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CONTROL SYSTEMS OF MOVING OBJECTS

Autonomous Longitudinal Motion of a Paraglider. Mathematical Simulation, Synthesis of Control

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Abstract—Mathematical model of motion of a paraglider in the longitudinal plane is constructed. The vehicle consists of a sail and a gondola. Both bodies are assumed to be perfectly rigid. They are connected by slings which are assumed to be perfectly rigid rods. Thus, the considered model of the paraglider represents one rigid body with three degrees of freedom. An engine, which develops thrust using a propeller, is mounted rigidly on the gondola of the vehicle. The orientation of the thrust vector with respect to the gondola is constant. The steady-state regimes of motion of the paraglider for the constant thrust are found. The law of automatic thrust control for which the flight of the vehicle is stabilized at the given altitude is designed. The domains of asymptotic stability of the paraglider motion at a constant altitude, including with account of delay, are constructed in the plane of the feedback coefficients. In this plane the domains in which a given stability factor is ensured are constructed. Some results of numerical simulation of the flight of the vehicle are presented.

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INTRODUCTION

At present, great attention is paid to development of remote-controlled and unmanned flying vehicles of different types. The creation of such vehicles is the topical direction of development of modern aviation [1, 2]. One of such promising flying vehicles is the paraglider.

In this paper, the mathematical model of the plane longitudinal motion of the vehicle is constructed for investigation of the paraglider flight dynamics and synthesis of control.

The considered single-link model of the vehicle consists of a wing (sail) and a gondola. Both bodies are assumed to be perfectly rigid; they are connected by slings which are simulated by perfectly rigid rods. Thus, the mechanical model of a paraglider represents one rigid body with three degrees of freedom in the plane longitudinal motion.

An engine is fixed rigidly to the gondola, which develops thrust using a propeller. The orientation with respect to the gondola of the thrust vector developed by this propeller is unchanged.

The sail, and so the whole vehicle, is influenced by the lifting force and the aerodynamic drag force. These forces and the force of gravity and the drag force of the gondola create the moments with respect to the center of mass which are balanced in the steady-state flight regime. For a special engine thrust value the paraglider in the steady-state regime moves horizontally, i.e., at a constant altitude. However, this horizontal flight at constant thrust is not asymptotically stable with respect to altitude. It can be stabilized by controlling the value of the thrust vector.

At the beginning of motion, at the start leg, the gondola of the paraglider moves on the ground (rolls on wheels). When some sufficiently high velocity is acquired, the vehicle takes off the ground. Thus, at the start leg, the unilateral constraint is imposed on the system; this constraint is taken into account in the mathematical model of the system.

The program for computer simulation of longitudinal motion of the paraglider, including the capability of flight animation, was developed.

1. MATHEMATICAL MODEL OF LONGITUDINAL MOTION OF THE VEHICLE

Figure 1 shows the schematic diagram of the paraglider. The wing (sail) in this figure is shown in the form of the straight segment BD with the center at the point A (side view). Let the point A be also the center of mass of the wing. It will be assumed that both the lifting force of the sail $P = C_y\frac{1}{2}\rho V_A^2 S$, and the aerodynamic drag force $Q = C_x\frac{1}{2}\rho V_A^2 S$ are applied at the center A of the plate. The quantities C_y and C_x represent the coefficients of the lifting force and the drag force [3, 4], ρ is the air density, V_A is the velocity of the point A with respect to the incoming air, and S is the area of the sail.

The point of application of the lifting force P and the drag force Q varies with variation of the velocity and orientation of the sail. This circumstance, however, will not be taken into account, since the moment of these forces with respect to the center of mass of the paraglider "weakly" depends on the position of the point of their application if the arm (the distance between the center of mass of the vehicle and the sail) is large, as compared to the sail size.

We denote by G the center of mass of the gondola and the distance GA by L. Let m_1 be the mass of the gondola and m_2, the mass of the sail, $m_1 + m_2 = M$. Then the distance l_1 from the center of mass C of the vehicle to the point G is

$$l_1 = \frac{m_2 L}{m_1 + m_2} = \frac{m_2 L}{M}.$$

The distance CA equal to $L - l_1$ will be denoted by l_2. If m_2 is considerably smaller than m_1, then the distance l_1 is considerably smaller than the distance $l_2 : l_1 \ll l_2$. In this case, the center of mass of the whole paraglider C is close to that of the gondola G. We denote by T the thrust developed by the engine. It will be assumed that the trust T is applied at the center of mass of the gondola G and its vector is perpendicular to the straight line AG. The quantity J denotes the moment of inertia of the vehicle with respect to its center of mass C, and g, the acceleration of gravity. If the lengths of the slings are different, the angle between the sail and the straight line AG is not the right angle; we denote the angle between the perpendicular to the sail and the straight line AG by σ (Fig. 1). The turn of the sail in the longitudinal plane by some angle σ can be used for changing its lifting force. In the model considered below, the angle σ is assumed constant.

