Universidade do Minho
Departamento de Matemática

LFIS e MIEFIS

Cálculo para Ciências

Folha 6 — outubro de 2021

Exercício 1. Calcule:

a) $\displaystyle\int (3x^2 - 2x^5)\,dx$;

b) $\displaystyle\int (\sqrt{x}+2)^2\,dx$;

c) $\displaystyle\int (2x+10)^{20}\,dx$;

d) $\displaystyle\int x^2 e^{x^3}\,dx$;

e) $\displaystyle\int x^4(x^5+10)^9\,dx$;

f) $\displaystyle\int \frac{2x+1}{x^2+x+3}\,dx$;

g) $\displaystyle\int \sqrt{2x+1}\,dx$;

h) $\displaystyle\int \frac{x}{3-x^2}\,dx$;

i) $\displaystyle\int \frac{1}{4-3\,x}\,dx$;

j) $\displaystyle\int \frac{1}{e^{3x}}\,dx$;

k) $\displaystyle\int \frac{-7}{\sqrt{1-5x}}\,dx$;

l) $\displaystyle\int \frac{\sqrt{1+3\ln x}}{x}\,dx$;

m) $\displaystyle\int x\ \operatorname{sen} x^2\,dx$;

n) $\displaystyle\int \frac{1}{x(\ln^2 x+1)}\,dx$;

o) $\displaystyle\int \left(\frac{2}{x}-3\right)^2 \frac{1}{x^2}\,dx$;

p) $\displaystyle\int \operatorname{sen}(\pi - 2\,x)\,dx$;

q) $\displaystyle\int \operatorname{th} x\,dx$;

r) $\displaystyle\int \operatorname{sen} x\ \cos x\,dx$;

s) $\displaystyle\int \operatorname{sen}(2x)\ \cos x\,dx$;

t) $\displaystyle\int \operatorname{sen}^2 x\,dx$;

u) $\displaystyle\int \operatorname{sen}^2 \tfrac{x}{2}\ \cos^2 \tfrac{x}{2}\,dx$;

v) $\displaystyle\int \cos^3 x\,dx$;

w) $\displaystyle\int \frac{x}{x^2-1}\,dx$;

x) $\displaystyle\int \frac{x}{\sqrt{x^2-1}}\,dx$;

y) $\displaystyle\int \frac{1}{x}\ \operatorname{sen}(\ln x)\,dx$;

z) $\displaystyle\int \frac{-3}{x\,(\ln x)^3}\,dx$.

Exercício 2. Calcule:

a) $\displaystyle\int \ln x\,dx$;

b) $\displaystyle\int x\ \operatorname{sen}(2x)\,dx$;

c) $\displaystyle\int \operatorname{arctg} x\,dx$;

d) $\displaystyle\int x\cos x\,dx$;

e) $\displaystyle\int \ln(1-x)\,dx$;

f) $\displaystyle\int x\ \ln x\,dx$;

g) $\displaystyle\int x^2 \operatorname{sen} x\,dx$;

h) $\displaystyle\int x\ \operatorname{sen} x\ \cos x\,dx$;

i) $\displaystyle\int \ln^2 x\,dx$;

j) $\displaystyle\int e^x \cos x\,dx$;

k) $\displaystyle\int \operatorname{arcsen} x\,dx$;

l) $\displaystyle\int e^{\operatorname{sen} x}\ \operatorname{sen} x\ \cos x\,dx$;

m) $\displaystyle\int \frac{\operatorname{arcsen}\sqrt{x}}{\sqrt{x}}\,dx$;

n) $\displaystyle\int x\ \operatorname{arctg} x\,dx$;

o) $\displaystyle\int x^2\ \ln x\,dx$;

p) $\displaystyle\int \operatorname{sen}(\ln x)\,dx$;

q) $\displaystyle\int \operatorname{ch} x\ \operatorname{sen}(3x)\,dx$;

r) $\displaystyle\int x^3\, e^{x^2}\,dx$.

