All roads lead to Math: Enhancing math learning with spatial thinking
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Abstract:
Various of evidences proved that spatial thinking training have a positive effect on improving students' math abilities. Based on this foundation, this paper reviewed two training methods, mental rotation activities and spatial visualization activities that educators can use to support 3-15 years old students learning of spatial thinking on math. What's more, different activities could also cause different gender effect, papers based on mental rotation did not report special effect on gender, while most papers based on spatial visualization reported positive implication on girls' interest and performance which is worth noting.

Key Words:
spatial thinking, mental rotation, spatial visualization, gender effect

Introduction

Definition and Importance

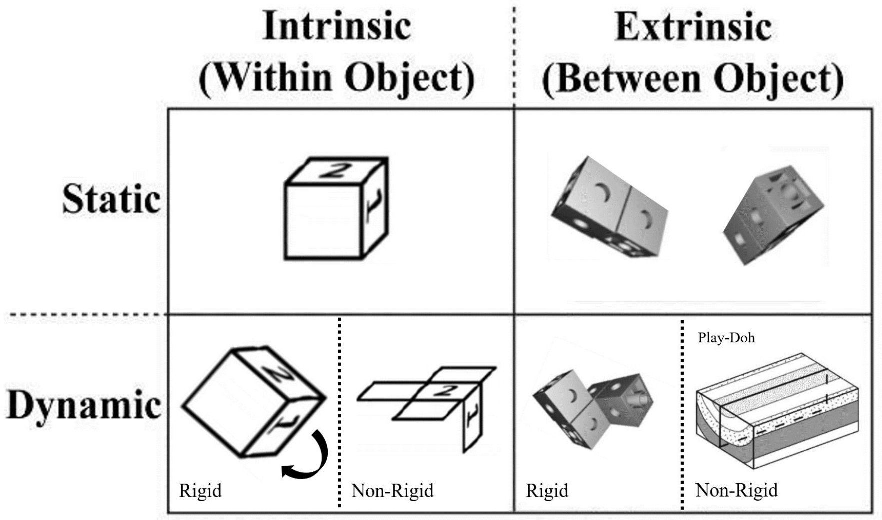
What is spatial thinking?

Spatial thinking, which also referred to as spatial reasoning and spatial perception (Kurt et al., 2023), represent the ability of constructing and manipulating the shape of objects, their locations, relationships, transformation and moving paths (Newcombe, 2009; Bruce et al., 2017; Cohen & Hegarty, 2012; Lee & Bednarz 2012). A widely used classification divided spatial thinking into three types, Spatial Visualization, Mental Rotation, and Spatial Orientation (Mix & Cheng, 2012; Linn & Petersen, 1985; McGee, 1979).

The typology of spatial thinking

A generic typology of spatial thinking proposed by Uttal et al. (2013), Newcombe and Shipley (2014) was based on two dichotomous factors, intrinsic/extrinsic and static/dynamic. Intrinsic and extrinsic depends on the spatial properties of an object and the object's location concerning others or a reference frame, while the static and dynamic factor can be distinguished by whether the object involves active transformations or not. According to this approach, spatial thinking can be divided into four broad categories: Intrinsic-Static, Intrinsic-Dynamic, Extrinsic-Static, and Extrinsic-Dynamic. Nonetheless, even in one type of spatial thinking, differentiation can hold significance. Adding Harris et al. (2013)'s finding might segmentation this typology, he pointed the importance of continuing identify the rigid body and non-rigid transformations. Although both of the two characteristics involve spatial transformations, but for rigid body, the structure of the object remains, while in non-rigid, the object itself is altered. Figure 1 is a categorization diagram based on the two categorization methods, and was formed with slight modifications on the diagram of Uttal et al. (2013).

Figure 1:



Uttal et al. (2013), Burte et al. (2019), Bruce & Hawes (2014) , Atit et al. (2015)

Spatial thinking and mathematics abilities

Sufficient studies have showed the malleability of spatial thinking and its prominent role in improving STEM learning (Uttal & Cohen, 2012), especially in mathematics education. According to students' mathematics performance in international assessments (e.g., PISA and TIMMS), weaker skills in shape and space domains were shown compared with abilities in other mathematics sub-domains (Gilligan-Lee et al., 2023), which demonstrated the need to improve such skills in students. Educators and researchers have been recognizing the importance of space education and calling for the increase of space instructions, since spatial thinking skills are direct and strongly correlated with the ability of solving such matters, and were proved benefit for several other types of mathematics problems (Sorby & Panther, 2020). According to extensive research, the connection between the two abilities, space and math, was proposed as the most mature and reliable one in cognitive psychology (Mix & Cheng, 2012).