Let us introduce the coordinate system XOY motionless with respect to the Earth; the axis OX of this coordinate system is directed horizontally, and the axis OY vertically. Let x and y be the coordinates of the center of mass C of the paraglider, let V be the velocity of the center of mass, and let θ be the angle between the velocity vector of the center of mass and the positive direction of the axis OX. The pitch angle, i.e., the angle between the vertical axis OY and the straight line GA counted counterclockwise is denoted by ϑ (see Fig. 1), and the pitch angular velocity $\dot{\vartheta}$, by ω.

If the paraglider is rotated about the center of mass, the absolute velocity V_A of the center A of the sail and the absolute velocity V_G of the center of mass G of the gondola are not equal to the velocity V of the center of mass C of the vehicle. The velocity vectors $\mathbf{V}_A$ and $\mathbf{V}_G$ of the points A and G are comprised of the velocity vector of the center of mass of the paraglider and the velocity vectors of these points with respect to the center of mass. The values of these velocities are calculated using the formulas

$$\begin{aligned} V_A &= \sqrt{V^2 + \omega^2 l_2^2 - 2V\omega l_2 \cos(\vartheta - \theta)}, \\ V_G &= \sqrt{V^2 + \omega^2 l_1^2 - 2V\omega l_1 \cos(\vartheta - \theta)}. \end{aligned} \quad (1.1)$$

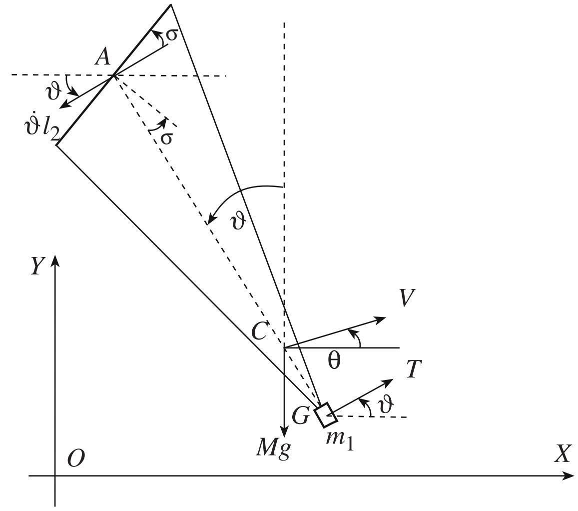

Fig. 1. Schematic diagram of paraglider: V is the velocity of the center of mass C of the vehicle; θ is the angle formed by the velocity vector and the horizon; ϑ is the pitch angle; $\omega = \dot{\vartheta}$ is the pitch angular velocity; T is the engine thrust; Mg is the weight of the vehicle; σ is the angle of rotation of the sail; A is the center of the sail; G is the center of mass of the gondola; $CG = l_1$; and $AC = l_2$.

The angle β_A between the vectors $\mathbf{V}$ and $\mathbf{V}_A$ is determined using the law of sines,

$$\sin\beta_A = \omega l_2 \sin(\vartheta - \theta)/V_A. \quad (1.2)$$

The angle β_G between the velocity vectors $\boldsymbol{\omega}\mathbf{l}_1$ and $\mathbf{V}_G$ can also be found using the law of sines,

$$\sin\beta_G = V\sin(\vartheta - \theta)/V_G. \quad (1.3)$$

The coefficient C_y of the lifting force of the sail in the linear approximation with respect to the angle of attack α can be represented in the form of the product $C_y = C_y^\alpha \alpha$, where C_y^α, similar to C_x, is constant. The expression for the reduced angle of attack of the sail has the form $\alpha = \vartheta - \theta + \beta_A + \sigma$. The term β_A in this sum appears due to the change of the direction of velocity of the incoming air on the sail if the sail is rotated with respect to the center of mass C of the vehicle. The sine of this angle (see relation (1.2)) is proportional to the pitch angular velocity ω and the distance l_2 from the point A to the center of mass C if the velocity V_A is close to the velocity V. At each point of the sail there is the "local" angle of attack, but it differs slightly from the angle $\vartheta - \theta + \beta_A + \sigma$ if the distance l_2 is large, as compared to the size BD of the sail. The distance l_2 is the larger, the larger the length of the slings and the closer the center of mass of the vehicle to that of the gondola. In this work it is assumed that the angle of attack at all points of the sail is equal to $\vartheta - \theta + \beta_A + \sigma$.

For taking into account the nonlinear dependence of the coefficients of the lifting and the drag forces on the angle of attack, the polar connecting the coefficients C_y

and C_x for different angles of attack [3, 4] should be known.

In the equations given below the drag force of the gondola $Q_G = C_{xG}\frac{1}{2}\rho V_G^2 S$, where $C_{xG} = const$ is the drag coefficient of the gondola, is taken into account.

For derivation of the equations of plane longitudinal motion of the paraglider, we use the general dynamics theorems of the system of material points, the theorem of motion of the center of mass and the theorem of variation of the ungular momentum of the system with respect to the center of mass [5].

