Universidade do Minho

Álgebra Linear

Exercícios 3 - Sistemas de equações lineares — 2020/2021

1. Use o método de eliminação de Gauss para resolver os seguintes sistemas de equações lineares e classifique-os quanto ao número de soluções:

a) $\begin{cases} x-2y+z &= 2 \\ x+5y-z &= 1 \\ x+\ y+\ z &= 3 \end{cases}$

$$[\mathrm{Ab}]=\begin{bmatrix}1&-2&1&2\\1&5&-1&1\\1&1&1&3\end{bmatrix}\xrightarrow{-L_1+l_2,-l_1+l_3}\begin{bmatrix}1&-2&1&2\\0&7&-2&-1\\0&3&0&1\end{bmatrix}\xrightarrow{-7L_3}\begin{bmatrix}1&-2&1&2\\0&7&-2&-1\\0&-21&0&-7\end{bmatrix}$$

$$\xrightarrow{3l_2+l_3}\begin{bmatrix}1&-2&1&2\\0&7&-2&-1\\0&0&-6&-10\end{bmatrix}\xrightarrow{(-\frac{1}{2})l_3}\begin{bmatrix}1&-2&1&2\\0&7&-2&-1\\0&0&3&5\end{bmatrix}.$$

$$\begin{cases} x-2y+z &= 2 \\ 7y-2z &= -1 \\ 3z &= 5 \end{cases}\iff\begin{cases} x &= 1 \\ y &= \frac{1}{3} \\ z &= \frac{5}{3}\end{cases}.\ \text{PD}$$

b) $\begin{cases} -x+\ y+z &= 1 \\ 3x+2y-z &= 2 \\ x+\ y+\ z &= 3 \end{cases}$

$$[\mathrm{Ab}]=\begin{bmatrix}-1&1&1&1\\3&2&-1&2\\1&1&1&3\end{bmatrix}\xrightarrow{3L_1+l_2,l_1+l_3}\begin{bmatrix}-1&1&1&1\\0&5&2&5\\0&2&2&4\end{bmatrix}\xrightarrow{\frac{1}{2}L_3,L_2\leftrightarrow L_3}\begin{bmatrix}-1&1&1&1\\0&1&1&2\\0&5&2&5\end{bmatrix}$$

$$\xrightarrow{-5l_2+l_3}\begin{bmatrix}-1&1&1&1\\0&1&1&2\\0&0&-3&-5\end{bmatrix}.$$

$$\begin{cases} -x+\ y+z &= 1 \\ y+z &= 2 \\ -3z &= -5 \end{cases}\iff\begin{cases} x &= 1 \\ y &= \frac{1}{3} \\ z &= \frac{5}{3}\end{cases}.\ \text{PD}$$

c) $\begin{cases} x_1+x_2+x_3 &= 1 \\ 2x_1-x_2+3x_3 &= 2 \\ 4x_1+x_2+x_3 &= 4 \end{cases}$

$$[\mathrm{Ab}]=\begin{bmatrix}1&1&1&1\\2&-1&3&2\\4&1&1&4\end{bmatrix}\xrightarrow{-2L_1+l_2,-4l_1+l_3}\begin{bmatrix}1&1&1&1\\0&-3&1&0\\0&-3&-3&0\end{bmatrix}\xrightarrow{-L_2+L_3}\begin{bmatrix}1&1&1&1\\0&-3&1&0\\0&0&-4&0\end{bmatrix}.$$

$$\begin{cases} x_1+x_2+x_3 &= 1 \\ 3x_2+x_3 &= 0 \\ -4x_3 &= 0 \end{cases},\quad [x_1=1,x_2=0,x_3=0]\ \text{PD}$$

d) $\begin{cases} x+y+z+u &= 0 \\ 2x-y+z-u &= 0 \\ 5x-y+3z &= 0 \\ -x+5y+z+2u &= 0 \end{cases}$

$$[\text{A}]=\begin{bmatrix}1&1&1&1\\2&-1&1&-1\\5&-1&3&0\\-1&5&1&2\end{bmatrix}\xrightarrow{-2L_1+l_2,-5l_1+l_3,L_1+L_4}\begin{bmatrix}1&1&1&1\\0&-3&-1&-3\\0&-6&-2&-5\\0&6&2&3\end{bmatrix}\xrightarrow{-2L_2+L_3,2L_2+L_4}$$

