Tutorial 1 - Exercises Lectures 1, 2 and 3
Computer Vision 1, Master AI, 2023-2024

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1 EXERCISE - pinhole camera

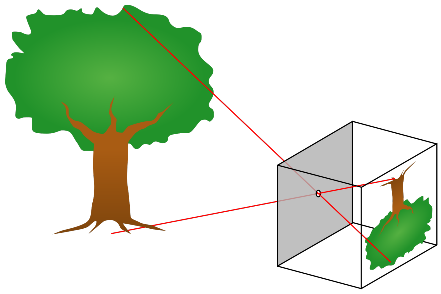

Figure 1: Pinhole camera model (source: Wikipedia)

The camera obscura or pinhole image leverages a natural optical phenomenon. The camera has the shape of a box, light from an object enters through a small hole (the pinhole) in the front and produces an image on the back camera wall.

Q.a Annotate the Figure 1 with the focal length, center of projection and image plane. What is the virtual image in this context, and where would you place it?

Q.b Assume that the pinhole is at the origin $C = (0, 0, 0)$ of a 3D coordinate system (x, y, z). To generate a projection P of a scene point O, we form the line between O and C and extend it to intersect it with the image plane at $z = -1$. Draw the coordinate system and add the point $O = (1, 1, 1)$, the pinhole C and the intersection at the image plane. Derive the equation for computing a point on the image plane.

Q.c Compute the image of the cube with corners in $(\pm 1, \pm 1, 1)$ and $(\pm 1, \pm 1, 3)$ according to the model from (Q.b).

2 EXERCISE - image representation

A digital image is an image composed of picture elements (pixels). Each pixel has a finite and discrete numeric quantity representing the local intensity. In

Figure 2: First picture to be scanned, stored and recreated in digital pixels (source: Wikipedia)

practice, the set of quantities is determined by the number of bits available per pixel. Depending on whether the image resolution is fixed, it may be a vector or raster type. Here we focus on the latter.

Q.a Consider $f : X \mapsto Y$. How are digital images represented as a function f? What are common sets for Y and their associated image types?

Q.b What are the differences between changes to the quantization level and changes to the spatial resolution? How do these changes affect the visual perception of the image?

3 EXERCISE - Camera Projection

3.1 Homogeneous Coordinates

Q.a Convert the following cartesian 2D vector into homogeneous coordinates: $v = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

Q.b Convert the following homogeneous coordinate into cartesian coordinates.

$$w_h = \begin{pmatrix} 3 \\ 5 \\ 4 \\ 1 \end{pmatrix}$$

Q.c In the lecture the extrinsic matrix [R t] is discussed. If we project a 3D point on an 2D image, what are the dimensions of this matrix [R t] ? And what are the dimensions of the matrices R and t?

Q.d Explain why homogeneous coordinates are necessary.

3.2 Camera Translations and Rotations

Write the homogeneous 3x4 matrices for the following transforms:

Q.a Camera is translated by +5 units in the X direction

Q.b Rotate by 30 degrees about the X axis.

Q.c The combination of the rotation followed by the translation above.

4 EXERCISE - color

To calculate the color of light sources, the following intuitive color models are used: intensity V, hue H, saturation S and chromaticity xy, Let's assume, for simplicity reasons, that sunlight S is given by the CIE values $X = Y = Z = 100$. Further, let $X = 100$, $Y = 100$ en $Z = 150$ be the values for a given artificial lamp A.

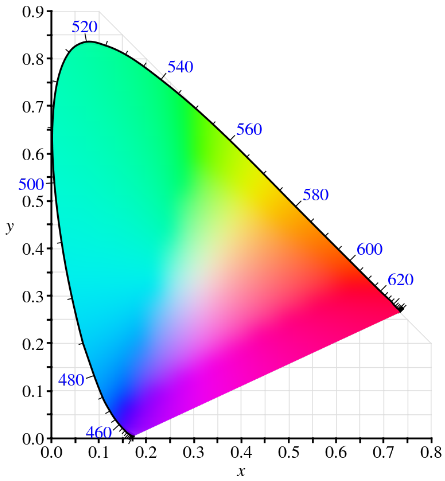

Figure 3: Chromaticity diagram

Q.a Calculate the intensity V (in same dimensions as the X, Y, Z values) of the two light sources S and A.

Q.b Calculate the chromaticy values $x = X/(X+Y+Z)$, $y = Y/(X+Y+Z)$ and plot these in the chromaticity diagram given in Figure 3.

Q.c) What is the estimated hue of light source A with S as reference white light.

Q.d Rank the light sources with respect to their saturation.

Q.e Plot the colors which are produced through the mixture of S and A.

