Assignment Q1

Steel Production Problem

Quantitative Methods for Logistics

ME44206

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Contents

a Mathematical model formulation

The notation used for this mathematical formulation is provided in Table 1.

Table 1: Notation

Sets and indices		
T	Set for the months	$t \in T$
P	Set for the products	$i \in P$
S	Set for the suppliers	$j \in S$
Parameters		
$D_{i,t}$	Demand for product i in month t	[kg]
c_j	Cost per kg of material from supplier j	[euro/kg]
h_i	Holding cost per kg for product i	[euro/kg]
P_{max}	Maximum production capacity per month	[kg]
$X_{j,\text{max}}$	Maximum supply from supplier j per month	[kg]
α_i	Chromium content in product i	[%]
α_j	Chromium content in material from supplier j	[%]
β_i	Nickel content in product i	[%]
β_j	Nickel content in material from supplier j	[%]
Variables		
$P_{i,t}$	Production quantity of product i in month t	[kg]
$S_{i,t}$	Inventory quantity of product i in month t	[kg]
$X_{i,j,t}$	Quantity of scrap material purchased from supplier j for product i in month t	[kg]

The mathematical formulation then follows as:

$$\min \sum_{t\in T} \left(\sum_{j\in S} c_j \cdot \sum_{i\in P} X_{i,j,t} \right) + \sum_{t\in T} \left(\sum_{i\in P} h_i \cdot S_{i,t} \right) \tag{1}$$

Subject to:

$$P_{i,t} + S_{i,t-1} = D_{i,t} + S_{i,t} \qquad \forall i \in P, \forall t \in T \tag{2}$$

$$\sum_{i\in P} P_{i,t} \le P_{\text{max}} \qquad \forall t \in T \tag{3}$$

$$\sum_{i\in P} X_{i,j,t} \le X_{j,\text{max}} \qquad \forall j \in S, \forall t \in T \tag{4}$$

$$P_{i,t} = \sum_{j\in S} X_{i,j,t} \qquad \forall i \in P, \forall t \in T \tag{5}$$

$$\alpha_i \cdot P_{i,t} = \sum_{j\in S} \alpha_j \cdot X_{i,j,t} \qquad \forall i \in P, \forall t \in T \tag{6}$$

$$\beta_i \cdot P_{i,t} = \sum_{j\in S} \beta_j \cdot X_{i,j,t} \qquad \forall i \in P, \forall t \in T \tag{7}$$

$$P_{i,t}, S_{i,t}, X_{i,j,t} \ge 0 \qquad \forall i \in P, \forall j \in S, \forall t \in T \tag{8}$$

The objective function (1) minimizes the total cost, which includes the purchasing cost of materials from suppliers and the holding cost of inventory across all months. Constraint (2) ensures that the production and inventory balance is maintained, ensuring that demand is met for each product in every month. Constraint (3) limits the total production capacity in each month. Constraints (4) impose limits on the amount of material that can be sourced from each supplier. Constraint (5) ensures that the mass of production matches the mass of procurement from suppliers. Constraints (6) and (7) ensure that the chromium and nickel content in each produced product matches the content of each purchased material. Finally, constraint (8) guarantees non-negativity for the production, inventory, and purchased material variables.

b Optimal Solution

In this part, the mathematical model is implemented in python and solved with Gurobi. Coding files are uploaded along with this report.
The optimal solution for the production scheduling problem is as follows:

Minimized Cost: 9646.78 euro
Production Table:

Table 2: Production Table (unit: kg)

Month	18/10 Production	18/8 Production	18/0 Production
1	25.00	10.00	8.09
2	25.00	10.00	65.00
3	0.00	10.00	86.96
4	0.00	10.00	86.96
5	0.00	14.37	85.62
6	50.00	5.63	44.38
7	12.00	10.00	78.00
8	0.00	10.00	80.00
9	10.00	10.00	62.00
10	10.00	10.00	80.00
11	45.00	19.00	36.00
12	99.00	1.00	0.00

Storage Table:

Table 3: Storage Table (unit: kg)

Month	18/10 Storage	18/8 Storage	18/0 Storage
1	0.00	0.00	3.09
2	0.00	0.00	48.09
3	0.00	0.00	55.04
4	0.00	0.00	117.00
5	0.00	4.37	152.62
6	0.00	0.00	72.00
7	0.00	0.00	0.00
8	0.00	0.00	0.00
9	0.00	0.00	22.00
10	0.00	0.00	67.00
11	0.00	9.00	100.00
12	0.00	0.00	0.00

Supplier Procurement Table:

Table 4: Supplier Procurement Table for Product 18/10 (unit: kg)