The motion equation of the center of mass C of the vehicle projected to the tangent line to its trajectory has the form

$$\begin{aligned} M\dot{V} &= -Mg\sin\theta + T\cos(\vartheta - \theta) \\ &+ C_y^\alpha(\vartheta - \theta + \beta_A + \sigma)\frac{1}{2}\rho V_A^2 S\sin\beta_A \\ &- C_x\frac{1}{2}\rho V_A^2 S\cos\beta_A - C_{xG}\frac{1}{2}\rho V_G^2 S\cos(\vartheta - \theta - \beta_G) \\ &+ R_y\sin\theta - R_x\cos\theta. \end{aligned} \tag{1.4}$$

The first term in the right-hand side of Eq. (1.4) describes the projection to the tangent line of the force of gravity of the vehicle Mg, the second term, the projection to the tangent line of the thrust force T, the third term, the projection of the lifting force, and the fourth and the fifth terms, the projections to the tangent line of the drag forces applied to the sail and the gondola, respectively; R_y is the vertical component of the reaction force of the supporting surface, which is nonzero at the start leg of the paraglider motion, i.e., during its take-off run when the gondola rolls on wheels on the ground, and R_x is the horizontal component of the force acting from the ground on the gondola. It describes the force of resistance to the gondola rolling on the ground. The force R_y is assumed to be positive if it is directed vertically upward. Upon simulation (numerical investigation) the vehicle takes off when this quantity changes sign. After the take-off of the paraglider from the ground, $R_y = R_x = 0$.

The motion equations of the center of mass C of the vehicle projected to the normal to its trajectory has the form

$$\begin{aligned} MV\dot{\theta} &= -Mg\cos\theta + T\sin(\vartheta - \theta) \\ &+ C_y^\alpha(\vartheta - \theta + \beta_A + \sigma)\frac{1}{2}\rho V_A^2 S\cos\beta_A \\ &+ C_x\frac{1}{2}\rho V_A^2 S\sin\beta_A - C_{xG}\frac{1}{2}\rho V_G^2 S\sin(\vartheta - \theta - \beta_G) \\ &+ R_y\cos\theta + R_x\sin\theta. \end{aligned} \tag{1.5}$$

The first term in the right-hand side of Eq. (1.5) describes the projection to the normal of the force of gravity of the vehicle, the second term, the projection of the thrust force, the third term, the projection of the lifting force applied to the sail, and the fourth and the fifth terms, the projections to the normal of the drag forces applied to the sail and the gondola, respectively.

The third dynamic equation, the equation for the moments of forces with respect to the center of mass, has the form

$$\begin{aligned} J\ddot{\vartheta} &= -C_y^\alpha\frac{1}{2}\rho V_A^2 S(\vartheta - \theta + \beta_A + \sigma)l_2\sin(\vartheta - \theta + \beta_A) \\ &+ C_x\frac{1}{2}\rho V_A^2 S l_2\cos(\vartheta - \theta + \beta_A) - C_{xG}\frac{1}{2}\rho V_G^2 S l_1\cos\beta_G \\ &+ Tl_1 + (R_y\sin\vartheta - R_x\cos\vartheta)l_1. \end{aligned} \tag{1.6}$$

For description of motion of the center of mass of the paraglider, we use the obvious kinematical relations,

$$\dot{x} = V\cos\theta, \quad \dot{y} = V\sin\theta. \tag{1.7}$$

The gondola of the paraglider during its take-off run moves (rolls) on the ground and then during the flight of the paraglider, above the ground. Therefore, the following constraint, the unilateral constraint, is imposed on the ordinate h of the center G of the gondola:

$$h = y - l_1\sin\vartheta \geq 0.$$

During the take-off run when the gondola rolls on the ground, the following identity is satisfied:

$$h = y - l_1\sin\vartheta \equiv 0. \tag{1.8}$$

By differentiating (1.8) two times we obtain the relation

$$\dot{V}\sin\theta + V\dot{\theta}\cos\theta + l_1\ddot{\vartheta}\sin\vartheta + l_1\dot{\vartheta}^2\cos\vartheta \equiv 0. \tag{1.9}$$

By substituting to (1.9) the expressions for $\dot{V}$ from Eq. (1.4), for $\dot{\theta}$ from (1.5), and for $\ddot{\vartheta}$ from (1.6), this equation can be resolved with respect to the reaction force R_y (for the given force R_x). The expression for the reaction R_y is not given here due to its bulky character. However, it is used in simulation of the motion of the vehicle at the start leg. If the value of this reaction is positive, by substituting it into relations (1.4)–(1.6), we obtain the motion equations of the paraglider when the gondola moves on the ground. At the time instant of "vanishing" of this reaction the phase of flight of the vehicle begins.

The thrust T is further considered as the control action. Let us first find the steady-state regimes of the paraglider motion assuming the thrust force constant.

