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EDO Capítulo 4

1.c) $y' + 3y = t + e^{2t}$

$P_{(T)} = 3$

$I = e^{\int 3 dT}$

$I = e^{3T}$

$e^{3T}\frac{dy}{dT} + e^{3T}y = e^{3T}T + e^{5T}$

$f \; g' \quad f' g \quad (f.g)' = e^{3T}.y \frac{dy}{dT} = e^{3T}.T + e^{5T}$

$\int e^{3T} y \, dy = \int e^{T}.T + e^{5T} dT$

$\frac{e^{3T}y^2}{2} = \frac{1}{9}e^{3T}.3T - e^{3T} + \frac{e^{5T}}{5} = y^2 = \frac{2}{e^{3T}9}(e^{3T}.3T - e^{3T} + \frac{e^{5T}}{5})$

$y^2 = \frac{6Te^{3T}}{9e^{3T}} - \frac{2e^{3T}}{9e^{3T}} + \frac{2e^{5T}}{9e^{3T}} = \frac{2T}{3} - \frac{2}{9} + \frac{2}{9}e^{2T} + Ce^{-3T}$

$y_1 = \frac{T}{3} - \frac{1}{9}$

3.c) $y' + y = Te^{-T} + 1$

$y' + P_{(T)}y = g_{(T)}$

$I = e^{\int P(T)}$

$I = e^{\int 1 dT} = e^{T}$

$\underset{f\,g'}{e^{T}\frac{dy}{dT}} + \underset{f'g}{e^{T}y} = e^{T}T + e^{T}$

$\int d(e^{T}y) = \int e^{T}T + e^{T} dT$

$e^{T}y = e^{T}.T - e^{T} + e^{T} + C \quad \to y = T + \frac{C}{e^{T}}$

$7.c)\ y' + 2Ty + 2Te^{-T^2} = 0$

$y' + 2Ty = -2Te^{-T^2}$ $\qquad y' + P_{(T)}y = g_{(T)}$

$I = e^{\int P(T)\,dT}$

$e^{T^2}y' + e^{T^2}y2T = -2e^{T}$ $\qquad I = e^{\int 2T\,dT} = e^{2\frac{T^2}{2}} = e^{T^2}$

$e^{T^2}y' + e^{T^2}2Ty = -2T$

$d(e^{T^2} \cdot 2Ty) = \int -2T\,dT + C$

$e^{T^2} \cdot y = \frac{-2T^2}{2} + C = T^2 + C$

$y_{(T)} = \frac{T^2}{e^{T^2}} + \frac{C}{e^{T^2}}$ //

$11.c)\ y' + y = 5\,sen\,2T$ $\qquad e^{T} = I$

$e^{T}y' + ye^{T} = 5\,sen(2T)e^{T}$

$(e^{T} \cdot y) = \int sen(2T)e^{T} \cdot dT = sen(2T)e^{T} - 2\cos(2T)e^{T} + C$

$u = 2T$

$du = 2$

$y = sen(2T) - 2\cos(2T) + \frac{C}{e^{T}}$ //

$13)\ y' - y = 2Te^{2T}$ $\qquad e^{-T}$

$e^{T}y' \ e^{T}y = 2Te^{2T}e^{-T}$ $\qquad y_{(0)} = 1$

$\int d(e^{T}y) = \int 2Te^{T} = e^{-T}y = T^2$ $\qquad y = T^2 + \frac{C}{e^{T}}$

$C = e^{T}$ $\qquad 1 =$

$17)\ y' - 2y = Te^{2T}$ $\qquad y_{(0)} = 2$ $\qquad e^{-2T}$

$e^{-2T}y' - 2e^{-2T}y = e^{2T}e^{-2T}$

$(e^{-2T} \cdot y) = 1 = e^{-2T}y = T + C =$

$y = \frac{T}{e^{-2T}} + \frac{C}{e^{-2T}} = 2e^{-2T} = C // = 2 //$

c

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19) $t^3y' + 4T^2y = e^{-T}$

$y' + \frac{4}{T}y = e^{T}T^{-3}$ $\quad$ $P_{(t)} = \frac{4}{T}$ $\quad$ $\int \frac{1}{T}\,dT = \ln|T|$

