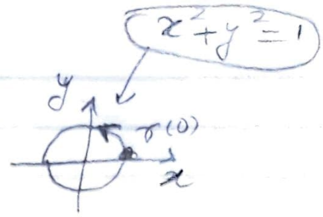
Folha 1 - correcção

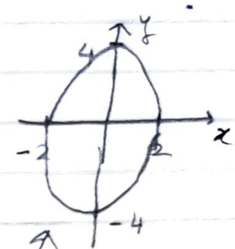


1) a) $\gamma : [0, 2\pi] \longrightarrow \mathbb{R}^2$

$t \longmapsto (\cos t, \operatorname{sen} t)$

Notem que $\cos^2 t + \operatorname{sen}^2 t = 1$, para todo o $t \in [0, 2\pi]$

A curva γ percorre a circunferência de centro em $(0,0)$ e raio 1, no sentido directo, uma vez, com ponto inicial $\gamma(0) = (1,0)$.



b) $\gamma : [0, 2\pi] \longrightarrow \mathbb{R}^2$

$t \longmapsto (2\operatorname{sen} t, 4\cos t)$

$$\frac{(2\operatorname{sen} t)^2}{4} + \frac{(4\cos t)^2}{16} = \operatorname{sen}^2 t + \cos^2 t = 1$$

$$\frac{x^2}{4} + \frac{y^2}{16} = 1$$

Então γ percorre a elipse $\frac{x^2}{4} + \frac{y^2}{16} = 1$, uma vez, no sentido retrógrado, começando em $\gamma(0) = (0, 4)$.

c) $c : \mathbb{R} \longrightarrow \mathbb{R}^3$

$t \longmapsto (2t-1, t+2, t)$

$c(t) = (2t-1, t+2, t) = (-1, 2, 1) + t(2, 1, 1), \quad t \in \mathbb{R}$

c percorre a recta de equação vectorial

$(x, y, z) = (-1, 2, 1) + \lambda(2, 1, 1), \quad \lambda \in \mathbb{R}$

d) $c : [1, 3] \longrightarrow \mathbb{R}^3$

$t \longmapsto (-t, 2t, 1/t)$

é difícil fazer o desenho. Mostrei a curva numa aula teórica, desenhada no software Maple.

2) a) $c'(t) = (6, 6t, 3t^2), \quad t \in \mathbb{R}$

b) $c'(t) = (3\cos(3t), -3\operatorname{sen}(3t), \cancel{2} \cdot \frac{3}{\cancel{2}} t^{1/2}), \quad t \in \mathbb{R}_0^+$

c) $r'(t) = (2\cos t(-\operatorname{sen} t), 3 - 3t^2, 1), \quad t \in \mathbb{R}$

d) $r'(t) = (4e^t, 24t^3, -\operatorname{sen} t), \quad t \in \mathbb{R}$

3) $c(t) = (6t, 3t^2, t^3), \quad t \in \mathbb{R}$

$c'(t) = (6, 6t, 3t^2)$

$c'(0) = (6, 0, 0)$ vector velocidade no instante 0

4 a) $c(t)=(\operatorname{sen}(3t), \cos(3t), 2t^{5/2})$

$c'(t)=(3\cos(3t), -3\operatorname{sen}(3t), \cancel{2}\cdot\frac{5}{\cancel{2}}t^{3/2})$

$c'(1)=(3\cos(3), -3\operatorname{sen}(3), 5)$

$c(1)=(\operatorname{sen}(3), \cos(3), 2)$

Recta tangente a c no instante $t=1$:

$(x,y,z)=c(1)+\lambda c'(1), \lambda\in\mathbb{R}$

$=(\operatorname{sen}(3), \cos(3), 2)+\lambda(3\cos(3), -3\operatorname{sen}(3), 5), \lambda\in\mathbb{R}$

b) $c(t)=(\cos^2 t, 3t-t^3, t)$

$c'(t)=(-2\cos t\operatorname{sen} t, 3-3t^2, 1)$

$c'(0)=(0,3,1)$

$c(0)=(1,0,0)$

Recta tangente a c no instante $t=0$:

