Lista 03 - OS

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1-a) $2x + y + a = 14$

$2x + 3y + b = 24$

Base	x	y	a	b	TI
a	2	1	1	0	14
b	2	3	0	1	24
$\bar{Z}$	-9	-3	0	0	0

(a escolhida p/ sair da base; x pivô)

Solução básica inicial:

$a = 14 \quad x = 0$

$b = 24 \quad y = 0$

$Z = 0$

1 escolhida p/ entrar na base (x entrando)

Dividir (TI, a) por (x, a) e (TI, b) por (x, b)

$14 \div 2 = 7 \quad 24 \div 2 = 12$

Sai o menor

Base	x	y	a	b	TI
x	1	1/2	1/2	0	7
b	0	2	-1	1	10
$\bar{Z}$	0	3/2	9/2	0	63

- $3 - (2 \cdot 1/2) = 2$
- $3 - (1 \times 9/2) = 3/2$
- $0 - (1 \times 2/2) = -1$
- $0 - (1 \times -9/2) = 9/2$
- $24 - (14 \times 2/2) = 10$
- $0 - (14 \times -9/2) = 63$

Solução ótima: $x = 7 \quad a = 0$

$b = 10 \quad y = 0$

$Z = 63$

- Pivô: elemento da coluna que entra com linha que sai
- Pivô = 2
- Zerar coluna do pivô, exceto pivô = 1.
- Dividir a linha do pivô pelo pivô (2)
- $\begin{bmatrix} P & D_1 \\ D_2 & x \end{bmatrix} \quad x_n = x_c \times \left(\frac{D_1 \times D_2}{P}\right)$

b) $8x + 4y + a = 32$

$x + 2y + b = 8$

Base	x	y	a	b	TI
a	8	4	1	0	32
b	1	2	0	1	8
Z	-5	-5	0	0	0

sai de base → a; ↑ entra na base

Base	x	y	a	b	TI
x	1	1/2	1/8	0	4
b	0	3/2	-1/8	1	4
Z	0	-20/8	5/8	0	20

sai de base ← b; ↑ entra na base

Base	x	y	a	b	TI
x	1	0	1/6	-1/3	8/3
y	0	1	-16/3	2/3	8/3
Z	0	0	5/6	5/3	80/3

Solução básica inicial:

$a = 32 \quad x = 0$

$b = 8 \quad y = 0$

$Z = 0$

- $2 - (4 \times 1/8) = 3/2$
- $0 - (1 \times 1/8) = -1/8$
- $-5 - (4 \times -5/8) = -20/8$
- $0 - (1 \times -5/8) = 5/8$
- $8 - (32 \times 1/8) = 4$
- $0 - (32 \times -5/8) = 20$

- $1 - (1/2 \times 0 / 3/2) = 1$
- $1/8 - (1/2 \times -1/8 / 3/2) =$

$\frac{1}{8} - \left(\frac{1}{2} \times -\frac{1}{8} \times \frac{2}{3}\right) =$

$\frac{1}{8} - \left(-\frac{2}{48}\right) = \frac{1}{8} + \frac{1}{24} =$

- $\frac{5}{8} - \left(-\frac{1}{8} \times \frac{-20}{8} \times \frac{2}{3}\right) =$

$\frac{5}{8} + \frac{5}{24} = \frac{5}{6}$

- $0 - \left(\frac{-20}{8} \times 1 \times \frac{2}{3}\right) = 10/6 + 5/3$
- $0 - \left(1 \times \frac{1}{2} \times \frac{2}{3}\right) = -1/3$
- $4 - \left(4 \times \frac{1}{2} \times \frac{2}{3}\right) = \frac{8}{3}$
- $20 - \left(4 \times -\frac{5}{2} \times \frac{2}{3}\right) = 20 + \frac{20}{3} =$

$\frac{80}{3}$

Solução ótima: $a, b = 0$

$x = 8/3 \quad y = 8/3 \quad Z = 80/3$

c) $2x + 3y + a = 12$

$2x + y + b = 8$

$2x + c = 8$

Base	x	y	a	b	c	TI
a	2	3	1	0	0	12
b	2	1	0	1	0	8
c	(2)	0	0	0	1	8
Z	-16	-12	0	0	0	0

Solução básica inicial:

$a = 12$

$b = 8 \quad x, y = 0$

$c = 8 \quad Z = 0$

↳ entra na base

Base	x	y	a	b	c	TI
a	0	(3)	1	-0	-1	4
b	0	1	0	1	-1	4
x	1	0	0	0	1/2	4
Z	0	-12	0	0	8	64

↳ entra na base

Base	x	y	a	b	c	TI
y	0	1	1/3	-0	-1/3	4/3
b	0	0	-1/3	1	+2/3	8/3
x	1	0	0	0	1/2	4
Z	0	0	0	0	-4	80

- $8 - (-1 \times -12/3) = 4$
- $4 - (4/3) = 8/3$
- $64 - (4 \times -5/3) = 80$

Solução ótima:

$x = 4$

$y = 4/3 \quad Z = 80 \quad a, c = 0$

$b = 8/3$

d) $2x + 4y + z + a = 16$

$6x + 2y + b = 24$

$2x + c = 6$

Base	x	y	z	a	b	c	TI
a	2	(4) pivô	1	1	0	0	16
b	6	2	0	0	1	0	24
c	2	0	0	0	0	1	6
$\bar{z}$	-3	-5	-1	0	0	0	0

sai ↙

↳ entra

Solução básica:

