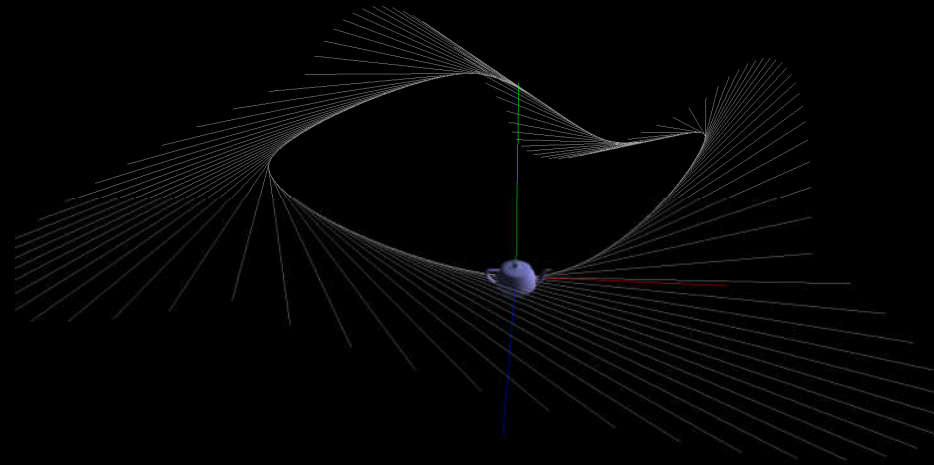




Animation with Catmull-Rom Curves

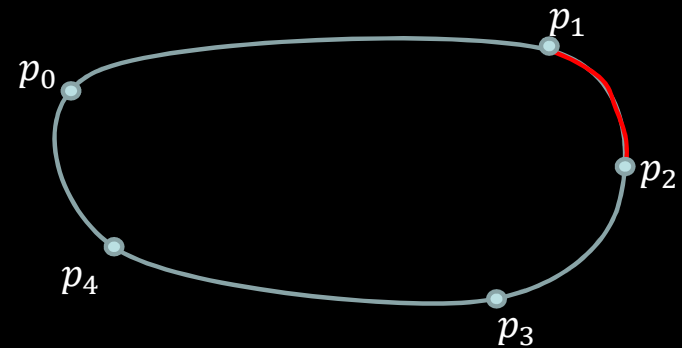




Cubic Curves – Catmull-Rom

- Matrix formulation

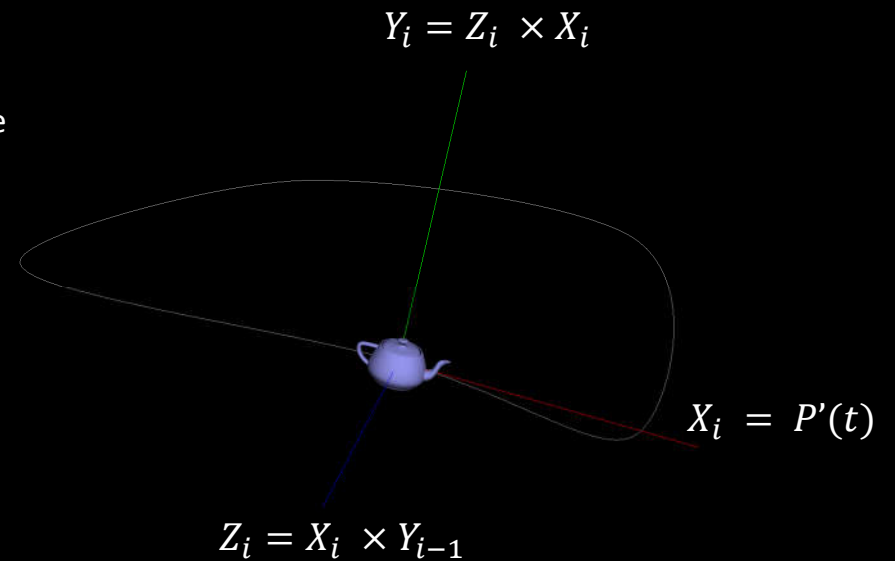
- $$P(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} -0.5 & 1.5 & -1.5 & 0.5 \\ 1 & -2.5 & 2 & -0.5 \\ -0.5 & 0 & 0.5 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$$
- $$P'(t) = \begin{bmatrix} 3t^2 & 2t & 1 & 0 \end{bmatrix} \begin{bmatrix} -0.5 & 1.5 & -1.5 & 0.5 \\ 1 & -2.5 & 2 & -0.5 \\ -0.5 & 0 & 0.5 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$$





Cubic Curves – Catmull-Rom

- Axis for Rotation Matrix
 - Available data at instant t
 - $P(t)$ - position of an object “walking” along the curve
 - $P'(t)$ - vector tangent to the curve
 - Transform for teapot
 - Translation to place teapot
 - Rotation to align with curve
 - $Y_0 = (0,1,0)$





