Better together: Facilitating math learning with gestures and questions

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Contribution list:

Importance	Lujie
Literature review	Lujie (Gesture part)
	Andrea (Question part)
Research Plan	Lujie (Intervention design, Research plan structure modify, Appendix A-C)
	Andrea (Intervention modify, Research plan structure design, Appendix D)
Data analysis plan & Proposed impact	Together

Importance

As the very gateway and key of scientific inquiry (Bacon, 1900), the significance of mathematics is beyond dispute. Learning and enhancing one's mathematics knowledge is crucial for achievement in academia, economics, and the broader life (Rittle-Johnson, 2017). Moreover, developing mathematical skills can increase the percentage of students pursuing STEM careers, thereby increasing a nation's global competitiveness (Maass et al.,2019). However, a disconcerting reality revealed by the outcomes of the PISA exam, paints a less-than-satisfactory picture of students' mathematical performance in many countries (Maass et al.,2019). Compounding this educational challenge is the pervasive impact of the COVID-19 pandemic (Meeter, 2021). This distressing trend underscores the urgency of implementing interventions to improve students' mathematical performance. The optimal solution for promoting students' math learning is to find a relatively better way of teaching in the classroom, since even slight improvements in teaching methodologies can have substantial positive outcomes for the education of countless students (Stigler & Hiebert, 2004). In this paper, we investigate a rather ideal teaching approach that integrates both nonverbal gestures and verbal questions in the realm of mathematics education.

Literature Review

Questions shape learning

It is common practice for teachers to use questions to guide learning - high school teachers ask three to four hundred questions in one day (Levin & Long, 1981). The effects and importance of questions in education cannot be overstated; as a form of active learning, they serve to engage students, stimulate critical thinking, and facilitate a deeper understanding of the subject matter (Huang et al., 2015). Questions as a form of verbal communication also help instructors assess the learners' comprehension and identify areas where further explanation may be needed. Throughout the processes of mathematical problem-solving, the use of questions can assist in identifying essential concepts and transferring problem-solving strategies to similar sets of problems, ultimately leading to the goal of generalization (Blanton & Kaput, 2002). However, as there is a variability among the question types teachers asked and the purpose for generating certain questions, the results may differ. Some may ask questions to check understanding, while others may elicit thinking and reflection. In the Franke et al. (2009) study, researchers did naturalistic observation and analysis on three elementary classrooms focusing on algebraic reasoning. For the question that involved an equation, subtraction and addition (e.g. $11 + 2 = 10 + \_$), after the initial question, the question styles varied a lot. Teachers might choose to ask a general question for explanation, specific question according to students' explanation, or guide students towards particular answers and give them a chance to respond. Leading questions, which are questions that serve guiding purposes largely reflect teachers' thoughts on the problem. The study indicated that the type of questions posed by teachers can influence students' responses, underscoring the importance of guiding students' thinking and providing them with appropriate questions in a timely manner.

Specific questions and general questions

Different formatting of questions can not only affect students' participation, but can also affect learning outcomes. Specific questions that target the center of the problem and point out the structural component, work more effectively than general questions that only provide procedural scaffolding. Lee & Chen (2009) presented a frog-jumping problem to junior high school in which an equal number of green and red frogs were depicted standing on lotus leaves on both sides of a riverbank. The frogs were arranged in a line and can only jump to vacant leaves in order to cross to the other side, but they cannot jump to a leaf that is farther away or has frogs between them. The number of frogs varied, and students were asked to predict the number of total jumps. In order to solve the problem, students needed to come up with a final equation between the number of frogs and the total jumps. In this case, a general question would ask for solving an equation with the number of frogs and its corresponding jumps, while a specific question would ask the students to identify the condition, draw down the pattern and the relationship between frogs and jumps. Once prior knowledge was accounted for, students who received specific questions demonstrated an improved ability to generalize the above frog-jumping questions to other questions that asked for forming equations from two variables and to justify their responses.

A similar result could be seen from the Kramarski et al. study (2013). This study compared two questioning approaches, a general approach which was based on the “IMPROVE” question for “comprehension, connection, strategy, and reflection” and a context-specific approach based on “WWWH” on what, when, why, and how. General questions include identifying the task and selecting strategies, while the context-specific questions point out the integration of tasks and strategies with specific elements included in the questions. The intervention was conducted for a total period of three weeks, and researchers administered a test to assess the algebraic skills of the students before and after the intervention. While there was no distinction observed in immediate algebraic procedural task performance, participants in the specific questions group demonstrated significantly greater improvement in long-term transfer to novel tasks, in comparison to the general questions group.

