Geometry Formulas

Area of rectangle	$A = lw$
Area of circle	$A = \pi r^2$
Area of triangle	$A = \frac{1}{2}bh$
Surface Area of sphere	$A = 4\pi r^2$
Lateral Surface Area of cylinder	$A = 2\pi rh$
Volume of box	$V = lwh$
Volume of sphere	$V = \frac{4}{3}\pi r^3$
Volume of cylinder	$V = \pi r^2 h$
Volume of cone	$V = \frac{1}{3}$ (area of base) $\times$ (height)

Trigonometric Identities

Pythagorean

$$\cos^2\theta + \sin^2\theta = 1,\ 1 + \tan^2\theta = \sec^2\theta,\ \cot^2\theta + 1 = \csc^2\theta$$

Parity

$$\sin(-\theta) = -\sin\theta,\ \cos(-\theta) = \cos\theta,\ \tan(-\theta) = -\tan\theta$$

$$\csc(-\theta) = -\csc\theta,\ \sec(-\theta) = \sec\theta,\ \cot(-\theta) = -\cot\theta$$

Co-relations

$$\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right),\ \csc\theta = \sec\left(\frac{\pi}{2} - \theta\right),\ \cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$$

Addition formulas

$$\sin(\theta + \phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$$

$$\sin(\theta - \phi) = \sin\theta\cos\phi - \cos\theta\sin\phi$$

$$\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$$

$$\cos(\theta - \phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$$

$$\tan(\theta + \phi) = \frac{(\tan\theta + \tan\phi)}{(1 - \tan\theta\tan\phi)}$$

$$\tan(\theta - \phi) = \frac{(\tan\theta - \tan\phi)}{(1 + \tan\theta\tan\phi)}$$

Double-angle formulas

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$\tan 2\theta = \frac{2\tan\theta}{(1 - \tan^2\theta)}$$

Half-angle formulas

$$\sin^2\frac{\theta}{2} = \frac{1 - \cos\theta}{2} \quad \text{or} \quad \sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

$$\cos^2\frac{\theta}{2} = \frac{1 + \cos\theta}{2} \quad \text{or} \quad \cos^2\theta = \frac{1 + \cos 2\theta}{2}$$

$$\tan\frac{\theta}{2} = \frac{\sin\theta}{1 + \cos\theta} = \frac{1 - \cos\theta}{\sin\theta} \quad \text{or} \quad \tan\theta = \frac{1 - \cos 2\theta}{\sin 2\theta}$$