$$\begin{bmatrix}1&1&1&1\\0&-3&-1&-3\\0&0&0&1\\0&0&0&-3\end{bmatrix}\xrightarrow{3L_3+L_4}\begin{bmatrix}1&1&1&1\\0&-3&-1&-3\\0&0&0&1\\0&0&0&0\end{bmatrix}.$$

$$\begin{cases}x+y+z+u&=&0\\-3y-z-3u&=&0\\u&=&0\end{cases}\Longleftrightarrow\begin{cases}x+y+z&=&0\\-3y-z&=&0\\u&=&0\end{cases}\overset{z=\alpha}{\Longleftrightarrow}\begin{cases}x+y&=&-\alpha\\-3y&=&\alpha\\u&=&0\end{cases}\Longleftrightarrow\begin{cases}x&=&-\frac{2\alpha}{3}\\y&=&-\frac{\alpha}{3}\\z&=&\alpha\\u&=&0\end{cases},$$

$\alpha\in\mathbb{R}$. PI

e) $$\begin{cases}2x+3y&=&y+3x\\x-3z&=&2y+1\\3y+z&=&2-2x\end{cases}\Longleftrightarrow\begin{cases}-x+2y&=&0\\x-2y-3z&=&1\\2x+3y+z&=&2\end{cases}$$

$$[\text{Ab}]=\begin{bmatrix}-1&2&0&0\\1&-2&-3&1\\2&3&1&2\end{bmatrix}\xrightarrow{L_1+l_2,2l_1+l_4}\begin{bmatrix}-1&2&0&0\\0&0&-3&1\\0&7&1&2\end{bmatrix}\xrightarrow{L_2\leftrightarrow l_3}\begin{bmatrix}-1&2&0&0\\0&7&1&2\\0&0&-3&1\end{bmatrix}$$

$$\begin{cases}2x+3y&=&y+3x\\x-3z&=&2y+1\\3y+z&=&2-2x\end{cases}\Longleftrightarrow\begin{cases}-x+2y&=&0\\7y+z&=&2\\-3z&=&1\end{cases}\Longleftrightarrow\begin{cases}x&=&\frac{2}{3}\\y&=&\frac{1}{3}\\z&=&-\frac{1}{3}\end{cases}.\ \text{PD}$$

f) $$\begin{cases}3x_1+x_2+x_3+x_4&=&0\\2x_1-x_3+2x_4&=&1\\x_1+x_2-2x_3-x_4&=&-1\\3x_1-x_2+5x_3&=&8\end{cases}\quad\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&-1&1&0\\0&-2&0&1\end{bmatrix}\begin{bmatrix}1&1&-2&-1&-1\\0&-2&3&4&3\\0&-2&7&4&3\\0&-4&11&3&11\end{bmatrix}=\begin{bmatrix}1&1&-2&-1&-1\\0&-2&3&4&3\\0&0&4&0&0\\0&0&5&-5&5\end{bmatrix}$$

$$[\text{Ab}]=\begin{bmatrix}3&1&1&1&0\\2&0&-1&2&1\\1&1&-2&-1&-1\\3&-1&5&0&8\end{bmatrix}\xrightarrow{L_1\leftrightarrow l_3}\begin{bmatrix}1&1&-2&-1&-1\\2&0&-1&2&1\\3&1&1&1&0\\3&-1&5&0&8\end{bmatrix}\xrightarrow{-2L_1+l_2,-3l_1+l_3,-3L_1+L_4}$$

$$\begin{bmatrix}1&1&-2&-1&-1\\0&-2&3&4&3\\0&-2&7&4&3\\0&-4&11&3&11\end{bmatrix}$$

$$\xrightarrow{-L_2+l_3,-2l_2+l_4}\begin{bmatrix}1&1&-2&-1&-1\\0&-2&3&4&3\\0&0&4&0&0\\0&0&5&-5&5\end{bmatrix}\xrightarrow{(\frac{1}{4})L_3,(\frac{1}{5})L_4}\begin{bmatrix}1&1&-2&-1&-1\\0&-2&3&4&3\\0&0&1&0&0\\0&0&1&-1&1\end{bmatrix}\xrightarrow{-L_3+L_4}$$

