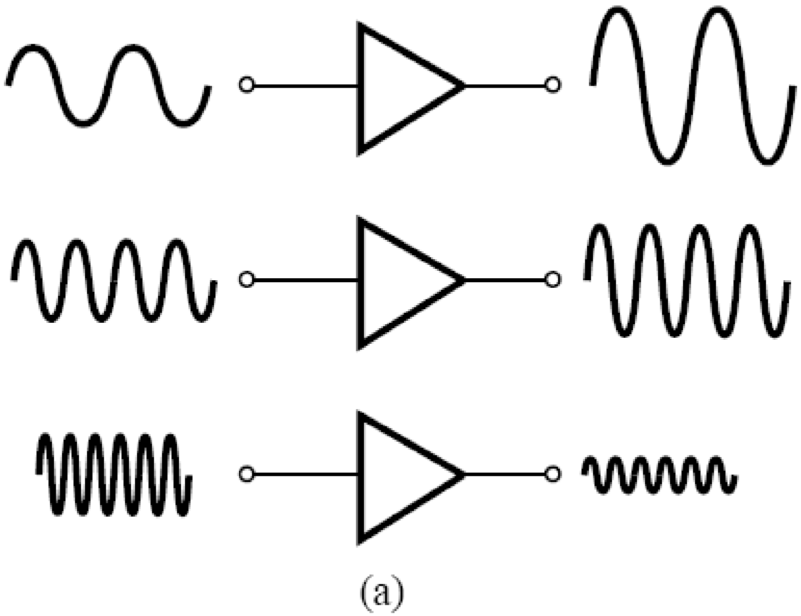
FREQUENCY RESPONSE

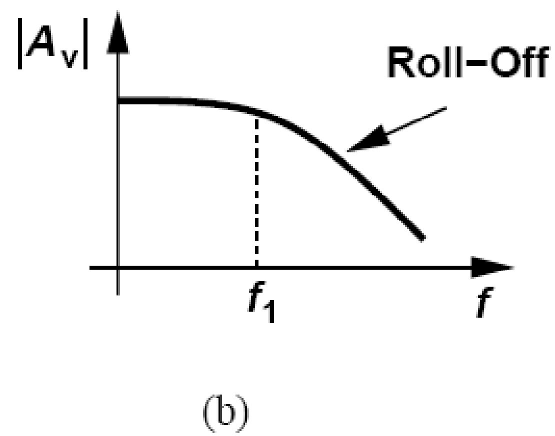
- Amplifier Gain vs Frequency
- Bode Plots
- Association of Poles with Nodes
- MOS Intrinsic Capacitances
- Transition Frequency of MOS Transistor
- Frequency Response of CS Stage
- Frequency Response of Source Follower
- Frequency Response of CG Stage
- Frequency Response of Cascode Stage
- Frequency Response of Differential Pairs

References

1. B. Razavi, ***Design of Analog CMOS Integrated Circuits***. McGraw-Hill, 2001.
2. B. Razavi, ***Fundamentals of Microelectronics***. Hoboken, NJ: Wiley, 2008.
3. P. Gray, P. Hurst, S. Lewis, R. Meyer, ***Analysis and Design of Analog Integrated Circuits***. Hoboken, NJ: Wiley, 2010.
4. Ali Sheikholeslami, ***Miller’s Approximation***. IEEE SSC Magazine, Fall 2015, p.7-8,13
5. W. Sansen, ***Analog Design Essentials***, Springer, 2005

Amplifier Gain vs Frequency





(a) Conceptual test of frequency response, (b) gain roll-off with frequency [2].

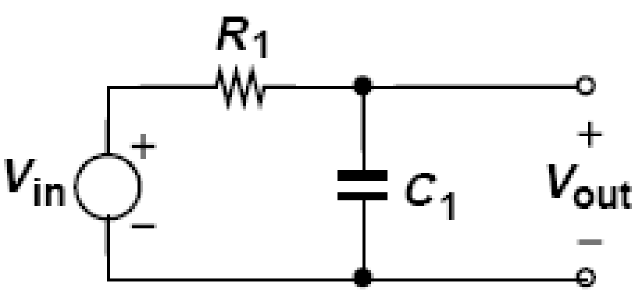
As frequency of operation increases, the gain of amplifier decreases.

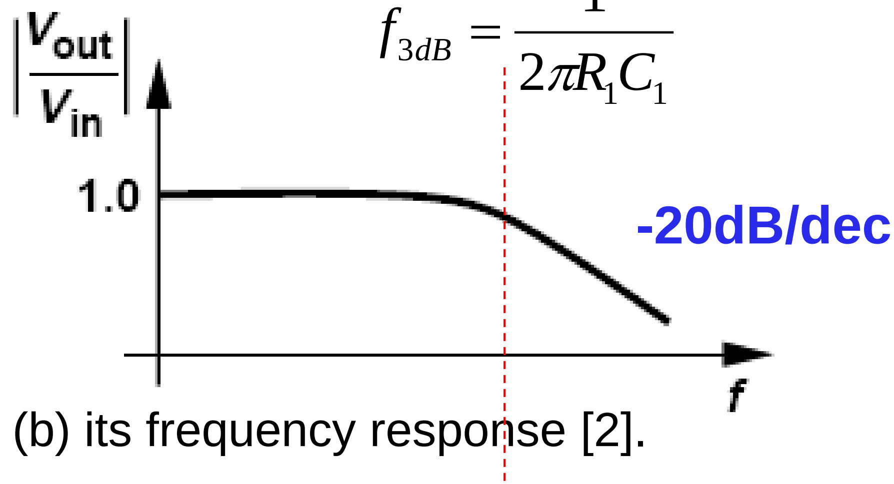
Bode Plot

$$H(s) = A_0 \frac{\left(1+\frac{s}{\omega_{z1}}\right)\left(1+\frac{s}{\omega_{z2}}\right)\cdots}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)\cdots}$$

- **A zero, $\omega_{zj} \Rightarrow$ the Bode magnitude rises with a slope of +20dB/dec.**
- **A pole, ω_{pj}, the Bode magnitude falls with a slope of -20dB/dec**

Example – simple low pass filter





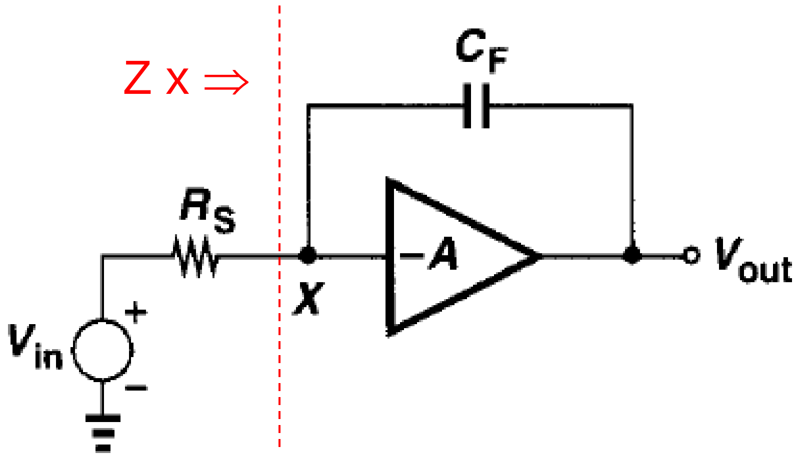
$$f_{3dB} = \frac{1}{2\pi R_1 C_1}$$

(a) Simple low-pass filter, and (b) its frequency response [2].