$(x,y,z)=c(0)+\lambda c'(0), \lambda\in\mathbb{R}$

$=(1,0,0)+\lambda(0,3,1), \lambda\in\mathbb{R}.$

5 a) $c(t)=(t^2, t^3-4t, 0)$, $t_0=2$ e $t_1=3$

$c'(t)=(2t, 3t^2-4, 0)$

$c(2)=(4,0,0)$

$c'(2)=(4,8,0)$

A partícula sai disparada em $t_0=2$, percorrendo uma trajectória rectilínea, com ponto inicial $(4,0,0)$ e vector director $(4,8,0)$. Então temos

$$(x(t), y(t), z(t)) = \underbrace{(4,0,0)}_{c(2)} + t\,\underbrace{(4,8,0)}_{c'(2)}$$

A partícula estará, quando $t_1=3$, no ponto

$$(x,y,z)=(4,0,0)+\underbrace{(t_1-t_0)}_{1}(4,8,0)=(8,8,0)$$

- As restantes alíneas são análogas, apresentarei apenas os cálculos

b) $c(t)=(e^t, e^{-t}, \cos t)$, $t_0=1$ e $t_1=2$

$c'(t)=(e^t, -e^{-t}, -\operatorname{sen} t)$

$c'(1)=(e, -\frac{1}{e}, -\operatorname{sen}1)$ $\quad c(1)=(e, \frac{1}{e}, \cos 1)$

$(x,y,z)=c(1)+(t_1-t_0)c'(1)$

$=(e, \frac{1}{e}, \cos 1)+(e, -\frac{1}{e}, -\operatorname{sen}1)=(2e, 0, \cos 1-\operatorname{sen}1)$

c) $c(t)=(4e^t, 6t^4, \cos t)$, $t_0=0$, $t_1=1$

$c'(t)=(4e^t, 24t^3, -\operatorname{sen} t)$

$c(0)=(4,0,1)$

$c'(0)=(4,0,0)$

$(x,y,z)=c(0)+(t_1-t_0)c'(0)$

$=(4,0,1)+(4,0,0)=(8,0,1)$

d) $c(t)=(\operatorname{sen}(e^t), t, 4-t^3)$, $t_0=1$, $t_1=2$

$c'(t)=(\cos(e^t)e^t, 1, -3t^2)$

$c'(1)=(e\cos(e), 1, -3)$

$c(1)=(\operatorname{sen}(e), 1, 3)$

$(x,y,z)=c(1)+(t_1-t_0)c'(1)$

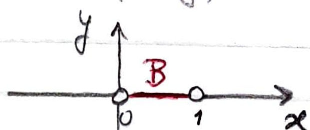
$=(\operatorname{sen}(e), 1, 3)+(e\cos(e), 1, -3)$

$=(\operatorname{sen}(e)+e\cos(e), 2, 0)$

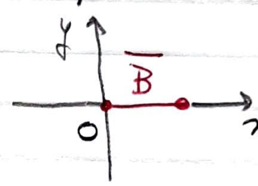
6) Sempre que possível, desenharei os conjuntos

a) $A=\mathbb{R}^2\setminus\{(0,0)\}=\mathring{A}$, $\overline{A}=\mathbb{R}^2$

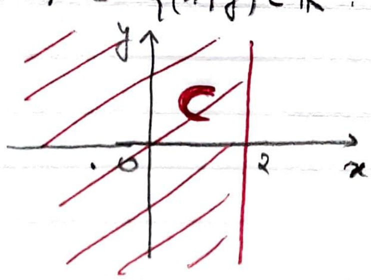
b) $B=\{(x,y)\in\mathbb{R}^2 : y=0, 0<x<1\}$



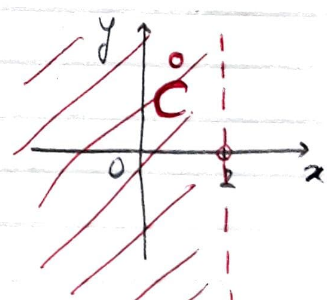
$\mathring{B}=\emptyset$



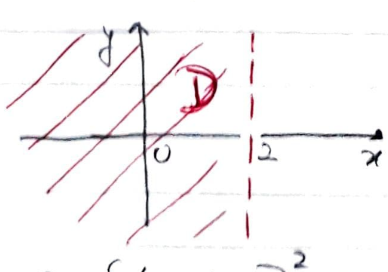
c) $C=\{(x,y)\in\mathbb{R}^2 : x\leq 2\}$