$$\begin{bmatrix}1&1&-2&-1&-1\\0&-2&3&4&3\\0&0&1&0&0\\0&0&0&-1&1\end{bmatrix}$$

$\left[x_1=\frac{3}{2},x_2=-\frac{7}{2},x_3=0,x_4=-1\right]$. PD

g) $$\begin{cases}x-3y+z&=&2\\2x-4y+3z&=&7\\-3x+y+2z&=&9\end{cases}$$

$$[\text{Ab}]=\begin{bmatrix}1&-3&1&2\\2&-4&3&7\\-3&1&2&9\end{bmatrix}\xrightarrow{-2L_1+l_2,3l_1+l_3}\begin{bmatrix}1&-3&1&2\\0&2&1&3\\0&-8&5&15\end{bmatrix}\xrightarrow{4l_2+l_3}\begin{bmatrix}1&-3&1&2\\0&2&1&3\\0&0&9&27\end{bmatrix}$$

$(x,y,z)=(-1,0,3)$. PD

h) $\begin{cases} x+\ y-z & = & 3 \\ 2x-y+3z & = & 13 \\ 3x-\ 3y+\ 7z & = & 23 \end{cases}$

$$[\text{Ab}]=\begin{bmatrix} 1 & 1 & -1 & 3 \\ 2 & -1 & 3 & 13 \\ 3 & -3 & 7 & 23 \end{bmatrix} \xrightarrow{-2L_1+l_2,-3l_1+l_3} \begin{bmatrix} 1 & 1 & -1 & 3 \\ 0 & -3 & 5 & 7 \\ 0 & -6 & 10 & 14 \end{bmatrix} \xrightarrow{-2l_2+l_3} \begin{bmatrix} 1 & 1 & -1 & 3 \\ 0 & -3 & 5 & 7 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

$\begin{cases} x+\ y-z & = & 3 \\ 2x-y+3z & = & 13 \\ 3x-\ 3y+\ 7z & = & 23 \end{cases} \Longleftrightarrow \begin{cases} x+y-z & = & 3 \\ -3y+5z & = & 7 \end{cases} \overset{z=\alpha}{\Longleftrightarrow} \begin{cases} x & = & \frac{16}{3}-\frac{2}{3}\alpha \\ y & = & \frac{5}{3}\alpha-\frac{7}{3} \\ z & = & \alpha \end{cases}$, $\alpha \in \mathbb{R}$

$(x,y,z)=(\frac{16}{3}-\frac{2}{3}\alpha,\frac{5}{3}\alpha-\frac{7}{3},\alpha)$,$\alpha\in\mathbb{R}$, PI

i) $\begin{cases} -2x+y+3z & = & 10 \\ 10x-5y-15z & = & 30 \\ x+y-3z & = & 25 \end{cases}$

$$[\text{Ab}]=\begin{bmatrix} -2 & 1 & 3 & 10 \\ 10 & -5 & -15 & 30 \\ 1 & 1 & -3 & 25 \end{bmatrix} \xrightarrow{(\frac{1}{5})L_2,L_1\leftrightarrow l_3} \begin{bmatrix} 1 & 1 & -3 & 25 \\ 2 & -1 & -3 & 6 \\ -2 & 1 & 3 & 10 \end{bmatrix} \xrightarrow{l_2+l_3} \begin{bmatrix} 1 & 1 & -3 & 25 \\ 2 & -1 & -3 & 6 \\ 0 & 0 & 0 & 16 \end{bmatrix}.$$

Imp.

j) $\begin{cases} 2x-5y+4z & = & -3 \\ x-2y+z & = & 5 \\ x-3y+3z & = & -8 \end{cases}$

$$[\text{Ab}]=\begin{bmatrix} 2 & -5 & 4 & -3 \\ 1 & -2 & 1 & 5 \\ 1 & -3 & 3 & -8 \end{bmatrix} \xrightarrow{L_1\leftrightarrow l_3} \begin{bmatrix} 1 & -3 & 3 & -8 \\ 1 & -2 & 1 & 5 \\ 2 & -5 & 4 & -3 \end{bmatrix} \xrightarrow{-l_1+l_2,-2l_1+l_3} \begin{bmatrix} 1 & -3 & 3 & -8 \\ 0 & 1 & -2 & 13 \\ 0 & 1 & -2 & 13 \end{bmatrix}$$

$$\xrightarrow{-l_2+l_3} \begin{bmatrix} 1 & -3 & 3 & -8 \\ 0 & 1 & -2 & 13 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