$$H(s) = \frac{1}{(1+sR_1C_1)}$$

Association of Poles with Nodes – ex. 3

Let's calculate the pole associated with node X [1]:

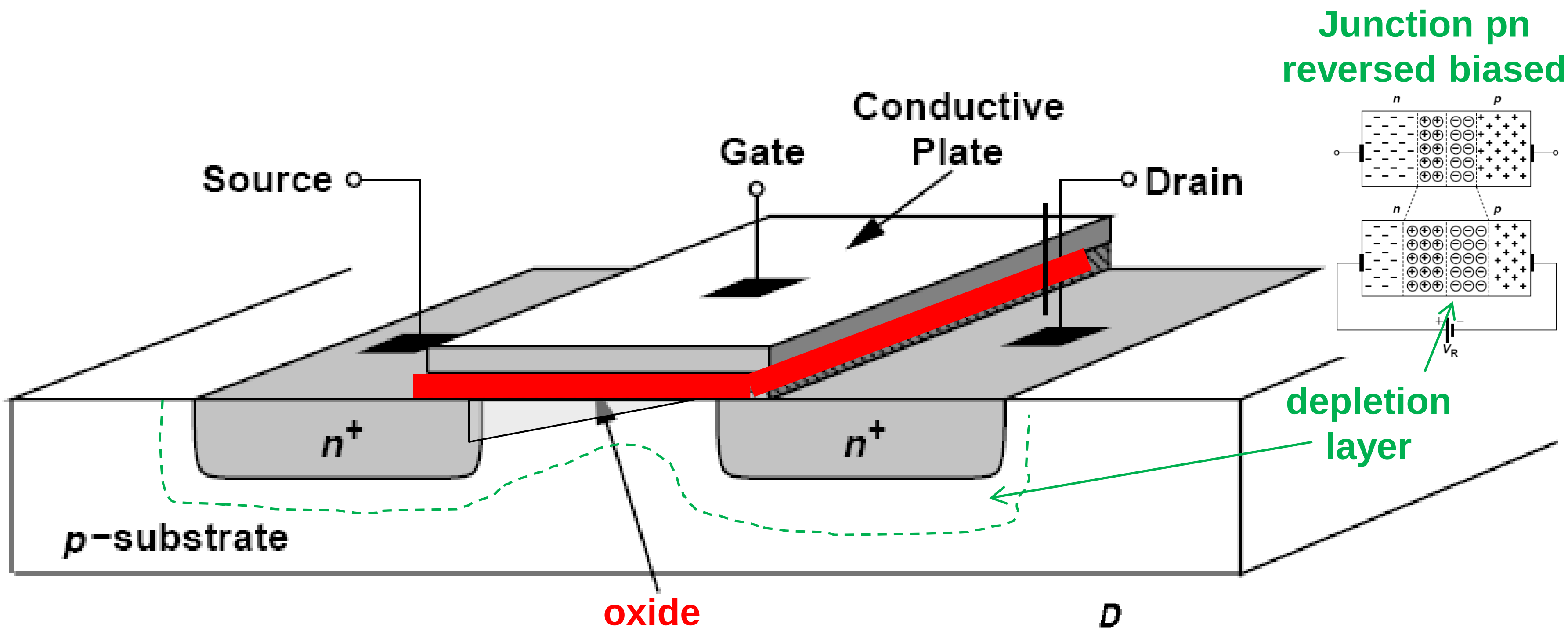


$$Z_X = \frac{V_X}{I_X} = \frac{V_X}{(V_X - V_{out})sC_F} = \frac{V_X}{(V_X + AV_X)sC_F} = \frac{1}{(1+A)sC_F}$$

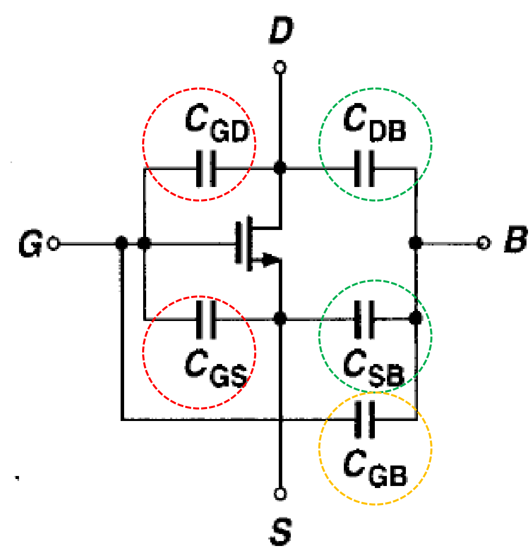
Effective capacitance to ground from node X: $$\boxed{C_X = (1+A)C_F}$$

Pole associated with node X: $$p_X = -\frac{1}{(1+A)C_F R_S}$$

MOS Intrinsic Capacitances

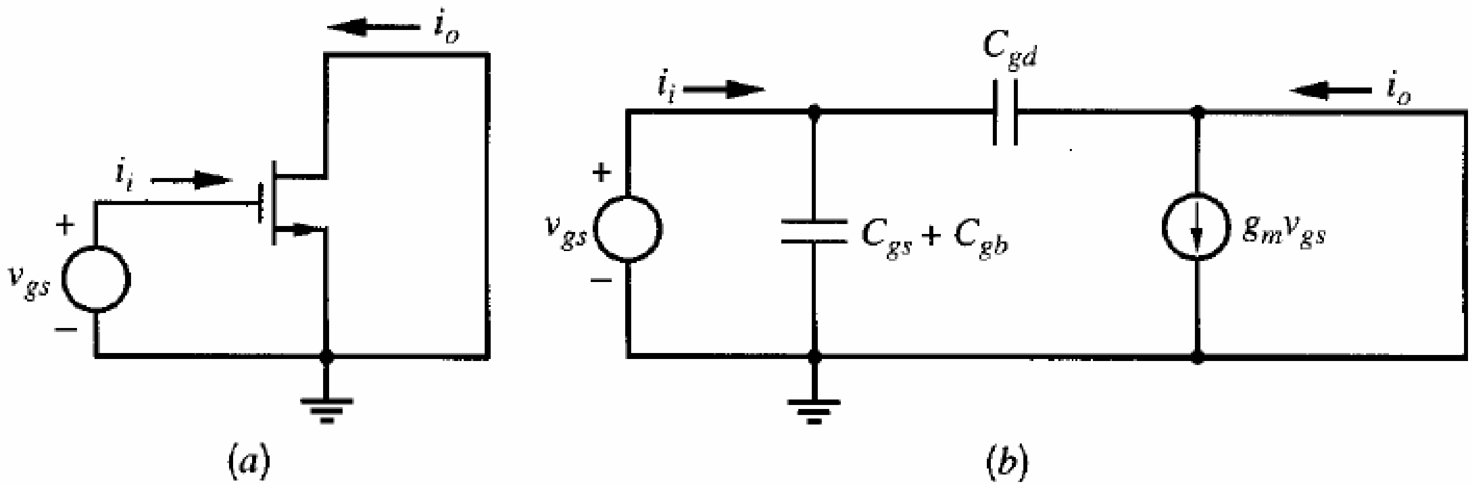


For a MOS, there exist oxide capacitance from gate to channel, junction capacitances from source/drain to substrate, and overlap capacitance from gate to source/drain.



