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EDO Parte 3

3.1) $\frac{d^2y}{dT^2} = g(t)$ | $\frac{dy}{dT} = \int_a^T g(T)dT = \int_{T0}^{T} g(t)dT + d$

$y(T_0) = C \quad y'(T_0) = d$

$y(T) = \int_{T_0}^{T}\int_{T_0}^{z} g(s)ds + d\,dz + c$

3.2) $\frac{dP}{dT} = 0,5P - 450$ | $y' = f(T,P)$ | $y^n = f(T, y, y', y'', \dots, y^{(n-1)})$

$y' = f(t,P) = 0,5P - 450$ //

3.3) $\frac{d^2y}{dT^2} = g(t) = \int\int \frac{d^2y}{dT^2}dTdT = \int\int g(T)\,dT\,dT + C_1 + C_2$ //

3.4) Exercícios da Lista

3) $\frac{dy}{dT} = -ay + b \quad = y' = -ay + b$

$y' - ay = b$

$\frac{dy}{dT} = -ay + b \quad \frac{dy}{dT} = dT = \int \frac{dy}{-ay+b} = \int dT$

$\int \frac{dy}{-ay+b} = T + c \quad = \frac{-\ln|ay+b|}{a} = T + c$

$u = -ay + b \quad -\ln|ay+b| = aT + c$

$du = -a\,dy \quad ay + b = e^{-aT+c}$

$\frac{du}{-a} = dy \quad -ay = e^{-aT+c} + b$

$y = Ce^{-aT} + \frac{b}{a}$ //

b) $S = \{a, b \in \mathbb{R} / a \neq 0\}$

c)

i) a aumenta

O ponto de equilíbrio se torna mais baixo e a função se aproxima mais rapidamente dele

ii) O ponto de equilíbrio é mais alto

iii) O equilíbrio é o mesmo.

4) a) $\frac{dy}{dT} = ay + b$ $\int \frac{dy}{ay-b} = \int dT$

$\frac{\ln|ay-b|}{a} = T + c \Rightarrow ay - b = e^{aT+c} \Rightarrow y = e^{aT} \cdot C + \frac{b}{a}$

$y_e = \frac{b}{a}$

b) $y(T) = \bar{y} - y_e = y' = aY$

7) $\frac{dP}{dT} = 0,5P - 450$

a) $\frac{dP}{0,5P-450} = dT = \frac{dP}{\frac{1}{2}P-450} = \frac{\frac{dP}{dT}}{P_{(T)}-900} = \frac{1}{2}$

$\frac{\ln|P_{(T)}-900|}{2} = \frac{T}{2} = P_{(T)} - 900 = e^{T/2} C$

$P_{(T)} = e^{\frac{T}{2}} C - 900$ $P_{(T)} = 900 - (900 - P_0) e^{\frac{1}{2}T}$

$P_0 = 850$ $P_{(T)} = 900 - 50e^{\frac{T}{2}} = 2\ln(18)$

b) $P_{(T)} = 0$

$T = 2\ln\left(\frac{-900}{-900+P_0}\right)$

c) $P_{(12)} = 0 \Rightarrow P_0 = 900\left(\frac{-1+e^6}{e^6}\right)$

$\log_x N = a \Rightarrow N = x^a$

8) a) $\frac{dP}{dT} = rP$ $\int \frac{dP}{rP} = \int dT$

$\frac{\ln|rP|}{r} = T$ $u = rP$

$du = r\,dP$

$rP = e^{rT+c}$ $P = Ce^{rT}$

$P(30) = 2P$ e $P(0) = C \Rightarrow 2P = Pe^{30r}$

$30r = \ln 2$ $r = \frac{\ln 2}{30}$ //

b) $P(n) = 2P$ $P = P(0)e^{rT} \Rightarrow 2P = Pe^{Nr}$

$r = \frac{\ln 2}{N}$

9) $\frac{dv}{dT} = 9{,}8 - \frac{V}{5}$ $v(0) = 0$

$dv = (9{,}8 - \frac{V}{5})\,dT = \frac{dv}{(V-5)\cdot 9{,}8} = \int -\frac{1}{5}\,dT$

$\ln|(v-5)\cdot 9{,}8| = -\frac{1}{5}T + C \Rightarrow (v-5)9{,}8 = e^{-\frac{1}{5}T}C$

$v = Ce^{-\frac{1}{5}T} + 49$

$0{,}98 \cdot 49 = 49e^{-\frac{1}{5}T} + 49 \Rightarrow 0{,}98 = 1 - e^{-\frac{1}{5}T}$