Meta-cognitive questions

In addition to context-specific questions that have a specific direction towards answers, the effectiveness of reflection questions designed to prompt meta-cognitive processes has also been investigated in the realm of mathematics education. Meta-cognitive capabilities are strongly correlated with numerical problem-solving abilities (Cornoldi, 1997). The definition of metacognitive prompting is the use of an external stimulus to activate reflective thinking or to encourage the use of strategies in order to improve the learning process (Hoffman & Spatariu, 2008). Putting emphasis on self-evaluation, meta-cognitive questions help to direct thought during learning processes (Bannert & Mengelkamp, 2013). Past work has outlined meta-cognitive questioning with 3 major purposes: planning, monitoring, and evaluating (Zepeda et al., 2015). Sample question would be “Does the solution make sense?” and “How can I solve it in another way?”. Kramarski and Friedman (2014) examined the differences between solicited questions, which gave participants the chance to choose if they wanted to see the meta-cognition questions, and unsolicited group, which questions would automatically appear, and the control group where no meta-cognition questions were given (Kramarski & Friedman, 2014). Eighth-grade students benefited most with automatic meta-cognitive questions, displayed less cognitive load, and showed a higher mathematical understanding.

Limitations of question-based learning

Several limitations can be identified within the literature on question-based learning in mathematical education. A significant proportion of these studies focuses on older children in middle school, often examining more intricate problems. For instance, while solving equations involving two variables students need to demonstrate a deep understanding and more advanced mathematical problem-solving skills. However, the concentration on older students and more complex problems leaves a gap in the literature regarding the effectiveness of question-based learning for younger children and simpler mathematical problems. As questions could provide direction and procedural scaffolding while eliciting thinking processes, they could be tailored to suit different age groups and vary problem complexities. Moreover, in actual classroom settings, many questions are posed verbally, yet a significant number of research implemented questions through online means, creating a deficiency in investigating the dynamics of traditional classroom environments.

While the limitations of question-based learning in mathematics education have been identified, it is essential to consider other complementary methods to enhance students' understanding and performance. One such approach involves the use of non-verbal communication, specifically gestures. By examining the role of gestures alongside questions, we aim to provide a more comprehensive understanding of effective teaching strategies in mathematics education.

Gestures in math teaching

From the studies mentioned above, it is evident that questions, as an effective verbal communication technique, can improve students' performance in mathematics. However, we do not advocate that it should lead to the neglect of other pedagogy. Just as the way human perception of the world is multifaceted, teachers can also help students comprehending and internalizing knowledge through diverse teaching ways. Therefore, it is prominent turning our attention to other non-verbal teaching methods. Gestures, as a compelling method, possessing the unique capacity to not only express the emotion of speakers, but also to complement verbal speech in order to enrich the conveyance of meanings (Goldin-Meadow, 1999). In the context of mathematics teaching, gestures embody a series of physical movements and actions to convey mathematical meaning, such as pointing, drawing, and hand motions, as defined by Aldugom (2020). While playing a crucial communicative role in teaching mathematics (Alibali, 2014), gestures have been proven to bring about profound benefits to students' learning.

Benefits of Gestures

Instruction-related gestures provided by math teachers are pivotal in facilitating students' comprehension. In Goldin-Meadow's study (1999), teachers individually taught students aged 8 to 11 mathematical equivalency concepts (such as $3 + 7 + 5 = \_ + 5$) in math classes. Two strategies were employed, one of which included using gestures to support verbal explanations, while the other did not involve gestures. According to the comparison between the two groups, students who received appropriate gestures along with speech could extract math problem-solving strategies (e.g., making both sides equal by adding 3 and 7) from the teacher's gestures and rephrase them in their own words. They also performed higher correctness rate when solving the basic mathematical equivalence problems. Goldin's research indicates that employing gestures appropriately to reinforce information may stands as an effective way to improve mathematical comprehension.

Transfer is an essential learning skill, which gestures can also contribute to students' ability on that as well. Students accepted gesture instructions performed better in other types of math problems (Cook et al., 2013). Cook and her colleagues used an animated instructional agent called Avatar to teach children with an average age of 9 mathematical equivalence lessons. The Avatar, acting as a teacher, either gestured or remained still while maintaining other consistent movements. Following exposure to a videotaped lesson on mathematical equivalence, students were asked to solve some transfer problems, for example, which have either equal addends located in different positions or no equal addends. The test result demonstrated that students in the gesture group exhibited superior knowledge transfer to different types of problems.