2. STEADY-STATE REGIMES OF UNCONTROLLED PARAGLIDER FLIGHT

Let us find the steady-state regimes of the paraglider motion at constant value of the engine thrust T. Using dynamic equations (1.4)–(1.6), we find the steady-state regime of flight of the vehicle in the form

$$V \equiv \text{const}, \quad \vartheta \equiv \text{const}, \quad \theta \equiv \text{const}. \tag{2.1}$$

Steady-state regime (2.1), if it exists, represents the uniform translation of the vehicle along the straight line which makes the angle θ with the horizontal axis OX. Under conditions (2.1) we have

$$\dot{V} = 0, \quad \dot{\vartheta} = \omega = 0, \quad \dot{\theta} = 0. \tag{2.2}$$

It follows from expressions (1.1)–(1.3) that under the condition $\omega = 0$,

$$V_A = V, \quad V_G = V, \quad \beta_A = 0, \quad \beta_G = \vartheta - \theta. \tag{2.3}$$

By substituting (2.1)–(2.3) to Eq. (1.4), we obtain the following relation connecting the thrust T, the angles ϑ, θ, and the velocity V (we recall that during the flight, $R_y = R_x = 0$):

$$\begin{aligned} &-Mg\sin\theta + T\cos(\vartheta-\theta) \\ &-(C_x + C_{xG})\frac{1}{2}\rho V^2 S = 0. \end{aligned} \tag{2.4}$$

Equality (2.4) implies the following expressions:

$$\begin{aligned} &\frac{1}{2}\rho V^2 S = \frac{T\cos(\vartheta-\theta) - Mg\sin\theta}{C_x + C_{xG}} \\ \text{or} \quad &T = \frac{(C_x + C_{xG})\rho V^2 S + 2Mg\sin\theta}{2\cos(\vartheta-\theta)}. \end{aligned} \tag{2.5}$$

By substituting (2.1)–(2.3) to Eq. (1.5), we obtain

$$\begin{aligned} &-Mg\cos\theta + T\sin(\vartheta-\theta) \\ &+ C_y^\alpha(\vartheta - \theta + \sigma)\frac{1}{2}\rho V^2 S = 0. \end{aligned} \tag{2.6}$$

Under conditions (2.1)–(2.3), Eq. (1.6) takes the form

$$\begin{aligned} &\frac{1}{2}\rho V^2 S[C_x l_2\cos(\vartheta-\theta) - C_{xG} l_1\cos(\vartheta-\theta) \\ &- C_y^\alpha(\vartheta-\theta+\sigma) l_2 \sin(\vartheta-\theta)] + T l_1 = 0. \end{aligned} \tag{2.7}$$

Equations (2.5)–(2.7) can be solved sufficiently rather easily if the steady-state flight regime at a constant altitude, i.e., when $\theta \equiv 0$, is sought. Under this condition with account of relations (2.5), we obtain from Eq. (2.7)

$$\begin{aligned} &\left(1 - \frac{C_{xG} l_1}{C_x l_2}\right)\cos^2\vartheta + \left(1 + \frac{C_{xG}}{C_x}\right)\frac{l_1}{l_2} \\ &- \frac{C_y^\alpha}{C_x}(\vartheta + \sigma)\sin\vartheta\cos\vartheta = 0. \end{aligned} \tag{2.8}$$

It follows from relation (2.8) that the steady-state value of the pitch angle ϑ depends on three ratios $\frac{C_{xG}}{C_x}$, $\frac{C_y^\alpha}{C_x}$, $\frac{l_1}{l_2}$, and the angle σ of orientation of the sail and does not depend on the thrust T and the velocity V. Equation (2.8) cannot be resolved analytically with respect to the unknown ϑ. However, the root of this equation can be found numerically by constructing the dependence of the left-hand side of Eq. (2.8) on ϑ for the given values of the vehicle parameters.

By assuming in relation (2.6) $\theta = 0$, the value of the thrust T for which the paraglider flies horizontally at a constant altitude can be found for the found value of the angle ϑ,

$$T = \frac{Mg}{\frac{C_y^\alpha}{C_x}\left(1 + \frac{C_{xG}}{C_x}\right)^{-1}(\vartheta+\sigma)\cos\vartheta + \sin\vartheta}. \tag{2.9}$$

For the known value of the angle ϑ, expressions (2.5) determine the relation between the thrust and the velocity. By calculating the thrust using formula (2.9), the corresponding value of the velocity V can be found using first relation (2.5).

Relations (2.5)–(2.7) provide the possibility of finding the steady-state flight regimes for which $\theta \neq 0$. For this purpose, it is convenient to solve Eqs. (2.5)–(2.7) using the "inverse" method. Let us take some value of the angle θ and substitute it to Eqs. (2.5)–(2.7). These three equations contain three unknown quantities: the pitch angle ϑ, the velocity V, and the thrust T which will be assumed unknown for the given angle θ. Then the airspeed head from first relation (2.5) is substituted to (2.6) and (2.7). In these two equations only two unknown quantities remain: the pitch angle ϑ and the thrust T; the latter is included in these equations linearly and therefore, is easily eliminated. One nonlinear equation obtained after elimination of the thrust contains only one unknown quantity, the pitch angle ϑ. This nonlinear equation can be solved numerically. After determination of the angle ϑ, the thrust T eliminated from the two equations can be found, and then, the velocity V can be found using formulas (2.5).

Figure 2 shows the plots of dependences of the flight velocity V, the pitch angle ϑ, and the thrust T on the angle of the velocity vector θ constructed from some hypothetic values of the paraglider parameters $m_1 = 100$ kg, $m_2 = 7$ kg, J = 358 kg m^2, $\rho = 1, 29$ kg/m^2, $L = 7.3$ m, $S = 30$ m^2, $C_y^\alpha = 1.2$, $C_x = 0.1$, $C_{xG} = 0.1$, and $C_{xG} = 0.1$). In spite of the fact that with increasing angle θ the thrust T in the steady-state regime is increased, the velocity V of the vehicle in this case is decreased. This decrease in the velocity is explained by the fact that with increasing angle θ the projection of the force of gravity of the vehicle to the direction of the tangent line to the trajectory $Mg\sin\theta$ which prevents the velocity growth is increased (see Eq. (1.4) and the first of relations (2.5)).