Product formulas

$$\sin\theta\sin\phi = \frac{1}{2}[\cos(\theta - \phi) - \cos(\theta + \phi)]$$

$$\cos\theta\cos\phi = \frac{1}{2}[\cos(\theta + \phi) + \cos(\theta - \phi)]$$

$$\sin\theta\cos\phi = \frac{1}{2}[\sin(\theta + \phi) + \sin(\theta - \phi)]$$

Derivatives

1. $\dfrac{d(au)}{dx} = a\dfrac{du}{dx}$

2. $\dfrac{d(u+v-w)}{dx} = \dfrac{du}{dx} + \dfrac{dv}{dx} - \dfrac{dw}{dx}$

3. $\dfrac{d(uv)}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

4. $\dfrac{d(u/v)}{dx} = \dfrac{v(du/dx) - u(dv/dx)}{v^2}$

5. $\dfrac{d(u^n)}{dx} = nu^{n-1}\dfrac{du}{dx}$

6. $\dfrac{d(u^v)}{dx} = vu^{v-1}\dfrac{du}{dx} + u^v(\ln u)\dfrac{dv}{dx}$

7. $\dfrac{d(e^u)}{dx} = e^u\dfrac{du}{dx}$

8. $\dfrac{d(e^{au})}{dx} = ae^{au}\dfrac{du}{dx}$

9. $\dfrac{da^u}{dx} = a^u(\ln a)\dfrac{du}{dx}$

10. $\dfrac{d(\ln u)}{dx} = \dfrac{1}{u}\dfrac{du}{dx}$

11. $\dfrac{d(\log_a u)}{dx} = \dfrac{1}{u(\ln a)}\dfrac{du}{dx}$

12. $\dfrac{d\sin u}{dx} = \cos u\dfrac{du}{dx}$

13. $\dfrac{d\cos u}{dx} = -\sin u\dfrac{du}{dx}$

14. $\dfrac{d\tan u}{dx} = \sec^2 u\dfrac{du}{dx}$

15. $\dfrac{d\cot u}{dx} = -\csc^2 u\dfrac{du}{dx}$

16. $\dfrac{d\sec u}{dx} = \tan u\sec u\dfrac{du}{dx}$

17. $\dfrac{d\csc u}{dx} = -(\cot u)(\csc u)\dfrac{du}{dx}$

18. $\dfrac{d\sin^{-1}u}{dx} = \dfrac{1}{\sqrt{1-u^2}}\dfrac{du}{dx}$

19. $\dfrac{d\cos^{-1}u}{dx} = \dfrac{-1}{\sqrt{1-u^2}}\dfrac{du}{dx}$

20. $\dfrac{d\tan^{-1}u}{dx} = \dfrac{1}{1+u^2}\dfrac{du}{dx}$

21. $\dfrac{d\cot^{-1}u}{dx} = \dfrac{-1}{1+u^2}\dfrac{du}{dx}$

22. $\dfrac{d\sec^{-1}u}{dx} = \dfrac{1}{u\sqrt{u^2-1}}\dfrac{du}{dx}$

23. $\dfrac{d\csc^{-1}u}{dx} = \dfrac{-1}{u\sqrt{u^2-1}}\dfrac{du}{dx}$

24. $\dfrac{d\sinh u}{dx} = \cosh u\dfrac{du}{dx}$

25. $\dfrac{d\cosh u}{dx} = \sinh u\dfrac{du}{dx}$

26. $\dfrac{d\tanh u}{dx} = \operatorname{sech}^2 u\dfrac{du}{dx}$

27. $\dfrac{d\coth u}{dx} = -(\operatorname{csch}^2 u)\dfrac{du}{dx}$

28. $\dfrac{d\operatorname{sech} u}{dx} = -(\operatorname{sech} u)(\tanh u)\dfrac{du}{dx}$

29. $\dfrac{d\operatorname{csch} u}{dx} = -(\operatorname{csch} u)(\coth u)\dfrac{du}{dx}$

30. $\dfrac{d\sinh^{-1}u}{dx} = \dfrac{1}{\sqrt{1+u^2}}\dfrac{du}{dx}$

31. $\dfrac{d\cosh^{-1}u}{dx} = \dfrac{1}{\sqrt{u^2-1}}\dfrac{du}{dx}$

32. $\dfrac{d\tanh^{-1}u}{dx} = \dfrac{1}{1-u^2}\dfrac{du}{dx}$

33. $\dfrac{d\coth^{-1}u}{dx} = \dfrac{1}{1-u^2}\dfrac{du}{dx}$

34. $\dfrac{d\operatorname{sech}^{-1}u}{dx} = \dfrac{-1}{u\sqrt{1-u^2}}\dfrac{du}{dx}$

35. $\dfrac{d\operatorname{csch}^{-1}u}{dx} = \dfrac{-1}{|u|\sqrt{1+u^2}}\dfrac{du}{dx}$

A Brief Table of Integrals

(*An arbitrary constant may be added to each integral.*)