5 EXERCISE - reflection

We consider the color of a matte, dull (not glossy) surface. The color at a specific location on the surface under white light illumination is given by the following simple reflection model $R = Ik_R \cos\theta$, $G = Ik_G \cos\theta$ and $B = Ik_B \cos\theta$, where I is the intensity of the white light source, k_R, k_G and k_B are the amount of red, green and blue reflected by the surface (i.e. color of the surface). Furthermore, $\cos\theta = \vec{n} \cdot \vec{l}$ is the dot product of the two-unit vectors $\vec{n}$ (i.e. surface normal) and $\vec{l}$ (i.e. direction of the light source), see Figure 4

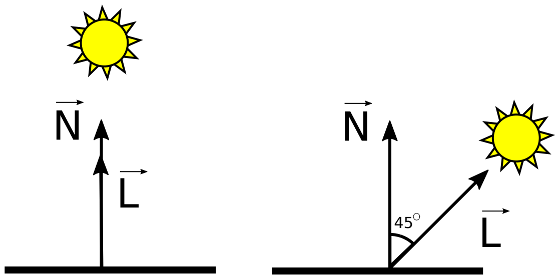

Figure 4: Light source position scenarios: coincidence with surface normal (left), and making an angle of 45° (right)

Q.a Assume that the surface is flat and homogeneously colored. Explain why the intensity is higher when the surface normal coincides with the direction of the light source than observed under an angle with respect to the direction of the light source.

Q.b Assume that the color of the surface is yellow i.e. $R = 100$, $G = 100$, and $B = 10$. Explain what will happen with the values R, G and B if (only) the intensity of the light source will diminish. Plot the positions of the colors in the RGB-color space.

Q.c In case of a curved (not flat) surface, indicate where the colors will be positioned in the RGB-color space. Explain your answer.

Q.d A simple color invariant is given by R/G. Proof that R/G is independent of the (intensity) light source I, object geometry and the direction of the light source.

Q.e The values of R, G and B will vary for a curved surface. Give the approximated shapes of the histograms for a homogeneously (curved) surface for R/G.

Q.f Consider the same surface. Assume that the surface is glossy (instead of matte). The reflection model is now given by $R = Ik_R cos\theta + Ik_s cos^n\alpha$, $G = Ik_G cos\theta + Ik_s cos^n\alpha$ and $B = Ik_B cos\theta + Ik_s cos^n\alpha$. k_s is the specular reflection coefficient and cos^n depends on the glossiness and α depends on the viewing condition. Plot the colors of the homogeneously colored (shiny) surface in RGB- and rgb-color space.
Proof that $\frac{R-G}{R-B}$ is a color invariant for shiny surfaces.

6 EXERCISE - Histograms

To calculate the histogram equalization of an image, we use the histogram of the image, binned with the gray level (L). We consider the distribution of pixel values in a 3-bit grayscale image given in table 1. The total number of gray level is 8 ($2^3 = 8$) ranging from 0 to 7.

Gray Level	0	1	2	3	4	5	6	7
No. of Pixels	770	1060	660	280	460	210	130	13

Table 1: Pixel distribution

Q.a Plot the histogram of the given 3-bit gray levels. What can you infer from the given gray level information? What do you think will be the type of image? Can you reconstruct the image from just the information given in table 1? Justify your answer.

Q.b Calculate the probabilities of pixels for each gray level.

Q.c Equalize the histogram and plot the histogram.

1	1	1	3	3	4	4	1
4	0	0	5	1	2	0	1
0	1	3	1	2	0	2	0
3	1	2	2	2	1	1	2
1	4	2	2	2	3	5	3
0	2	1	0	2	1	3	0
0	2	1	1	0	0	4	1
0	4	0	0	0	2	1	1

Table 2: 3-bit grayscale image patch

Q.d Consider the 3-bit grayscale image patch given in table 2. Calculate the pixel distribution like in table 1 and plot the histogram.

Q.e Equalize the histogram and then back project the equalized histogram to obtain the new image. Plot the new histogram.

Q.f So far you have equalized only grayscale image. How would you apply the algorithm to an RGB image? Discuss your answer.

Q.g Consider the histogram visualized in Figure 5. We have seen that histogram equalization works well, spreading the distribution when the distribution is clustered in one region. What would happen if we applied the algorithm when there are multiple clusters? How could the shortcomings be solved? Discuss your answer.

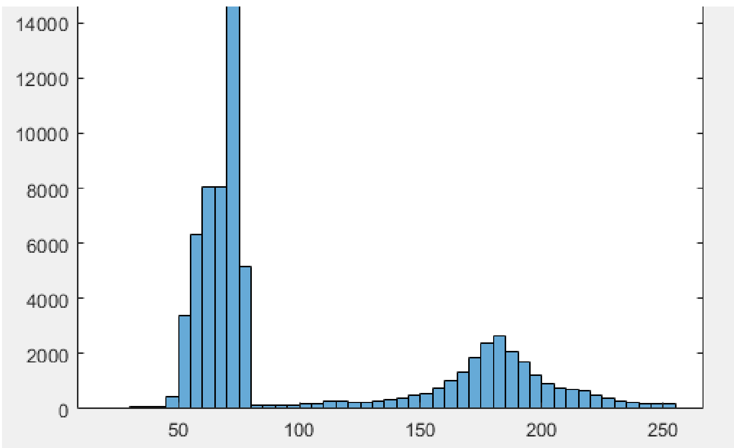

Figure 5: Double cluster histogram