e Electrolysis Problem

Mathematical model

The company wants the amount of copper in its products to be below a certain value, called the CopperLimit. In order to remove copper from a mix, electrolysis can be used. To take the use of electrolysis into consideration, the mathematical model needs to be extended. The notation used for the new mathematical formulation is provided in the table below.

Table 8: Notation

Sets and indices		
T	Set for the months	$t \in T$
P	Set for the products	$i \in P$
S	Set for the suppliers	$j \in S$
Parameters		
$D_{i,t}$	Demand for product i in month t	[kg]
c_j	Cost per kg of material from supplier j	[euro/kg]
h_i	Holding cost per kg for product i	[euro/kg]
P_{max}	Maximum production capacity per month	[kg]
$X_{j,\text{max}}$	Maximum supply from supplier j per month	[kg]
α_i	Chromium content in product i	[%]
α_j	Chromium content in material from supplier j	[%]
β_i	Nickel content in product i	[%]
β_j	Nickel content in material from supplier j	[%]
γ_j	Copper content in material from supplier j	[%]
$CuLimit_t$	Copper content limit in month t	[%]
E_c	Fixed cost of copper electrolysis	[euro]
e_c	cost of copper electrolysis per kg	[euro/kg]
M	A large number	[-]
Variables		
$P_{i,t}$	Production quantity of product i in month t	[kg]
$S_{i,t}$	Inventory quantity of product i in month t	[kg]
$X_{i,j,t}$	Quantity of scrap material purchased from supplier j for product i in month t	[kg]
B_t	Binary variable of whether electrolysis is used in month t	[binary]
$m_{i,t}$	Electrolysis quantity of copper of product i in month t	[kg]

The mathematical formulation then follows as:

$$\min \sum_{t \in T} \left(\sum_{j \in S} c_j \cdot \sum_{i \in P} X_{i,j,t} \right) + \sum_{t \in T} \left(\sum_{i \in P} h_i \cdot S_{i,t} \right) + \sum_{t \in T} E_c \cdot B_t + \sum_{t \in T} \left(\sum_{i \in P} e_c \cdot m_{i,t} \right) \tag{9}$$

Subject to:

$$P_{i,t} + S_{i,t-1} - m_{i,t} = D_{i,t} + S_{i,t} \qquad \forall i \in P, \forall t \in T \tag{10}$$

$$\sum_{i \in P} P_{i,t} \leq P_{\text{max}} \qquad \forall t \in T \tag{11}$$

$$\sum_{i \in P} X_{i,j,t} \leq X_{j,\text{max}} \qquad \forall j \in S, \forall t \in T \tag{12}$$

$$P_{i,t} = \sum_{j \in S} X_{i,j,t} \qquad \forall i \in P, \forall t \in T \tag{13}$$

$$\alpha_i \cdot (P_{i,t} - m_{i,t}) = \sum_{j \in S} \alpha_j \cdot X_{i,j,t} \qquad \forall i \in P, \forall t \in T \tag{14}$$

$$\beta_i \cdot (P_{i,t} - m_{i,t}) = \sum_{j \in S} \beta_j \cdot X_{i,j,t} \qquad \forall i \in P, \forall t \in T \tag{15}$$

$$\sum_{j \in S} \gamma_j \cdot X_{i,j,t} - m_{i,t} \leq CuLimit_t \cdot (P_{i,t} - m_{i,t}) \qquad \forall i \in P, \forall t \in T \tag{16}$$

$$m_{i,t} \leq B_t \cdot M \qquad \forall i \in P, \forall t \in T \tag{17}$$

$$P_{i,t}, S_{i,t}, X_{i,j,t}, m_{i,t} \geq 0 \qquad \forall i \in P, \forall j \in S, \forall t \in T \tag{18}$$