Month	From A	From B	From C	From D	From E
1	9.01	5.81	0.00	10.17	0.00
2	9.01	5.81	0.00	10.17	0.00
3	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	0.00	0.00	0.00
5	0.00	0.00	0.00	0.00	0.00
6	18.02	11.63	0.00	20.35	0.00
7	4.33	2.79	0.00	4.88	0.00
8	0.00	0.00	0.00	0.00	0.00
9	3.60	2.33	0.00	4.07	0.00
10	3.60	2.33	0.00	4.07	0.00
11	16.22	10.47	0.00	18.31	0.00
12	35.69	23.02	0.00	40.29	0.00

Table 5: Supplier Procurement Table for Product 18/8 (unit: kg)

Month	From A	From B	From C	From D	From E
1	4.88	1.86	0.00	3.26	0.00
2	4.88	1.86	0.00	3.26	0.00
3	3.04	2.09	4.87	0.00	0.00
4	3.04	2.09	4.87	0.00	0.00
5	4.37	3.00	7.00	0.00	0.00
6	2.75	1.05	0.00	1.83	0.00
7	4.88	1.86	0.00	3.26	0.00
8	4.88	1.86	0.00	3.26	0.00
9	4.88	1.86	0.00	3.26	0.00
10	4.88	1.86	0.00	3.26	0.00
11	9.28	3.53	0.00	6.19	0.00
12	0.49	0.19	0.00	0.33	0.00

Table 6: Supplier Procurement Table for Product 18/0 (unit: kg)

Month	From A	From B	From C	From D	From E
1	8.09	0.00	0.00	0.00	0.00
2	65.00	0.00	0.00	0.00	0.00
3	86.96	0.00	0.00	0.00	0.00
4	86.96	0.00	0.00	0.00	0.00
5	85.62	0.00	0.00	0.00	0.00
6	44.38	0.00	0.00	0.00	0.00
7	78.00	0.00	0.00	0.00	0.00
8	80.00	0.00	0.00	0.00	0.00
9	62.00	0.00	0.00	0.00	0.00
10	80.00	0.00	0.00	0.00	0.00
11	36.00	0.00	0.00	0.00	0.00
12	0.00	0.00	0.00	0.00	0.00

The optimal solution achieved a minimized cost of 9646.78 euro, along with a comprehensive production plan, inventory plan, and supplier procurement plan for each product. However, further verification and sensitivity analysis of the model are necessary for additional validation.

c Verification

In this part, the implemented mathematical model is tested through three types of verification experiments: objective function parameters, functional parameters, and RHS parameters.

Objective function parameters: Objective function parameters verification tests involve adjusting the holding costs of inventory and procurement costs to observe their impact on the overall minimized cost and storage levels, aiming to find out whether the objective function behaves as expected. In this report, storage cost of 18/0 product is increased to see whether overall cost is increased and storage level of 18/0 product is decreased. Also, procurement cost of supplier B is decreased to find out whether overall cost is decreased and procurement from supplier B is increased.

Functional parameters: Functional parameters verification tests modify the demand and supply configurations, such as limiting suppliers or demands, to verify the model's ability to handle functional constraints and their impact on the optimization results. In this report, tests are conducted to see whether the model can handle the "unmix" problem and produce "pure" metal.

RHS parameters: RHS parameters verification tests modify production capacity or supply limits, to check whether changes in minimized cost and production schedule due to fluctuation in capacity and supply behave as expected. Additionally, the given example verification is conducted in this part.

Table 7: Verification

Parameter	Description	Expected	Result	Pass?
Objective function parameter	Increase the storage cost of 18/0 product from 5 to 20	minimized cost > 9646.78 euro; storage decrease of 18/0 product	minimized cost = 17148.28 euro; storage decrease of 18/0 product in every month	OK
	Decrease the procurement cost of supplier B from 10 to 1	minimized cost < 9646.78 euro; increased procurement from supplier B	minimized cost = 8489.83 euro; increased procurement from supplier B for product 18/10 and 18/8	OK
Functional parameter	Adjust demand to produce only 1kg 25/0 product in the first	No optimal result	No optimal result	OK
	Adjust supply to have only two suppliers supplying pure metal	No optimal result	No optimal result	OK
RHS parameter	Increase the maximum production capacity from 100 to 1000	minimized cost > 9646.78 euro; decreased storage	minimized cost = 8005.89 euro; storage decrease of all products	OK
	Increase the maximum supply from supplier A from 90 to 900	minimized cost < 9646.78 euro	minimized cost = 9515.35 euro	OK
	Verification with a demand of Jan [10,10,10] and Feb-Dec zero	minimized cost = 185.58 euro	minimized cost = 185.58 euro	OK

Table 7 provides a comprehensive overview of the verification process. A total of seven verification tests were conducted to assess whether the mathematical model was implemented correctly. The table presents the test description, expected result, actual result, and the pass/fail outcome. As shown, the model successfully passed all seven tests, demonstrating that the code matches the theoretical model. This successful verification confirms the reliability of the implementation and supports its validity for further analysis.

d Experiment

In this part, experiments with different values of the maximum production capacity and the holding costs are conducted in order to get insights for the trade-off between these in the production and inventory holding decisions for products.