1. $\int x^n\,dx = \frac{1}{n+1}\,x^{n+1}\ (n \neq -1)$
2. $\int \frac{1}{x}\,dx = \ln|x|$
3. $\int e^x\,dx = e^x$
4. $\int a^x\,dx = \frac{a^x}{\ln a}$
5. $\int \sin x\,dx = -\cos x$
6. $\int \cos x\,dx = \sin x$
7. $\int \tan x\,dx = -\ln|\cos x|$
8. $\int \cot x\,dx = \ln|\sin x|$
9. $\int \sec x\,dx = \ln|\sec x + \tan x|$
   $= \ln\left|\tan\left(\frac{1}{2}x + \frac{1}{4}\pi\right)\right|$
10. $\int \csc x\,dx = \ln|\csc x - \cot x|$
    $= \ln\left|\tan\frac{1}{2}x\right|$
11. $\int \sin^{-1}\frac{x}{a}\,dx = x\sin^{-1}\frac{x}{a} + \sqrt{a^2 - x^2} \qquad (a > 0)$
12. $\int \cos^{-1}\frac{x}{a}\,dx = x\cos^{-1}\frac{x}{a} - \sqrt{a^2 - x^2} \qquad (a > 0)$
13. $\int \tan^{-1}\frac{x}{a}\,dx = x\tan^{-1}\frac{x}{a} - \frac{a}{2}\ln(a^2 + x^2) \qquad (a > 0)$
14. $\int \sin^2 mx\,dx = \frac{1}{2m}(mx - \sin mx\cos mx)$
15. $\int \cos^2 mx\,dx = \frac{1}{2m}(mx + \sin mx\cos mx)$
16. $\int \sec^2 x\,dx = \tan x$
17. $\int \csc^2 x\,dx = -\cot x$
18. $\int \sin^n x\,dx = -\frac{\sin^{n-1}x\cos x}{n} + \frac{n-1}{n}\int \sin^{n-2}x\,dx$
19. $\int \cos^n x\,dx = \frac{\cos^{n-1}x\sin x}{n} + \frac{n-1}{n}\int \cos^{n-2}x\,dx$
20. $\int \tan^n x\,dx = \frac{\tan^{n-1}x}{n-1} - \int \tan^{n-2}x\,dx \qquad (n \neq 1)$
21. $\int \cot^n x\,dx = -\frac{\cot^{n-1}x}{n-1} - \int \cot^{n-2}x\,dx \qquad (n \neq 1)$
22. $\int \sec^n x\,dx = \frac{\tan x\sec^{n-2}x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2}x\,dx \qquad (n \neq 1)$
23. $\int \csc^n x\,dx = -\frac{\cot x\csc^{n-2}x}{n-1} + \frac{n-2}{n-1}\int \csc^{n-2}x\,dx \qquad (n \neq 1)$
24. $\int \sinh x\,dx = \cosh x$
25. $\int \cosh x\,dx = \sinh x$
26. $\int \tanh x\,dx = \ln|\cosh x|$
27. $\int \coth x\,dx = \ln|\sinh x|$
28. $\int \operatorname{sech} x\,dx = \tan^{-1}(\sinh x)$

A Brief Table of Integrals, continued.

29. $\int \operatorname{csch} x\,dx = \ln\left|\tanh\frac{x}{2}\right| = -\frac{1}{2}\ln\frac{\cosh x + 1}{\cosh x - 1}$

30. $\int \sinh^2 x\,dx = \frac{1}{4}\sinh 2x - \frac{1}{2}x$

31. $\int \cosh^2 x\,dx = \frac{1}{4}\sinh 2x + \frac{1}{2}x$

32. $\int \operatorname{sech}^2 x\,dx = \tanh x$

33. $\int \sinh^{-1}\frac{x}{a}\,dx = x\sinh^{-1}\frac{x}{a} - \sqrt{x^2 + a^2} \qquad (a > 0)$

34. $\int \cosh^{-1}\frac{x}{a}\,dx = \begin{cases} x\cosh^{-1}\frac{x}{a} - \sqrt{x^2 - a^2} & \left[\cosh^{-1}\left(\frac{x}{a}\right) > 0,\ a > 0\right] \\ x\cosh^{-1}\frac{x}{a} + \sqrt{x^2 - a^2} & \left[\cosh^{-1}\left(\frac{x}{a}\right) < 0,\ a > 0\right] \end{cases}$

35. $\int \tanh^{-1}\frac{x}{a}\,dx = x\tanh^{-1}\frac{x}{a} + \frac{a}{2}\ln|a^2 - x^2|$

36. $\int \frac{1}{\sqrt{a^2 + x^2}}\,dx = \ln(x + \sqrt{a^2 + x^2}) = \sinh^{-1}\frac{x}{a} \qquad (a > 0)$

37. $\int \frac{1}{a^2 + x^2}\,dx + \frac{1}{a}\tan^{-1}\frac{x}{a} \qquad (a > 0)$

38. $\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} \qquad (a > 0)$

39. $\int (a^2 - x^2)^{3/2}\,dx = \frac{x}{8}(5a^2 - 2x^2)\sqrt{a^2 - x^2} + \frac{3a^4}{8}\sin^{-1}\frac{x}{a} \qquad (a > 0)$

40. $\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a} \qquad (a > 0)$

41. $\int \frac{1}{a^2 - x^2}\,dx = \frac{1}{2a}\ln\left|\frac{a + x}{a - x}\right|$

42. $\int \frac{1}{(a^2 - x^2)^{3/2}}\,dx = \frac{x}{a^2\sqrt{a^2 - x^2}}$

43. $\int \sqrt{x^2 \pm a^2}\,dx = \frac{x}{2}\sqrt{x^2 \pm a^2} \pm \frac{a^2}{2}\ln|x + \sqrt{x^2 \pm a^2}|$

44. $\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln|x + \sqrt{x^2 - a^2}| = \cosh^{-1}\frac{x}{a} \qquad (a > 0)$

