1.c

We start again by doing the multiplication with index notation to make it easier to compute the derivatives. Since $X \in \mathbb{R}^{n \times n}$ and $w \in \mathbb{R}^{n \times 1}$, the multiplication $w^T X w$ should give us a real number. So we do:

$$w^T X w = \sum_{p=1}^{n} \sum_{q=1}^{n} w_p X_{pq} w_q$$

Applying the derivatives w.r.t w, we get:

$$\frac{\partial w^T X w}{\partial w_i} = \frac{\partial}{\partial w_i} \sum_{p=1}^{n} \sum_{q=1}^{n} w_p X_{pq} w_q =$$

By the sum rule and also using the fact that X_{pq} is just a constant:

$$= \sum_{p=1}^{n} \sum_{q=1}^{n} X_{pq} \frac{\partial}{\partial w_i} w_p w_q =$$

By the product rule:

$$= \sum_{p=1}^{n} \sum_{q=1}^{n} X_{pq} (\frac{\partial w_p}{\partial w_i} w_q + w_p \frac{\partial w_q}{\partial w_i}) =$$

Using the Kronecker Delta once again:

$$= \sum_{p=1}^{n} \sum_{q=1}^{n} X_{pq}(\delta_{ip} w_q + w_p \delta_{iq}) = \sum_{p=1}^{n} \sum_{q=1}^{n} (X_{pq}\delta_{ip} w_q + X_{pq} w_p \delta_{iq}) =$$

$$= \sum_{q=1}^{n} X_{iq} w_q + \sum_{p=1}^{n} X_{pi} w_p = Xw + w^T X =$$

$$= w^T X^T + w^T X = w^T (X^T + X)$$

Thus the derivative $\frac{\partial}{\partial w} f$ with $f = w^T X w$ give us exactly $\frac{\partial}{\partial w} f = w^T (X^T + X)$.