Communication Network Using Graph theory

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Math Discrete

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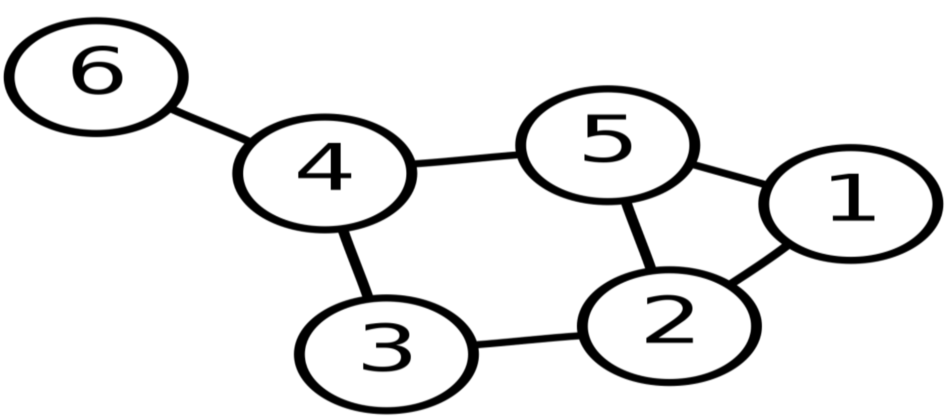
Abstract :

Graph theory is an important area in mathematics. This paper talks about the use of graphs for modelling communication networks. It represents the communication networks as binary tree, 2-D array, and butterfly network. All three representations have been compared on their diameter, switch size, switch count and congestion.

Introduction :

Firstly, Graph theory is the main study of graphs in mathematics. We use graphs to model relations between objects. Graphs consists of sets of vertices" nodes - points" that connected by edges" links -lines". In the picture below: Vertices are the numbers in the circles, but Edges are the lines which connected them.



$$G = (V, E)$$

In the above picture: $V = \{1,2,3,4,5,6\}$ **and :**
$E = \{\{1,2\},\{1,5\},\{2,3\},\{2,5\},\{3,4\},\{4,5\},\{4,6\}\}.$

There are many types of graphs based on edges, direction, weight, and these types are:

- **Simple graph: each edge connects two different vertices.**
- **Multi graph: many edges that connect the same pair of vertices.**
- **Complete graph, directed and undirected graphs, etc.**

- Graph theory is used to allocate- calculate the best path between two random points on maps. However, there are many studies approve the positive and direct impact role of graph theory on its performance. Graph theory has been widely utilized in many fields and become a significant filed of robotic application in our daily life. Also, there are lots of application that describe the graph theory such as social networks, Communication networks, Information networks, biological networks, and transportation networks.

- In this research, we will talk about *communication networks* which is used to enable communications between users of the terminals that is collected from collection of links, nodes, and terminals. Also, terminal should have a distinguished address, so that messages are sent to the correct receivers. In addition, communication networks consist of 3 main elements which are:

1*) processors*: it is used to provide data and transfer control functions

2*) transmission channels:* it is used to support data transfer

***3) terminals*: it is used to control the start and end points of the network.**

-Also, communication networks are intended to transfer data packets between processors, computers, or other devices. The term packet refers to a nearly constant amount of data: *256 bytes - 4096 bytes*. In addition to the fact that packets are sent from the input to the output through various switches.

- Different mathematical structures are used to represent communication networks. Also, it's useful for comparing different representations that based on switch size, number of

switches, and congestion. It is possible to link between the graph and the communication networks as:

- **Terminals → Vertices, Edges "processors " →Transmission channels.**

Therefore, the data packet hops from the input terminal over the network through a series of switches connected to the output terminal by the directed edge.

Methodology:

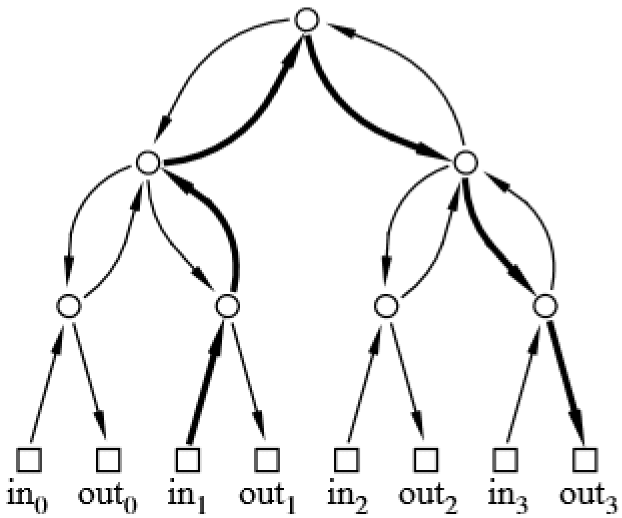
Instead of communication channels, nodes - switches are discussed. The following are some of the terminologies used in the binary tree context:

