Electron Specific Charge

Luis A. Flores & Rosemary Cortés
University of Puerto Rico at Río Piedras
FISI 4076 Intermediate Laboratory
Dr. Ratnakar Palai

Abstract
The purpose of our experiment was to determine the specific charge of the electron from the path of an electron beam in crossed electric and magnetic fields of variable strength. The highest experimental error was of 3.47% by calculating the mean of the charge to mass ratio of distinct radii. The error was compared to the theoretical value of the charge to mass ratio of the electron of 1.759×10^{11}.

1 Introduction

In the late nineteenth century English physicist Sir Joseph John Thomson (1856-1940) suggested that atoms were divisible. However, it was strongly believed that atoms were the fundamental building blocks of all matter. Scientists began studying the properties of atoms in large electric fields until it was demonstrated that atoms are composed of aggregates of charged particles. In 1897, Thomson carried a set of experiments which proved that cathode rays were streams of negatively charged particles which he called corpuscles. Thomson determined that these corpuscles had much smaller bodies than atoms, a large charge to mass ratio, and a measured large amount of negative energy. Thomson's corpuscle theory became accepted and transmuted with the contributions of other scientists into what we know today as the electron. In our experiment, we aim to obtain the charge to mass ratio of the electron by accelerating electrons in an electric field which entered a magnetic field at right angles to the direction of motion. The specific charge of the electron was then determined from the accelerating voltage, magnetic field strength, and the radius of the electron orbit.

2 Historical Experimentation

Cathode rays are streams of electrons observed in vacuum or crookes tubes. Thomson's experiment consisted of a gas sample inside a crookes tube which was introduced into a region between two charged plates. He observed that a current flow was produced due to a heated cathode that caused the atoms of the sample to ionize (***fig1***). What he observed came to be known as cathode rays. Thomson studied Maxwell's theory and knew that charged particles could be deflected in a magnetic field. Following the sketch of ***(fig1)***, electrons entered the region between the plates with an unknown velocity in the x-direction. To determine the velocity, electric and magnetic fields were both applied, and each gave rise to a force on the electron in the y-direction. The magnetic field was opposed to the force on the electric field.

F_E = eE, E: Magnitude electric field
F_H = -evH, H: Magnitude magnetic field

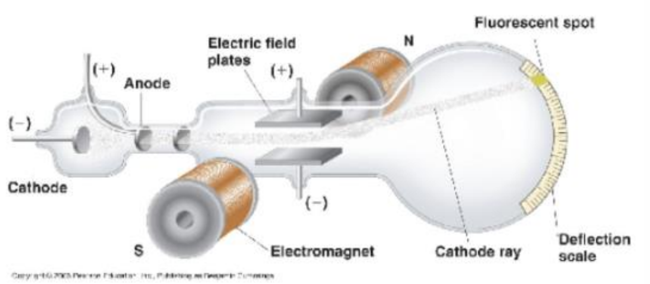

FIGURE 1. Sketch of Thomson's cathode ray second experiment conducted in 1897.

If these forces balance, then there will be no deflection of the electron in the y-direction and all the motion would be along the initial x-direction. Furthermore, the total force on the electrons would be zero, that is $F_E + F_H = 0$ from which the unknown velocity was determined as $v = E/H$. The magnetic field was then turned off so that the total force was due entirely to the electric field. With only the electric field, there was a nonzero force F_E in the y-direction but no force in the x-direction. Since the charge carriers were deflected by the force, this provided evidence for them being fundamental particles. To determine the charge to mass ratio the problem is like that of a projectile in a gravitational field. The ratio is characteristic of particles that move in vacuum along the same path when subjected to the same electric and magnetic fields. In the x-direction the electrons move with constant velocity $v = E/H$. The position equations in the x and y-directions expressed as a function of time and considering that the constant force gives rise to an acceleration are

$$x(t) = vt = \frac{E}{H}t$$

$$y(t) = \frac{1}{2}at^2 = \frac{1}{2}\frac{F_E}{m}t^2 = \frac{eE}{2m}t^2.$$

The electric field was tuned such that a particle would traverse the entire plate region in the time required for it to strike the positive plate. The total distance travelled in the y-direction in the time T required to traverse the plate region $x(T) = l$ was determined to be

$$s = \frac{eE}{2m}\left(\frac{lH}{E}\right)^2 \text{, where } l = \frac{ET}{H} \text{ and } T = \frac{lH}{E}.$$

Solving the above equation for the ratio e/m is how Thomson was able to determine the charge to mass ratio of the electron which has a theoretical value of $1.7588196x10^{11}\ C\ kg$.

