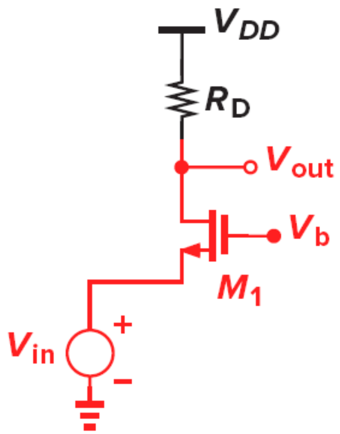
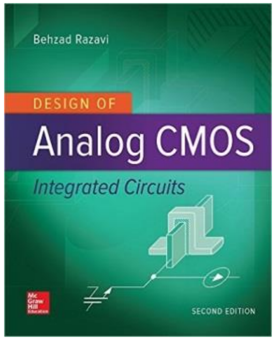
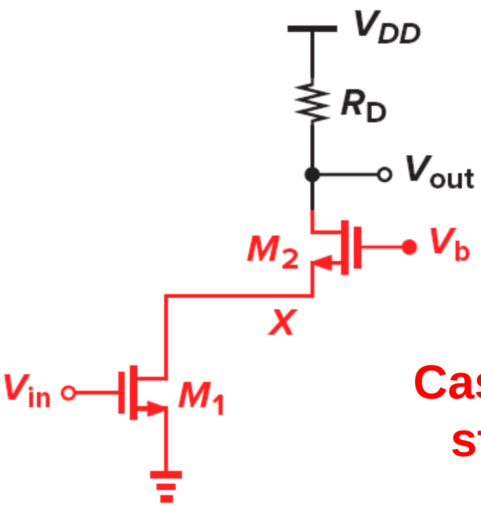
Common gate configuration
Cascode



