Luiz Eduardo Caldas Kramer RA: 2599665

EDO - Capítulo 5

5.1) 1) $y' = \frac{x^2}{y}$ $\quad \frac{dy}{dx} = \frac{x^2}{y}$ $\quad y\,dy = x^2\,dx$

$y^2 = \sqrt{\frac{x^3}{3}}$

3) $y' + y^2 \operatorname{sen} x = 0$ $\quad \to \int y^{-2}\,dy = \int -\operatorname{sen} x\,dx$

$-y^{-1} = \cos x + C$ $\quad = (C - \cos x)\,y = 1$ $\quad = y(x) = \frac{1}{C - \cos x}$

5) $y' = (\cos^2 x)(\cos^2 2y)$ $\quad = \int \cos^{-2} 2y \cdot dy = \int \cos^2 x\,dx$

$\tan 2y = \operatorname{sen} x \cos x + x + C$

7) $\frac{dy}{dx} = \frac{x - e^{-x}}{y + e^{y}}$ $\quad = \int (y + e^{y})\,dy = \int (x - e^{-x})\,dx$

$\frac{y^2}{2} + e^{y} = \frac{x^2}{2} + e^{x} + C$

9) a) $y' = (1 - 2x)\,y^2$ $\quad = \int y^{-2}\,dy = \int 1 - 2x\,dx$

$-y^{-1} = x - x^2 + C$

$y(0) = -\frac{1}{6}$ $\quad 6 = 0 - 0^2 + C$ $\quad C = 6$

$y = (x - x^2 + 6)^{-1}$

13) $\frac{2x}{(y + x^2 y)} = y'$ $\quad y' \cdot y\,(1 + x^2) = 2x$

$y' \cdot y = \frac{2x}{1 + x^2}$ $\quad = \int y\,dy = \int \frac{2x}{1 + x^2}\,dx = \int \frac{du}{u}$

$u = 1 + x^2$ $\quad y = \sqrt{\ln|u|} = \sqrt{\ln|1 + x^2| + C}$

$du = 2x\,dx$ $\quad 4 = \ln|1| + C$ $\quad C = 4$

$y(0) = -2$

17) a) $\frac{3x^2 - e^x}{2y-5} = y' = \int 3x^2 - e^x\,dx = \int (2y-5)\,dy$

$y(0)=1$

$x^3 - e^x + c = y^2 - 5y \Rightarrow 0 - 1 + c = 0 \quad c = 1$

19) a) $\operatorname{sen} 2x\,dx + \cos 3y\,dy = 0$

$\int \operatorname{sen} 2x\,dx + \int \cos 3y\,dy = 0$

$u = 2x \quad u = 3y \quad \int \frac{\operatorname{sen} u\,du}{2} + \int \frac{\cos u\,du}{3}$

$du = 2 \quad du = 3$

$\frac{-\cos 2x + c}{2} + \frac{\operatorname{sen} 3y}{3} = 0 \quad \frac{-\cos 2(\pi)}{-2} + \frac{\operatorname{sen}(3\pi)}{0\ 3} = \frac{1}{3}$

$y = \frac{-\pi}{3} + \frac{\operatorname{sen}^{-1}(3\cos^2 x)}{3}$

29) $\frac{dy}{dx} = \frac{ay+b}{cy+d} \quad \int \frac{dy\,(cy+d)}{ay+b} = \int dx$

$\int \frac{cy+d}{ay+b}\,dy = x + c$

$\int \frac{cy}{ay+b}\,dy + \int \frac{d}{ay+b}\,dy = c\int \frac{y}{ay+b}\,dy + d\int \frac{1}{ay+b}\,dy$

$u = ya+b \quad y = \frac{u-b}{a}$

$du = 1$

$c\int \frac{u-b}{a^2 u}\,du = \frac{c}{a^2}\int \frac{u-b}{u}\,du = \left(\int u\,du - \int \frac{b}{u}\,du\right)\frac{c}{a^2}$

$\frac{c}{a^2}(u - b\ln|u|) \to \frac{c}{a^2}(ay+b - b\ln|ay+b|)$

$\frac{c}{a^2}(ay+b - \ln|ay+b|) + \frac{d}{a}(\ln|ay+b|) = x + c$

30) $\dfrac{dy}{dx} = \dfrac{y-4x}{x-y}$

a) $\dfrac{dy}{dx} = \dfrac{\left(\frac{y}{x} - 4\right)x}{\left(1 - \frac{y}{x}\right)x} = \dfrac{\frac{y}{x} - 4}{1 - \frac{y}{x}}$

b) $y = xv \therefore \dfrac{dy}{dx} = \dfrac{d(xv)}{dx} = v + x\dfrac{dv}{dx}$

c) $v + x\dfrac{dv}{dx} = \dfrac{dy}{dx} = \dfrac{\frac{y}{x} - 4}{1 - \frac{y}{x}} = \dfrac{v-4}{1-v}$

$x\dfrac{dv}{dx} = \dfrac{v-4}{1-v} - v = \dfrac{v-4}{1-v} - v\left(\dfrac{1-v}{1-v}\right) = \left(\dfrac{v-4}{1-v}\right) - \left(\dfrac{v-v^2}{1-v}\right)$

$= \left(\dfrac{v-4-v+v^2}{1-v}\right) = \left(\dfrac{v^2-4}{1-v}\right) \therefore \left(\dfrac{x\,dv}{dx}\right) = \dfrac{v^2-4}{1-v}$

d) $\dfrac{1-v}{(v-2)(v+2)}\dfrac{dv}{dx} = \dfrac{1}{x} = \dfrac{Av+2A+Bv-2B}{(v-2)(v+2)}$

$\displaystyle\int \left(\frac{1-v}{(v-2)(v+2)}\right)\frac{dv}{dx}\,dx = \int \frac{1}{x}$

$\displaystyle -\frac{1}{4}\int \frac{1}{v-2}\,dv - \frac{3}{4}\int \frac{1}{v+v}\,dv = \ln(x) + c$

$-\dfrac{1}{4}\ln|v-2| - \dfrac{3}{4}\ln|v+2| = \ln|x| + c$

$\ln|(v-2)(v+2)^3x^4| = -4c$

$(v-2)(v+2)^3 = e^{-4c} = K$

e) $\left(\dfrac{y}{x} - 2\right)\left(\dfrac{y}{x} + 2\right)^3 x^4 = K$

$(y-2x)\dfrac{1}{x}(y+2x)^3\dfrac{1}{x^3}x^4 = K$

$(y-2x)(y+2x)^3 = K$

31) a) $\frac{x^2 + xy + y^2}{x^2} = \frac{\cancel{x^2}\left(1 + \frac{y}{x} + \frac{y^2}{x}\right)}{\cancel{x^2}}$

$\frac{y^2}{x} + \frac{y}{x} = -1$

b) $y = xv \quad \frac{dxv}{dx} = \frac{x^2v^2 + v - 1}{x} = xv^2 + v - 1$

35) $\frac{dy}{dx} = \frac{x + 3y}{x - y} = \frac{x\left(1 - \frac{3y}{x}\right)}{x\left(1 - \frac{y}{x}\right)} = \frac{1 - \frac{3y}{x}}{1 - \frac{y}{x}}$

b) $\frac{dxv}{dx} = \frac{1 - 3v}{1 - v}$

37) $\frac{x^2 - 3y^2}{2xy} + \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{2}\left(\frac{y}{x}\right)^{-1} - \frac{3y}{2x}$ b) $\frac{2v\,dv}{1 - 5v^2} = \frac{1}{x}dx$

$-\frac{1}{5}\ln(1 - 5v^2) = \ln|x| + C$