For $T = 0$, there exists the steady-state gliding regime which corresponds to the constant values of the velocity V, the pitch angle ϑ, and the angle of the velocity vector θ (Fig. 2). In this regime, the center of mass of the paraglider moves along the inclined straight line downward toward the ground, since $\theta < 0$. The pitch

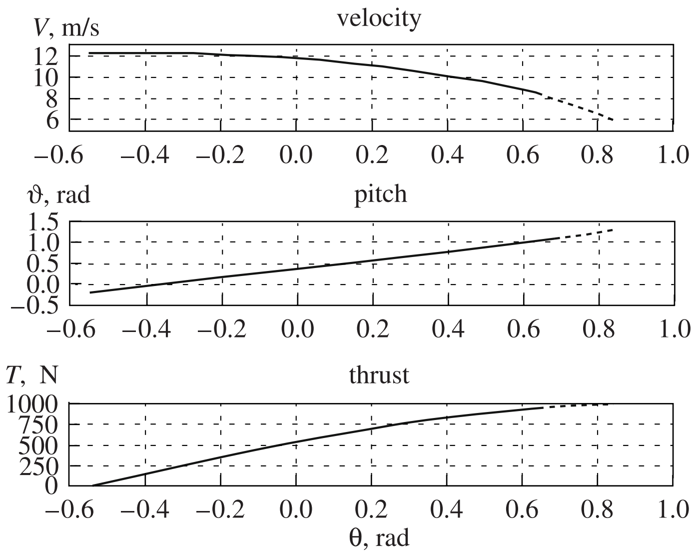

Fig. 2. Flight velocity V, pitch angle ϑ, and thrust T as functions of the angle of the velocity vector θ for steady-state flight regimes. Dashed lines show unstable steady-state regimes.

angle ϑ is also a negative quantity. Thus, in the gliding regime, the vehicle moves along the straight trajectory toward the ground, and its axis GA is inclined in the direction of motion.

The constructed steady-state regimes contain the regime of the horizontal flight when $\theta = 0$. This regime is performed at constant values of V, ϑ, and T. Let T_* be the value of the thrust corresponding to the horizontal flight of the paraglider. This value is calculated using formula (2.9) for the known pitch angle ϑ. For the constant thrust $T \neq T_*$ in the steady-state regime the vehicle flies along the straight trajectory gaining or losing altitude. The velocity of the horizontal flight of the paraglider cannot be increased or decreased by increasing or decreasing the thrust of the engine. This velocity can be changed only by changing the aerodynamic, geometric, and mass–inertial characteristics of the vehicle.

Using Eqs. (1.4)–(1.6), the variational equations with respect to steady-state values obtained from (2.5)–(2.7) can be obtained in a usual way. The system of variational equations has the fourth order. This system is not presented here due to its bulky character. Using variational equations the stability of steady-state regimes of the paraglider motion with respect to the four variables V, ω, ϑ, and θ for the given constant thrust T can be investigated. In Fig. 2, solid line shows the steady-state regimes which are asymptotically stable with respect to the above variables, and dashed lines show the unstable steady-state regimes. It follows from Fig. 2 that the paraglider gliding regime is asymptotically stable.

For low paraglider flight altitudes the dependence of the air density ρ on altitude can be neglected. In the mathematical model presented above the density ρ is assumed to be constant. In this case, the motion of the paraglider is independent of its altitude. In other words, the coordinate y of the center of mass C is a cyclic variable. Therefore, the horizontal uncontrolled (for $T = T_*$ *const*) motion of the vehicle is indifferent to the coordinate y and thus, is not asymptotically stable with respect to the flight altitude.

3. FLIGHT ALTITUDE CONTROL

The paraglider flight stability at a desired altitude can be provided by controlling the thrust of the engine. The thrust control stabilizing the vehicle flight at the given altitude will be constructed in the form of the feedback with respect to the deviation of the altitude h of the flight of the gondola G from the desired (given) value and the angle of the velocity vector θ,

$$T = T_s - k_h(h - h_d) - k_\theta \theta. \tag{3.1}$$

Here, T_s is the given constant thrust value equal to T_* or close to it,

$$h = y - l_1 \cos\vartheta, \tag{3.2}$$

h_d is the desired gondola G flight alititude, and k_h, k_θ are the constant feedback coefficients. The coefficient k_h should be positive, which follows from physical considerations. Indeed, if the flight altitude is larger than the desired value, the thrust should be decreased in order to decrease the velocity of the gondola motion, the angle of attack; therefore, the lifting force of the sail. If the flight altitude is smaller than the desired value, the thrust should be increased. The numerical investigations presented below for which the domains of asymptotic stability were constructed prove the above conclusion. Note that it follows from second