$T^4y' + \frac{T^4 4}{T}y = e^{T}.T$ $\quad$ $e^{4\ln|t|} = T^4$

$T^4.y = \int e^{T}.T\,dT = -(T+1)e^{T} + c$

$y = -\frac{(T+1)e^{-T}}{T^4} + \frac{c}{T^4}$ $\quad$ $y_{(-1)} = 0$

$0 = -\frac{((-1)+1)e^{1}}{T^4} + \frac{c}{-1^4} = 0$ $\quad$ $c = 0$ //

20) $ty' + (T+1)y = T$ $\quad$ $y(\ln(2)) = 1$

$Ty' + yT + y = T$

$y' + y + \frac{y}{T} = 1 \to y' + y\left(1 + \frac{1}{T}\right) = 1$

$I = e^{\int 1 + \frac{1}{T}dT} = e^{T + \ln|T|} = e^{T}.e^{\ln|T|} = e^{T}.T$

$e^{T}Ty' + \left(1 + \frac{1}{T}\right)e^{T}y = e^{T}.T$

$e^{T}.T.y = e^{T}.T - e^{T}$ $\quad$ $y = \frac{e^{T}.T}{e^{T}.T} - \frac{1}{T} + \frac{c}{e^{T}.T}$

$\ln|2| = 1 + 1 + \frac{c}{e}$ $\quad$ $e(\ln|2| - 2) = c$ //

30) $y' - y = 1 + 3\,\text{sen}\,T$ $\quad$ $y(0) = y_0$

$e^{-T}y' - e^{-T}y = e^{-T} + 3e^{-T}\text{sen}\,T$

$e^{-T}.y = \int e^{-T}dT + 3\int \frac{\text{sen}\,T}{e^{T}}\,dT$

$y = \frac{-1e^{-T}}{e^{-T}} + \frac{3\left(-\frac{1}{2}e^{-T}\cos(t) - \frac{1}{2}e^{-T}\text{sen}(t)\right)}{e^{-T}} + \frac{C}{e^{-T}}$

$y = -1 - \frac{3}{2}\left(\cos(t) + \text{sen}(T)\right) + Ce^{t} = -1 - 3 + C$

$y_0 = -4 + C$ //

$\frac{3}{2}+1 \quad \frac{5}{2}$

31) $y' - \frac{3}{2}y = 3T + 2e^{T}$ $\quad I = e^{\int \frac{3}{2}} = e^{\frac{3}{2}T}$

$e^{\frac{3}{2}T}y' - e^{\frac{3}{2}T}y = 3Te^{\frac{3}{2}T} + 2e^{\frac{5}{2}T}$

$e^{\frac{3}{2}T} \cdot y = 3\int Te^{\frac{3}{2}T}\,dT + 2\int e^{\frac{5}{2}T}\,dT$

$2e^{\frac{3}{2}T}T - \frac{\cancel{4}}{9}e^{2\frac{T}{2}} + C = e^{\frac{3}{2}T} \cdot y$

$2T - \frac{2}{9e^{T}} = y$

33) $y' + ay = be^{-\lambda T}$ $\quad I = e^{\int (T+1)dT} = e^{\int a\,dT} = e^{Ta}$

$e^{aT}y' + e^{aT}ay = be^{-\lambda T} \cdot e^{aT}$

$(e^{aT} \cdot y)' = be^{T(a-\lambda)}$

$\lambda = a$

$(e^{aT} \cdot y)' = b \rightarrow e^{aT} \cdot y = bT + C = y = \frac{bT}{e^{aT}} + \frac{C}{e^{aT}}$

$T \rightarrow \infty \quad y = 0$

$\lambda \neq a \rightarrow e^{aT} \cdot y = b\int e^{(a-\lambda)T}\,dT = \frac{b}{a-\lambda}e^{(a-\lambda)T} + C$

$y = \frac{b}{(a-\lambda)} \cdot e^{-\lambda T} + c \cdot e^{-aT} \quad t \rightarrow \infty \quad y = 0$