The age of 3-15 years is the critical period for training spatial skills, due to the greatest potential for improvements in spatial reasoning skills at this age range (Maresch & Sorby, 2021). More over, the results of two meta-analysis included 29 studies (Hawes et al.,2022) and 217 studies (Uttal et al., 2013) proved evidence that during the period of 3-15 years old, the training effect of students' spatial thinking abilities can also better transfer to their mathematical performance.

Research Question：
Thus, this literature review paper is aiming at exploring:
How do educators support 3-15 years old students learning of spatial thinking on math?

Method

Study Inclusion Criteria

1. Students' age range must be focused on 3-15, including elementary school, middle school, and part of high school.
2. The selected studies must be supporting student learning of spatial thinking, and learning on math.
3. Only peer reviewed articles are included.

Literature Search Process

The initial research results by searching with several search terms from ERIC and PsycINFO for peer reviewed papers are shown in Table 1. Then, after reading the abstract of them, those researches do not fit the criteria were eliminated, 7 papers are found. Since some authors were noticed that been frequently cited in the literature in this field, I also found their Google Scholar homepages and read their 5 most cited papers related to the research question. 12 papers are found at last.

Results	Database	Search terms	Qualifies
7 results	ERIC, APA PsycInfo	"spatial thinking" or, "spatial reasoning" or, "spatial perception" and "math"	peer reviewed only

Findings

According to the review, educator can use two training methods to improve 3-15 years old students' math performance by enhancing their spatial thinking: training based on Mental Rotation, and based on Spatial Visualization.

Mental Rotation

Mental rotation is defined as one's ability to rotate stimuli in general (Uttal & Cohen, 2012), which is a rigid body, intrinsic-dynamic reasoning.

In order to test the effect of mental rotation on enhancing students' spatial thinking abilities within a classroom learning context, Bruce & Hawes (2014) implemented a 4-month Lesson Study intervention. Their focus was on training students aged 4-8 years in both 2D and 3D mental rotation activities, including compose, decompose, and mentally rotate some 2D and 3D figures. Additionally, students also engaged in constructing of 3D figures, for example, rotating and fitting cube formations together by referring to 2D pictures and photos. The findings illuminated that students aged over 5 years old in this study had a significant improvement in spatial thinking skills after the interventions in an authentic classroom context, and 2D mental rotation tasks yielded better enhancements compared to their 3D counterparts. This study also suggested that students across a diverse ability levels can participate in and benefit from mental rotation activities integrated into classroom settings.

After exploring the impact of mental rotation on enhancing spatial thinking, Hawes and colleagues (2017) explored further into the effect of using mental rotation activities as an intervention for training spatial reasoning and improve students' mathematical performance. Their experiment was implemented in dynamic classrooms, engaging students from 4 to 7 years old. Teachers and students were divided into experimental groups and control group, teachers in the control group used an inquiry-based approach to environmental science intervention, while teachers in the experimental group implemented an extensive spatial intervention for 32 weeks that integrated spatial lessons and activities instead of ordinarily mathematics instructions. This intervention featured geometry lessons, concentrating on 2D congruence, tasks involving composing, decomposing, and transforming areas, as well as quick challenge lessons entailed drawing, building, copying, and visualization exercises. The outcome illuminated that students in the spatial intervention group exhibited substantial advancements in solving symbolic number comparison problems compared to those in the control group. This study suggests that mental rotation based spatial training may benefits to fundamental numerical skills, which offer a glimpse into the potential broad impacts of spatial interventions in shaping mathematical competencies among young learners.

