

1.b

Let's start by doing the multiplication with index notation to make it easier to compute the derivatives. Since $X \in \mathbb{R}^{n \times n}$ and $w \in \mathbb{R}^{n \times 1}$, multiplying both should give us $Xw \in \mathbb{R}^{n \times 1}$. So we do:

$$[Xw]_i = \sum_{p=1}^n X_{ip} w_p$$

Applying the derivatives w.r.t w on the i -th row, we get:

$$\frac{\partial}{\partial w_j} [Xw]_i = \frac{\partial}{\partial w_j} \sum_{p=1}^n X_{ip} w_p =$$

By the sum rule and also using the fact that X_{ip} is just a constant:

$$1$$

$$= \sum_{p=1}^n X_{ip} \frac{\partial w_p}{\partial w_j}$$

We now end up with a Kronecker Delta:

$$\frac{\partial w_p}{\partial w_j} = \delta_{jp} = \begin{cases} 1, & \text{if } j = p \\ 0, & \text{otherwise} \end{cases}$$

And then we can do:

$$\sum_{p=1}^n X_{ip} \frac{\partial w_p}{\partial w_j} = \sum_{p=1}^n X_{ip} \delta_{jp} = X_{ij}$$

Thus the derivative $\frac{\partial}{\partial w} f$ with $f = Xw$ give us exactly $\frac{\partial}{\partial w} f = X$.