23) $uM + uNy' = 0$ $\quad I = (uM)_y = (uN)_x$

$u_y M - u_x N = uN_x - uM_y$

$N_x - M_y = k(xM - yN)$, $\quad k$

$z = xy$

Eb é exato se: $u_y M - u_x N = uk(xM - yN) =$

$k(u_x M - u_y N)$

$u_y = (u_x)k$ e $u_x = (u_y)k$

$u = u(xy)$

$u_x = u'y \quad u_y = u'x \quad u' = \frac{du}{dz}$

$k = k(xy)$

$u = u(xy)$

32) $u = [xy(2x+y)^{-1}]$

$$\left[\frac{(3x+y)}{(2x^2+xy)}\right]dx + \left[\frac{1}{y} + \frac{1}{2x+y}\right]dy = 0$$

$M_y = N_x$

$\psi(x,y) = 2\ln|x| + \ln|2x+y| + h(y)$

$\psi(x,y)_y = (2x+y)^{-1} + h'(y)$

$\psi_y = N \quad h'(y) = \frac{1}{y} \quad h(y) = \ln(y)$

$$\psi(x,y) = 2\ln|x| + \ln|2x+y| + \ln|y|$$

$S = 2x^3y + x^2y^2 = C$

25) $(3x^2y + 2xy + y^3)dx + (x^2+y^2)dy = 0$

$I = \frac{1}{N}\, M_y - N_x = \frac{1}{x^2+y^2}\left(3x^2 + 2x + 3y^2 - 2x\right)$

$I = 3 \qquad e^{\int 3dx} = e^{3x}$

$e^{3x}(3x^2y + 2xy + y^3)dx + e^{3x}(x^2+y^2)dy$

$\psi(x,y) = \int e^{3x}(3x^2y + 2xy + y^3)dx$

$e^{3x}\left(\frac{3x^2y^2}{2} + xy^2 + \frac{y^4}{4}\right) + h(x)$

$3e^{3x}(3xy^2 + y^2 + 4y^3) + h'x = N$

$h(x) = \int -3e^{3x}(3xy^2) + y^3e^{3x} + x^2\, dx$

26) $y' = e^{2x} + y - 1$

$y' - y - e^{2x} + 1 = 0 \Rightarrow \frac{dy}{dx} - y - e^{2x} + 1 = 0$

$M = -y - e^{2x} + 1 \qquad = M_y = -1$

$N = 1 \qquad = N_x = 0$

$\frac{1}{1} \cdot -1 = e^{-x}$

$e^{-x}\frac{dy}{x} - e^{-x} - e^{x} + e^{-x}$

$e^{-x}\frac{dy}{dx} - e^{x}$

$\psi(x,y) - \int e^{x}dx = e^{x} + h(y)$

$h'(y) = e^{x} \qquad h(y) = e^{x}$

$\psi(x,y) = -e^{-x} + e^{-x} = 0$

27) $dx + \left(\frac{x}{y} - \operatorname{sen} y\right) dy$

$M = 1 \qquad M_y = 0$

$N = \frac{x}{y} - \operatorname{sen} y \quad N_x = \frac{1}{y}$

$I = e^{\wedge \int \frac{1}{\frac{x}{y} - \operatorname{sen} y} \frac{x}{y} - \operatorname{sen} y \, dx} = e^{x}$

$e^{x} = M$

$\psi(x,y) = e^{x} + h(y)$

$\psi(x,y)_y = h'(y) = \frac{x}{y} - \operatorname{sen} y \qquad h(y) = \frac{x^2}{2y} + \cos y + c$

$\psi(x,y) = e^{x} + \frac{x^2}{2y} + \cos y + c$ //

28) $y\,dx + (2xy - e^{-2y})\,dy$

$M = y \qquad M_y = 1$

$N = 2xy - e^{-2y} \quad N_x = 2y$

$I = e^{\wedge \int \frac{1}{2xy - e^{-2y}} (1 - 2y) \, dx}$

θu

$I = \frac{1 \; 2y - 1}{y} = e$

$\int \frac{2y - 1}{y} dy$

$u = 2$

30) $N_x - M_y = \frac{8x^3}{y^3} + \frac{6}{y^2}$

de modo que

$\frac{(N_x - M_y)}{M} = \frac{2}{y}$ $\quad \mu = \mu(y)$

$\mu'(y) = \frac{2}{y}\mu$

$I = y^2$

$(4x^3 + 3y)dx + (3x + 4y^3)dy = 0$

$\psi(x,y) = x^4 + 3xy + y^4 = C$