MOS transistor frequency response

Transition frequency f_T is definied as the frequency where the magnitude of the short-circuit, commom source current gain falls to unity.



Circuit for calculating f_T: a) AC schematic, b) small signal equivalent [3].

Small signal input current

$$i_i = s\left(C_{GS} + C_{GB} + C_{GD}\right)v_{gs}$$

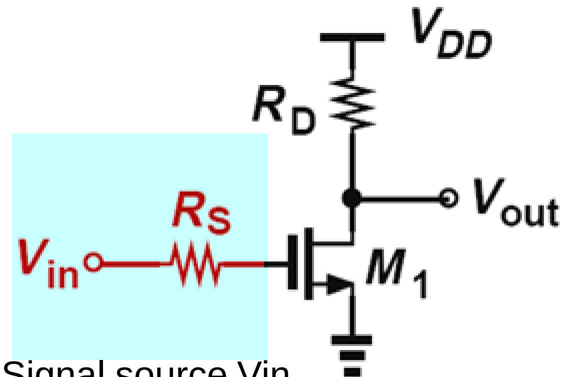
$$\frac{i_o}{i_i} \approx \frac{g_m}{s\left(C_{GS} + C_{GB} + C_{GD}\right)}$$

If the current fed forward through C_{GD} is neglected

$$i_o \approx g_m v_{gs}$$

$$f_T \approx \frac{g_m}{2\pi\left(C_{GS} + C_{GB} + C_{GD}\right)} \approx \boxed{\frac{g_m}{2\pi\ C_{GS}}}$$

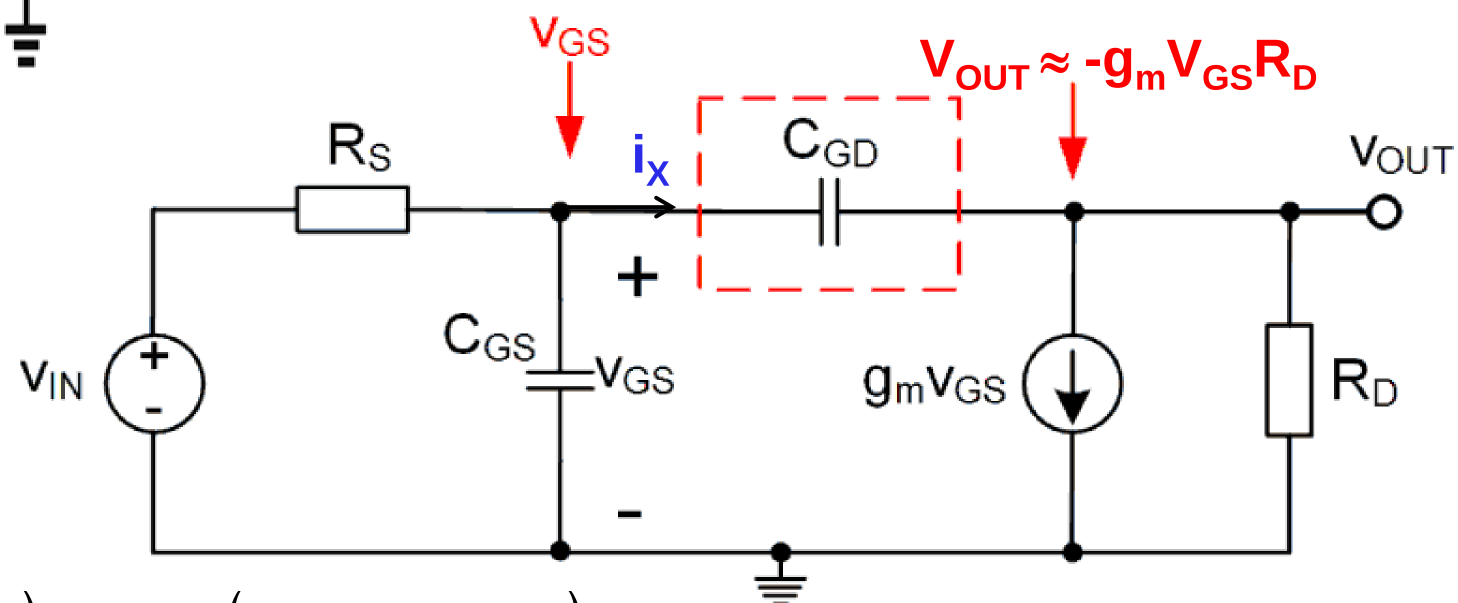
Frequency response of CS stage
Miller approximation [3]



Signal source Vin
with resistance Rs

$$C_M = (1 - A_{v0})C_{GD}$$

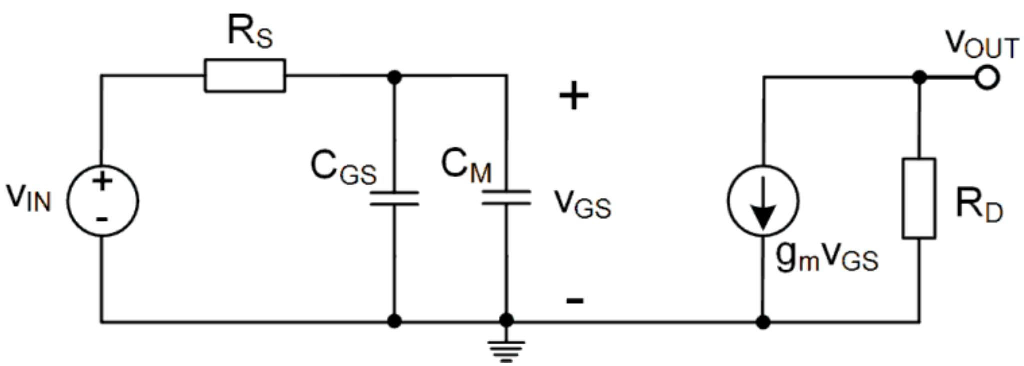
LOW FREQUENCY



$$\frac{i_x}{v_{gs}} = \frac{(v_{gs} - v_{out})sC_{GD}}{v_{gs}} \approx \frac{(v_{gs} + g_m v_{gs} R_D)sC_{GD}}{v_{gs}} = (1 + g_m R_D)sC_{GD}$$

$$C_M = (1 + g_m R_D)C_{GD}$$

Frequency response of CS stage
Miller approximation [3]