Exercício 3. Usando o método de substituição, calcule:

a) $\displaystyle\int x\ (x+3)^{1/3}\ dx;$

b) $\displaystyle\int \frac{1}{\operatorname{sen} x}\, dx;$

c) $\displaystyle\int \frac{x}{\sqrt{2-3x}}\, dx;$

d) $\displaystyle\int \frac{\operatorname{sen}\sqrt{x}}{\sqrt{x}}\, dx;$

e) $\displaystyle\int \frac{e^{2x}}{3+e^{x}}\, dx;$

f) $\displaystyle\int \frac{x^2}{\sqrt{1-x^2}}\, dx;$

g) $\displaystyle\int \frac{\sqrt{x}}{x-\sqrt[3]{x}}\, dx;$

h) $\displaystyle\int \sqrt{1+x^2}\, dx.$

Exercício 4. Calcule:

a) $\displaystyle\int \frac{2x^2+x+1}{(x-1)(x+1)^2}\, dx;$

b) $\displaystyle\int \frac{3x^2-4x-1}{(x^2-1)(x-2)}\, dx;$

c) $\displaystyle\int \frac{2x^2-x-2}{x^2(x-2)}\, dx;$

d) $\displaystyle\int \frac{2x^3+5x^2+6x+2}{x(x+1)^3}\, dx;$

e) $\displaystyle\int \frac{x^2-x+2}{x(x^2-1)}\, dx;$

f) $\displaystyle\int \frac{4x^2+x+1}{x^3-x}\, dx;$

g) $\displaystyle\int \frac{27}{x^4-3\,x^3}\, dx;$

h) $\displaystyle\int \frac{x^4-8}{x^3-2\,x^2}\, dx;$

i) $\displaystyle\int \frac{x+3}{(x-2)\,(x^2-2\,x+5)}\, dx;$

j) $\displaystyle\int \frac{x+1}{x(x^2+1)^2}\, dx;$

k) $\displaystyle\int \frac{x+2}{2x(x-1)^2(x^2+1)}\, dx;$

l) $\displaystyle\int \frac{3x^3+x^2-x-1}{x^2(x^2-1)}\, dx.$

Exercício 5. Calcule:

a) $\displaystyle\int \frac{1}{(2+\sqrt{x})^7\,\sqrt{x}}\, dx;$

b) $\displaystyle\int \operatorname{tg}^2 x\, dx;$

c) $\displaystyle\int \frac{x+(\operatorname{arcsen}(3x))^2}{\sqrt{1-9\,x^2}}\, dx;$

d) $\displaystyle\int \frac{x\, e^{\sqrt{1-x^2}}}{\sqrt{1-x^2}}\, dx;$

e) $\displaystyle\int \frac{1}{\cos^2 x\ \operatorname{sen}^2 x}\, dx;$

f) $\displaystyle\int \cos^2 x\ \operatorname{sen}^2 x\, dx;$

g) $\displaystyle\int \frac{1}{1+e^{x}}\, dx;$

h) $\displaystyle\int \frac{1}{x^2\,\sqrt{4-x^2}}\, dx.$

Exercício 6. Sendo $f:\mathbb{R}\to\mathbb{R}$ definida por $f(x)=x^2\ \operatorname{sen} x$, calcule a primitiva de f cujo gráfico passa pelo ponto $(\frac{\pi}{2},\pi)$.

Exercício 7. Em cada alínea, determine a única função $f:\mathbb{R}\to\mathbb{R}$, duas vezes derivável, tal que:

a) $f''(x)=4x-1, \quad x\in\mathbb{R}, \quad f(1)=3 \quad \text{e} \quad f'(2)=-2;$

b) $f''(x)=\operatorname{sen} x\cos x, \quad x\in\mathbb{R}, \quad f(0)=0 \quad \text{e} \quad f'(0)=1.$

Exercício 8. Calcule os seguintes integrais:

a) $\displaystyle\int_0^1 e^{\pi x}\,dx$;

b) $\displaystyle\int_{-\pi/2}^{\pi/2} |\operatorname{sen} x|\,dx$;

c) $\displaystyle\int_{-3}^{5} |x-1|\,dx$;

d) $\displaystyle\int_0^2 |(x-1)(3x-2)|\,dx$;

e) $\displaystyle\int_0^3 \sqrt{9-x^2}\,dx$;

f) $\displaystyle\int_{-5}^{0} 2x\sqrt{4-x}\,dx$;

g) $\displaystyle\int_{3/4}^{4/3} \frac{1}{x^2\sqrt{x^2+1}}\,dx$;

h) $\displaystyle\int_0^1 \log(x^2+1)\,dx$;

i) $\displaystyle\int_0^2 x^3\,e^{x^2}\,dx$;

j) $\displaystyle\int_0^\pi x\,\operatorname{sen}\,x\,dx$;