45. $\int \frac{1}{x(a + bx)}\,dx = \frac{1}{a}\ln\left|\frac{x}{a + bx}\right|$

46. $\int x\sqrt{a + bx}\,dx = \frac{2(3bx - 2a)(a + bx)^{3/2}}{15b^2}$

47. $\int \frac{\sqrt{a + bx}}{x}\,dx = 2\sqrt{a + bx} + a\int \frac{1}{x\sqrt{a + bx}}\,dx$

48. $\int \frac{x}{\sqrt{a + bx}}\,dx = \frac{2(bx - 2a)\sqrt{a + bx}}{3b^2}$

49. $\int \frac{1}{x\sqrt{a + bx}}\,dx = \frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{a + bx} - \sqrt{a}}{\sqrt{a + bx} + \sqrt{a}}\right| \qquad (a > 0)$

$= \frac{2}{\sqrt{-a}}\tan^{-1}\sqrt{\frac{a + bx}{-a}} \qquad (a < 0)$

50. $\int \frac{\sqrt{a^2 - x^2}}{x}\,dx = \sqrt{a^2 - x^2} - a\ln\left|\frac{a + \sqrt{a^2 - x^2}}{x}\right|$

51. $\int x\sqrt{a^2 - x^2}\,dx = -\frac{1}{3}(a^2 - x^2)^{3/2}$

52. $\int x^2\sqrt{a^2 - x^2}\,dx = \frac{x}{8}(2x^2 - a^2)\sqrt{a^2 - x^2} + \frac{a^4}{8}\sin^{-1}\frac{x}{a} \qquad (a > 0)$

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53. $\displaystyle\int \frac{1}{x\sqrt{a^2 - x^2}}\,dx = -\frac{1}{a}\ln\left|\frac{a + \sqrt{a^2 - x^2}}{x}\right|$

54. $\displaystyle\int \frac{x}{\sqrt{a^2 - x^2}}\,dx = -\sqrt{a^2 - x^2}$

55. $\displaystyle\int \frac{x^2}{\sqrt{a^2 - x^2}}\,dx = -\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} \qquad (a > 0)$

56. $\displaystyle\int \frac{\sqrt{x^2 + a^2}}{x}\,dx = \sqrt{x^2 + a^2} - a\ln\left|\frac{a + \sqrt{x^2 + a^2}}{x}\right|$

57. $\displaystyle\int \frac{\sqrt{x^2 - a^2}}{x}\,dx = \sqrt{x^2 - a^2} - a\cos^{-1}\frac{a}{|x|}$

$\displaystyle= \sqrt{x^2 - a^2} - a\sec^{-1}\left(\frac{x}{a}\right) \qquad (a > 0)$

58. $\displaystyle\int x\sqrt{x^2 \pm a^2}\,dx = \frac{1}{3}(x^2 \pm a^2)^{3/2}$

59. $\displaystyle\int \frac{1}{x\sqrt{x^2 + a^2}}\,dx = \frac{1}{a}\ln\left|\frac{x}{a + \sqrt{x^2 + a^2}}\right|$

60. $\displaystyle\int \frac{1}{x\sqrt{x^2 - a^2}}\,dx = \frac{1}{a}\cos^{-1}\frac{a}{|x|} \qquad (a > 0)$

61. $\displaystyle\int \frac{1}{x^2\sqrt{x^2 \pm a^2}}\,dx = \mp\frac{\sqrt{x^2 \pm a^2}}{a^2 x}$

62. $\displaystyle\int \frac{x}{\sqrt{x^2 \pm a^2}}\,dx = \sqrt{x^2 \pm a^2}$

63. $\displaystyle\int \frac{1}{ax^2 + bx + c}\,dx = \frac{1}{\sqrt{b^2 - 4ac}}\ln\left|\frac{2ax + b - \sqrt{b^2 - 4ac}}{2ax + b + \sqrt{b^2 - 4ac}}\right| \qquad (b^2 > 4ac)$

$\displaystyle= \frac{2}{\sqrt{4ac - b^2}}\tan^{-1}\frac{2ax + b}{\sqrt{4ac - b^2}} \qquad (b^2 < 4ac)$

64. $\displaystyle\int \frac{x}{ax^2 + bx + c}\,dx = \frac{1}{2a}\ln|ax^2 + bx + c| - \frac{b}{2a}\int \frac{1}{ax^2 + bx + c}\,dx$

65. $\displaystyle\int \frac{1}{\sqrt{ax^2 + bx + c}}\,dx = \frac{1}{\sqrt{a}}\ln|2ax + b + 2\sqrt{a}\sqrt{ax^2 + bx + c}\,| \qquad (a > 0)$