$$C_M = (1 + g_m R_D) C_{GD}$$

Input impedance

$$1/Z_{in} = s\underbrace{\left[C_{GS} + (1 + g_m R_D) C_{GD}\right]}_{C'_M}$$

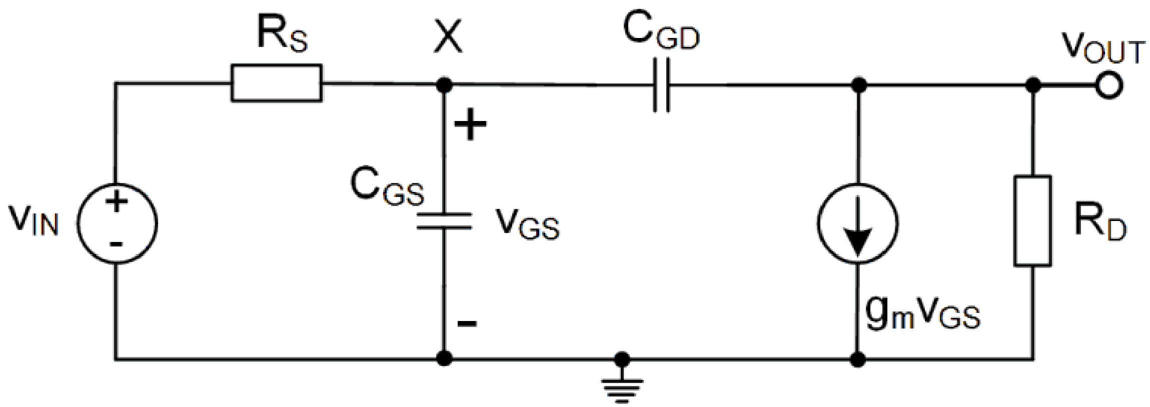
Transfer function

$$v_{out} = -g_m R_D v_{gs} = -g_m R_D \frac{\dfrac{1}{sC'_M}}{\dfrac{1}{sC'_M} + R_S} v_{in} = -g_m R_D \frac{1}{1 + sC'_M R_S} v_{in}$$

Dominant pole:

$$p_1 = -\frac{1}{R_S \left(C_{GS} + (1 + g_m R_D) C_{GD}\right)}$$

Frequency CS stage
Exact gain expression [3]



KCL for node X:

$$\frac{v_{in}-v_{gs}}{R_S}=v_{gs}sC_{GS}+\left(v_{gs}-v_{out}\right)sC_{GD} \Rightarrow v_{in}=v_{gs}\left[1+s\left(C_{GS}+C_{GD}\right)R_S\right]-v_{out}sC_{GD}R_S$$

KCL for node OUT:

$$\left(v_{gs}-v_{out}\right)sC_{GD}=g_m v_{gs}+\frac{v_{out}}{R_D} \Rightarrow v_{gs}=\frac{1+sC_{GD}R_D}{sC_{GD}R_D-g_mR_D}v_{out}$$

Transfer function:

$$\frac{v_{out}}{v_{in}}=\frac{-g_mR_D\left(1-sC_{GD}/g_m\right)}{s^2C_{GD}C_{GS}R_DR_S+s\left\{R_S\left[C_{GS}+\left(1+g_mR_D\right)C_{GD}\right]+C_{GD}R_D\right\}+1}$$

Positive real zero and two poles

Frequency CS stage
Exact gain expression [3]

If the poles are p1 and p2
we can write the dominator

$$D(s)=\left(1-\frac{s}{p_1}\right)\left(1-\frac{s}{p_2}\right)=1-s\left(\frac{1}{p_1}+\frac{1}{p_2}\right)+\frac{s^2}{p_1p_2}$$

If the poles are real and
widely separted |p1| << |p2 |

$$D(s)\approx 1-\frac{s}{p_1}+\frac{s^2}{p_1p_2}$$

Transfer function:

$$\frac{v_{out}}{v_{in}}=\frac{-g_mR_D(1-sC_{GD}/g_m)}{s^2C_{GD}C_{GS}R_DR_S+s\{R_S[C_{GS}+(1+g_mR_D)C_{GD}]+C_{GD}R_D\}+1}$$

Poles:

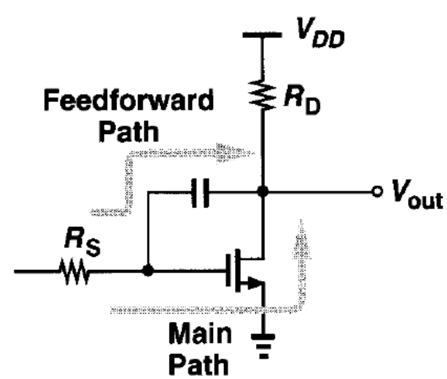
$$p_1=\frac{-1}{\underbrace{R_S[C_{GS}+(1+g_mR_D)C_{GD}]}_{\text{Miller approx.}}+C_{GD}R_D}$$

$$p_2=-\left(\frac{1}{R_DC_{GD}}+\frac{1}{R_DC_{GS}}+\frac{1}{R_SC_{GS}}+\underbrace{\frac{g_m}{C_{GS}}}_{\text{Transfer frequency (very high)}}\right)$$

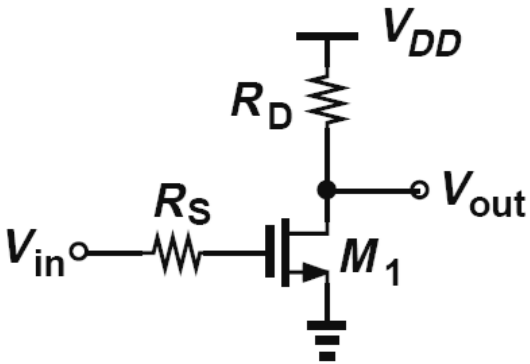
Positive real zero

$$z=\frac{g_m}{C_{GD}}$$

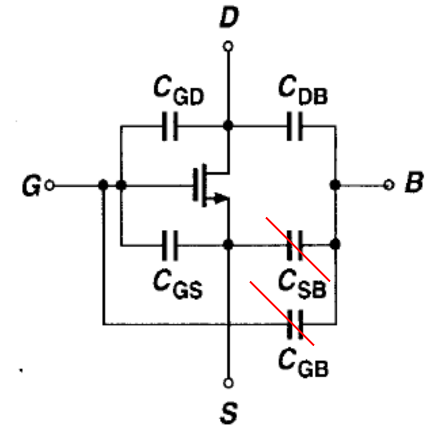
Located in the right half plane introduces **stabili**ty issues in feedback amplifier)