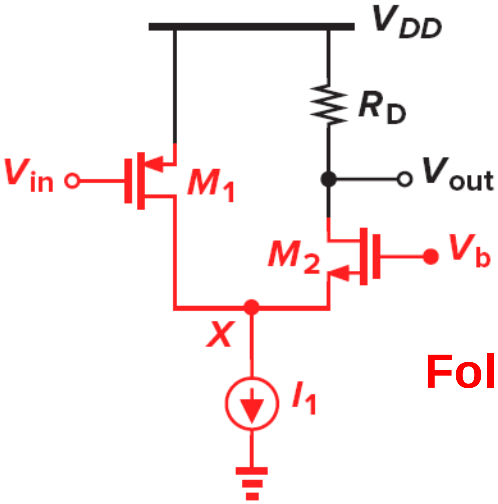
CG stage



Behzad Razavi:
Design of Analog Integrated Circuit, McGraw-Hill, 2016



Cascode stage



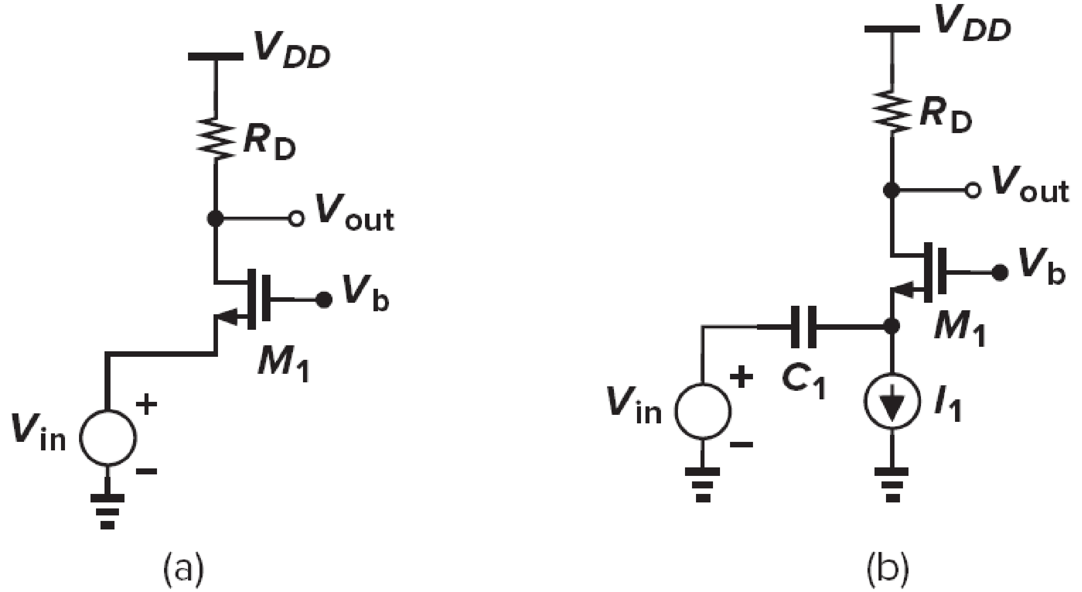
Folded cascode stage

Common gate stage

Common gate configuration

Shown in Fig. (a), a common-gate (CG) stage senses the input at the source and produces the output at the drain. The gate is connected to a dc voltage to establish proper operating conditions. Note that the bias current of *M*1 flows through the input signal source.

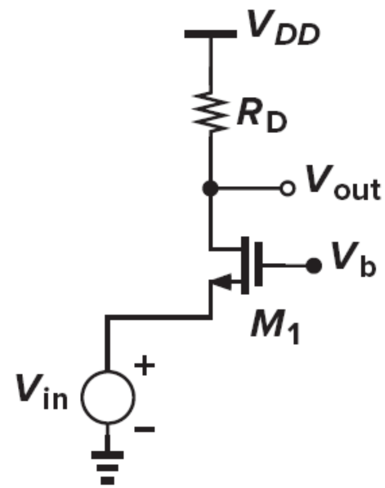
Alternatively, as depicted in Fig.(b), *M*1 can be biased by a constant current source, with the signal capacitively coupled to the circuit.



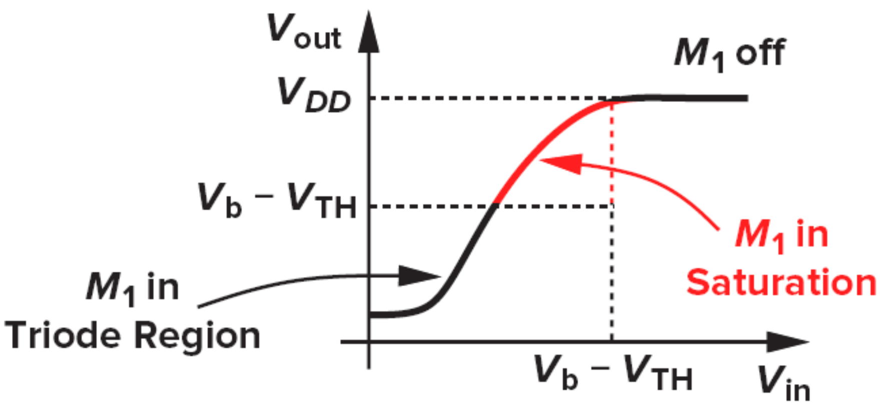
(a) Common-gate stage with direct coupling at input; (b) CG stage with capacitive coupling at input.

CG stage
(large-signal behaviour)

We first study the large-signal behavior of the circuit in Fig. (a). For simplicity, let us assume that Vin decreases from a large positive value. Also, λ = 0. For Vin ≥ $V_b - V_{TH}$, M1 is off and Vout = V_{DD}. For lower values of Vin, we can write (if M1 is in saturation):



$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_b - V_{in} - V_{TH})^2$$



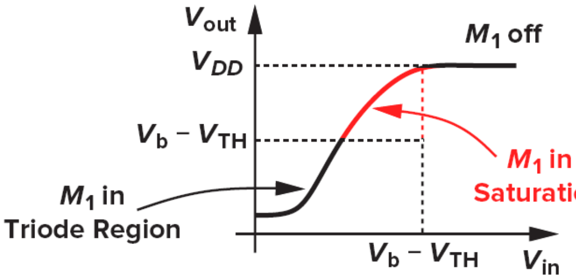
As Vin decreases, so does Vout (=$V_{DD} - R_D I_D$), eventually driving M1 into the triode region if:

$$V_{DS} = V_{GS} - V_{TH} \Leftrightarrow V_{out} - V_{in} = V_b - V_{in} - V_{TH} \Leftrightarrow V_{out} = V_b - V_{TH}$$