Cubic Curves – Catmull-Rom

- Assuming an initial specification of an \vec{Y}_0 vector, to align the object with the curve, we need to build a rotation matrix for the object:

$$\begin{aligned}\vec{X}_i &= P'(t) \\ \vec{Z}_i &= X_i \times \vec{Y}_{i-1} \\ \vec{Y}_i &= \vec{Z}_i \times \vec{X}_i\end{aligned} \quad M = \begin{bmatrix} X_x & Y_x & Z_x & 0 \\ X_y & Y_y & Z_y & 0 \\ X_z & Y_z & Z_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

*Note: All
vectors
need to be
normalized*

```
glMultMatrixf(float *m)
```

- Current OpenGL MODEL_VIEW matrix gets multiplied by m

Note: OpenGL matrices are column major => compute the transpose instead



Assignment

- Complete the function

```
void getCatmullRomPoint(float t,
                       float *p0, float *p1, float *p2, float *p3,
                       float *pos, float *deriv) {

    // catmull-rom matrix
    float m[4][4] = {
        {-0.5f, 1.5f, -1.5f, 0.5f},
        { 1.0f, -2.5f, 2.0f, -0.5f},
        {-0.5f, 0.0f, 0.5f, 0.0f},
        { 0.0f, 1.0f, 0.0f, 0.0f}};

    // For each component i: // x, y, z
    //   Compute vector A = M * P // use function multMatrixVector
    //   in component i P is the vector (p0[i], p1[i], p2[i], p3[i])
    //   Compute pos[i] = T * A
    //   compute deriv[i] = T' * A
}
```



Assignment – cont.

- Write the function

```
void renderCatmullRomCurve() {  
  
    // draw the curve using line segments - GL_LINE_LOOP (see slides near the end)  
}
```

- To get the points for the full curve call

```
void getGlobalCatmullRomPoint(float gt, float *pos, float *deriv)
```

with `gt` in `[0,1[`.

- To draw the curve select a tessellation level, for instance 0.01. This will get a curve with 100 line segments.



Assignment – cont.

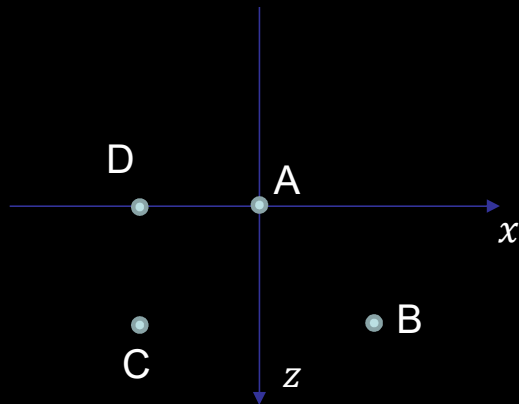
- In function `renderScene`, apply the required transformations to have the teapot travelling along the curve oriented accordingly to the derivative.
 - Get the position and derivative in the curve for current `t` value - `getGlobalCatmullRomPoint(t, pos, deriv)`
 - Use `glTranslate` to position the teapot along the curve
 - Build the rotation matrix to align the teapot with the curve
 - Compute the axis
 - functions, `cross` and `normalize`, are provided in the code.
 - Use `buildRotMatrix` provided in the source code
 - Apply the matrix using `glMultMatrix`



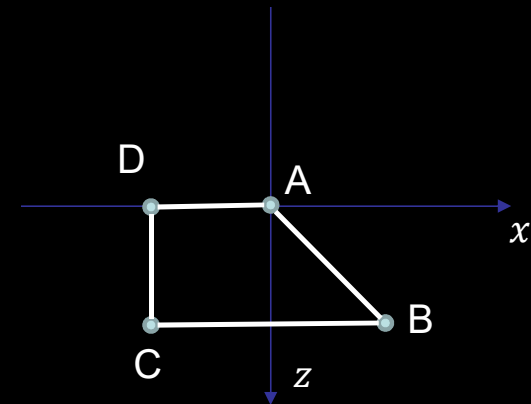
Drawing with line loops

Consider the following points:

- $A = (0,0,0)$, $B = (1,0,1)$, $C = (-1,0,1)$, $D = (-1,0,0)$



A line loop receives all the points and draws a line between each two points and connects the last to the first point:





Drawing with line loops

- Line loops in OpenGL – code example considering an array of 4 points

```
float k[4][3] = {{ 0.0f,  0.0f, 0.0f}, // A
                 { 1.0f,  0.0f, 1.0f}, // B
                 {-1.0f,  0.0f, 1.0f}, // C
                 {-1.0f,  0.0f, 0.0f}}; // D

glBegin(GL_LINE_LOOP);
for (int i = 0; i < 4; ++i) {
    glVertex3f(k[i][0], k[i][1], k[i][2]);
}
glEnd();
```



Questions

- Replace the teapot with the cone from GLUT.
 - What happens?
 - How to fix it?
- What would be required to use Bezier or Hermite curves?
 - What parts of the code would require change?