3 Experimental Theory

In our experiment, an electron of mass m_o and charge e is accelerated by a potential difference U. The kinetic energy of the electron can be defined as

$$eU = \frac{1}{2}\, m_0 v^2. \qquad \text{(Eq.1)}$$

The Lorentz force $\boldsymbol{F} = q\boldsymbol{E} + q\boldsymbol{v} \times \boldsymbol{B}$ is the force exerted on a charged particle q moving with velocity v through an electric field $\boldsymbol{E}$ and a magnetic field $\boldsymbol{B}$. In the uniform magnetic field seen in (***fig.2***), if $\boldsymbol{v} \perp \boldsymbol{B}$ then the particle follows a circular trajectory with radius $r = mv/qB$. However, if $\boldsymbol{v}\angle\boldsymbol{B} < 90°$ the orbit becomes a helix with an axis parallel to the field lines. The last case is when $\boldsymbol{v}\angle\boldsymbol{B}=0°$ in which the particle will continue to move undeflected along the field lines. In a magnetic field of strength B, the Lorentz force acting on an electron with velocity v is $F = e \cdot \boldsymbol{v} \times \boldsymbol{B}$. In our arrangement we used Helmholtz coils (***fig.3***) to obtain a uniform magnetic field. Due to the cross product in Lorentz formula, the force is always at right angles to the motion and the magnitude of the velocity never changes. The magnitude of the Lorentz force will then equal the centripetal force needed to keep the particle in circular motion defined as $F_c = m_0 \frac{v^2}{r}$. Equating the magnitudes of the forces and using (***Eq.1***) we solve for the charge to mass ratio,

$$v = \frac{e}{m_0}Br\ ,\ \frac{e}{m_0} = \frac{2U}{(Br)^2}. \qquad \text{(Eq.2)}$$

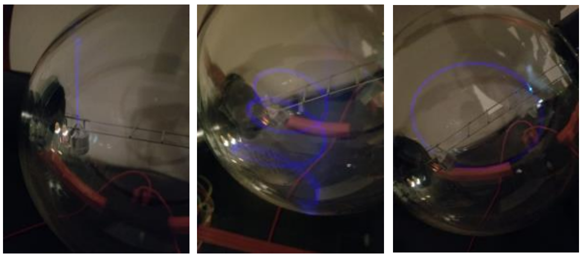

FIGURE 2. PHYWE's beam tube in which the beam of electrons can be seen following different trajectories depending on the parameters following Lorentz law.

In our experiment we can control U (***Eq.2***) through the voltage applied to the electron source and $\boldsymbol{B}$ trough the current in the Helmholtz coils. By measuring the radius r for different values of these parameters we can determine the charge to mass ratio of the electron. The B field at the center of the coils is proportional to the coil current I and has a theoretical value of

$$B = \mu_0 \left(\frac{4}{5}\right)^{\frac{3}{2}} \frac{n}{R} I, \text{ with } \mu_0 = 4\pi\times10^{-7}\ NA^{-2},$$

where n=154 is the number of turns in each coil, R=0.2m the coil radius, and I the applied current. The formula above corresponds to the magnetic field produced by the Helmholtz coils at a distance z along the axis of current loop of radius R lying in the x-y plane.

4 Equipment & Experimental Procedure

The equipment used for the experiment (***fig.3***) consisted of a narrow beam tube, a pair of Helmholtz coils, two power supplies, a digital multimeter, and various connecting cords. The equipment was already mounted with the two coils turned towards each other and the connections partially made. The magnetic field was varied through current and the velocity of the electrons through accelerating voltage to obtain the radius of orbit that coincided with luminous traces inside the beam tube (***fig.2***). Following the PHYWE instruction manual, it was seen that when the electron beam coincides with the luminous traces only half of a circle was observed. The radii used for the experiment were of 2, 3, 4, and 5cm.

FIGURE 3. PHYWE Equipment utilized to determine the specific charge of the electron.

Once the equipment was set up and ready, we proceeded to vary the current and the accelerating voltage to values close to those seen in (***table1***) provided by the PHYWE instruction manual. These values were obtained using the same setup as ours. However, the circular trajectory of the electron beam did not necessarily coincide with the luminous traces, thus we had to make changes to the values of (***table1***). Once the circular beam coincided with the luminous trace (***fig.2***), we would annotate the data of the current and voltage and proceeded to increase the voltage by increments of 20V until reaching 300V. The data was utilized to calculate the magnitude of the electric field ***B*** and ultimately the electron mass to charge ratio.

	$r = 0.02$ m		$r = 0.03$ m	
$\frac{U}{V}$	I	$\frac{e/m_0}{10^{11}\ \frac{As}{kg}}$	I	$\frac{e/m_0}{10^{11}\ \frac{As}{kg}}$
100	2.5	1.7	1.6	1.8
120	2.6	1.9	1.7	1.9
140	2.8	1.9	1.9	1.8
160	–	–	2.0	1.9
180	–	–	2.2	1.7
200	–	–	2.3	1.8
220	–	–	2.4	1.8
240	–	–	2.5	1.8
260	–	–	2.6	1.8
280	–	–	2.7	1.8
300	–	–	2.8	1.8

TABLE 1. PHYWE Instruction manual table containing approximations to some of the data or parameters needed to verify and determine the charge to mass ratio of the electron.