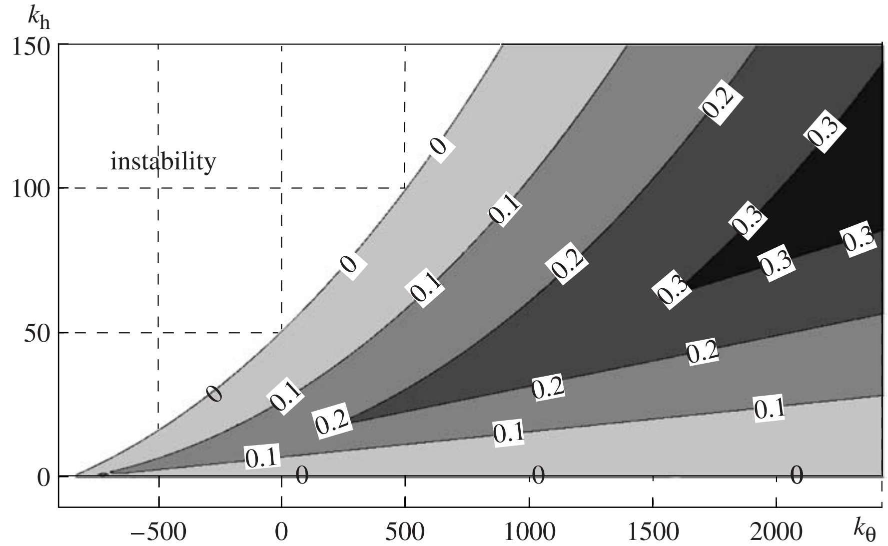

Fig. 3. Domains of asymptotic stability and stability with a factor of 0.1, 0.2, and 0.3 constructed in the plane of the coefficients k_h, k_θ.

relation (1.7) that for small values of θ the derivative of the altitude y of the center of mass C of the vehicle (the lifting velocity) $\dot{y} \approx V\theta$; i.e., it is proportional to the angle of the velocity vector θ.

For control (3.1), system (1.4)–(1.7) has the steady-state regime of motion,

$$V \equiv \text{const}, \quad \theta \equiv 0, \quad \vartheta \equiv \text{const}, \quad \omega \equiv 0, \\ y \equiv \text{const}, \quad h \equiv \text{const}, \quad T \equiv \text{const}. \tag{3.3}$$

The values of ϑ, T, and V are determined successively from relations (2.8), (2.9), and (2.5). For the horizontal flight, the steady-state value of the thrust is T_*. According to equality (3.1), the steady-state value of the altitude h of the gondola flight is found from the relation

$$h = h_d + (T_s - T_*)/k_h. \tag{3.4}$$

It follows from expression (3.4) that the error $\Delta h = |h - h_d|$ in tracking the given altitude value h_d is the smaller the closer to zero the difference $|T_s - T_*|$ and (or) the larger the altitude feedback coefficient k_h. However, it is known from the control theory [6], that if the position feedback coefficient k_h is chosen too large, steady-state regime (3.3) may become unstable. This statement also follows from the stability domains constructed below. The error $\Delta h = |h - h_d| = 0$ if and only if $T_s = T_*$. If $T_s > T_*$, it follows from expression (3.4) that the flight altitude h of the paraglider in the steady-state regime is larger than the given h_d; this altitude h is smaller than the given h_d if $T_s < T_*$. Note that since the vehicle characteristics are known only approximately, the thrust value T_* can be found only approximately.

Using Eqs. (1.4)–(1.6), second kinematic equation (1.7), and relations (3.1), (3.2), the variational equations with respect to steady-state values (3.3) can be written. The system of variational equations has the fifth order. This system is not given here due to its bulky character. The stability of steady-state regimes of the paraglider motion with respect to five variables V. ω, ϑ, θ, and h can be studied using the variational equations. Figure 3 shows the domain of asymptotic stability with respect to these variables constructed numerically using the variational equations in the plane of feedback coefficients k_h, k_θ. It follows from this figure, in particular, that k_h which is positive is bounded from above for each value of the coefficient k_θ. This figure also shows the boundaries of the domains with a given stability reserve of 0.1, 0.2, and 0.3. These are the domains in which all eigenvalues of the system λ_i ($i = 1…5$), are situated to the left of the imaginary axis by 0.1, 0.2, and 0.3 as a minimum. With increasing the desired stability reserve the corresponding domain is, of course, decreased.