CG stage
(large-signal behaviour in saturation)

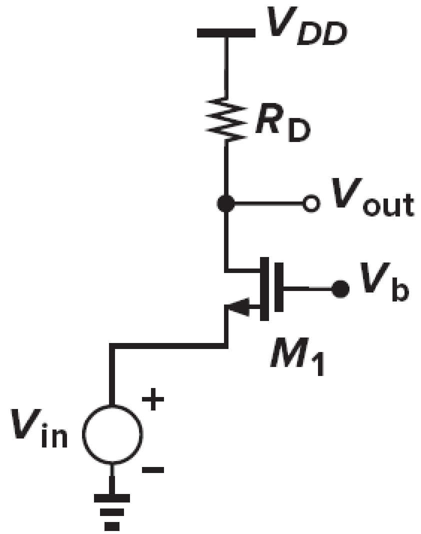
If M1 is in saturation:

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_b - V_{in} - V_{TH})^2$$



Output voltage $V_{out} = V_{DD} - I_D R_D$:

$$V_{out} = V_{DD} - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_b - V_{in} - V_{TH})^2 R_D$$



Obtaining the small-signal gain:

$$\frac{\partial V_{out}}{\partial V_{in}} = -\mu_n C_{ox}\frac{W}{L}(V_b - V_{in} - V_{TH})\left(-1 - \frac{\partial V_{TH}}{\partial V_{in}}\right) R_D$$

Since $\partial V_{TH}/\partial V_{in} = \partial V_{TH}/\partial V_{SB} = \eta,$ we have:

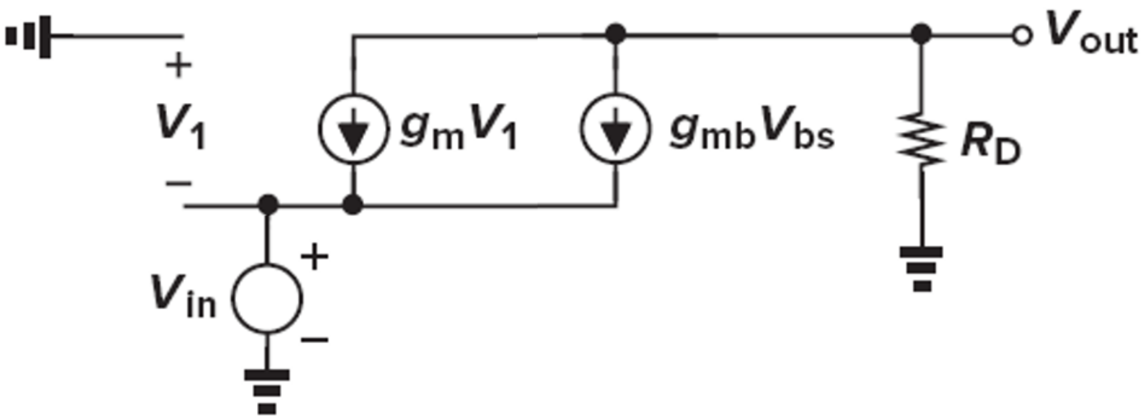
$$\frac{\partial V_{out}}{\partial V_{in}} = \mu_n C_{ox}\frac{W}{L} R_D (V_b - V_{in} - V_{TH})(1+\eta)$$
$$= g_m (1+\eta) R_D$$

Note that the gain is positive. Interestingly, body effect increases the equivalent transconductance of the stage.