$T = 19{,}56$ s

b) $V = \frac{d}{t}$ $d = V \cdot T$ $d = 49\frac{m}{s} \cdot 19{,}56 = 958{,}44m$

12) $\frac{dQ}{dT} = -rQ$ $T = 1620$ anos

Taxa de decaimento $= \frac{\ln 121}{1600}$ ano

$Q(t) = Q(0)e^{-0{,}00043 = r}$ $e \cdot T = \frac{3}{4}$

$-r \cdot T = \ln\left(\frac{3}{4}\right) \Rightarrow T = 672.36$ anos //

13) $\frac{dQ}{dt} = -\frac{1}{RC}Q$

$Q(0) = 0$

$Q(T) = CV(1 - e^{-T/CR})$

$T \to \infty \quad Q_L = CV$

14)

$m(0) = M_0$

$m(T) = m(0) \cdot e^{KT}$

$T = 1620$ años

$m(1620) = \frac{1}{2} m_0 \rightarrow m_0 \cdot e^{K \cdot 1620} = m_0$

$e^{K \cdot 1620} = \frac{1}{2} \Rightarrow \ln \frac{1}{2} = K \cdot 1620$

$K = -\frac{\ln(2)}{1620} \qquad K \approx -4{,}28 \cdot 10^{-4} \cdot (\text{años}^{-1})$

$m(T) = \frac{3}{4} m_0 \qquad e^{KT} = \frac{3}{4}$

$\ln\left(\frac{3}{4}\right) = KT$

$\ln(3) - 2\ln(2) \cdot \frac{1}{K} = T$

$T = 673$ años

15) $\frac{du}{dT} = -K(u-T) \Rightarrow \frac{du}{u-T} = -K\,dT$

$\int \frac{du}{u-T} = \int K\,dT \rightarrow \ln|u-T| = -KT + C$

$C = \ln u_0 - T$

$\ln \frac{u-T}{u_0-T} = -KT \Rightarrow u = T + (u_0 - T)e^{-KT}$

a) $T = (u_0 - T)e^{KT} - u$ //

16) como $u = T + (u_0 - T)e^{KT}$

e $u_{(0)} = 20°C$

$\ln \frac{0+12}{20+12} = 0{,}15 \cdot T \Rightarrow T = 6{,}54h$ //

17) a) $R\frac{dQ}{dT} + \frac{Q}{C} = V$

$\frac{du(t)}{dT} = \frac{u(t)}{RC} = \int \ln(u) \Rightarrow \ln|u| = \frac{T}{RC}$

$u_{(T)} = e^{T/RC} = \int \left(e^{T/RC} \cdot \frac{V}{R}\right) / dT = e^{T/RC} + \frac{V}{R}$

$e^{T/RC} \times Q = \frac{V}{R}\int e^{T/RC}\,dT$

$Q_{(T)} = \left(\frac{V}{R} \cdot \left(RC \cdot e^{T/RC}\right) + C_1\right) = CV + \frac{C_1}{e^{T/RC}}$

$e^{T/RC}$

$C_1 = -CV$

$Q(T) = CV(1 - e^{-T/RC})$

b) $t \rightarrow \infty \Rightarrow Q(T) = CV = Q_L$

18) a) $(0.01).(300)\ g/h$

$Q(t) \cdot 10^{-6}\ g/6l \quad \therefore (3.10^{-4}).Q(t)\ g/h$

$\frac{dQ}{dT} = 3 - 0.0003\,Q(t)\ gm/hr$

$Q(0) = 0$ //

b) $\int \frac{dQ}{10000 - Q} = \int 0.0003\,dT$

$-\ln|10000 - Q| = 3.10^{-4}T + C$

$10.10^3 - Q = Ce^{-3.10^{-4}T}, \quad Q(0) = 0$

$C = 10000$

$T = 8760h \qquad Q(8760) = 9277.77\ g$ //

19) a) $200.\frac{Q(t)\ Kg}{60000\ min} = 200.\left(\frac{60}{1000}\right)\frac{Q(t)\ gm}{60\ h}$

b) $\frac{dq}{dt} = -0.2q\ (gm/h), \quad Q(0) = 5000\ g$

b) $q(t) = 5000e^{-0.2T}$ //

c) $T = 4, \quad q(4) = 5000e^{-0,8} = 2246.69\ g$

$\frac{2246}{6.10^9} = 0.0379\ g/gal.$

d) $5000.e^{-0,2T} = 1200 \therefore T = 7.14h$ //

e) $\frac{dq}{dT} = -r \cdot q$ / $1200 = 5000 e^{-r/50}$

$Q(t) = 5000 e^{-T r/1000}$ / $r = 3.57$ gal/min

$T = 4$ $q = 1200$ /