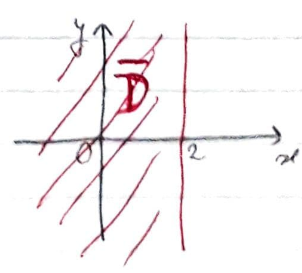
$C=\overline{C}$



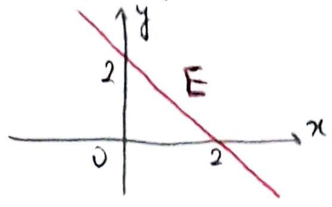
d) $D=\{(x,y)\in\mathbb{R}^2 : x<2\}$



$D=\mathring{D}$

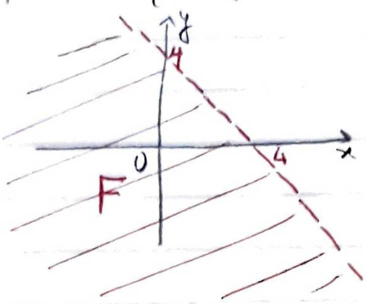


e) $E=\{(x,y)\in\mathbb{R}^2 : x+y=2\}$

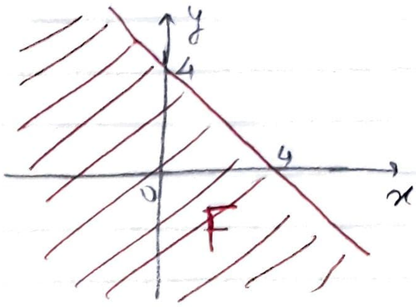


$E=\overline{E}$, $\mathring{E}=\emptyset$

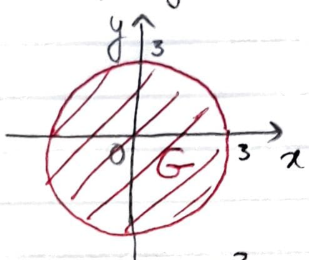
f) $F = \{(x,y) \in \mathbb{R}^2 : x + y < 4\}$



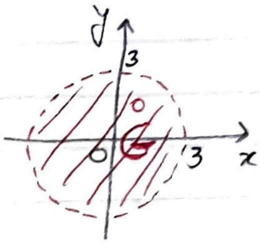
$F = \overset{\circ}{F}$



g) $G = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 9\}$

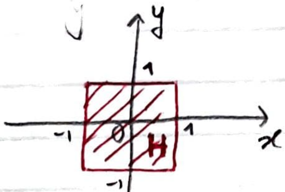


$G = \overline{G}$

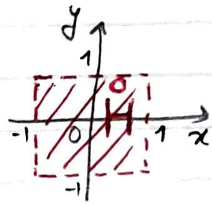


h) $\{(x,y) \in \mathbb{R}^2 : \max\{|x|, |y|\} \leq 1\} = H$

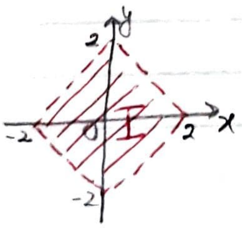
$\max\{|x|, |y|\} \leq 1 \iff |x| \leq 1 \;\wedge\; |y| \leq 1$



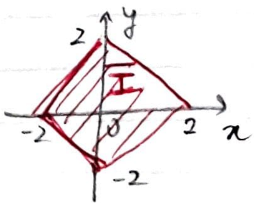
$\overline{H} = H$



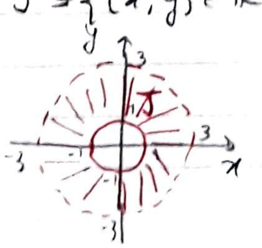
i) $I = \{(x,y) \in \mathbb{R}^2 : |x| + |y| < 2\}$ (visto na aula com detalhe)