As a key role for students' future learning, the influence of gestures in retaining math learning can not be underestimated. Students immersed in lessons enriched by gestures exhibited not only a more profound understanding of the y-intercept, but also retained this knowledge more effectively than their peers who received instruction devoid of enhanced gestures (Alibali, 2013).

Collectively, these findings furnish compelling evidence that incorporating gestures into instruction serves as a powerful pedagogy for strengthening students' capacity to comprehension, transfer and retain their mathematical knowledge.

Gestures with speech

Previous studies showed using gestures imparts tangible benefits for math learning. Based on that foundation, subsequent studies explored the effect of combing gesture and speech. According to Cook's (2013) study, incorporating gestures can improve children's learning of abstract concepts compared to using speech only. They conducted an experiment involving students from 7 to 10 years old to compare the learning outcomes of speech-alone and gesture-speech classes. The result of mathematical equivalence tests through addition showed that children exposed to speech and gesture had a higher accuracy rate compared to their counterparts who relied on auditory input only. This study suggests the combination effect of gesture and speech plays a more dynamic role in enhancing students' mathematical performance than using speech alone.

Moreover, the same study showed that the performance of students who learned from both speech and gesture improved even after a 24-hour delay, while the role of speech was minimized when it happened alone; those who only heard speech did not show any improvement after the delay (Cook, 2013). According to the findings of previous studies, memory consolidation happens within the first 24 hours after learning (Diekelmann & Born, 2010). The result of this study also showed the possibility that adding gestures is helpful in enhancing the learning of abstract concepts. Cook noted that in the speech-alone condition, teachers merely gave out rather descriptive and general instructions in a plain tone. This served as more of a way of informing than actively teaching, which could potentially hinder students' learning.

Simultaneously, Alibali and colleagues (2013) focused toward establishing a shared ground of teachers' gestures and speech by resolving trouble spots in mathematics classes. The researchers investigated how teachers implement micro-interventions during middle-school math lessons when students demonstrated signs of struggling or a lack of comprehension. These interventions characterized by their small-scale and was spontaneity acted as the lesson progresses. The outcome revealed the benefit for math teachers using adaptive gestures in these micro-interventions to establish a common ground when instructional communication breaks down, which contributed to integrating gestures and language in the classroom.

Limitations of gesture-integrated teaching

In conclusion, for elementary school students, more often in the 9-10 year old age group, gestures have been shown to facilitate their performance on mathematical equivalence problems. Further, combining gestures with verbal forms has a more significant impact on students' mathematics performance. However, the literature also shows some unexplored directions. Firstly, the significance of spoken language was underemphasized in gesture literature. In past studies, teachers only combined their gestures with plain descriptive speech, simply describing the procedure toward different strategies for solving the target problem. It might, to some extent, undermine the goal of explanation and attention to the direction among gestures. A second limitation is that prior studies lack an active teaching style. These studies did not strongly encourage student engagement, foster motivation, or stimulate thinking, which might not maximize the gesture teaching effectiveness.

Research Gap

In addition to their individual limitations, the literature reveals some shared constraints when combining question-based learning and gesture-integrated learning. First, most studies on question-based learning as well as gesture-integrated learning are conducted in lab or online settings. Students either receive questions on mathematics equation problems or receive instructions with gestures on equivalence problems alone. However, most learning occurs in a classroom environment. Further research could look at how lab studies can be replicated in real classroom settings. Furthermore, based on the test-enhanced learning theory proposed by McDaniel et al. (2013), past research has not thoroughly investigated the integration of optimal teaching practices. Most research employs and studies a single mechanism, without incorporating several teaching practices for an addition or interaction effect.

Study overview

To tackle the existing gaps in current research, we would propose using questions as an active teaching strategy, which can help direct students' selective attention on essential constructs (Reynolds & Anderson, 1982). Combining questions with gestures may lead to more effective outcomes, such as a better performance in targeted problems as well as in novel transfer problems.

Research Plan

Study 1

The aim of Study 1 is to replicate the effect of gestures from previous literature and test our hypothesis that gestures and questions can enhance math performance and transfer compared to questions only.

Participants

We will recruit participants from public schools in Tennessee, specifically 6 classes of students between 9 and 10 years old. To ensure standardization of the socioeconomic background of our participants, we will select public schools where the student population comes from middle socioeconomic status families as eligible to participate.