Experiment description

In the experiment, the production capacity of the system and the holding costs for the three product types (18/10, 18/8, and 18/0) are modified.

The initial step is to determine the range of values to be tested, which means a "pre-test" is needed. Various random combinations of production capacity and holding costs are examined. The results show that production capacity should be at least 93 kg/month, while a capacity above 200 kg/month has minimal impact on minimizing costs and the production plan. Based on these, the production capacity is set at [95, 100, 120, 150, 200] kg/month. For holding costs, a cost multiplier set is applied to assess model performance under different conditions. The final values of the cost multiplier set are determined to be [0.1, 0.5, 1.0, 2.0, 5.0], with each value multiplying the current holding cost. This results in a total of 625 tests (5*5*5*5) to be conducted.

In short, the experiment involve the following parameter change:

- **Production capacity [kg/month]**: [95, 100, 120, 150, 200]
- **18/10 holding cost [euro/kg]**: [2, 10, 20, 40, 100]
- **18/8 holding cost [euro/kg]**: [1, 5, 10, 20, 50]
- **18/8 holding cost [euro/kg]**: [0.5, 1, 5, 10, 25]

As the number of tests increases to 625, the output is adjusted accordingly. Instead of providing detailed inventory and procurement plans for all suppliers, the model will primarily output the total cost, total storage cost, and total procurement cost for each experiment. In the subsequent analysis, plots are used to illustrate and discuss the experiment results. Complete experiment data can be found on GitHub via the link in statement if needed.

Experiment analysis

Based on the experimental data, a thorough analysis of the outcomes is conducted to gain insights into the trade-offs involved in production and inventory decisions for the products. The statistical results include key metrics such as total cost, total storage cost, and total procurement cost. In the next phase, data visualization methods including heatmap, scatter plot, and box plot are employed to clearly identify patterns and trends, illustrating how variations in parameters influence both the inventory and procurement plans. This analysis further highlights how these changes impact the overall optimization of production strategies.