- **Latency: The time it takes for a packet to transit from its source to its destination. The number of switches that a packet must pass through when travelling between the furthest distant input and output is one measure of delay.**
- **Diameter: A network's diameter is the number of switches on the shortest path between the farthest-apart input and output. As a result, diameter is a rough estimate of worst-case latency.**
- **Congestion: a statistic that is used to quantify communication network bottlenecks. It refers to the total number of packets that transit through any switch.**

-A complete binary tree can be used to represent a communication network.

The squares **→ data packet terminals, sources, and destinations.**

Switches – route direct packets through the network → circles symbolize them. a switch accepts packets and transfers them to the outgoing edges on the incoming edges. In an undirected tree, every pair of vertices has a unique path between them. As a result, the equivalent directed path is the obvious approach to transport a packet of data from an input terminal to an output in the whole binary tree.



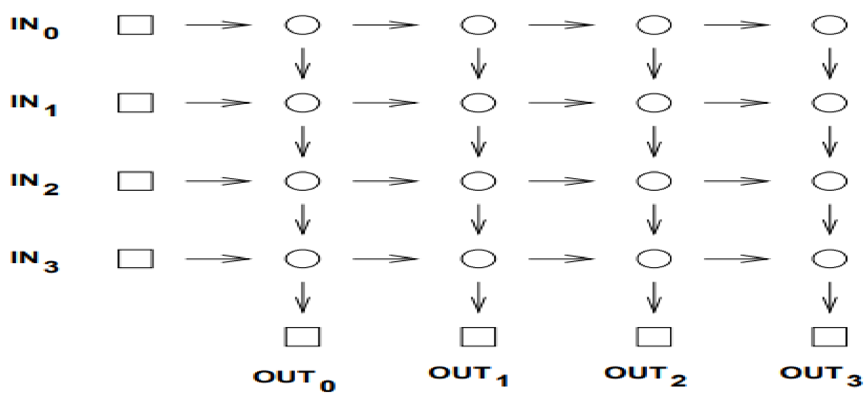
(Binary tree in communication networks " 4 input and 4 output)

- **Diameter: A complete binary tree with N inputs and outputs will have a diameter of 2 log N +1. So, if 2^10 = 1024 inputs and outputs are concatenated using the full binary tree, then the latency is 2 log (2^10) + 1 = 21.**
- **Switch size: Each network tends to have the smallest diameter. Using a larger switch is one way to achieve: that. In the whole binary tree, most switches have 2 inbound edges and 2 outbound edges, making it a 3x3 switch. If you have a 4×4 switch with a smaller diameter, you can make a complete ternary tree. Basically, all inputs and outputs can be connected through a single monster switch which acts like an N x N switch. This approach doesn't seem very productive as the original network design issues are hidden inside large switches. Therefore, the network must be designed to provide the functionality of an N x N switch using a basic device such as a 3x3 switch.**
- **Switch count: Another problem in network design is the number of switches. On quantity Switches have higher hardware costs. So, the number of switches should be as few as possible. the total number of switches in a complete binary tree is 2N – 1 which is as close as possible with a 3×3 switch**

- **The basic function of communication networks considered today is to transfer data packets between processors, telephones, or other devices. The term packet refers to a nearly fixed amount of data (such as 256 bytes or 4096 bytes) as we mentioned before. In the picture above, switch receives the packet at the incoming edge and forwards it along the outgoing edge. This is a way to imagine a data packet hopping from an input terminal through a set of switches connected to the output terminal by a directional edge over the network.**

- **Noting that there is a unique path between each pair of nodes in the undirected tree. Therefore, the natural way to route a data packet from an input terminal to the output of the complete binary tree is an analog directed path. As an example, the route of a packet traveling from input 1 to output 3 is shown in bold lines .**

- A 2-D Array: communication network can be represented as in 2 d-array. Also, this is called “crossbar – grid”.