Procedure

A match group randomized design will be used to assign 6 classrooms to one of the two conditions, the gesture-question groups and the question only groups.

Before the intervention, a pre-test will be done to test students' initial math proficiency. Students will be given 6 problems that assess the knowledge of mathematical equivalence in two formats, + C format and P + format (e.g., a + b + c = ? + c; p + q + r = p + ?).

During our intervention, the researcher will take the role of a guest lecturer instead of the regular teachers. This is due to concerns that teachers might feel uneasy about abstaining from using gestures, become distracted, or inadvertently use gestures without realizing it.

For the gesture-question groups, questions include specific questions and meta-cognitive questions to promote students' thinking (Zepeda et al., 2015) (see Appendix A and B for detailed wording on the questions that include the use of gestures). The meta-cogitative questions in our study paradigm is adapted from the study of Fyfe & Nelson (2022), and the two strategies to solve the mathematical equivalence problem is adapted from the study of Congdon et al. (2017): Equalizer (EQ) strategy and Add-Subtract (AS) strategy. The EQ strategy is a conceptual strategy; its core principle is that the two sides of an equation must be equal. The AS strategy is a procedural strategy that adds addends on the left side of the equation and then subtracts the number on the right side from that total to get the correct answer. The experimenter will explain how to solve the math problem by using the EQ strategy in question and simultaneously performing the AS strategy in gesture. The question groups will only contain the above intervention (including questions and explanations) with no gestures.

After the intervention, the post-test will consist of two parts, immediate test and follow-up tests. Immediate tests will include 6 similar basic problem sets from the pre-test and 5 generalization problems that use a more varied format of mathematical equivalence. Follow-up tests will proceed 1 day after instruction, including 4 basic problems and 5 generalization problems. The basic problems will include + C format question and P + format question. The generalization problems will include three problem types. Some of the generalization problems will have a blank space on the left side of the equation rather than on the right, some will not contain a pair of equal addends, and others will have both of these characteristics (see Appendix C for generalization problems examples).

Study 2

The aim of Study 2 is to examine the effect of questions and test our hypothesis that gestures and questions can enhance math performance and transfer compared to gestures and speech.

Participants

We will recruit participants from public schools in Tennessee, specifically 6 classes of students between 9 and 10 years old from middle SES families.

Procedure

A similar procedure as Study 1 will be adopted, assigning six classrooms to one of the two conditions, the gesture-question groups and the gesture-speech groups. Pre-tests and post-tests will be the same in Study 1.

During our intervention, the gesture-question groups work similarly as the Study 1, with revisions if necessary, while in the gesture-speech groups, students will receive class combining gestures with speech simultaneously (see Appendix A and B for detailed wording). The researcher will take on the role of teacher in a manner similar to that of Study 1.

Pilot Study

The pilot study will be implemented after Study 1 and Study 2 shown that combining gestures with appropriate questions is effective when implemented by researcher. Our aim of the pilot study is to measure whether the new technique can still promote students' performance in mathematics in a real classroom outside lab settings.

Participants

We will recruit participants from an elementary school in Tennessee, specifically a class of students ages between 9 and 10 years old, who come from middle-class families.

Procedure

Pre-tests and post-tests will be the same in Study 1.

The intervention will involve a real teacher using the teaching style that incorporates a combination of gestures and questions, and work similarly to the the gesture-question groups in Study 1, with revisions if necessary.

After the pilot, we will interview the teacher and ask what she/he thinks about this new teaching style, what difficulties did she/he encountered during implementation, and if there are any suggestions for the researchers to improve (see Appendix D for complete interview questions).

Data analysis plan

For both pre-and post-test, we will calculate the percentage of correct answers among all answers for immediate +C and P+ format questions and generalization questions. For the pilot study, we will conduct a paired sample t-test to examine if the means of the post-test scores were significantly higher than the pretest scores, meaning that if the teaching method of gesture and question could lead to a significant improvement in student performance. For both Study 1 and

Study 2, we will conduct two linear mixed effects models on gesture and question group comparing to question only group (Study 1) as well as on gesture and question group comparing to gesture and speech group (Study 2). One model will predict the performance on immediate posttest +C and P+ format questions and the other will predict the performance on generalization questions. For both models, pre-test scores will be controlled as covariates, with random effects by students.