Beyond improving number comparison problems performance, mental rotation training also influence to foster students' numeracy skills. A cohort of 6 to 8 years old students who are developing basic calculation skills but not yet mastered, were conducted in Cheng and Mix's study (2013), either undertook conventional crossword puzzle tasks training in control group, or trained by mental rotation tasks in treatment group. Within the mental rotation task, children were presented with two segments of a flat shape and then prompted to identify one of four pictures that can demonstrate the shape integrated. Students also got feedback, they were provided with the two parts on separate pieces of card stock and asked to verify or revise their choices after aligning them, thus reconstructing the entirety of the shape. The results of the post-test exhibited that students in the experiment group performed a significant improvement in calculation problem-solving, especially in missing term problems (e.g., 4 + ____ = 11), while their counterparts in the control group who did not have observable improvement. This study underscores the potential of mental rotation as a spatial training way that enhance students numeracy skills.

Spatial Visualization

Spatial visualization is another important branch of spatial thinking skills which refers to the ability of perceiving complex spatial patterns and comprehending imaginary movements in space (Mix & Cheng, 2012). As a broad category, there are many different training methods for spatial visualization, including video games, mental paper folding, and ways to combine a range of activities.

Video games

As a way of extrinsic-dynamic training, sufficient evidence proved that video games have positive effect on enhancing spatial visualization abilities (Gagnon, 1985; Dorval & Pépin, 1986; Milani et al., 2019). A number of researchers have further explored its enhancement on math.

With the aim of exploring if students' performance in mathematics can be developed through spatial visualization training by video games, students in grades 4 to grade 5, aged from 9 to 11 years old were engaged in a two-year project named "A me gli occhi" ("Eyes on Me") (Freina et al.,2017). In addition to attended regular classes with control-group students, students in the game group were treated to attached activities involving individual interaction with Android tablets. Their training encompassed a various of activities, including engaging with digital games and participating in a Minecraft contest. After training, students took standardized math tests, evaluating their progress across a variety of mathematical skills. The assessments encompassed written number operations, identifying the larger value between two numbers with decimals, deducing a number from word names using place value rules, ordering numbers with decimals, and word problems demanding a two-step reasoning process. The comparison of pre test and post test indicated that game group not only outperformed control group in tasks involving ordering and word problems, but the extent of improvement was only statistically significant exclusively within the game group. This study indicates that video game, as a way of spatial visualization training, have positive effect on students math performance, especially in ordering and word problems.

Compelling evidence was provided that digital games exerts a noteworthy influence on learners' spatial thinking skills and math performance, its benefits go far beyond that. Spatial thinking with digital games interventions is also related with gender

difference. TOPTAŞ and colleagues (2012) explored the relationship by undertaking an experiment within classroom context, focusing on 8th-grade students.
In the experimental settings, students in the computer group engaged in identical activities as the control group, followed by immersive lessons using a 3D modeling program called Google SketchUp (GSU) for 5 weeks. The test results revealed a significant performance boost in the experimental group. What's more, girls demonstrated higher total performances at the post-test when contrasted with boys, which provided potential evidence that girls might gain more effect in digital games as a spatial visualization training.

Mental Paper Folding

In addition to video games, the researchers found more types of training that appeals to gender effect. Mental paper folding, as a non-rigid, intrinsic-dynamic reasoning, is another training method that can also improve both spatial visualization and girls' interest.

Origami is the most iconic activity in mental paper folding. Taylor and Hutton's (2013) study introduced a program called Think3d!, which stands for "Transforming Paper Into 3D Objects". This program has three units and was specifically designed to enhance students' spatial thinking. In order to test the effect of Think3d!, they recruited fourth-grade students to participate in interventions and tested them by three spatial thinking assessments and feedback questionnaires after the training. The results revealed that students who accepted the intervention demonstrated greater improvements in mental paper folding. Furthermore, students' feedback, as captured in the questionnaires, indicated that girls found the program more enjoyable than boys. This study suggests that the origami program led to spatial thinking improvements and increased engagement among the students, especially for girls. Similarly, Burte and colleagues (2015) used this origami activity as an intervention to test their new-proposed measurement of mental paper folding skills, which is called Make-A-Dice. And they came to same conclusion, proving that origami can enhance students' mental paper folding abilities.

After confirming origami's unique appeal to students' spatial thinking skills and to girls' interest, researchers went on providing that origami could also enhance students' performance on mathematics. Burte and colleagues (2017) examined the impact of this origami program on 3 to 6 grade elementary students' spatial and mathematical thinking. Students were asked to tested by a math assessments consisted of eight to ten mathematics problems sourced from Common Core mathematics worksheets, including visual problems, word problems, real-world problems and abstract problems. Their performance indicated that the two upper grades improved on visual problems, and the accuracy of real-world problems have improved in all grades, math accuracy gains were overall more evident in higher grades. The origami training provide some positive indications on enhancing older grades students' math performance.