CG stage

(Homework: small-signal model without r_O)

Calculate the gain and Rin using small-signal model



$$\frac{V_{out}}{V_{in}} = g_m(1+\eta)R_D$$

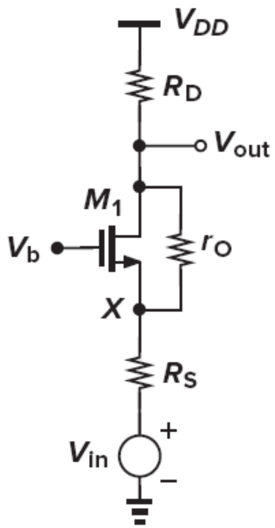
$V_{out} = -(g_m V_1 + g_{mb} V_{bs}) R_D$

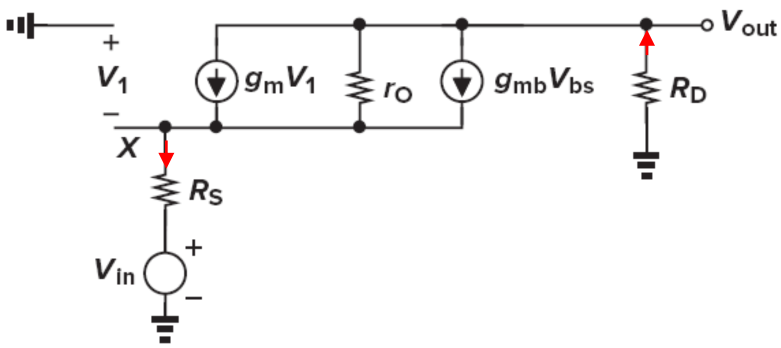
$V_{in} = -V_1 = -V_{bs}$

$$R_{in} = \frac{1}{g_m(1+\eta)}$$

CG stage – study in a more general case

(small-signal model with r_O and source resistance Rs)





Noting that the current flowing through R_S is equal to $-V_{out}/R_D$, we have:

$$V_1 - \frac{V_{out}}{R_D}R_S + V_{in} = 0 \qquad (*)$$

Since the current through r_O is equal to $-V_{out}/R_D - g_mV_1 - g_{mb}V_1$, we can write:

$$r_O\left(\frac{-V_{out}}{R_D} - g_mV_1 - g_{mb}V_1\right) - \frac{V_{out}}{R_D}R_S + V_{in} = V_{out}$$

Substitution V_1, from (*):

$$r_O\left[\frac{-V_{out}}{R_D} - (g_m + g_{mb})\left(V_{out}\frac{R_S}{R_D} - V_{in}\right)\right] - \frac{V_{out}R_S}{R_D} + V_{in} = V_{out}$$

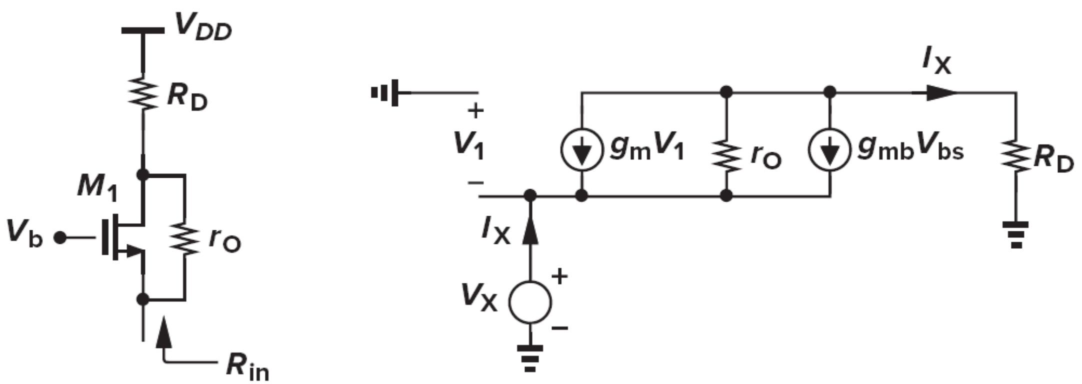
It follows that:

$$\frac{V_{out}}{V_{in}} = \frac{(g_m + g_{mb})r_O + 1}{r_O + (g_m + g_{mb})r_OR_S + R_S + R_D}R_D \approx \frac{R_D}{R_S}$$

Similar to CS stage with degeneration

CG stage – input resistance
(small-signal model with r_O and source resistance Rs)

To obtain the impedance seen at the source, we use the equivalent circuit in Fig. (b).