5 Experimental Data & Results

To obtain our results we used the values in (***table1***) and then varied the voltage and the current until the circular beam of electrons coincided with the luminous traces at different radii. These experimental values can be seen in the following data tables (***table2***), (***table3***), (***table4***), and (***table5***) along with the calculated values of the magnetic field **B** and the mass to charge ratio from (***Eq2***). The theoretical value of the ratio is 1.759×10^{11} and was used to calculate the percentages of error of our experimental data. The mean values for the four different radii were calculated to obtain a better approximation of our results to the theoretical value. For the values in (***table2***) the mean of the charge to mass ratio was of 1.7755×10^{11}giving an error of 0.94%. The values in (***table3***) gave a mean of 1.801×10^{11} with an error of 2.39%. The values in (***table4***) gave a mean of $1.7980 \times 10^{11}$and an error of 2.22%. Lastly, for the values of (***table5***) we obtain a mean of $1.82 \times 10^{11}$with an error of 3.47%.

radius 0.02m			
U/V	I	B x10^-3	e/m₀ x 10^11
106.9	2.45	1.6968	1.8565
119.8	2.634	1.8242	1.8000
140.4	2.96	2.0499	1.6700

TABLE 2. Experimental values obtained when the circular beam of electrons coincided with the luminous trace at r=0.02m.

radius 0.03m			
U/V	I	B x10^-3	e/m₀ x 10^11
106.4	1.582	1.0956	1.9697
120	1.709	1.18359	1.9035
140.3	1.906	1.32	1.7893
160.2	2.013	1.3941	1.8316
180.1	2.203	1.5257	1.7193
200	2.307	1.5977	1.741
220	2.408	1.6677	1.7578
240	2.506	1.7356	1.7706
260	2.602	1.8021	1.7792
280	2.702	1.8713	1.7769
304	2.804	1.942	1.7914
350	3.02	2.0915	1.778

TABLE 3. Experimental values obtained when the circular beam of electrons coincided with the luminous trace at r=0.03m.

radius 0.04m			
U/V	I	B x10^-3	e/m0 x 10^11
103.6	1.126	7.798x10^-4	2.1295
120.9	1.286	8.9064x10^-4	1.9052
140.4	1.444	1.0000	1.7548
161.0	1.538	1.0652	1.7738
180.2	1.630	1.1289	1.7675
202.0	1.740	1.2051	1.7388
219.0	1.809	1.2528	1.7440
241.0	1.904	1.3186	1.7325
260.0	1.953	1.3526	1.7765
280.0	2.041	1.4135	1.7517
300.0	2.111	1.4620	1.7544
350.0	2.285	1.5825	1.7470

TABLE 4. Experimental values obtained when the circular beam of electrons coincided with the luminous trace at r=0.04m.

radius 0.05m			
U/V	I	B	e/m0 x 10^11
105.6	0.922	6.3855x10^-4	2.0719
119.9	1.027	7.1126x10^-4	1.8960
140.6	1.124	7.784x10^-4	1.8562
160.9	1.211	8.3869x10^-4	1.8299
181.0	1.296	8.9756x10^-4	1.7974
201.0	1.374	9.5158x10^-4	1.7758
219.0	1.430	9.9037x10^-4	1.7862
240.0	1.485	1.0285x10^-3	1.8152
262.0	1.570	1.0873x10^-3	1.7728
281.0	1.633	1.131x10^-3	1.7575
303.0	1.713	1.1864x10^-3	1.7220
350.0	1.827	1.2653x10^-3	1.7489

TABLE 5. Experimental values obtained when the circular beam of electrons coincided with the luminous trace at r=0.05m.

From these results the lowest error of 0.94% for the charge to mass ratio of the electron was obtain from the values of (***table2***) corresponding to a radius of 0.02m. The errors for the other radii were between 2.22% and 3.47% which is still within an acceptable range. However, taken the values of the charge to mass ratio individually without calculating the mean, our highest error was of 21.06% and the lowest of 0.07%. High errors were observed in all radii in the first entry of each of the data tables but they rapidly decreased and stabilized as we continued incrementing the voltage by 20V.

6 Conclusion

The purpose of our experiment was to obtain a valid measurement of the charge to mass ratio of the electron. The highest mean error was of 3.47% which falls within the acceptable range of error and therefore we conclude that our data is a good approximation of the charge to mass ratio of the electron. We note that a high error was observed in the first entry of all the data tables corresponding to a voltage of around 105V. However, the error quickly dropped as we increased the voltage. Furthermore, we observed that the path of the electron beam (***fig.2***) followed Lorentz law as we changed the electric and magnetic fields.

7 References

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