Combining activities

With the exception of video games and origami, most interventions that promote spatial visualization are collections of multiple activities together.

In a study conducted by Lowrie and colleagues (2019), a cohort of grade 5 and 6 students, spanning the age range of 10 to 12, was divided into control and treatment groups. The three-week intervention program taught by classroom teachers was combined by several ranges of spatial visualization activities, including paper-folding, reflection, and symmetry, and assembled different materials, including gesture, student encoding and the digital games. In parallel, students in the control classes took standard mathematics instruction. The analysis revealed that participants in the intervention group exhibited significant improvement in their spatial reasoning performance than the control group. Moreover, the intervention group also demonstrated a marked enhancement in their mathematics test performance, particularly in the domains of geometry and word problems. The results add to evidence that spatial visualization enrichment program in classroom context improved students' math abilities.

Further studies have shown that the combination of spatial visualization activities with educational courses have more robust improvement results compared to standalone spatial visualization training. In a recent study, Lowrie and Logan (2023) implemented spatial visualization training in a learning program combined with digital games for students at grade 4. The first treatment group honed in an isolated spatial training that completed with digital spatial exercises, while the second one took a more integrative approach with spatial visualization activities embedded into regular math classes, coupled with the inclusion of digital spatial training. Meanwhile, the third group serving as a control group that maintained ordinary classes. The training effect revealed that the embedded intervention group showed substantial additive effects. The isolated intervention program exhibited less transfer effect on math, but still better when compared to the business-as-usual control group. This study supports the claim that integrated spatial materials and digital tools with daily math lessons can enrich students' mathematics performance.

Furthermore, studies of combining activities have also shown their particular effect on girls' spatial thinking abilities and performance in mathematics.
In a study conducted by Sorby and Veurink (2019), their 7th-grade participant students were immersed in an intervention that encompassed several spatial visualization activities. Students engaged in activities that was blended of combining solids, object reflections, planes of symmetry and isometric sketching. Subsequently, they underwent the state's annual standardized mathematics test as a measurement of their math abilities. The outcome revealed that students who trained by the spatial visualization activities exhibited higher gains on standardized math tests compared to

their counterparts without such training. Notably, the study also suggested spatial visualization training playing a pivotal role in fostering success among girls in passing the math assessment, and the success was at least partially attributed to improvements in geometry performance among girls. This study underscores the significant positive impact of spatial skills training in shaping educational outcomes, with a distinct emphasis on its particularly benefit for girls.

Numerous studies have demonstrated the contribution of spatial visualization to student achievement in mathematics. Beyond its direct impact on mathematical performance, spatial visualization has been provided with additional benefits for students engaged in the learning of math. Rabab and Veloo (2015) delved into the relationship between mathematics attitude and mathematics achievement through spatial visualization. Their study involved the collection of data through three instruments: questionnaire, math assessments, and test of spatial visualization. The instruments spanned five critical sections: mathematics attitude, mathematics motivation, mathematics self-regulation, math self-efficacy, and mathematics anxiety. Their analysis suggest that spatial visualization emerged as a potent mediator, fully mediating the relationship between motivation, math anxiety, and mathematics achievement, while also partially mediating the relationship between attitude and mathematics achievement. Thus, spatial visualization can also be considered as a dynamic bridge that enhances students engagement with mathematics.

Discussion

Overall, this paper reviewed two training ways based on mental rotation and spatial visualization in classroom context to support 3-15 years old students learning of spatial thinking on math.

A notable finding surfaced that despite girls exhibit higher scores of spatial anxieties (Wei et al., 2018), the review unveiled that spatial visualization training could potentially serve as an effective treatment, and most of the articles on spatial visualization reported unique interest appeal and performance-enhancing effects for girls' spatial thinking and math learning. Yet this particular facilitation effect is not reported according to the review of literature on mental rotation. Conversely, a subset of literature suggests that girls may not perform as well as boys in mental rotation (Voyer et al., 1995; Ramirez et al., 2012; Moè, 2018). Future research could focus on what makes the difference.