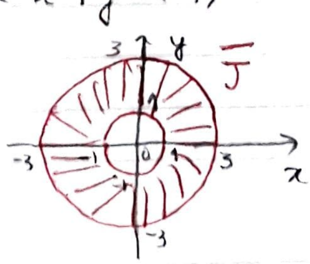


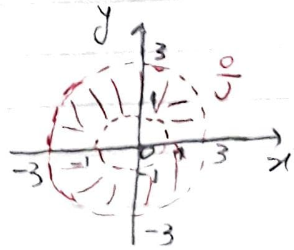
$I = \overset{\circ}{I}$



j) $J = \{(x,y) \in \mathbb{R}^2 : 1 \leq x^2 + y^2 < 9\}$

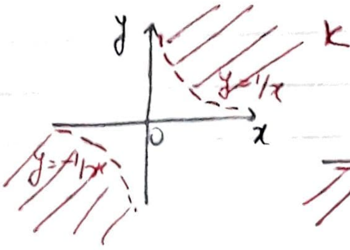




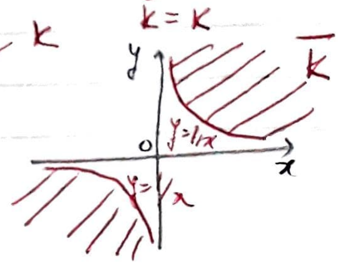


k) $K = \{(x,y) \in \mathbb{R}^3 \; xy > 1\}$

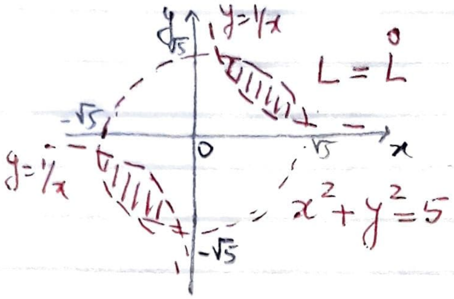
$$xy > 1 \iff \begin{cases} x > 0 \;\wedge\; y > 1/x \\ \vee \\ x < 0 \;\wedge\; y < 1/x \end{cases}$$

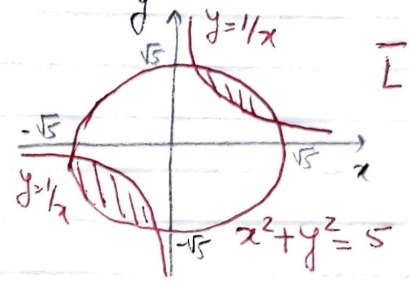


$\overset{\circ}{K} = K$



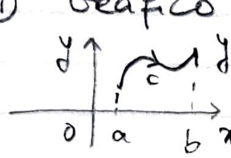
e) $\{(x,y)\in\mathbb{R}^2 : xy>1\} \cap \{(x,y)\in\mathbb{R}^2 : x^2+y^2<5\} = L$





PARAMETRIZAÇÃO DE CURVAS – breve revisão para aproveitar a folha

① Gráfico de uma função f



$f: [a,b] \to \mathbb{R}$

$c: [a,b] \longrightarrow \mathbb{R}^2$
$t \longmapsto (t, f(t))$

② Gráfico percorrido no sentido inverso

$\varphi: [a,b] \longrightarrow [a,b]$ $\quad \varphi(a)=b$
$t \longmapsto -t+a+b$ $\quad \varphi(b)=a$

$d = c\circ\varphi : [a,b] \longrightarrow [a,b]$
$t \longmapsto c(-t+a+b)$

$d(a)=c(b)$ e $d(b)=c(a)$

③ Circunferência de centro em (x_0,y_0) e raio r, percorrida uma vez no sentido directo

$c: [0,2\pi] \longrightarrow \mathbb{R}^2$
$t \longmapsto (x_0,y_0) + r(\cos t, \operatorname{sen} t)$

no sentido retrógrado

$d: [0,2\pi] \longrightarrow \mathbb{R}^2$
$t \longmapsto (x_0,y_0) + r(\cos(2\pi-t), \operatorname{sen}(2\pi-t))$

④ Elipse de equação $\dfrac{(x-x_0)^2}{a^2} + \dfrac{(y-y_0)^2}{b^2} = 1$, percorrida uma vez no sentido directo

$c: [0,2\pi] \longrightarrow \mathbb{R}^2$
$t \longmapsto (x_0,y_0) + (a\cos t, b\operatorname{sen} t)$

sentido retrógrado

$d: [0,2\pi] \longrightarrow \mathbb{R}^2$
$t \longmapsto (x_0,y_0) + (a\cos(2\pi-t), b\operatorname{sen}(2\pi-t))$

⑤ Hipérbole de equação $\dfrac{(x-x_0)^2}{a^2} - \dfrac{(y-y_0)^2}{b^2} = 1$

$c: \mathbb{R} \longrightarrow \mathbb{R}^2$
$t \longmapsto (x_0,y_0) + (a\,\mathrm{ch}\,t, b\,\mathrm{sh}\,t)$

⑥ Segmento de recta que une o ponto A ao ponto B

$c: [0,1] \longrightarrow \mathbb{R}^2$
$t \longmapsto A + t(B-A)$

$d: [0,1] \longrightarrow \mathbb{R}^2$ (de B para A)
$t \longmapsto B + t(A-B)$