Heatmap analysis

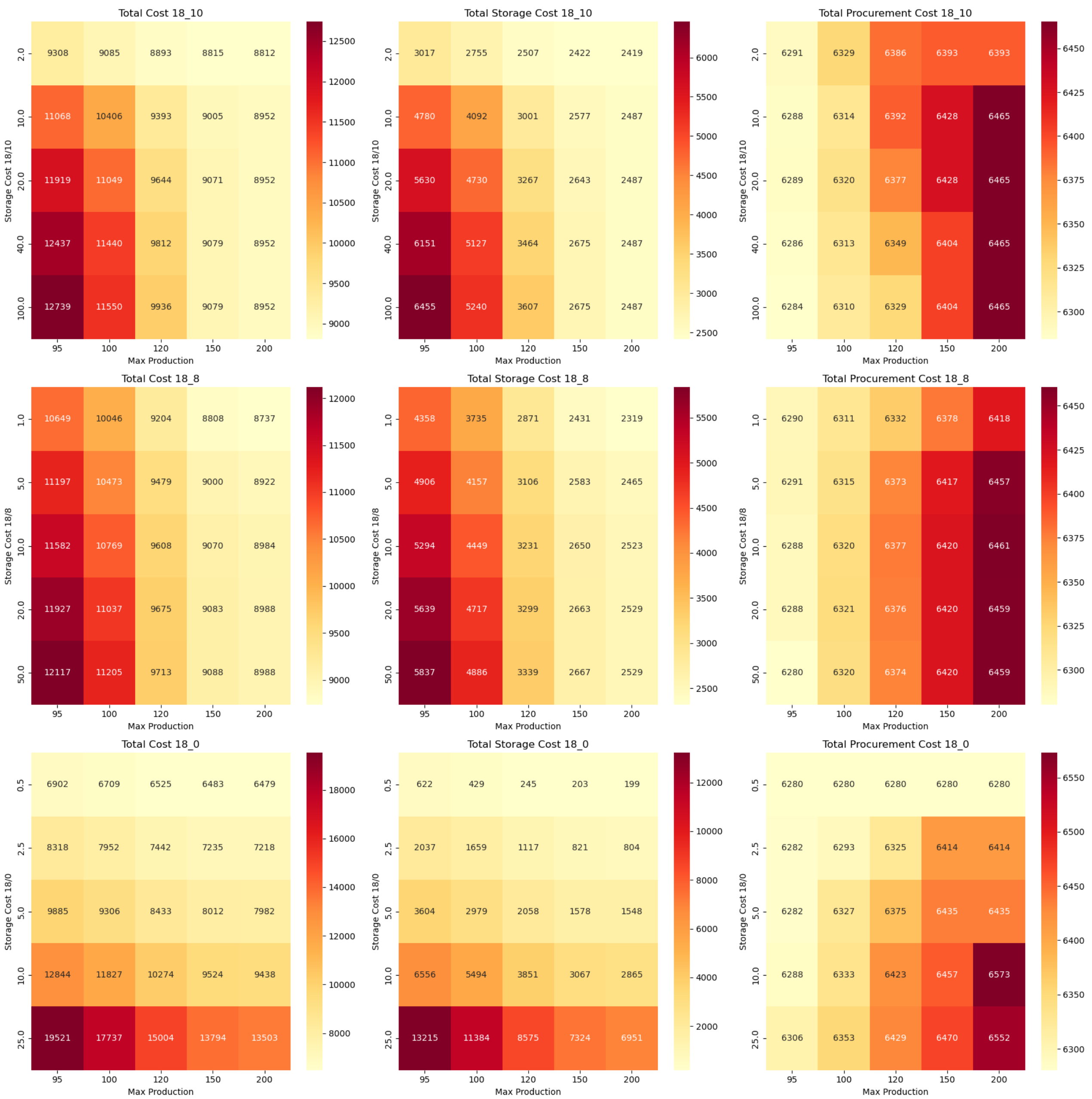

Figure 1: Heatmap of total cost, total storage cost and total procurement cost

Figure 1 shows the influence of various combinations of max production capacity and storage cost of different products on total cost, total storage cost and total procurement cost. A yellow color represents a low cost, and red and darker colors represent higher costs. In the first column, it shows that lower total costs tend to occur at higher production levels combined with lower storage costs. In the second volumn, it is shown that lower production levels and lower storage cost scenarios lead to significantly reduced storage costs. The third column shows the procurement costs, which seem to be relatively stable across different scenarios. In the whole plot, the best results in terms of minimizing costs seem to be achieved when max production is high, and storage costs are low, especially in the 18/0 scenario. To justify this assessment, further analysis is needed.

