookrightarrow y > \frac{1}{x}$



$A \neq \bar{A}$

$\bar{A} = \{(x,y) \in \mathbb{R}^2 : xy \geq 1\}$