In control law (3.1) the possible delay in receiving the information on the current altitude h of the vehicle and (or) the angle of the velocity vector θ is not taken into account. Figure 4 shows the domain of asymptotic stability constructed in the plane of the coefficients k_h, k_θ with account of a delay of 0.5 s in receiving information on the current altitude h; the stability domain constructed in Fig. 3 without account of any delay is shown for comparison. The delay is simulated using the aperiodic link. The comparison of the constructed domains

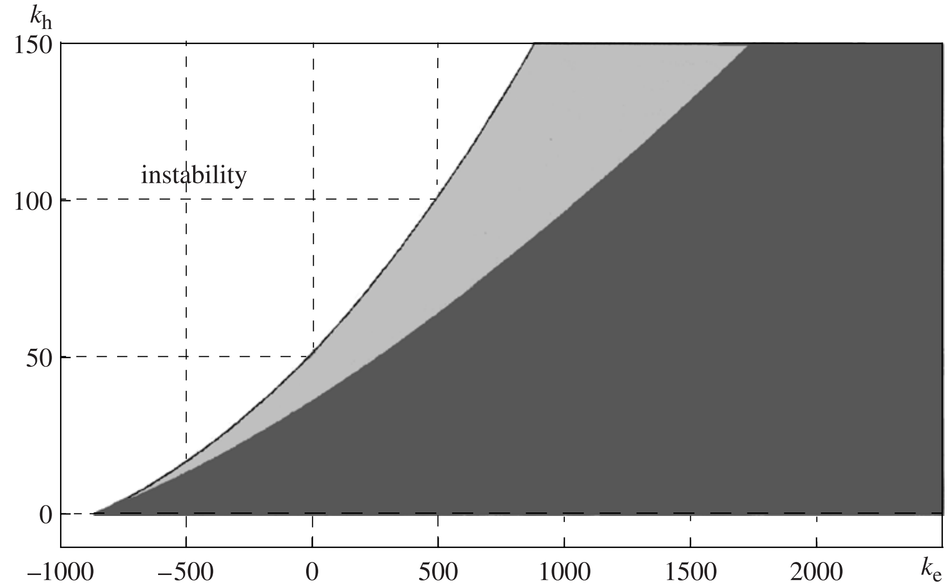

Fig. 4. Domain of asymptotic stability without account of delay and (inside it) stability domain with account of delay.

shows that the delay narrows the stability domain. In this case, the bound from above for the coefficient k_h becomes stricter. The decrease in the stability domain if the delay is taken into account seems natural, since the delay is usually the destabilizing factor.

The possible thrust values are bounded from above by some value T_m; moreover, the thrust cannot be negative. Therefore, instead of "purely" linear feedback (3.1), the feedback with the saturation of the following form should be considered:

$$T = \begin{cases} T_m & \text{for } T_s - k_h(h - h_d) - k_\theta\theta \geq T_m, \\ T_s - k_h(h - h_d) - k_\theta\theta & \text{for } 0 \leq T_s - k_h(h - h_d) - k_\theta\theta \leq T_m. \\ 0 & \text{for } T_s - k_h(h - h_d) - k_\theta\theta \leq 0. \end{cases} \quad (3.5)$$

The program for solution of system of equations (1.4)–(1.7), (3.5) developed using "MATLAB" provides the possibility of investigation of the regimes of motion of the paraglider with the feedback. The numerical investigation was performed for the above hypothetic values of the vehicle parameters.

Figure 5 shows the plots of time variation of the velocity V of the vehicle, the angle of the velocity vector θ, the pitch angle ϑ, the altitude h of the gondola, the thrust T, and the reduced angle of attack $\alpha = \vartheta - \theta + \beta_A + \sigma$. The velocity of motion V of the paraglider, as follows from the figure, is first increased and then after several oscillations takes the constant value. The angle of the velocity vector θ and the pitch angle ϑ after the surge at the beginning of motion approach (asymptotically) their steady-state values. At the take-off run the gondola moves on the ground. Then the paraglider takes off from the ground. After the take-off, the altitude of the gondola h is increased monotonically and reaches a constant value of 22 m. At the beginning of motion the thrust takes the maximum possible value T_m and then after several oscillations approaches the steady-state value T_*. The reduced angle of attack after the surge at the take-off run approaches "promptly" the steady-state value. It follows from the figure that the paraglider in the steady-state regime flies at the constant altitude h = 22 m, while in control (3.1) the desired altitude h_d = 20 m is determined. Thus, the static error Δh makes 2 m. It can be reduced by increasing the coefficient k_h. It should not be forgotten, however, that with increasing this coefficient the transition process may become oscillatory and even unstable; it is possible to suppress the oscillations or the instability by increasing the coefficient k_θ.

The simulation shows that for the small initial velocity $V(0)$ of the paraglider the reduced angle of attack $\alpha = \vartheta - \theta + \beta_A + \sigma$ at the beginning of motion takes "large" values for which the linearization of the coefficient of the lifting force C_y with respect to the angle of attack ($C_y = C_y^\alpha \alpha$) is incorrect. The mathemat-