Frequency CS stage - example

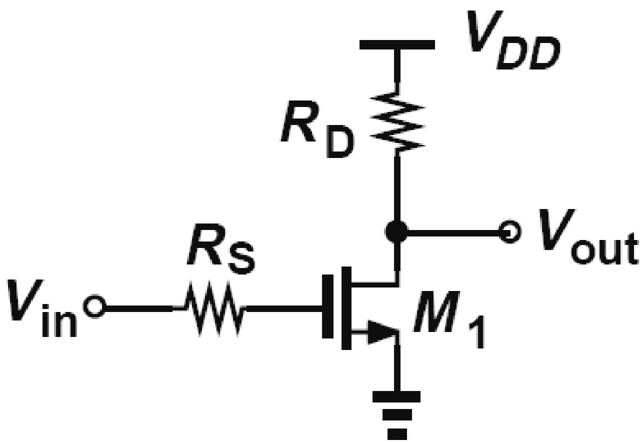


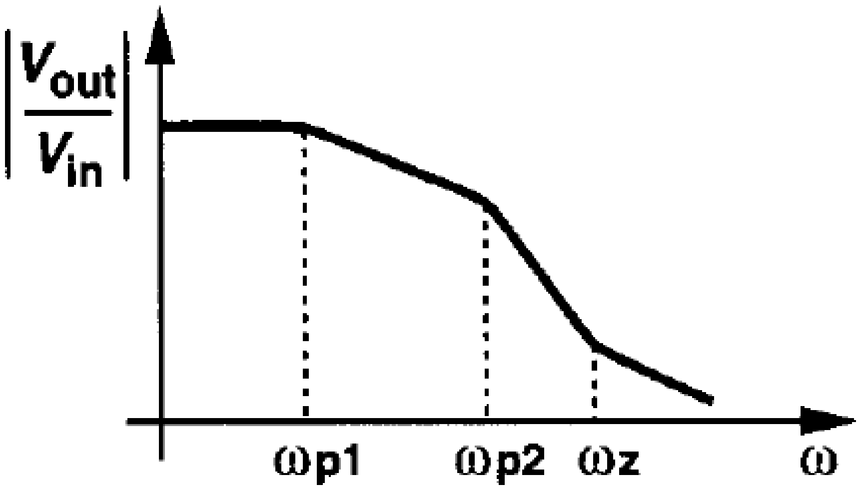
Example:
$R_S=200\Omega$, $R_D=2k\Omega$,
$g_m=(150\Omega)^{-1}$, $\lambda=0$
$C_{GS}=250fF$, $C_{GD}=80fF$,
$C_{DB}=100fF$



	Miller's approx.	Calculation with $\lvert p1\rvert \ll \lvert p2\rvert$	Calculation with $\lvert p1\rvert \ll \lvert p2\rvert$ and C_{DB}	Exact calculation and C_{DB}
f_{p1}	568 MHz	362 MHz	249 MHz	264 MHz
f_{p2}	-	8.73 GHz	4.79 GHz	4.53 GHz
f_{zero}	-	13.2 GHz	13.2 GHz	13.2 GHz

Frequency CS stage - conclusions



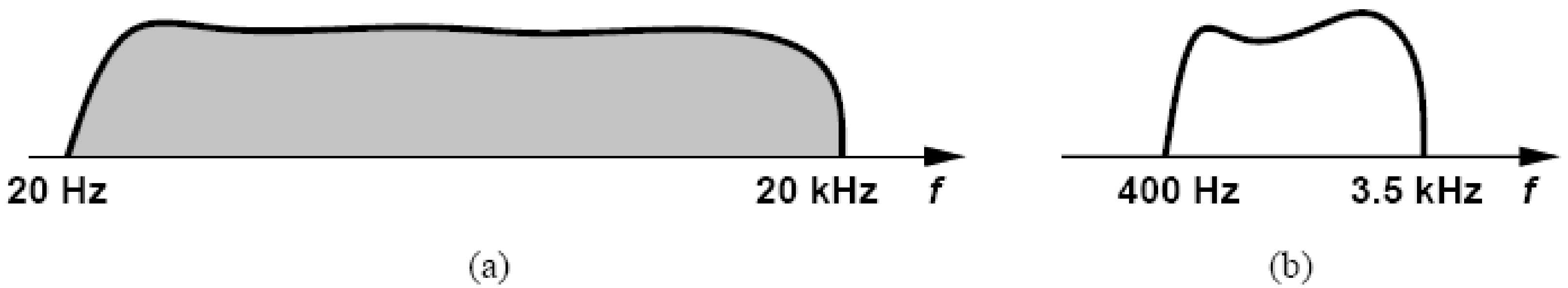


- **Miller's approx:** fast and very effective for dominant pole
- **Exact calculaton:** positive zero and two widely separted poles
- **Input impedance:** capacitive - Miller multiplication
  (for higher frequency see [1])

ADDITIONAL SLIDES

Example: Human Voice

Why people's voice over the phone sounds different from their voice in face-to-face conversation?

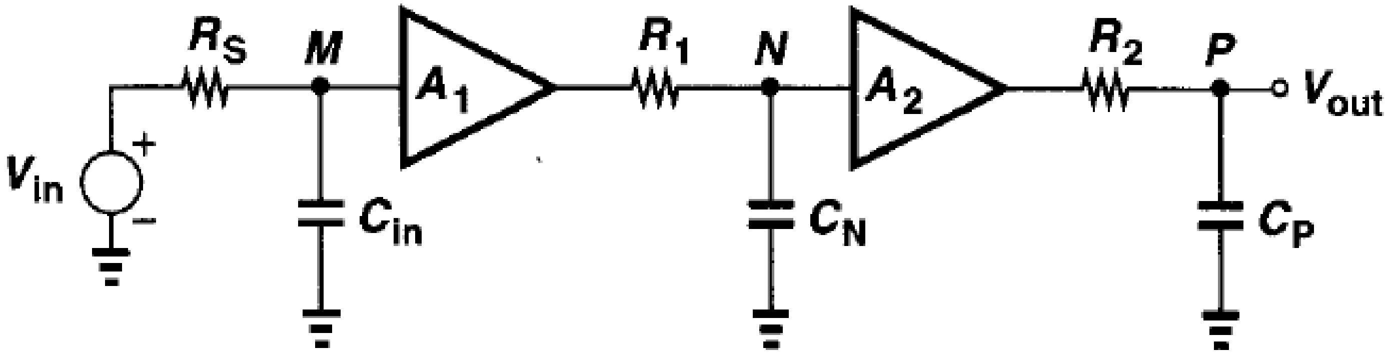


(a) Natural voice, (b) telephone system [2].

Natural human voice spans a frequency range from 20 Hz to 20 kHz, however conventional telephone system passes frequencies from 400Hz to 3.5 kHz.

Association of Poles with Nodes – ex. 1

Example - a cascade amplifiers: A_1 and A_2 are ideal volatge amplifier, R_s is source resistance, R_1 and R_2 model the output resistance of each stage, C_{in} and C_N represents the input capacitance of each stage and C_p is the load capacitance.