Additionally, according to the reviewed, current studies are mainly focused on preschool and elementary school students, only three studies reference to the training of secondary school students. However, spatial thinking is not only important for young children, in light of a recent meta-analysis (Hawes et al.,2022), the impact of spatial training on mathematics proficiency expands with age. Thus, future studies can shift more attention to secondary school students to contribute more teaching methods suitable for secondary school classrooms.

Reference:

Atit, K., Gagnier, K., & Shipley, T. F. (2015). Student gestures aid penetrative thinking. *Journal of Geoscience Education*, *63*(1), 66–72. https://doi.org/10.5408/14-008.1

Bruce, C. D., Davis, B., Sinclair, N., McGarvey, L., Hallowell, D., Drefs, M., Francis, K., Hawes, Z., Moss, J., Mulligan, J., Okamoto, Y., Whiteley, W., & Woolcott, G. (2017). Understanding gaps in research networks: using "spatial reasoning" as a window into the importance of networked educational research. *Educational Studies in Mathematics*, *95*(2), 143–161. https://doi.org/10.1007/s10649-016-9743-2

Bruce, C. D., & Hawes, Z. (2014). The role of 2D and 3D mental rotation in mathematics for young children: What is it? why does it matter? and what can we do about it? *ZDM*, *47*(3), 331–343. https://doi.org/10.1007/s11858-014-0637-4

Burte, H., Gardony, A. L., Hutton, A., & Taylor, H. A. (2019). Make-A-dice test: Assessing the intersection of mathematical and spatial thinking. *Behavior Research Methods*, *51*(2), 602–638. https://doi.org/10.3758/s13428-018-01192-z

Cheng, Y.-L., & Mix, K. S. (2013). Spatial training improves children's Mathematics ability. *Journal of Cognition and Development*, *15*(1), 2–11. https://doi.org/10.1080/15248372.2012.725186

Cohen, C. A., & Hegarty, M. (2012). Inferring cross sections of 3D objects: A new spatial thinking test. *Learning and Individual Differences*, *22*(6), 868–874. https://doi.org/10.1016/j.lindif.2012.05.007

Dorval, M., & Pépin, M. (1986). Effect of playing a video game on a measure of spatial visualization. *Perceptual and Motor Skills*, *62*(1), 159–162. https://doi.org/10.2466/pms.1986.62.1.159

Freina, L., Bottino, R., Ferlino, L., & Tavella, M. (2017). Training of spatial abilities with Digital Games: Impact on mathematics performance of Primary School Students. *Lecture Notes in Computer Science*, 25–40. https://doi.org/10.1007/978-3-319-71940-5_3

Gagnon, D. (1985). Videogames and spatial skills: An exploratory study. *ECTJ*, *33*(4), 263–275. https://doi.org/10.1007/bf02769363

Gilligan-Lee, K. A., Hawes, Z. C., & Mix, K. S. (2023). Spatial thinking as the missing piece in mathematics curricula. *Npj Science of Learning*, *7*(1). https://doi.org/10.1038/s41539-022-00128-9

Hawes, Z., Moss, J., Caswell, B., Naqvi, S., & MacKinnon, S. (2017). Enhancing children's spatial and numerical skills through a dynamic spatial approach to early geometry instruction: Effects of a 32-week intervention. *Cognition and Instruction*, *35*(3), 236–264. https://doi.org/10.1080/07370008.2017.1323902

Hawes, Z. C., Gilligan-Lee, K. A., & Mix, K. S. (2022). Effects of spatial training on mathematics performance: A meta-analysis. *Developmental Psychology*, *58*(1), 112–137. https://doi.org/10.1037/dev0001281

Lee, J., & Bednarz, R. (2011). Components of spatial thinking: Evidence from a spatial thinking ability test. *Journal of Geography*, *111*(1), 15–26. https://doi.org/10.1080/00221341.2011.583262

Linn, M. C., & Petersen, A. C. (1985). Emergence and characterization of sex differences in spatial ability: A meta-analysis. *Child Development*, *56*(6), 1479. https://doi.org/10.2307/1130467

Lowrie, T., Logan, T., & Hegarty, M. (2019). The influence of spatial visualization training on students' spatial reasoning and Mathematics Performance. *Journal of Cognition and Development*, *20*(5), 729–751. https://doi.org/10.1080/15248372.2019.1653298