Scatter plot analysis

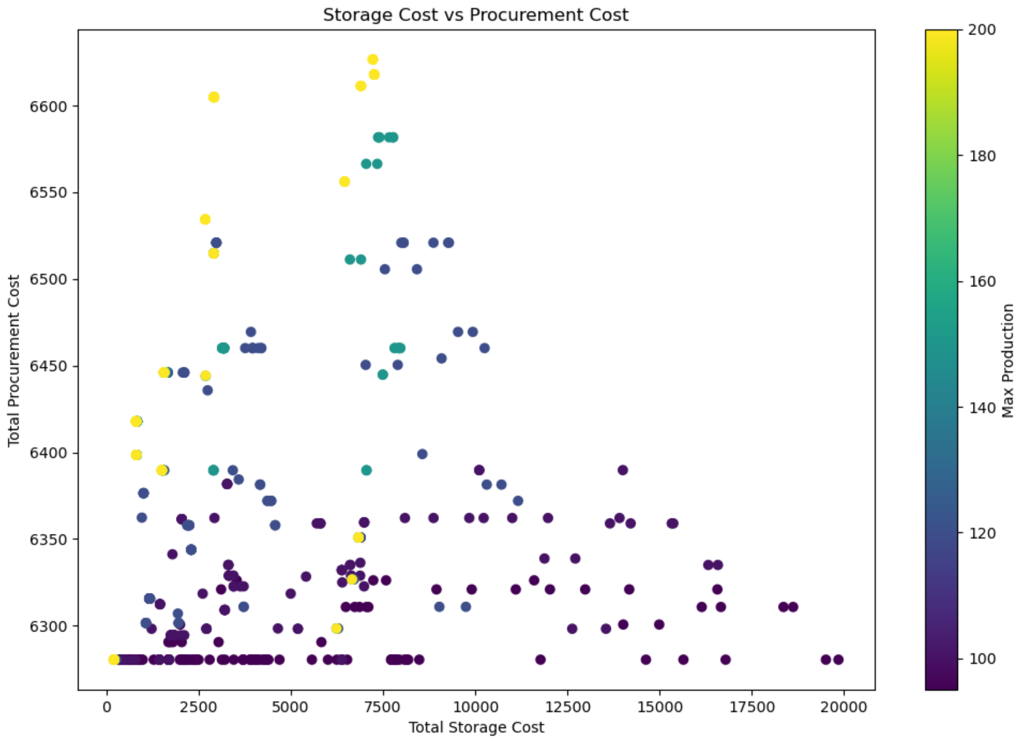

Figure 2: Scatter plot of storage cost vs procurement cost

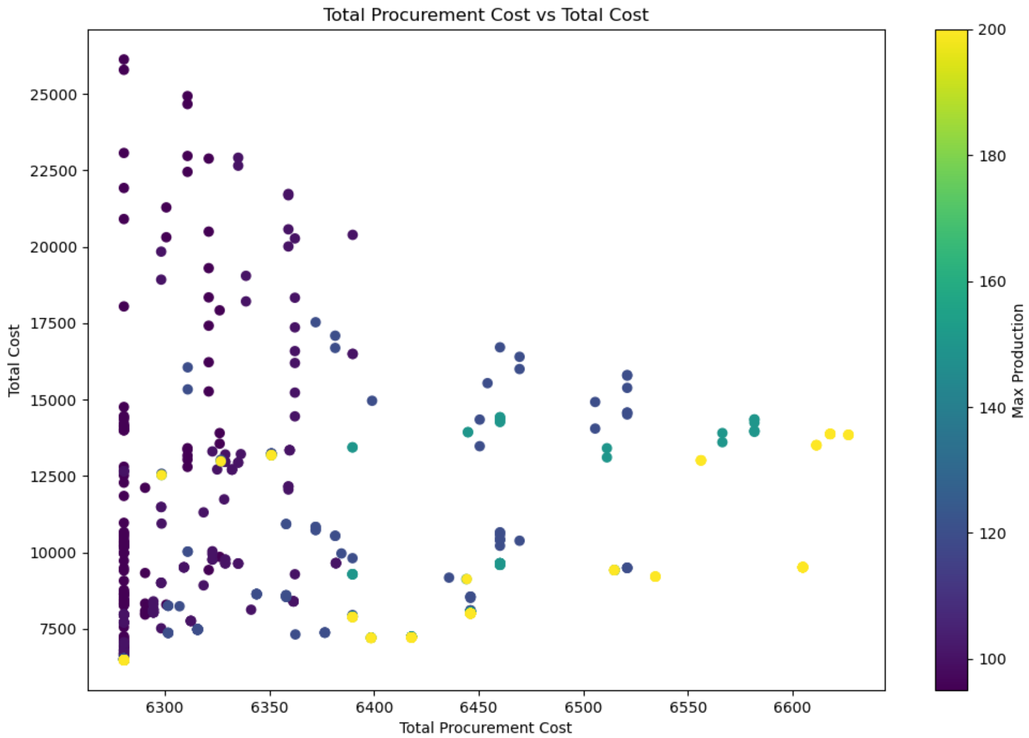

Figure 3: Scatter plot of total procurement cost vs total cost

Figure 4: Scatter plot of total storage cost vs total cost

To further analyze the direct impact of production capacity on total cost, total procurement cost, and total storage cost, scatter plots were drawn. The points in the figures are color-coded according to the maximum production capacity, with blue and darker points representing higher production capacities, and yellow and green points indicating lower production capacities.

In Figure 2 and Figure 3, it is evident that total procurement cost remains relatively stable, with no significant fluctuations as the parameters change. This suggests that procurement costs are relatively insensitive to variations in production capacity. In contrast, total storage cost shows dramatic fluctuations, particularly in scenarios with higher production capacities, where storage costs increase significantly. This trend indicates that higher production levels require more storage space to accommodate larger inventories, leading to substantial increases in storage costs. Additionally, as seen in Figure 4, there is a clear linear relationship between total storage cost and total cost. As storage costs rise, total costs increase correspondingly. This linearity supports the deductions made in the previous heatmap analysis, where storage costs were identified as a key factor driving overall cost increases.

From this scatter plot analysis, it can be concluded that while procurement costs remain relatively stable, managing storage costs is critical to controlling overall costs. Specifically, at higher production levels, the rise in storage costs can significantly drive up total costs.