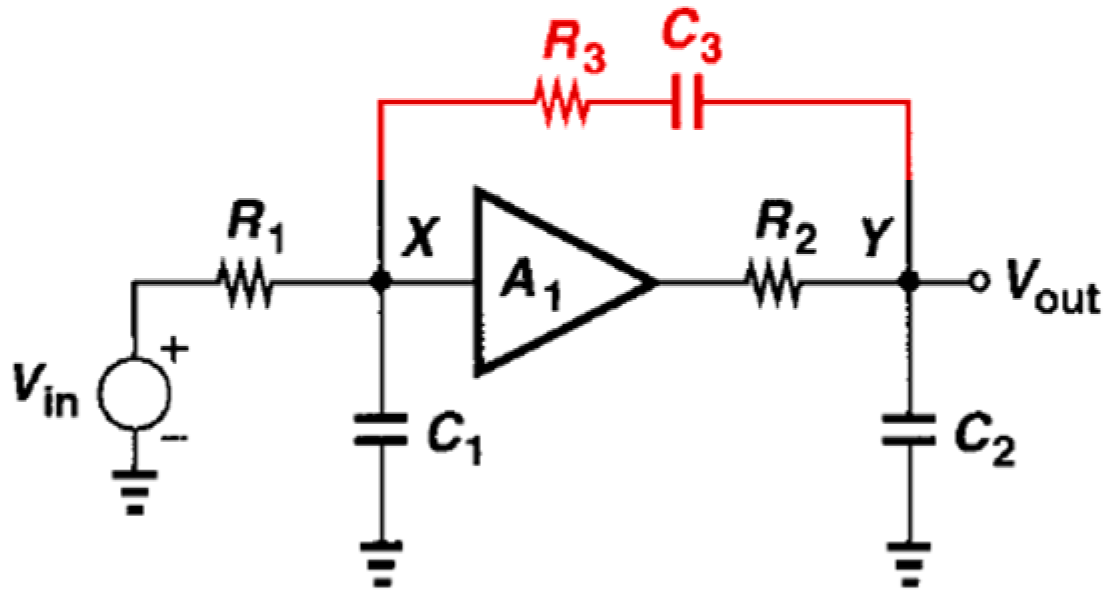
Cascade of amplifiers [1].

$$\frac{V_{out}}{V_{in}}(s) = \frac{A_1}{1 + R_S C_{in} s} \cdot \frac{A_2}{1 + R_1 C_N s} \cdot \frac{1}{1 + R_2 C_P s}$$

Three poles – each of which is determined by the total capacitance seen from each node to ground multiplied by the total resistance seen at the node to ground

Association of Poles with Nodes – ex. 2

Example of interaction between two nodes [1]:



The location of the poles is difficult to calculate because R_3 and C_3 create interaction between X and Y.

Miller's Theorem

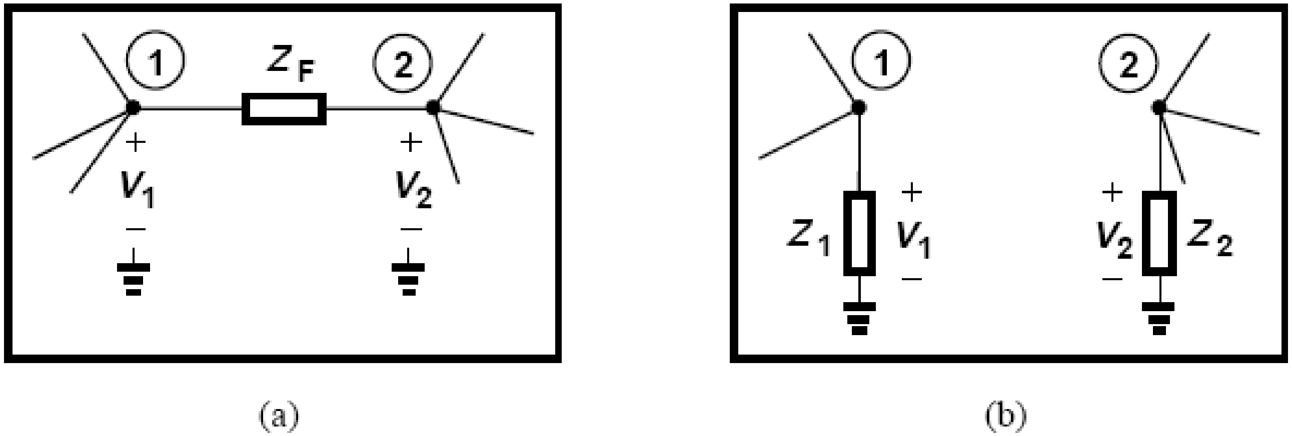


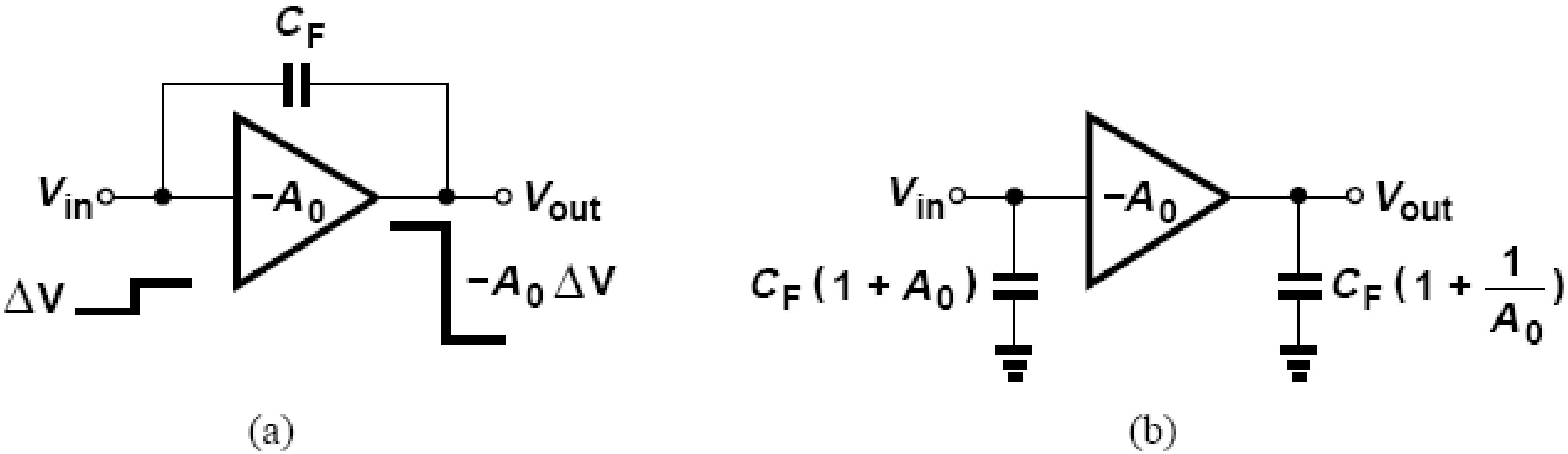
Fig. 1 (a) General circuit including a floating impedance, (b) equivalent of (a) as obtained from Miller's theorem [2].

