a Mathematical model formulation

The notation used for this mathematical formulation is provided in Table 1.

Table 1: Notation

Sets and indices		
T	Set for the months	$t \in T$
P	Set for the products	$i \in P$
S	Set for the suppliers	$j \in S$
Parameters		
$D_{i,t}$	Demand for product i in month t	[kg]
c_j	Cost per kg of material from supplier j	[euro/kg]
h_i	Holding cost per kg for product i	[euro/kg]
P_{max}	Maximum production capacity per month	[kg]
$X_{j,\text{max}}$	Maximum supply from supplier j per month	[kg]
α_i	Chromium content in product i	[%]
α_j	Chromium content in material from supplier j	[%]
β_i	Nickel content in product i	[%]
β_j	Nickel content in material from supplier j	[%]
Variables		
$P_{i,t}$	Production quantity of product i in month t	[kg]
$S_{i,t}$	Inventory quantity of product i in month t	[kg]
$X_{i,j,t}$	Quantity of scrap material purchased from supplier j for product i in month t	[kg]

The mathematical formulation then follows as:

$$\min \sum_{t \in T} \left(\sum_{j \in S} c_j \cdot \sum_{i \in P} X_{i,j,t} \right) + \sum_{t \in T} \left(\sum_{i \in P} h_i \cdot S_{i,t} \right) \tag{1}$$

Subject to:

$$P_{i,t} + S_{i,t-1} = D_{i,t} + S_{i,t} \qquad \forall i \in P, \forall t \in T \tag{2}$$

$$\sum_{i \in P} P_{i,t} \leq P_{\text{max}} \qquad \forall t \in T \tag{3}$$

$$\sum_{i \in P} X_{i,j,t} \leq X_{j,\text{max}} \qquad \forall j \in S, \forall t \in T \tag{4}$$

$$P_{i,t} = \sum_{j \in S} X_{i,j,t} \qquad \forall i \in P, \forall t \in T \tag{5}$$

$$\alpha_i \cdot P_{i,t} = \sum_{j \in S} \alpha_j \cdot X_{i,j,t} \qquad \forall i \in P, \forall t \in T \tag{6}$$

$$\beta_i \cdot P_{i,t} = \sum_{j \in S} \beta_j \cdot X_{i,j,t} \qquad \forall i \in P, \forall t \in T \tag{7}$$

$$P_{i,t}, S_{i,t}, X_{i,j,t} \geq 0 \qquad \forall i \in P, \forall j \in S, \forall t \in T \tag{8}$$

The objective function (1) minimizes the total cost, which includes the purchasing cost of materials from suppliers and the holding cost of inventory across all months. Constraint (2) ensures that the production and

inventory balance is maintained, ensuring that demand is met for each product in every month. Constraint (3) limits the total production capacity in each month. Constraints (4) impose limits on the amount of material that can be sourced from each supplier. Constraint (5) ensures that the mass of production matches the mass of procurement from suppliers. Constraints (6) and (7) ensure that the chromium and nickel content in each produced product matches the content of each purchased material. Finally, constraint (8) guarantees non-negativity for the production, inventory, and purchased material variables.