Box plot analysis

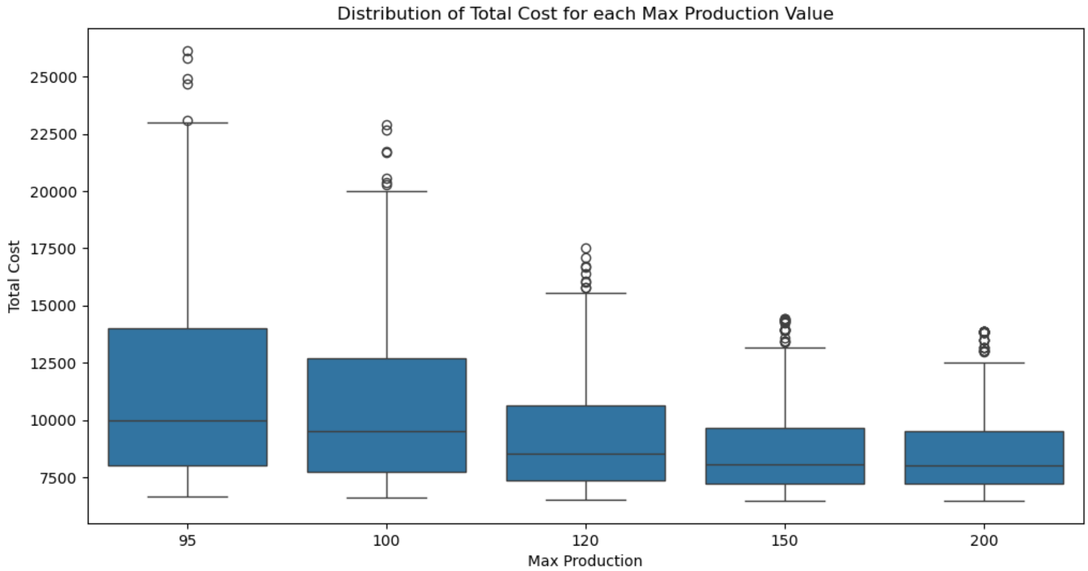

Figure 5: Total cost distribution by max production capacity

To better understand the impact of storage cost fluctuations on total cost under varying maximum production capacity scenarios, a box plot is created. Figure 5 clearly illustrates the range of total cost variations across different production capacity levels. As shown in the plot, as the maximum production capacity increases, the range of total cost variation becomes narrower. This implies that the impact of storage cost changes diminishes as production capacity increases.

The reduction in total cost variation with higher production capacities suggests that increasing production capacity can mitigate the effects of fluctuating storage costs. In other words, a larger production capacity can absorb storage cost volatility more effectively, leading to more stable overall costs. Thus, if stability in total cost is a key priority, increasing production capacity can be an effective strategy to minimize the variability introduced by changing storage costs.

Discussion

In conclusion, as max production capacity and storage cost of each product changes, the analysis highlights that storage cost is a key factor driving total cost variation, while procurement costs remain stable. Higher production capacities lead to decreased storage costs and can help reduce total cost variability. This suggests that increasing production capacity can help stabilize overall costs by mitigating the impact of fluctuating storage costs. Effective cost management should prioritize controlling storage expenses, especially in scenarios with higher production levels, to maintain a balanced and predictable total cost structure.

e Electrolysis Problem

Mathematical model

The company wants the amount of copper in its products to be below a certain value, called the CopperLimit. In order to remove copper from a mix, electrolysis can be used. To take the use of electrolysis into consideration, the mathematical model needs to be extended. The notation used for the new mathematical formulation is provided in the table below.

Table 8: Notation

Sets and indices		
T	Set for the months	$t \in T$
P	Set for the products	$i \in P$
S	Set for the suppliers	$j \in S$
Parameters		
$D_{i,t}$	Demand for product i in month t	[kg]
c_j	Cost per kg of material from supplier j	[euro/kg]
h_i	Holding cost per kg for product i	[euro/kg]
P_{max}	Maximum production capacity per month	[kg]
$X_{j,\text{max}}$	Maximum supply from supplier j per month	[kg]
α_i	Chromium content in product i	[%]
α_j	Chromium content in material from supplier j	[%]
β_i	Nickel content in product i	[%]
β_j	Nickel content in material from supplier j	[%]
γ_j	Copper content in material from supplier j	[%]
$CuLimit_t$	Copper content limit in month t	[%]
E_c	Fixed cost of copper electrolysis	[euro]
e_c	cost of copper electrolysis per kg	[euro/kg]
M	A large number	[-]
Variables		
$P_{i,t}$	Production quantity of product i in month t	[kg]
$S_{i,t}$	Inventory quantity of product i in month t	[kg]
$X_{i,j,t}$	Quantity of scrap material purchased from supplier j for product i in month t	[kg]
B_t	Binary variable of whether electrolysis is used in month t	[binary]
$m_{i,t}$	Electrolysis quantity of copper of product i in month t	[kg]

The mathematical formulation then follows as:

$$\min \sum_{t\in T}\left(\sum_{j\in S} c_j \cdot \sum_{i\in P} X_{i,j,t}\right) + \sum_{t\in T}\left(\sum_{i\in P} h_i \cdot S_{i,t}\right) + \sum_{t\in T} E_c \cdot B_t + \sum_{t\in T}\left(\sum_{i\in P} e_c \cdot m_{i,t}\right) \tag{9}$$

Subject to:

$$
\begin{aligned}
P_{i,t} + S_{i,t-1} - m_{i,t} &= D_{i,t} + S_{i,t} && \forall i \in P, \forall t \in T && (10)\\
\sum_{i \in P} P_{i,t} &\leq P_{\text{max}} && \forall t \in T && (11)\\
\sum_{i \in P} X_{i,j,t} &\leq X_{j,\text{max}} && \forall j \in S, \forall t \in T && (12)\\
P_{i,t} &= \sum_{j \in S} X_{i,j,t} && \forall i \in P, \forall t \in T && (13)\\
\alpha_i \cdot (P_{i,t} - m_{i,t}) &= \sum_{j \in S} \alpha_j \cdot X_{i,j,t} && \forall i \in P, \forall t \in T && (14)\\
\beta_i \cdot (P_{i,t} - m_{i,t}) &= \sum_{j \in S} \beta_j \cdot X_{i,j,t} && \forall i \in P, \forall t \in T && (15)\\
\sum_{j \in S} \gamma_j \cdot X_{i,j,t} - m_{i,t} &\leq CuLimit_t \cdot (P_{i,t} - m_{i,t}) && \forall i \in P, \forall t \in T && (16)\\
m_{i,t} &\leq B_t \cdot M && \forall i \in P, \forall t \in T && (17)\\
P_{i,t}, S_{i,t}, X_{i,j,t}, m_{i,t} &\geq 0 && \forall i \in P, \forall j \in S, \forall t \in T && (18)
\end{aligned}
$$