$\displaystyle= \frac{1}{\sqrt{-a}}\sin^{-1}\frac{-2ax - b}{\sqrt{b^2 - 4ac}} \qquad (a < 0)$

66. $\displaystyle\int \sqrt{ax^2 + bx + c}\,dx = \frac{2ax + b}{4a}\sqrt{ax^2 + bx + c} + \frac{4ac - b^2}{8a}\int \frac{1}{\sqrt{ax^2 + b + c}}\,dx$

67. $\displaystyle\int \frac{x}{\sqrt{ax^2 + bx + c}}\,dx = \frac{\sqrt{ax^2 + bx + c}}{a} - \frac{b}{2a}\int \frac{1}{\sqrt{ax^2 + bx + c}}\,dx$

68. $\displaystyle\int \frac{1}{x\sqrt{ax^2 + bx + c}}\,dx = \frac{-1}{\sqrt{c}}\ln\left|\frac{2\sqrt{c}\sqrt{ax^2 + bx + c} + bx + 2c}{x}\right| \qquad (c > 0)$

$\displaystyle= \frac{1}{\sqrt{-c}}\sin^{-1}\frac{bx + 2c}{|x|\sqrt{b^2 - 4ac}} \qquad (c < 0)$

69. $\displaystyle\int x^3\sqrt{x^2 + a^2}\,dx = \left(\frac{1}{5}x^2 - \frac{2}{15}a^2\right)\sqrt{(a^2 + x^2)^3}$

70. $\displaystyle\int \frac{\sqrt{x^2 \pm a^2}}{x^4}\,dx = \frac{\mp\sqrt{(x^2 \pm a^2)^3}}{3a^2x^3}$

71. $\displaystyle\int \sin ax \sin bx\,dx = \frac{\sin(a - b)x}{2(a - b)} - \frac{\sin(a + b)x}{2(a + b)} \qquad (a^2 \neq b^2)$

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A Brief Table of Integrals, continued.

72. $\int \sin ax \cos bx\, dx = -\frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)} \qquad (a^2 \neq b^2)$

73. $\int \cos ax \cos bx\, dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} \qquad (a^2 \neq b^2)$

74. $\int \sec x \tan x\, dx = \sec x$

75. $\int \csc x \cot x\, dx = -\csc x$

76. $\int \cos^m x \sin^n x\, dx = \frac{\cos^{m-1}x \sin^{n+1}x}{m+n} + \frac{m-1}{m+n}\int \cos^{m-2}x \sin^n x\, dx$

$$= -\frac{\sin^{n-1}x \cos^{m+1}x}{m+n} + \frac{n-1}{m+n}\int \cos^m x \sin^{n-2}x\, dx$$

77. $\int x^n \sin ax\, dx = -\frac{1}{a}x^n \cos ax + \frac{n}{a}\int x^{n-1}\cos ax\, dx$

78. $\int x^n \cos ax\, dx = \frac{1}{a}x^n \sin ax - \frac{n}{a}\int x^{n-1}\sin ax\, dx$

79. $\int x^n e^{ax}\, dx = \frac{x^n e^{ax}}{a} - \frac{n}{a}\int x^{n-1}e^{ax}\, dx$

80. $\int x^n \ln ax\, dx = x^{n+1}\left[\frac{\ln ax}{n+1} - \frac{1}{(n+1)^2}\right]$

81. $\int x^n (\ln ax)^m\, dx = \frac{x^{n+1}}{n+1}(\ln ax)^m - \frac{m}{n+1}\int x^n (\ln ax)^{m-1}\, dx$

82. $\int e^{ax}\sin bx\, dx = \frac{e^{ax}(a \sin bx - b \cos bx)}{a^2 + b^2}$

83. $\int e^{ax}\cos bx\, dx = \frac{e^{ax}(b \sin bx + a \cos bx)}{a^2 + b^2}$

84. $\int \operatorname{sech} x \tanh x\, dx = -\operatorname{sech} x$

85. $\int \operatorname{csch} x \coth x\, dx = -\operatorname{csch} x$

Greek Alphabet

α	alpha	ι	iota	ρ	rho
β	beta	κ	kappa	σ	sigma
γ	gamma	λ	lambda	τ	tau
δ	delta	μ	mu	υ	upsilon
ϵ	epsilon	ν	nu	ϕ	phi
ζ	zeta	ξ	xi	χ	chi
η	eta	o	omicron	ψ	psi
θ	theta	π	pi	ω	omega