The objective function (9) minimizes the total cost, which includes the procurement cost, holding cost, fixed electrolysis cost, and electrolysis cost. Constraint (10) ensures that the production and inventory balance is maintained, and electrolysis quantity is taken into consideration. Constraint (11) limits the total production capacity in each month. Constraints (12) impose limits on the amount of material that can be sourced from each supplier. Constraint (13) ensures that the mass of production matches the mass of procurement from suppliers. Constraints (14) and (15) ensure that the chromium and nickel content in each produced product matches the content of each purchased material. Constraint (16) ensures that the copper content purchased per product per month is within the copper limit of the product. Constraint (17) applies big M number to guarantee the relationship between electrolysis quantity and the binary variable. Constraint (18) guarantees non-negativity for all the variables.

Model changes

The mathematical model is transferred to an MILP problem, after the binary variable is introduced. Specific changes are stated below.

New variables

- B_t: Binary variable B_t is introduced to determine whether electrolysis is used in each month.
- $m_{i,t}$: $m_{i,t}$ is introduced to determine the quantity of copper to electrolysis in each month.

New parameters

- γ_j: γ_j describes the copper content in material from supplier j.
- $CuLimit_t$: $CuLimit_t$ describes the copper limit in month t.
- E_c: E_c represents the fixed cost of using electrolysis in a month, which is equal to 100 euro.
- e_c: e_c represents the cost of using electrolysis per kg, which is equal to 5 euro/kg.
- M: M is introduced in as a very large number.

New objective function

In the new objective function, the two terms shown below are included.

- : $\sum_{t \in T} E_c \cdot B_t$: This term represents the fixed cost of using electrolysis.
- : $\sum_{t \in T} \left(\sum_{i \in P} e_c \cdot m_{i,t} \right)$: This term represents the cost of copper electrolysis based on quantity.

New constraints

Constraint (10), (14) and (15) are adjusted to take copper electrolysis quantity into account. New constraints are presented below:

$$\sum_{j\in S}\gamma_j \cdot X_{i,j,t} - m_{i,t} \leq CuLimit_t \cdot (P_{i,t} - m_{i,t}) \qquad \forall i \in P, \forall t \in T$$

This constraint ensures that copper content in material is limited by the copper limit when electrolysis is done.

$$m_{i,t} \leq B_t \cdot M \qquad \forall i \in P, \forall t \in T$$

This constraint ensures that copper removal is dependent on whether to conduct electrolysis.

Result

In the solution for (b), the minimized cost is 9646.78 euro. Based on this, the lowest CopperLimit use is determined without exceeding the minimized cost. In order to find the lowest CopperLimit, binary search method is implemented. A loop is created to reduce the possible range of CopperLimit and maintain original cost in the meantime.
The final result of the lowest CopperLimit is 2.9651%, which maintains the total cost at 9646.78 euro.

Experiment

In this part, experiments with other values of the CopperLimit are conducted to see its impact on costs. As the lowest CopperLimit is found to be 2.9651%, the range of CopperLimit is determined to be [0, 3%], with the step of 0.1%. So the total number of experiments is 30, and a figure is presented below to show how different costs are affected by the value of CopperLimit.