Lowrie, T., & Logan, T. (2023). Spatial Visualization supports students' math: Mechanisms for spatial transfer. *Journal of Intelligence*, *11*(6), 127. https://doi.org/10.3390/jintelligence11060127

Milani, L., Grumi, S., & Di Blasio, P. (2019). Positive effects of videogame use on visuospatial competencies: The impact of visualization style in preadolescents and adolescents. *Frontiers in Psychology*, *10*. https://doi.org/10.3389/fpsyg.2019.01226

Moè, A. (2018). Mental rotation and mathematics: Gender-stereotyped beliefs and relationships in primary school children. *Learning and Individual Differences*, *61*, 172–180. https://doi.org/10.1016/j.lindif.2017.12.002

Kurt, G., Önel, F., & Çakıoğlu, Özge. (2023). An investigation of Middle School Students' Spatial Reasoning Skills. *International Electronic Journal of Elementary Education, 16(1),* 123–141. Retrieved from https://iejee.com/index.php/IEJEE/article/view/2109

Maresch, G., & Sorby, S. A. (2021). *Perspectives on spatial thinking*. *Journal for Geometry and Graphics Volume 25,* 271–293 https://www.heldermann-verlag.de/jgg/jgg25/j25h2mare.pdf

McGee, M. G. (1979). Human spatial abilities: Psychometric Studies and Environmental, genetic, hormonal, and neurological influences. *Psychological Bulletin*, *86*(5), 889–918. https://doi.org/10.1037/0033-2909.86.5.889

Mix, K. S., & Cheng, Y.-L. (2012). The relation between space and math. *Advances in Child Development and Behavior Volume 42*, 197–243. https://doi.org/10.1016/b978-0-12-394388-0.00006-x

Newcombe, N. S. (2009). *Picture this: Increasing math and science learning by improving spatial thinking.* American Educator. https://eric.ed.gov/?id=EJ889152

Rabab'h, B., & Veloo, A. (2015). Spatial visualization as mediating between mathematics learning strategy and mathematics achievement among 8th grade students. *International Education Studies*, *8*(5). https://doi.org/10.5539/ies.v8n5p1

Ramirez, G., Gunderson, E. A., Levine, S. C., & Beilock, S. L. (2012). Spatial anxiety relates to spatial abilities as a function of working memory in children. *Quarterly Journal of Experimental Psychology*, *65*(3), 474–487. https://doi.org/10.1080/17470218.2011.616214

Sorby, S. A., & Veurink, N. (2019). Preparing for stem: Impact of spatial visualization training on Middle School Math Performance. *Journal of Women and Minorities in Science and Engineering*, *25*(1), 1–23. https://doi.org/10.1615/jwomenminorscieneng.2018024516

Sorby, S. A., & Panther, G. C. (2020). Is the key to better Pisa math scores improving spatial skills? *Mathematics Education Research Journal*, *32*(2), 213–233. https://doi.org/10.1007/s13394-020-00328-9

Taylor, H. A., & Hutton, A. (2013). Think3d!: Training spatial thinking fundamental to stem education. *Cognition and Instruction*, *31*(4), 434–455. https://doi.org/10.1080/07370008.2013.828727

Uttal, D. H., & Cohen, C. A. (2012). Spatial thinking and STEM education. *The Psychology of Learning and Motivation*, 147–181. https://doi.org/10.1016/b978-0-12-394293-7.00004-2

Uttal, D. H., Meadow, N. G., Tipton, E., Hand, L. L., Alden, A. R., Warren, C., & Newcombe, N. S. (2013). The malleability of spatial skills: A meta-analysis of training studies. *Psychological Bulletin*, *139*(2), 352–402. https://doi.org/10.1037/a0028446

Voyer, D., Voyer, S., & Bryden, M. P. (1995). Magnitude of sex differences in spatial abilities: A meta-analysis and consideration of critical variables. *Psychological Bulletin*, *117*(2), 250–270. https://doi.org/10.1037/0033-2909.117.2.250

Wei, W., Budakova, A., Bloniewski, T., Matsepuro, D., & Kovas, Y. (2018). Spatial anxiety and spatial performance in university students in Russia and China. In The European Proceedings of Social & Behavioural Sciences, 771–781. https://www.europeanproceedings.com/article/10.15405/epsbs.2018.11.02.89