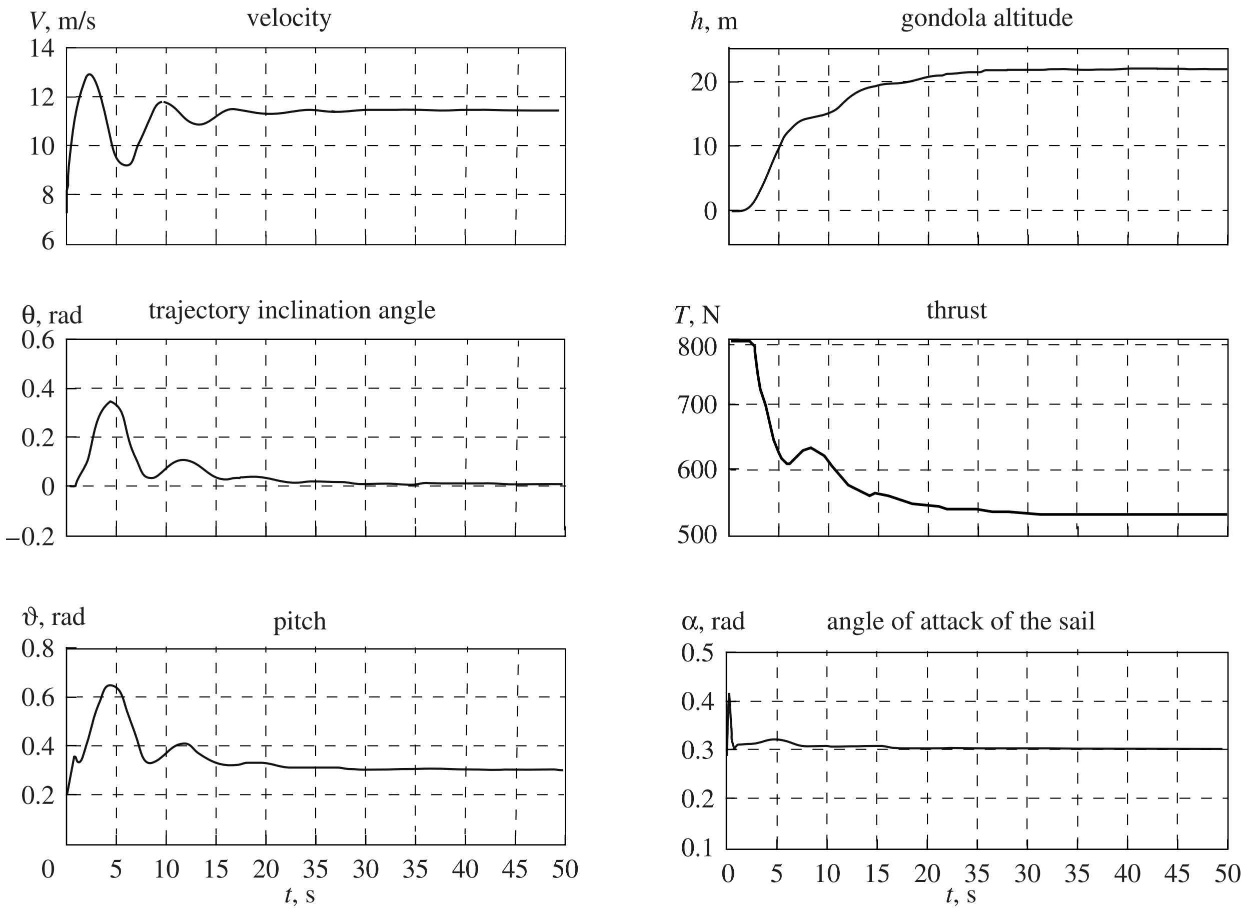

Fig. 5. Paraglider take-off and flight at constant altitude.

ical model constructed above is correct beginning from the time instant at which the paraglider has already gained sufficient velocity; in addition, upon further motion on the ground it does not perform large pitch oscillations.

The flight of the paraglider at the given altitude can be stabilized not only using control (3.5), but also using the feedback with respect to the altitude h and its derivative $\dot{h}$,

$$T = \begin{cases} T_m & \text{for } T_s - k_h(h - h_d) - k_{\dot{h}}\dot{h} \geq T_m, \\ T_s - k_h(h - h_d) - k_{\dot{h}}\dot{h} & \text{for } 0 \leq T_s\,(k_h(h - h_d) - k_{\dot{h}}\dot{h} \leq T_m), \\ 0 & \text{for } T_s - k_h(h - h_d) - k_{\dot{h}}\dot{h} \leq 0. \end{cases} \quad (3.6)$$

Expression (3.6) describes the feedback with respect to the position and its derivative, which corresponds to the so called PD regulator. For implementation of control law (3.6), it is necessary to measure both the altitude of the gondola and the rate of altitude variation. Note that it follows from expressions (1.7), (3.2) that for small values of the angles θ, ϑ and the angular velocity ω, the value of $\dot{h}$ is proportional to the angle θ. Thus, control laws (3.1) and (3.6) are close. Up to quantities of the second order of smallness, it can be assumed that the coefficients $k_{\dot{h}}$ and k_θ are proportional to each other with the factor V, $k_\theta \cong V k_{\dot{h}}$.

Note that by determining the quantity h_d in the form of the function of time, $h_d = h_d(t)$ or range, $h_d = h_d(x)$, the take-off trajectory, the cruise, and the landing trajectory of the vehicle can be planned.

CONCLUSIONS

A mathematical model of the autonomous controlled motion of the paraglider in the longitudinal plane was constructed. The steady-state regimes of the paraglider motion at constant thrust developed by the engine mounted on the gondola were found and the problem of their stability was studied. The thrust control law in the form of the feedback for which the vehicle flight is stabilized at the given altitude was constructed. The domains of asymptotic stability of the paraglider motion at the constant altitude were constructed in the plane of the feedback coefficients. Some results of numerical simulation of the flight of the vehicle were presented.

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