Proposed impact

Overall, this present work assesses the combined approach of gesture-integrated learning and question-based learning within a real-world classroom environment. Most existing research has been conducted in lab settings, therefore, investigating the efficacy of this combined approach in an authentic classroom context is crucial. Ideally, it may provide us some potential evidence on the influence of these teaching methods under the unique social and attentional constraints present in the classroom, and will probably helpful to future studies. Moreover, it will enable us to evaluate whether these lab-tested approaches can be successfully implemented as an effective intervention in educational settings. Thus, this work has the potential to create a more dynamic and interactive learning environment that encourages student engagement and promotes better understanding and retention of the material, by incorporating both questioning techniques and gestures.

Appendix A

Guide for using gestures

When referring to “ equal sign”.	Gesture to draw circles on the equal sign.
When mentioning “two sides”.	Moves hands in a back-and-forth motion beneath the two portions of the equation.
When mentioning “left side”.	Move the left hand in a back-and-forth motion beneath the left portion of the equation.
When mentioning “right side”.	Move the right hand in a back-and-forth motion beneath the right portion of the equation.
When mentioning numbers in the equation.	Gesture to point to each addend.
When mentioning the unknown number (?) in the equation.	Gesture to point to the question mark.

Appendix B

Guide for combining gestures with question and speech

Example question: 7+8+5 = ?+5.

	Gesture + Question (simultaneously)		Gesture + Speech (simultaneously)
Using EQ strategy	Question 1 : (metacognition question)	“What information is given?” “There are two sides [gesture] to this problem.” “One on the left side [gesture] of the equal sign [gesture].” “And one on the right side [gesture] of the equal sign	“The information given is:” “There are two sides [gesture] to this problem.” “One on the left side [gesture] of the equal sign.” “And one on the right side [gesture] of the equal

		[gesture]." "The left side is 7+8+5 [gesture], and the right side is ? + 5 [gesture]."	sign." "The left side is 7+8+5 [gesture], and the right side is ? + 5."
	Question 2: (specific question)	"As there is an equal sign [gesture] here, what does that mean for the two sides [gesture] of the sign? " "Yes, the left side [gesture] and right side [gesture] are equal."	"There is an equal sign [gesture] here, it means that the left side [gesture] and right side [gesture] are equal."
Using AS strategy	Question 3: (metacogniti on question)	"Now, what step do we need to take to get the right answer?" "First, there is no unknown number on the left side [gesture] of the equal sign [gesture]. So, we add up the numbers on the left side [gesture] to get a result." "Then, find the unknown number [gesture] that will make the right sign [gesture] of the equal sign [gesture] have the same result."	"The step do we need to take to get the right answer is." "First, there is no unknown number on the left side [gesture] of the equal sign [gesture]. So, we add up the numbers on the left side [gesture] to get a result." "Then, find the unknown number [gesture] that will make the right sign of the equal sign [gesture] have the same result."
	Question 4: (specific question)	"So, for this problem, what is the result of the left side [gesture]?" "Correct, it is 20." [draw underline below the left side and write 20] " What number will make the right side [gesture] have 20 also?" "Right, 15, because 15 + 5 is also 20." "Now we get the answer 15.	"So, for this problem, the result of the left side[gesture] is 20." [draw underline below the left side and write 20] "15 will make the right side [gesture] have 20 also." "Now we get the answer 15.
	Question 5: (meta-cognitive question)	"After getting the answer, how can we check the solution is correct?" "We have 20 on the left side [gesture] of the equal sign [gesture], and 20 on the right side [gesture] of the equal sign [gesture]. So we know	"After getting the answer, we check if the solution is correct." "We have 20 on the left side [gesture] of the equal sign [gesture], and 20 on the right side [gesture] of the equal sign [gesture]. So

		we solved it correctly.”	we know we solved it correctly.”

Appendix C

Generalization problems in follow-up tests

have blank space on the left side	e.g., __ + 7 = 4 + 8 + 7
not contain a pair of equal number	e.g., 6 + 2 + 7 = 4 + __
have both of the two characteristics	e.g., 4+__ = 8 + 3 + 2

Appendix D

Interview question for the pilot study

1. How often did you use the gestures and the question outlined in the guide during teaching?
2. Did you find the combination of gestures and question helpful in facilitating math instruction?
3. Were there any parts of the guide that you found difficult to follow or implement?
4. Did you have any questions or concerns about the guide that were not addressed?
5. Were there any external factors that made it difficult for you to follow the guide as closely as you would have liked?
6. Would you recommend this guide to others? Why or why not?

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