The objective function (9) minimizes the total cost, which includes the procurement cost, holding cost, fixed electrolysis cost, and electrolysis cost. Constraint (10) ensures that the production and inventory balance is maintained, and electrolysis quantity is taken into consideration. Constraint (11) limits the total production capacity in each month. Constraints (12) impose limits on the amount of material that can be sourced from each supplier. Constraint (13) ensures that the mass of production matches the mass of procurement from suppliers. Constraints (14) and (15) ensure that the chromium and nickel content in each produced product matches the content of each purchased material. Constraint (16) ensures that the copper content purchased per product per month is within the copper limit of the product. Constraint (17) applies big M number to guarantee the relationship between electrolysis quantity and the binary variable. Constraint (18) guarantees non-negativity for all the variables.

Model changes

The mathematical model is transferred to an MILP problem, after the binary variable is introduced. Specific changes are stated below.

New variables

- B_t: Binary variable B_t is introduced to determine whether electrolysis is used in each month.
- $m_{i,t}$: $m_{i,t}$ is introduced to determine the quantity of copper to electrolysis in each month.

New parameters

- γ_j: γ_j describes the copper content in material from supplier j.
- $CuLimit_t$: $CuLimit_t$ describes the copper limit in month t.
- E_c: E_c represents the fixed cost of using electrolysis in a month, which is equal to 100 euro.
- e_c: e_c represents the cost of using electrolysis per kg, which is equal to 5 euro/kg.
- M: M is introduced in as a very large number.

New objective function

In the new objective function, the two terms shown below are included.

- : $\sum_{t \in T} E_c \cdot B_t$: This term represents the fixed cost of using electrolysis.
- : $\sum_{t \in T} \left(\sum_{i \in P} e_c \cdot m_{i,t}\right)$: This term represents the cost of copper electrolysis based on quantity.

New constraints

Constraint (10), (14) and (15) are adjusted to take copper electrolysis quantity into account. New constraints are presented below:

$$\sum_{j \in S} \gamma_j \cdot X_{i,j,t} - m_{i,t} \leq CuLimit_t \cdot (P_{i,t} - m_{i,t}) \qquad \forall i \in P, \forall t \in T$$

This constraint ensures that copper content in material is limited by the copper limit when electrolysis is done.

$$m_{i,t} \leq B_t \cdot M \qquad \forall i \in P, \forall t \in T$$

This constraint ensures that copper removal is dependent on whether to conduct electrolysis.

Result

In the solution for (b), the minimized cost is 9646.78 euro. Based on this, the lowest CopperLimit use is determined without exceeding the minimized cost. In order to find the lowest CopperLimit, binary search method is implemented. A loop is created to reduce the possible range of CopperLimit and maintain original cost in the meantime.
The final result of the lowest CopperLimit is 2.9651%, which maintains the total cost at 9646.78 euro.

Experiment

In this part, experiments with other values of the CopperLimit are conducted to see its impact on costs. As the lowest CopperLimit is found to be 2.9651%, the range of CopperLimit is determined to be [0, 3%], with the step of 0.1%. So the total number of experiments is 30, and a figure is presented below to show how different costs are affected by the value of CopperLimit.