$\begin{cases} 2x-5y+4z & = & -3 \\ x-2y+z & = & 5 \\ x-3y+3z & = & -8 \end{cases} \Longleftrightarrow \begin{cases} x-3y+3z & = & -8 \\ y-2z & = & 13 \end{cases} \overset{z=\alpha}{\Longleftrightarrow} \begin{cases} x & = & 3\alpha+31 \\ y & = & 2\alpha+13 \\ z & = & \alpha \end{cases}$,$\alpha \in \mathbb{R}$

$(x,y,z)=(3\alpha+31,2\alpha+13,\alpha)$,$\alpha\in\mathbb{R}$ PI

k) $\begin{cases} x+y+z & = & 6 \\ 2x+y-2z & = & -2 \\ x-y-z & = & -4 \\ 5x-2y+2z & = & 7 \end{cases}$

$$[\text{Ab}]=\begin{bmatrix} 1 & 1 & 1 & 6 \\ 2 & 1 & -2 & -2 \\ 1 & -1 & -1 & -4 \\ 5 & -2 & 2 & 7 \end{bmatrix} \xrightarrow{-2l_1+l_2,-l_1+l_3,-5l_1+l_4} \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -1 & -4 & -14 \\ 0 & -2 & -2 & -10 \\ 0 & -7 & -3 & -23 \end{bmatrix} \xrightarrow{-2l_2+l_3,-7l_2+l_4}$$

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -1 & -4 & -14 \\ 0 & 0 & 6 & 18 \\ 0 & 0 & 25 & 75 \end{bmatrix} \xrightarrow{(\frac{1}{6})l_3,(\frac{1}{3})l_4} \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -1 & -4 & -14 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 1 & 3 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -1 & -4 & -14 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

$\begin{cases} x+y+z & = & 6 \\ -y-4z & = & -14 \\ z & = & 3 \end{cases} \Longleftrightarrow \begin{cases} x & = & 1 \\ y & = & 2 \\ z & = & 3 \end{cases}$. PD

1) $\begin{cases} 3x_1+5x_2+2x_3 &= 5\\ x_1+x_2+x_3 &= 2\\ 2x_1+3x_2 &= 2\\ 3x_1-2x_3 &= 6 \end{cases}$

$$[Ab]=\begin{bmatrix}3&5&2&5\\1&1&1&2\\2&3&0&2\\3&0&-2&6\end{bmatrix}\xrightarrow{L_1\leftrightarrow l_2}\begin{bmatrix}1&1&1&2\\3&5&2&5\\2&3&0&2\\3&0&-2&6\end{bmatrix}\xrightarrow{-3l_1+l_2,-2l_1+l_3,-3l_1+l_4}\begin{bmatrix}1&1&1&2\\0&2&-1&-1\\0&1&-2&-2\\0&-3&-5&0\end{bmatrix}$$

$$\xrightarrow{L_2\leftrightarrow l_3}\begin{bmatrix}1&1&1&2\\0&1&-2&-2\\0&2&-1&-1\\0&-3&-5&0\end{bmatrix}\xrightarrow{-2l_2+l_3,3l_2+l_4}\begin{bmatrix}1&1&1&2\\0&1&-2&-2\\0&0&3&3\\0&0&-11&-6\end{bmatrix}\xrightarrow{(\frac{1}{3})L_3}\begin{bmatrix}1&1&1&2\\0&1&-2&-2\\0&0&1&1\\0&0&-11&-6\end{bmatrix}\xrightarrow{11l_3+l_4}$$

$$\begin{bmatrix}1&1&1&2\\0&1&-2&-2\\0&0&1&1\\0&0&0&5\end{bmatrix}\text{Imp.}$$

2. Calcule a característica das seguintes matrizes:

$$A=\begin{bmatrix}1&2&-1\\3&1&2\\1&-3&4\end{bmatrix},\ \text{C(A)=2} \qquad B=\begin{bmatrix}0&1&2\\3&4&2\\6&3&2\end{bmatrix},\ \text{C(A)=3}$$

$$C=\begin{bmatrix}1&1&1&2&0&-1\\1&0&2&-1&1&1\\1&1&0&0&1&-1\end{bmatrix},\ \text{C(A)=3} \qquad D=\begin{bmatrix}2&1&0\\1&3&4\\1&2&0\\0&1&2\\1&2&2\end{bmatrix},\ \text{C(A)=3.}$$

