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EDO - Capítulo 6

1) $(2x+3)(2y-2)y' = 0$

$M = 2x+3 \quad M_y = 0$ → A EDO é EXATA

$N = 2y-2 \quad N_x = 0$

$\psi(x,y) = \int M_x \, dx = \int 2x+3 \, dx = x^2 + 3x + h(y)$

$\psi(x,y)_y = h'(y) = 2y-2 \quad h(y) = y^2 - 2y + c$

$\psi(x,y) = x^2 + 3x + y^2 - 2y = C$

5) $\frac{dy}{dx} = -\frac{(ax+by)}{bx+cy} \rightarrow (ax+by) + (bx+cy)dy = 0$

$M = ax + by \quad M_y = b$ → a EDO é EXATA

$N = bx + cy \quad N_x = b$

$\psi(x,y) = \int M \, dx = \int ax + by \, dx = \frac{ax^2}{2} + byx + h(y)$

$\psi(x,y)_y = bx + h'(y) = bx + cy \quad h'(y) = cy \quad h(y) = \frac{cy^2}{2}$

$\psi(x,y) = \frac{ax^2}{2} + bxy + \frac{cy^2}{2} = 0$

9) $(ye^{xy}\cos 2x - 2e^{xy}\operatorname{sen} 2x + 2x)dx + (xe^{xy}\cos 2x - 3)dy = 0$

$M = ye^{xy}\cos 2x - 2e^{xy}\operatorname{sen} 2x + 2x$

$M_y = e^{xy}\cos 2x - 2xe^{xy}\operatorname{sen} 2x$

$N = xe^{xy}\cos 2x - 3$

$N_x = e^{xy}\cos 2x + e^{xy}x \cdot \cos 2x + -2\operatorname{sen} 2x e^{xy}x$

$M_y \neq N_x$

A EDO não é exata

11) $(x \ln y + xy)dx + (y \ln x + xy)dy$

$M = x \ln y + xy \quad My = \frac{x}{y} + x$

$N = y \ln x + xy \quad Nx = \frac{y}{x} + y$

A EDO não é exata

12) $\frac{x\,dx}{(x^2+y^2)^{3/2}} + \frac{y\,dy}{(x^2+y^2)^{3/2}} = 0$

$M = \frac{x}{x^2+y^2} \quad My = \frac{2y \cdot x}{(x^2+y^2)^2}$ A EDO É EXATA

$N = \frac{y}{x^2+y^2} \quad Nx = \frac{2x \cdot y}{(x^2+y^2)^2}$

$\psi(x,y) = \int M\,dx = \int \frac{x}{x^2+y^2}\,dx \quad u = x^2+y^2$

$du = 2x\,dx$

$2\int \frac{1}{u}\,du = 2 \ln|x^2+y^2| + h(y)$

$\psi(x,y)y = 2 \cdot \frac{1}{x^2+y^2} \cdot 2y + h'(y) = y$

$\frac{2y}{(x^2+y^2)}$

$\psi(x,y)y = 4y + h'y = y$

$\psi(x,y) = \ln(x^2+y^2)^2 + 3y = C$

13) $(2x-y)dx + (2y-x)dy = 0 \quad y(1) = 3$

$M = 2x - y \quad My = -1$

$N = 2y - x \quad Nx = -1$

$\psi(x,y) = \int 2x - y\,dx = x - yx + h(y)$

$\psi(x,y)y = -x + h'y = 2y - x \quad h'x = 2y$

$h = y$

$\psi(x,y) = x - yx + y^2 = C$

$1 - 3 + 9 = C \quad C = 7$

14) $(9x^2+y-1)dx - (4y-x)dy = 0 \quad y(1)=0$

$M = 9x^2+y-1 \quad My = 1$

$N = -4y+x \quad Nx = 1$

$\Psi(x,y) = \frac{9x^3}{3} + yx - x + h(y)$

$\Psi(x,y)y = x + h(y) = -4y+x \quad h'(y) = -4y$

$h(y) = -2y^2$

$3x^3 + yx - x - 2y^2 = C$

$3 + 0 - 1 = C \quad C = 2$

15) $(xy^2 + bx^2y)dx + (x+y)x^2\,dy$

$M = x^2y^2 + bx^2y \quad My = 2xy + bx^2$

$N = x^3 + yx^2 \quad Nx = 3x^2 + 2xy \quad b = 3$

$\Psi(x,y) = \int xy^2 + 3x^2y\,dx = \frac{y^2x^2}{2} + x^3 + h(y)$

$\Psi(x,y)y = 2yx^2 + h'(y) = x^3 + yx^2$

$h'y = x^3y + y^2x^2 - y^2x^2$

$hy = x^3y$

$\Psi(x,y) = \frac{y^2x^2}{2} + x^3 + x^3y = C$

16) $(ye^{2xy} + x)dx + bxe^{2xy}dy = 0$

$M = ye^{2xy} + x \quad My = e^{2xy}2xy + e^{2xy}$

$N = bxe^{2xy} \quad Nx = b2yxe^{2xy} + be^{2xy}$

$b = 1$

$\Psi(x,y) = \frac{e^{xy2}}{2} + h(x) \quad \Psi x = ye^{2xy} + h' = M$

$h'(x) = x \quad h(x) = \frac{x^2}{2}$

$\Psi(x,y) \frac{e^{2y2}}{2} + \frac{x^2}{2} = C$

17) $\psi(x,y) = \int_{x_0}^{x} M(s,y_0)\,ds + \int_{y_0}^{y} N(x,t)\,dt$

$\psi(x,y) = \int N(x,y)\,dy + h(x)$

$\psi = \int \frac{\partial}{\partial x} N(x,y)\,dy + h'x \qquad \psi_x = M(x,y)$

$h'(x) = M(x,y) = \int \frac{\partial}{\partial x} N(x,y)\,dy \quad \therefore M_y = N_x$

$\frac{\partial}{\partial y}[h'(x)] = 0 \qquad \int dx$

$h(x) = \int M(x,y)\,dx - \int \left[\int \frac{\partial}{\partial x} N(x,y)\,dy\right] dx$

18) $M(x) + N(y)y' = 0$

$\frac{\partial M(x)}{\partial y} = \frac{\partial N(y)}{\partial y} = 0$