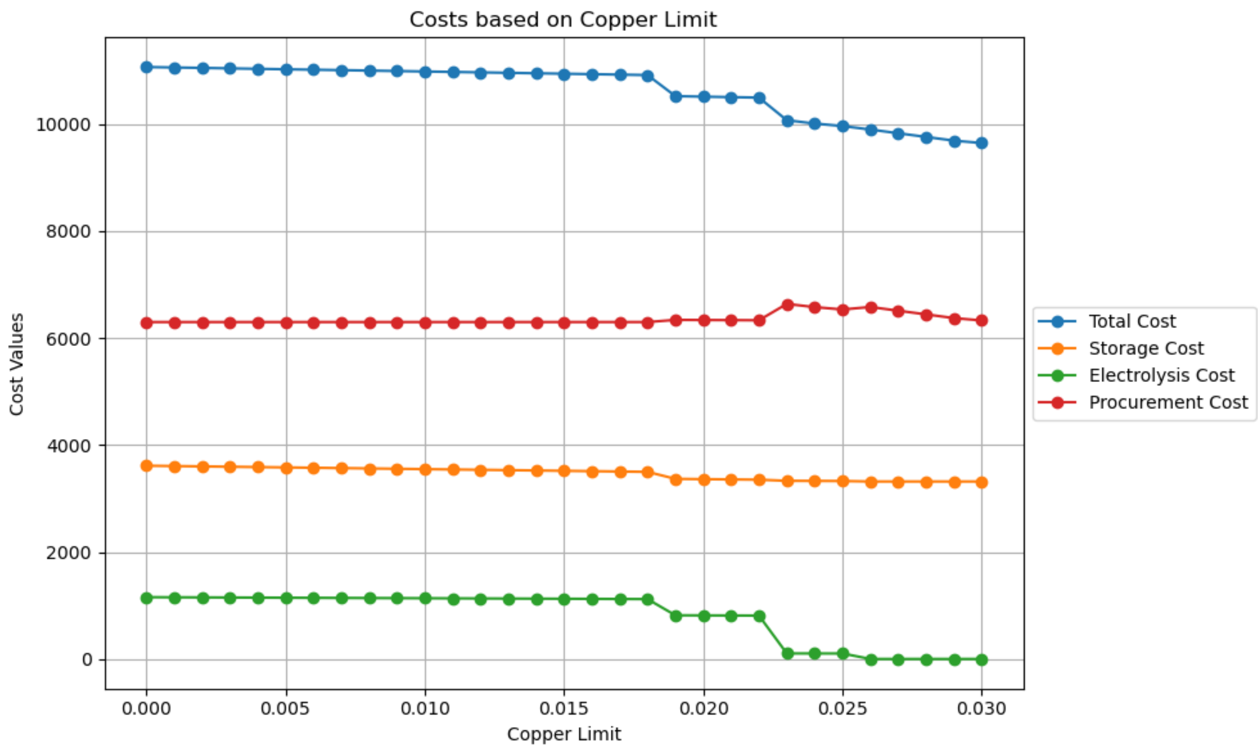

Figure 6: Costs based on Copper Limits

Based on figure 6, it is clearly shown how different costs change according to CopperLimit.

Electrolysis cost

During the decrease of CopperLimit, electrolysis is not immediately used. When CopperLimit falls to around 2.5%, eletrolysis is activated. The electrolysis cost remains a steady, gradual increase in three separate stages, within the CopperLimit ranges of [0, 1.8%], [1.9%, 2.2%], and [2.3%, 2.5%], with sharp increases between these stages. Further analysis of changes in other costs can help explain this trend.

Storage cost

Storage cost remains relatively stable throughout the whole process, suggesting inventory levels are not significantly affected by the use of electrolysis. However, there is a slight change when Copper Limit reaches 1.8%, indicating the increase in electrolysis quantity still has a small impact on inventory management.

Procurement cost

The procurement cost line saw an increase when CopperLimit starts to decrease. This increase shows the initial procurement method taken to deal with CopperLimit: Just buy more materials, especially those with low copper content. But as procurement cost rises, electrolysis is eventually activated, which reduces the need to buy more materials. Thus, the procurement cost starts to drop, and finally drop to almost the initial level.

Total cost

As Copper Limit drops from 3% to 0, total cost increases. The increase can be divided into three stages. Initially, the rise in procurement cost is the main drive, as electrolysis has not yet been used. In the latter two stages, the increase in total cost is driven by rising electrolysis costs.

Conclusion

In summary, as CopperLimit begins to decrease, the system initially responds by purchasing more materials from suppliers, as electrolysis costs can be too high at this stage. As procurement cost keeps rising, electrolysis is eventually activated, reducing the procurement amount. Inventory levels also experience a slight increase in response to the increasing use of electrolysis. Throughout the process, total costs increase as CopperLimit decreases.