If the circuit of Fig. 1(a) can be converted to that of Fig. 1(b) , then

$$Z_1 = \frac{Z_F}{1 - A_v} \qquad\qquad Z_2 = \frac{Z_F}{1 - 1/A_v}$$

where $$A_v = \frac{V_Y}{V_X}$$

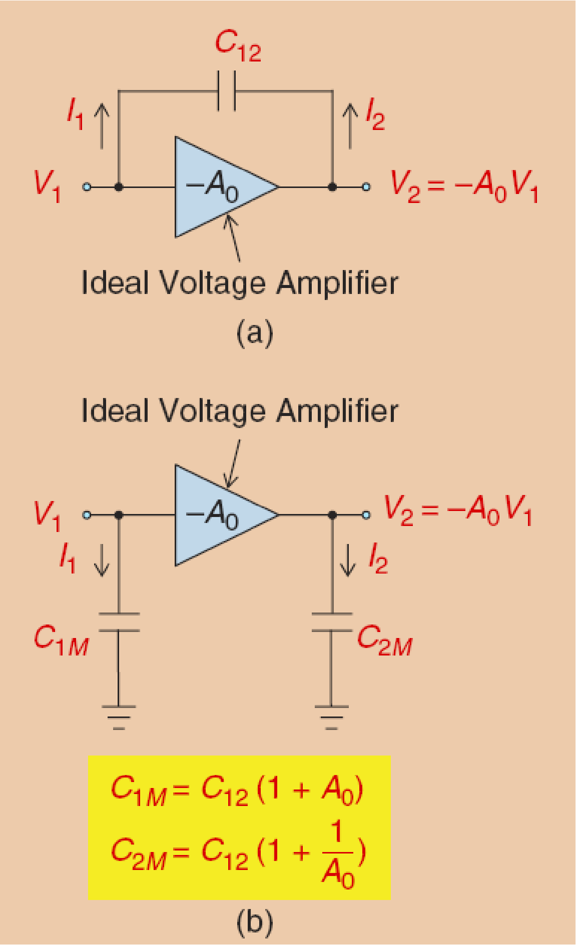
Miller Multiplication – example



(a) Inverting circuit with floating capacitor, (b) equivalent circuit as obtained from Miller's theorem [2].

With Miller's theorem, we can separate the floating capacitor. However, the input capacitor is larger than the original floating capacitor. We call this Miller multiplication.

Miller's Aproximation [4]



1. How to apply Miller's theorem when its gain is frequency dependent (i.e., not constant)?

$$A_V(j\omega) = \frac{-A_0}{1 + \frac{j\omega}{\omega_P}} \Rightarrow C_{1M} = C_{12}\left(1 + \frac{A_0}{1 + \frac{j\omega}{\omega_P}}\right)$$

C1M is fequency dependent **!**

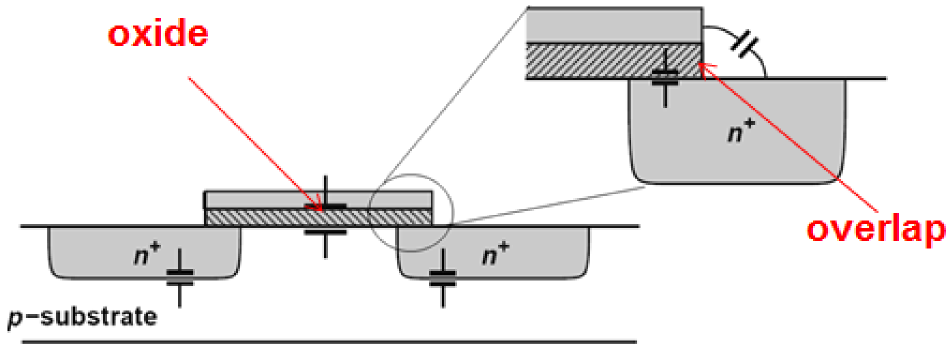
2. How to apply Miller's theorem when the amplifier is nonideal (e.g., has a finite output impedance)?

see: [4] Ali Sheikholeslami: Miller's Approximation. IEEE SSC Magazine, Fall 2015, p.7-8,13

In summary, Miller's approximation uses the **dc gain of the amplifier to provide a relatively accurate estimation of its dominant pole** (i.e., the circuit bandwidth). This approximation, however, becomes inaccurate when determining the second pole of the amplifier; other intuitive methods exist for this purpose [4].

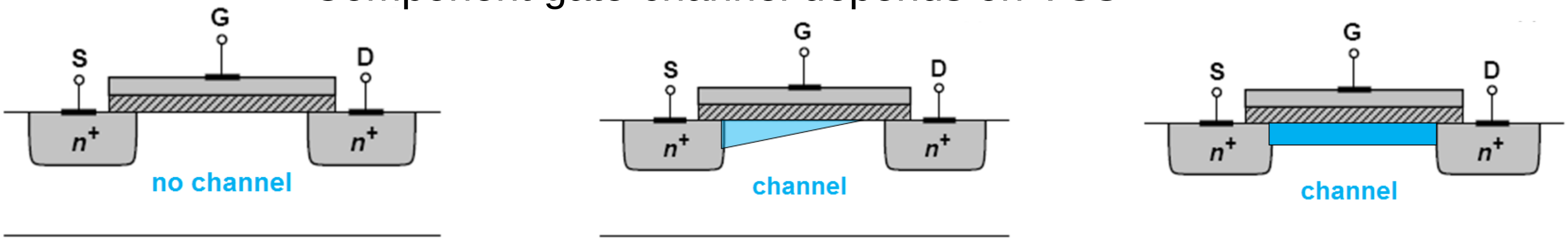
Variation of C_{GS} and C_{GD} versus V_{GS} [1].

Two components of C_{GS} and C_{GD}: channel + overlap

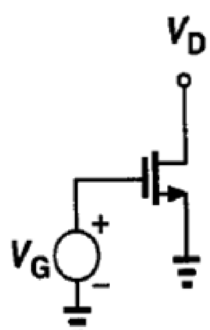


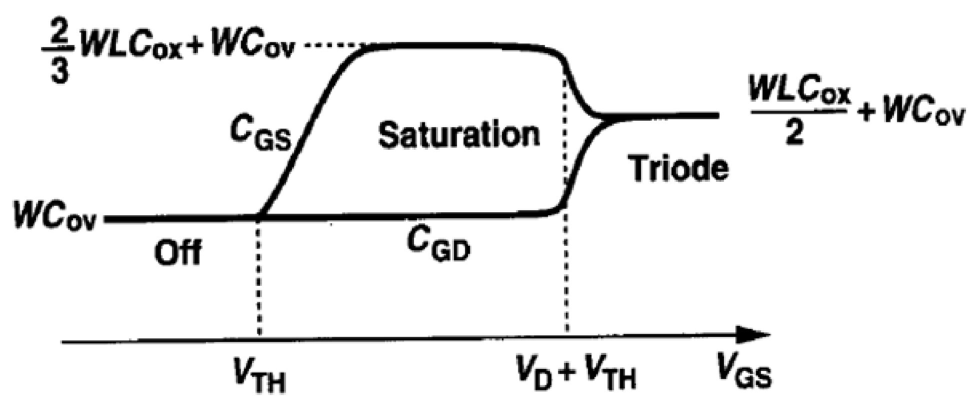
Example - CMOS 130nm
tox = 2.7 nm
Cox=12.8 fF/μm^2
Cov=0.35 fF/μm