3. Discuta, em função dos valores de α e β, a característica das seguintes matrizes:

$$A=\begin{bmatrix}1&0&2\\0&\alpha-1&\alpha\\\alpha&0&\alpha\\\alpha+1&0&4\end{bmatrix}\xrightarrow{\alpha\neq0,(\frac{1}{\alpha})L_3}\begin{bmatrix}1&0&2\\0&\alpha-1&\alpha\\1&0&1\\\alpha+1&0&4\end{bmatrix}\xrightarrow{-L_1+L_3}\begin{bmatrix}1&0&2\\0&\alpha-1&\alpha\\0&0&-1\\\alpha+1&0&4\end{bmatrix}$$

$$\xrightarrow{\alpha\neq-1,-(\alpha+1)L_1+L_4}\begin{bmatrix}1&0&2\\0&\alpha-1&\alpha\\0&0&-1\\0&0&-2(\alpha-1)\end{bmatrix}\xrightarrow{\alpha\neq1}\begin{bmatrix}1&0&2\\0&\alpha-1&\alpha\\0&0&-1\\0&0&0\end{bmatrix}$$

$\alpha\neq1,\ \ C(A)=3.$

$$\alpha=1,A=\begin{bmatrix}1&0&2\\0&0&1\\1&0&1\\2&0&4\end{bmatrix}\xrightarrow{-l_1+l_3,-2l_1+l_4}\begin{bmatrix}1&0&2\\0&0&1\\0&0&-1\\0&0&0\end{bmatrix}\xrightarrow{l_2+l_3}\begin{bmatrix}1&0&2\\0&0&1\\0&0&0\\0&0&0\end{bmatrix}.C(A)=2.$$

$$\alpha=0,A=\begin{bmatrix}1&0&2\\0&-1&0\\0&0&0\\1&0&4\end{bmatrix}\xrightarrow{-l_1+l_4}\begin{bmatrix}1&0&2\\0&-1&0\\0&0&0\\0&0&2\end{bmatrix}\xrightarrow{l_3\leftrightarrow l_4}\begin{bmatrix}1&0&2\\0&-1&0\\0&0&2\\0&0&0\end{bmatrix}.C(A)=3.$$

$\alpha=-1,C(A)=3.$

$$B=\begin{bmatrix}\beta&-\alpha&0\\0&0&\alpha\\0&\beta&0\end{bmatrix}\xrightarrow{l_2\leftrightarrow l_3}\begin{bmatrix}\beta&-\alpha&0\\0&\beta&0\\0&0&\alpha\end{bmatrix}$$

$\alpha \neq 0, \beta \neq 0, C(A) = 3.$

$\alpha = 0, \beta \neq 0, C(A) = 2.$

$\alpha \neq 0, \beta = 0, C(A) = 2.$

4. Considere os seguintes sistemas, nas incógnitas x, y e z, e classifique-os quanto ao número de soluções, em função dos valores dos parâmetros reais α e β (em cada caso, indique a característica da matriz dos coeficientes e da matriz ampliada do sistema).

a) $\begin{cases} x - y + z = -1 \\ 2x + z = 2 \\ x - y + \alpha z = \beta \end{cases}$

$$[Ab] = \begin{bmatrix} 1 & -1 & 1 & -1 \\ 2 & 0 & 1 & 2 \\ 1 & -1 & \alpha & \beta \end{bmatrix} \overset{Gauss}{\longrightarrow} \begin{bmatrix} 1 & -1 & 1 & -1 \\ 0 & 2 & -1 & 4 \\ 0 & 0 & \alpha - 1 & \beta + 1 \end{bmatrix}$$

$\alpha \neq 1, C(A) = C(Ab) = n = 3, \quad \text{PD}$

$\alpha = 1, \beta = -1, C(A) = C(Ab) = 2 < n = 3, \;\; \text{PI}$

$\alpha = 1, \beta \neq -1, C(A) \neq C(Ab), \;\; \text{Imp.}$

b) $\begin{cases} x - y + z = -3 \\ -x + 4y - z = 3\alpha \\ \beta x + z = 3 \end{cases}$

$$[Ab] = \begin{bmatrix} 1 & -1 & 1 & -3 \\ -1 & 4 & -1 & 3\alpha \\ \beta & 0 & 1 & 3 \end{bmatrix} \overset{Gauss}{\longrightarrow} \begin{bmatrix} 1 & -1 & 1 & -3 \\ 0 & 3 & 0 & 3\alpha - 3 \\ 0 & 0 & 1 - \beta & 4\beta - \alpha\beta + 3 \end{bmatrix}$$