$x, y, z = 0$

$z = 0$

$a = 16$ $c = 6$

$b = 24$

Base	x	y	z	a	b	c	TI
y	1/2	1	1/4	1/4	0	0	4
b	5	0	-1/2	-1/2	1	0	16
c	(2) pivô	0	0	0	0	1	6
$\bar{z}$	-1/2	0	1/4	5/4	0	0	20

sai ↙

↳ entra

. $6 - (0 \times 2/4) = 5$

. $-3 - (-5 \times 2/4) = -3 + 5/2 = -1/2$

. $-1 - (1 \cdot -5/4) = -\frac{4}{4} + \frac{5}{4} = \frac{1}{4}$

. $24 - \left(\frac{16 \times 0}{4}\right) = 16$

. $0 - \left(\frac{16 \times -5}{4}\right) = 20$

Base	x	y	z	a	b	c	TI
y	0	1	1/4	1/4	0	-1/2	5/2
b	0	0	-1/2	-1/2	1	-5/2	1
x	1	0	0	0	0	1/2	3
$\bar{z}$	0	0	1/4	5/4	0	1/4	43/2

. $0 - \left(\frac{1 \times -1/2}{2}\right) = 1/4$

. $20 - \left(\frac{16 \times -1/2}{2}\right) = \frac{43}{2}$

. $4 - \left(\frac{16 \times 1/2}{8}\right) =$

$\frac{8-3}{2} = \frac{5}{2}$

. $16 - \left(\frac{8 \times 5}{8}\right) = 1$

Solução ótima:

$y = \frac{5}{2}$ $a, z, c = 0$

$x = 3$ $z = 43/2$

$b = 1$

2- Pior dos casos: complexidade exponencial

Melhor dos casos: nula (base ótima)

3-

variáveis artificiais

$$2x_1 - 1x_2 + 3x_3 - 1x_4 + X_1^a = 4$$

$$+ \quad 1x_1 + 2x_2 - 1x_5 + X_2^a = 6$$

$$3x_1 - 1x_2 + 2x_3 - 1x_6 + X_3^a = 7$$

$$Z_a = -(6x_1 + 0x_2 + 5x_3 - 1x_4 - 1x_5 - 1x_6 + 0X_1^a + 0X_2^a + 0X_3^a = 17)$$

Base	x_1	x_2	x_3	x_4	x_5	x_6	X_1^a	X_2^a	X_3^a	b
X_1^a	(2)	-1	3	-1	0	0	1	0	0	4
X_2^a	1	2	0	0	-1	0	0	1	0	6
X_3^a	3	-1	2	0	0	-1	0	0	1	7
Z	-2	3	-5	0	0	0	0	0	0	0
Z_a	-6	0	-5	1	1	1	0	0	0	-17

sai ← X_1^a

↳ entra

Base	x_1	x_2	x_3	x_4	x_5	x_6	X_1^a	X_2^a	X_3^a	b
x_1	1	-1/2	3/2	-1/2	0	0	1/2	0	0	2
X_2^a	0	5/2	-3/2	1/2	-1	0	-1/2	1	0	4
X_3^a	0	(1/2)	-5/2	3/2	0	-1	-3/2	0	1	1
Z_1	0	2	-2	-1	0	0	1	0	0	4
Z_a	0	-3	4	-2	1	1	3	0	0	-5

sai ← X_3^a

↳ entra

Base	x_1	x_2	x_3	x_4	x_5	x_6	X_1^a	X_2^a	X_3^a	b	
x_1	1	0	-1	1	0	-1	-1	0	1	3	-3
X_2^a	0	0	(11)	-7	-1	7	7	1	-5	-1	$-1/11$
x_2	0	1	-5	3	0	-1	-3	0	2	2	$-2/5$
z	0	0	18	-7	0	4	7	0	-4	0	
z_a	0	0	-11	7	1	-5	-6	0	6	1	

sai

↳ entra

$\cdot 3 - \left(\frac{-7 \times -5}{11}\right)$

$\frac{33}{11} - \frac{35}{11} = \frac{-2}{11}$

$\cdot -7 - \left(\frac{-7 \times 18}{11}\right) =$

$\frac{-77}{11} + \frac{126}{11} = \frac{49}{11}$

Base	x_1	x_2	x_3	x_4	x_5	x_6	X_1^a	X_2^a	X_3^a	b
x_1	1	0	0	$4/11$	$-1/11$	$-4/11$				$32/11$
x_3	0	0	1	$-7/11$	$-1/11$	$7/11$	$7/11$	$1/11$	$-5/11$	$-1/11$
x_2	0	1	0	$-2/11$	$15/11$	$24/11$				$17/11$
z	0	0	0	$49/11$	$18/11$	$-82/11$				$18/11$
z_a	0	0	0	0	0	2	1	1	1	0

Fase 2 - Simplex normal

Base	x_1	x_2	x_3	x_4	x_5	x_6	b
x_1	1	0	0	$4/11$	$-1/4$	$-4/11$	$32/11$
x_3	0	0	1	$-7/11$	$15/11$	$7/11$	$-1/11$
x_2	0	1	0	$-2/11$	$18/11$	$24/4$	$17/11$
z	0	0	0	$49/11$	$18/11$	$-82/11$	$18/11$

⋮