k) $\displaystyle\int_0^{\sqrt{2}/2} \operatorname{arcsen}\,x\,dx$;

l) $\displaystyle\int_{-3}^{2} \sqrt{|x|}\,dx$;

m) $\displaystyle\int_0^2 f(x)\,dx$, com
$$f(x) = \begin{cases} x^2 & \text{se} \quad 0 \le x \le 1, \\ 2-x & \text{se} \quad 1 < x \le 2; \end{cases}$$

n) $\displaystyle\int_0^1 g(x)\,dx$, com
$$g(x) = \begin{cases} x & \text{se} \quad 0 \le x \le 1/2, \\ -x & \text{se} \quad 1/2 < x \le 1. \end{cases}$$

Exercício 9. Dado $a \in \mathbb{R}^+$, seja $f : [-a,a] \to \mathbb{R}$ uma função integrável. Mostre que:

a) se f é par então $\displaystyle\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$;

b) se f é ímpar então $\displaystyle\int_{-a}^{a} f(x)\,dx = 0$.

Exercício 10. Dados $a < b \in \mathbb{R}$, mostre que se $f : [a,b] \to \mathbb{R}$ é uma função contínua e $\displaystyle\int_a^b f(x)\,dx = 0$, então existe $c \in \,]a,b[$ tal que $f(c) = 0$.

Exercício 11. Em cada uma das alíneas, calcule a função derivada de F, sendo F definida por:

a) $F(x) = \displaystyle\int_0^x (1+t^2)^{-3}\,dt$, $x \in \mathbb{R}$;

b) $F(x) = \displaystyle\int_0^{x^2} (1+t^2)^{-3}\,dt$, $x \in \mathbb{R}$;

c) $F(x) = \displaystyle\int_{x^3}^{x^2} \frac{t^6}{1+t^4}\,dt$, $x \in \mathbb{R}$.

Exercício 12. Sabendo que $f : \mathbb{R}_0^+ \to \mathbb{R}$ é uma função contínua e satisfaz a igualdade abaixo para $x \ge 0$, calcule f em cada um dos seguintes casos:

a) $\displaystyle\int_0^x f(t)\,dt = x^2(1+x)$;

b) $\displaystyle\int_0^{x^2} f(t)\,dt = x^3e^x - x^4$.

Exercício 13. Em cada alínea calcule a área da região limitada pelas curvas de equações:

a) $x = 1, \quad x = 4, \quad y = \sqrt{x}, \quad y = 0;$

b) $x = 0, \quad x = 1, \quad y = 3x, \quad y = -x^2 + 4;$

c) $x = 0, \quad x = 2, \quad x^2 + (y - 2)^2 = 4, \quad x^2 + (y + 2)^2 = 4;$

d) $x = 0, \quad x = \pi/2, \quad y = \operatorname{sen} x, \quad y = \cos x;$

e) $x = -1, \quad y = |x|, \quad y = 2x, \quad x = 1;$

f) $y = -x^3, \quad y = -(4x^2 - 4x);$

g) $y = -x^2 + \dfrac{7}{2}, \quad y = x^2 - 1;$

h) $y = 0, \quad x = -\ln 2, \quad x = \ln 2, \quad y = \operatorname{sh} x.$

Exercício 14. Escreva uma expressão integral que permita calcular a área de cada uma das seguintes regiões:

a) $\{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 2 \wedge -x \leq y \leq x^2\};$

b) $\{(x, y) \in \mathbb{R}^2 : (x - 2)^2 + y^2 \leq 4 \wedge 0 \leq y \leq x\};$

c) $\{(x, y) \in \mathbb{R}^2 : |x| + |y| \leq 1\};$

d) $\{(x, y) \in \mathbb{R}^2 : x^2 - 1 \leq y \leq x + 1\};$

e) $\{(x, y) \in \mathbb{R}^2 : -1 \leq x \leq 2 \wedge 0 \leq y \leq e^x \wedge 0 \leq y \leq e^{-x}\};$

f) $\{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 2 \wedge 0 \leq y \leq x^2 \wedge 0 \leq y \leq 2 - x\};$

g) $\{(x, y) \in \mathbb{R}^2 : y \geq 0 \wedge y \geq x^2 - 2x \wedge y \leq 4\};$

h) $\{(x, y) \in \mathbb{R}^2 : x \leq 3 \wedge y \geq x^2 - 4x + 3 \wedge y \leq -x^2 + 5x - 4\}.$