$\beta \neq 1, C(A) = C(Ab) = n = 3, \quad \text{PD}$

$\beta = 1, \alpha = 7, C(A) = C(Ab) = 2 < n = 3, \;\; \text{PI}$

$\alpha = 1, \beta \neq 7, C(A) \neq C(Ab), \;\; \text{Imp.}$

c) $\begin{cases} x - \alpha y + z = -\beta \\ x - y + (\beta + 1)z = 1 \\ x - y + z = 3 \end{cases}$

$$[Ab] = \begin{bmatrix} 1 & -\alpha & 1 & -\beta \\ 1 & -1 & (\beta + 1) & 1 \\ 1 & -1 & 1 & 3 \end{bmatrix} \overset{Gauss}{\longrightarrow} \begin{bmatrix} 1 & -\alpha & 1 & -\beta \\ 0 & \alpha - 1 & \beta & \beta + 1 \\ 0 & 0 & -\beta & 2 \end{bmatrix}.$$

$\alpha \neq 1, \beta \neq 0, C(A) = C(Ab) = n = 3, \quad \text{PD}$

$$\alpha = 1, \begin{bmatrix} 1 & -\alpha & 1 & -\beta \\ 0 & 0 & \beta & \beta + 1 \\ 0 & 0 & -\beta & 2 \end{bmatrix} \overset{L_2 + L_3}{\longrightarrow} \begin{bmatrix} 1 & -\alpha & 1 & -\beta \\ 0 & 0 & \beta & \beta + 1 \\ 0 & 0 & 0 & \beta + 3 \end{bmatrix}.$$

1) $\beta = -3$, $C(A) = C(Ab) = 2 < n = 3$, PI

2) $\beta \neq -3, C(A) \neq C(Ab)$, Imp.

$$\beta = 0, \begin{bmatrix} 1 & -\alpha & 1 & 0 \\ 0 & \alpha - 1 & 0 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}.$$

1) $\alpha = 1$, $C(A) = 1 \neq C(Ab) = 2$, Imp.

2) $\alpha \neq 1, C(A) = 2 \neq C(Ab) = 3$, Imp.

d) $\begin{cases} -2x + \alpha y - \beta z = -3 \\ x + \beta z = 1 \\ 2x + 4y + 3\beta z = -\beta \end{cases}$

$$[Ab] = \begin{bmatrix} -2 & \alpha & -\beta & -3 \\ 1 & 0 & \beta & 1 \\ 2 & 4 & 3\beta & -\beta \end{bmatrix} \xrightarrow{l_1 \leftrightarrow l_2} \begin{bmatrix} 1 & 0 & \beta & 1 \\ -2 & \alpha & -\beta & -3 \\ 2 & 4 & 3\beta & -\beta \end{bmatrix} \xrightarrow{2l_1+l_2,-2l_1+l_3} \begin{bmatrix} 1 & 0 & \beta & 1 \\ 0 & \alpha & \beta & -1 \\ 0 & 4 & \beta & -\beta-2 \end{bmatrix}$$

$$\xrightarrow{l_2 \leftrightarrow l_3, 4L_3} \begin{bmatrix} 1 & 0 & \beta & 1 \\ 0 & 4 & \beta & -\beta-2 \\ 0 & 4\alpha & 4\beta & -4 \end{bmatrix} \xrightarrow{-\alpha l_2+l_3} \begin{bmatrix} 1 & 0 & \beta & 1 \\ 0 & 4 & \beta & -\beta-2 \\ 0 & 0 & 4\beta - \alpha\beta & \alpha(\beta+2)-4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & \beta & 1 \\ 0 & 4 & \beta & -\beta-2 \\ 0 & 0 & \beta(4-\alpha) & \alpha(\beta+2)-4 \end{bmatrix}.$$

$\alpha \neq 4, \beta \neq 0, C(A) = C(Ab) = n = 3, \quad \text{PD}$

$\alpha = 4, \beta = -1, C(A) = C(Ab) = 2 < n = 3, \quad \text{PI}$

$\alpha = 4, \beta \neq -1, C(A) \neq C(Ab)$ Imp.

$\beta = 0, \alpha = 2,\ C(A) = C(Ab) = 2 < n = 3, \quad \text{PI}$

$\beta = 0, \alpha \neq 2,\ C(A) \neq C(Ab)$ Imp.