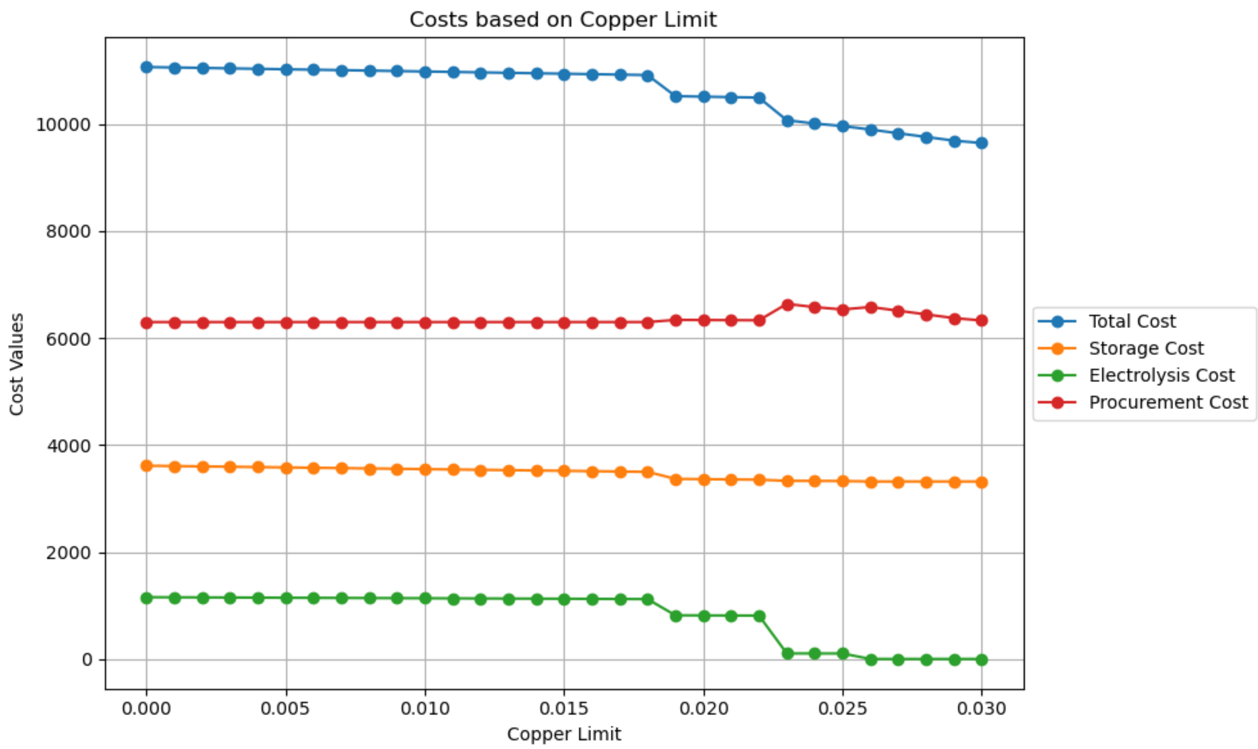

Figure 6: Costs based on Copper Limits

Based on figure 6, it is clearly shown how different costs change according to CopperLimit.

Electrolysis cost

During the decrease of CopperLimit, electrolysis is not immediately used. When CopperLimit falls to around 2.5%, eletrolysis is activated. The electrolysis cost remains a steady, gradual increase in three separate stages, within the CopperLimit ranges of [0, 1.8%], [1.9%, 2.2%], and [2.3%, 2.5%], with sharp increases between these stages. Further analysis of changes in other costs can help explain this trend.

Storage cost

Storage cost remains relatively stable throughout the whole process, suggesting inventory levels are not significantly affected by the use of electrolysis. However, there is a slight change when Copper Limit reaches 1.8%, indicating the increase in electrolysis quantity still has a small impact on inventory management.

Procurement cost

The procurement cost line saw an increase when CopperLimit starts to decrease. This increase shows the initial procurement method taken to deal with CopperLimit: Just buy more materials, especially those with low copper content. But as procurement cost rises, electrolysis is eventually activated, which reduces the need to buy more materials. Thus, the procurement cost starts to drop, and finally drop to almost the initial level.

Total cost

As Copper Limit drops from 3% to 0, total cost increases. The increase can be divided into three stages. Initially, the rise in procurement cost is the main drive, as electrolysis has not yet been used. In the latter two stages, the increase in total cost is driven by rising electrolysis costs.

Conclusion

In summary, as CopperLimit begins to decrease, the system initially responds by purchasing more materials from suppliers, as electrolysis costs can be too high at this stage. As procurement cost keeps rising, electrolysis is eventually activated, reducing the procurement amount. Inventory levels also experience a slight increase in response to the increasing use of electrolysis. Throughout the process, total costs increase as CopperLimit decreases.

AI Statement

In this assignment, ChatGPT and Copilot are used.

ChatGPT is used to help adjust the report layout in latex form. Sometimes the latex output is not presented as expected, then ChatGPT is asked to offer help. In table 7, the original text layout was a little messy, so ChatGPT was asked, "How do I put table with too many texts in order?" ChatGPT suggested that "setlength{tabcolsep}{length}" and "renewcommand{arraystretch}{height}" command could be used to adjust column padding and row height, which helped me improve the layout.

Copilot is used in solving a Gurobi license problem. At first, "gurobi.lic" was not saved in the path "C:/Users/username". This caused a problem in solving question (e), stating "gurobipy.GurobiError: Model too large for size-limited license". Copilot was asked about this error and suggested that the missing license document in the specific path could be a reason, which helped me solve the problem.

All the coding files, plots, experiment result excels and report files can be accessed on Github via the following link:
https://github.com/LuizLi/QML-Q1-Assignment.git