Component gate-channel depends on V_{GS}

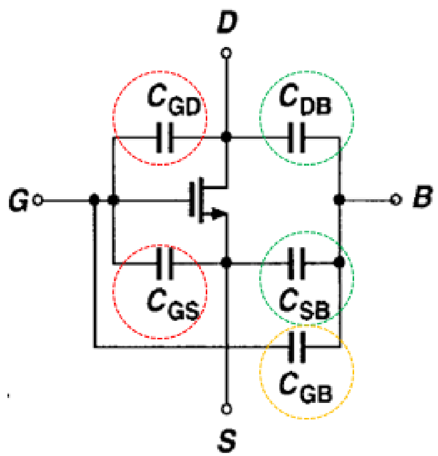


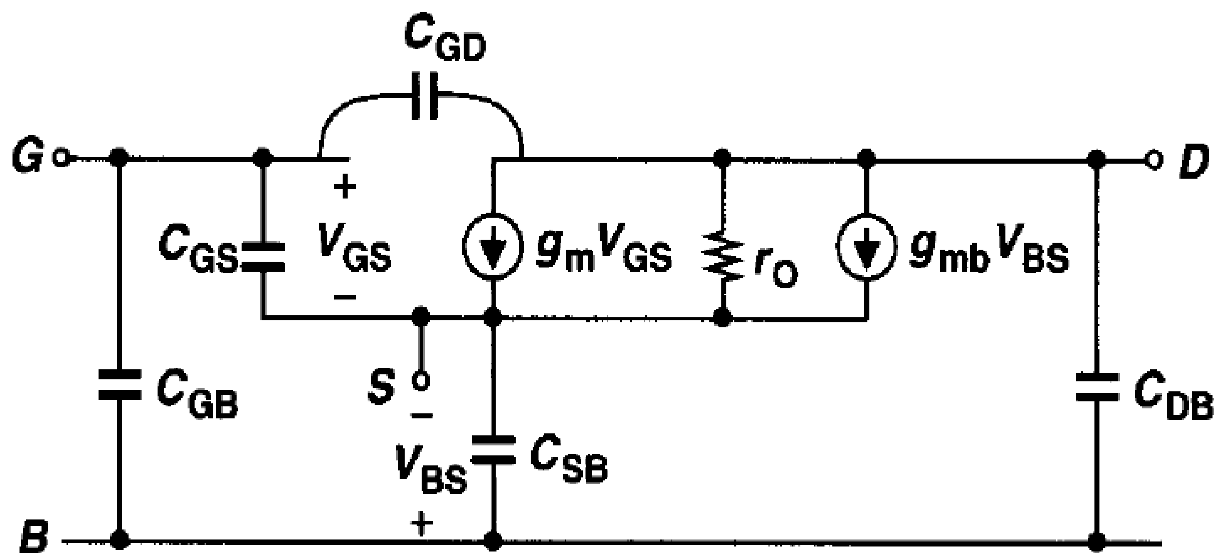
$C_{GS} = f(V_{GS})$
$C_{GD} = f(V_{GS})$





Complete MOS small-signal model [1]





Technology	Drain current (VGS=VDS)	W/L [μm/μm]	CGS [F]	CGD [F]	CSB [F]	CDB [F]	CGB [F]
130 nm	100 μA 10 μA	100 /0.2	55 f 45 f	44 f 44 f	2.1 f 0.16 f	103 f 106 f	32 f 25 f
45 nm	100 μA 10 μA	100 /0.2	77 f 34 f	15 f 15 f	8.9 f 2.9 f	0.048 f 0.026 f	19 f 25 f
130 nm	10 μA 1 μA	0.15 /0.13 (min)	0.181 f 0.103 f	0.071 f 0.072 f	0.637 f 0.627 f	0.540 f 0.565 f	0.008 f 0.018 f
45 nm	10 μA 1 μA	0.12 /0.04 (min)	0.042 f 0.028 f	0.002 f 0.019 f	- -	- -	- -

Maximum f_T versus channel length L [5]

$$C_{GS} = \frac{2}{3} WLC_{ox} \quad g_m = 2K' \frac{W}{L}(V_{GS}-V_T) \quad K' = \frac{\mu C_{ox}}{2n}$$

$$\boxed{f_T = \frac{g_m}{2\pi C_{GS}} = \frac{1}{2\pi}\frac{3}{2n}\frac{\mu}{L^2}(V_{GS}-V_T)} \quad \text{or} \quad \boxed{\approx \frac{v_{sat}}{2\pi L}}$$

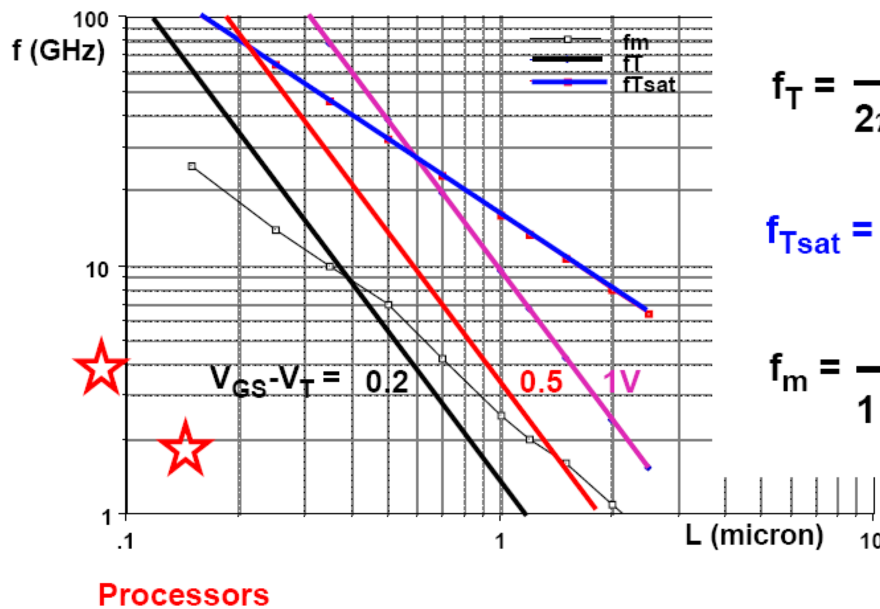
W. Sansen [5] uses the formula for drain current:

$$I_{DS} = \frac{\mu C_{ox}}{2n}\frac{W}{L}(V_{GS}-V_T)^2$$

$$n = 1 + \frac{C_D}{C_{ox}} \approx 1.2 - 1.5$$

This frequency is proportional to VGS−VT and inversely proportional to L^2. Decreasing the channel length allows an higher frequency performance. In velocity saturation however, the time that the electrons need to cross the channel length is L/v_{sat}. The frequency f_T in velocity saturation is, to put it simply $v_{sat}/2\pi L$. This is the highest frequency that can be obtained with a MOS transistor.

1. For technology with L≈100nm fT of 100 GHz is available.
2. Experimental upper frequencies of VCO and LNA marked as fm is about 1/5 of fT.
3. Clock frequency of processors is about 1/100 of fT.



$$f_T = \frac{\mu}{2\pi L^2}\underbrace{(V_{GS}-V_T)}_{0.2\ \dots\ 1\ V}$$

$$f_{Tsat} = \frac{v_{sat}}{2\pi L}$$

$$f_m = \frac{f_T}{1+\alpha_{BD}}$$

$$\alpha_{BD} \approx \frac{C_{BD}}{C_{ox}}$$