5. Use o método de Gauss para calcular a inversa das seguintes matrizes:

$$A = \begin{bmatrix} 1 & 4 & 0 \\ 2 & 2 & 0 \\ 0 & 2 & -2 \end{bmatrix}$$

$$[AI] = \begin{bmatrix} 1 & 4 & 0 & 1 & 0 & 0 \\ 2 & 2 & 0 & 0 & 1 & 0 \\ 0 & 2 & -2 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{-2L_1+L_2} \begin{bmatrix} 1 & 4 & 0 & 1 & 0 & 0 \\ 0 & -6 & 0 & -2 & 1 & 0 \\ 0 & 2 & -2 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{l_2 \leftrightarrow l_3} \begin{bmatrix} 1 & 4 & 0 & 1 & 0 & 0 \\ 0 & 2 & -2 & 0 & 0 & 1 \\ 0 & -6 & 0 & -2 & 1 & 0 \end{bmatrix}$$

$$\xrightarrow{3L_2+L_3} \begin{bmatrix} 1 & 4 & 0 & 1 & 0 & 0 \\ 0 & 2 & -2 & 0 & 0 & 1 \\ 0 & 0 & -6 & -2 & 1 & 3 \end{bmatrix} \xrightarrow{-\frac{1}{3}L_3+L_2} \begin{bmatrix} 1 & 4 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & \frac{2}{3} & -\frac{1}{3} & 0 \\ 0 & 0 & -6 & -2 & 1 & 3 \end{bmatrix} \xrightarrow{-2L_2+L_1} \begin{bmatrix} 1 & 0 & 0 & -\frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 2 & 0 & \frac{2}{3} & -\frac{1}{3} & 0 \\ 0 & 0 & -6 & -2 & 1 & 3 \end{bmatrix}$$

$$\xrightarrow{\frac{1}{2}L_2, \frac{-1}{6}L_3} \begin{bmatrix} 1 & 0 & 0 & -\frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 1 & 0 & \frac{1}{3} & -\frac{1}{6} & 0 \\ 0 & 0 & 1 & \frac{1}{3} & -\frac{1}{6} & -\frac{1}{2} \end{bmatrix}.$$

A inversa de A: $\begin{bmatrix} -\frac{1}{3} & \frac{2}{3} & 0 \\ \frac{1}{3} & -\frac{1}{6} & 0 \\ \frac{1}{3} & -\frac{1}{6} & -\frac{1}{2} \end{bmatrix}$.

$$B = \begin{bmatrix} 2 & 1 & -2 \\ -2 & 1 & 2 \\ -2 & -4 & 4 \end{bmatrix}$$

$$[BI] = \begin{bmatrix} 2 & 1 & -2 & 1 & 0 & 0 \\ -2 & 1 & 2 & 0 & 1 & 0 \\ -2 & -4 & 4 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{L_1+L_2, L_1+L_3} \begin{bmatrix} 2 & 1 & -2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 1 & 1 & 0 \\ 0 & -3 & 2 & 1 & 0 & 1 \end{bmatrix} \xrightarrow{\frac{1}{2}L_2} \begin{bmatrix} 2 & 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & -3 & 2 & 1 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{3L_2+L_3} \begin{bmatrix} 2 & 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 2 & \frac{5}{2} & \frac{3}{2} & 1 \end{bmatrix} \xrightarrow{L_3+L_1} \begin{bmatrix} 2 & 1 & 0 & \frac{7}{2} & \frac{3}{2} & 1 \\ 0 & 1 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 2 & \frac{5}{2} & \frac{3}{2} & 1 \end{bmatrix} \xrightarrow{-L_2+L_1} \begin{bmatrix} 2 & 0 & 0 & 3 & 1 & 1 \\ 0 & 1 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 2 & \frac{5}{2} & \frac{3}{2} & 1 \end{bmatrix}$$

$$\xrightarrow{\frac{1}{2}L_1, \frac{1}{2}L_3} \begin{bmatrix} 1 & 0 & 0 & \frac{3}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 & \frac{5}{4} & \frac{3}{4} & \frac{1}{2} \end{bmatrix}.$$

A inversa de B: $\begin{bmatrix} \frac{3}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{5}{4} & \frac{3}{4} & \frac{1}{2} \end{bmatrix}$.