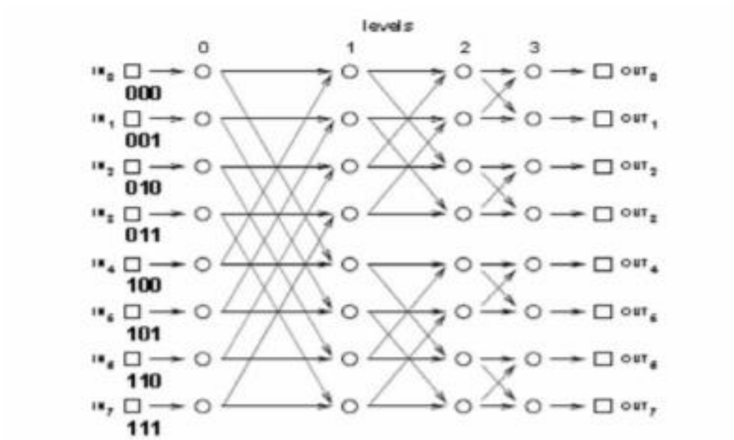


(2D-array in communication networks)

some of the terminologies used in the 2D-array context:

- **Diameter:** **is 2N-1 as: number of switches on the shortest path between the farthest input - output, for N inputs - outputs**
- **Switch size:** **is 2×2**
- **Switch count:** **number of switches is required when *communication network* in 2-D array is *N2*. So, if there is a network that its size N=1000, it would order a million 2×2 switches.**
- **Congestion:** **Use π as a substitute. Next, let π (i) be a path that extends from the input i to the right in column j and then to the output π (i). Therefore, the switch in row i and column j sends up to two packets. Packets sent from input i and packets destined for column j. Therefore, the maximum congestion in this case is 2.**

A Butterfly network : **All terminals and switches in the network are located in column N. In particular, the input i is on the left edge of line i and the output i is on the right edge of line i. Lines are labeled in binary, so line i is labeled in binary b1b2 ... A b log N that represents the integer i. There is a log (N) + 1 switch level between the input and output, numbered from 0 to log N. Each level consists of columns of N switches, one for each row. This means that every switch in the network is uniquely identified by a sequence (b1, b2, … blog N, L), here b1b2 ...... blogN which is the binary line of the "switch" and L is the level of the switch. There is a directed edge from the switch (b1, b2, ..., B logN, L) to the next level of two switches. One edge leads to a switch in the same row, the other edge leads to a series of switches obtained by inverting bit L + 1.**



(Butterfly → communication network)

- **Diameter :** Between input and output, log(N) + 1 transition, numbered 0 to log N. Each level consists of a column of N switches, one switch per row. Therefore, the diameter for this case is log(N) + 1.
- **Switch size :** is 2 × 2
- **Switch count :** because the network consists of level (N) +1 switches , *each level* → *N switches*. So, the sum of number of switches is N(log(N)+1).
- Congestion: Because each input and output have their own path, the maximum amount of messages that may transit through a vertex for any routing determines the congestion. There is a path from exactly 2i input vertices to v and a path from v to exactly 2n-i output vertices if v is a vertex in column I of the butterfly network. As a result, the butterfly network's congestion is about N if N is an even power of 2 and around N/2 if N is an odd power of 2.

Results of : Comparison of various representation :

The butterfly network has less congestion than the entire binary tree, and it utilizes fewer switches and has a smaller diameter than the 2-D array, as shown in Table 1. The congestion for a 2-D array is constant and independent of the number of inputs and outputs, however this is not the true for a binary tree or a 2-D array. The structure of a binary tree, which is otherwise simpler than that of a butterfly, grows larger and more complex as the number of inputs and outputs grows. For binary tree representation, the root functions as a bottleneck. Despite the intricacy of butterfly networks, owing to the binary labelling of rows, routing a packet from input to output is fairly straightforward. At each step, one bit is repaired.

Network	*Diameter*	*Switch size*	*Switch count*	*Congestion*
Complete Binary tree	2logN+1	3×3	2N-1	N
Butterfly	logN+1	2×2	N(log(N)+1)	√N or √N/2
2-D array	2N-1	2×2	N ^2	2

Conclusion :

In this research, we explained the applications of graph theory in communication networks. Communication networks were represented as binary trees, 2D arrays, and butterfly networks. We compared all three representations in terms of diameter, switch size, number of switches and overload. Butterflies are less crowded than full binary trees. It also uses fewer switches and is smaller in diameter than the array. However, the butterfly is not a capture of the best features of each network, but a compromise between two. Congestion is minimized in 2D arrays, and congestion remain constant as the number of switches increases. Binary trees are not scalable because the complexity increases as the number of inputs and outputs increases.

References :

Volume 1, issue 2, October 2012 - ijaiem.org. (n.d.). Retrieved December 27, 2021, from https://www.ijaiem.org/volume1Issue2/IJAIEM-2012-10-11-017.pdf

Wikimedia Foundation. (2021, December 22). Graph theory. Wikipedia. Retrieved December 27, 2021, from https://en.wikipedia.org/wiki/Graph_theory

http://web.mit.edu/neboat/Public/6.042/communicationnetworks.pdf

https://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-042j-mathematics-for-computer-science-fall-2010/video-lectures/lecture-9-communication-networks/