(a) (b)

Since $V_1 = -V_X$ and the current through r_O is equal to $I_X + g_mV_1 + g_{mb}V_1 = I_X - (g_m + g_{mb})V_X$, we can add up the voltages across r_O and R_D and equate the result to:

$$R_D I_X + r_O[I_X - (g_m + g_{mb})V_X] = V_X$$

Thus,

$$\frac{V_X}{I_X} = \frac{R_D + r_O}{1 + (g_m + g_{mb})r_O}$$

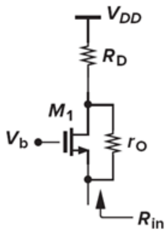
$$\approx \frac{R_D}{(g_m + g_{mb})r_O} + \frac{1}{g_m + g_{mb}}$$

if $(g_m + g_{mb})r_O \gg 1$

This result reveals that the drain impedance is divided by $(g_m + g_{mb})r_O$ when seen at the source. This is particularly important in short-channel devices because of their low intrinsic gain.

CG stage: input resistance – two cases

(small-signal model with r_O and source resistance Rs)



$$\frac{V_X}{I_X} = \frac{R_D + r_O}{1 + (g_m + g_{mb})r_O}$$

$$\approx \frac{R_D}{(g_m + g_{mb})r_O} + \frac{1}{g_m + g_{mb}}$$

Two special cases of the above eq. are worth studying.

First, suppose ***R_D* = 0**. Then,

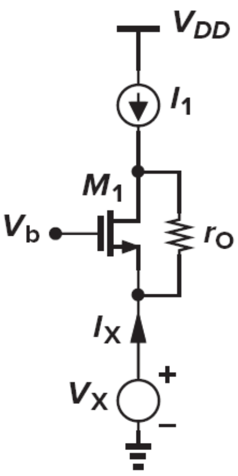
$$\frac{V_X}{I_X} = \frac{r_O}{1 + (g_m + g_{mb})r_O}$$

$$= \frac{1}{\frac{1}{r_O} + g_m + g_{mb}}$$

which is simply the impedance seen at the source of a source follower, a predictable result because if R_D = 0, the circuit configuration is the same as in source folower output resistance calculation.

Second, let us replace *R_D* with an ideal current source. Equation (above) predicts that the input impedance approaches *infinity*.

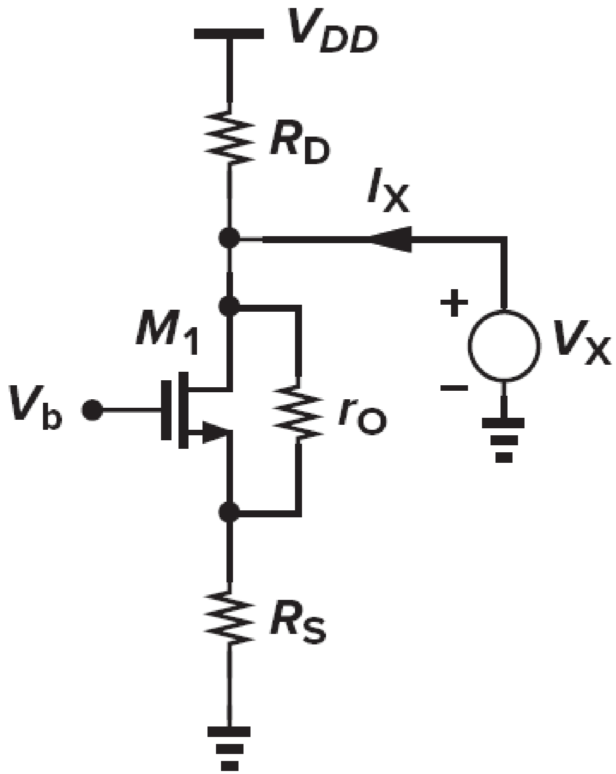
While somewhat surprising, this result can be explained with the aid of Fig. (right). Since the total current through the transistor is fixed and equal to I_1, a change in the source potential cannot change the device current, and hence I_X = 0.



In other words, the input impedance of a common-gate stage is relatively low only if the load impedance connected to the drain is small.

CG stage – output resistance
(small-signal model with r_O and source resistance Rs)

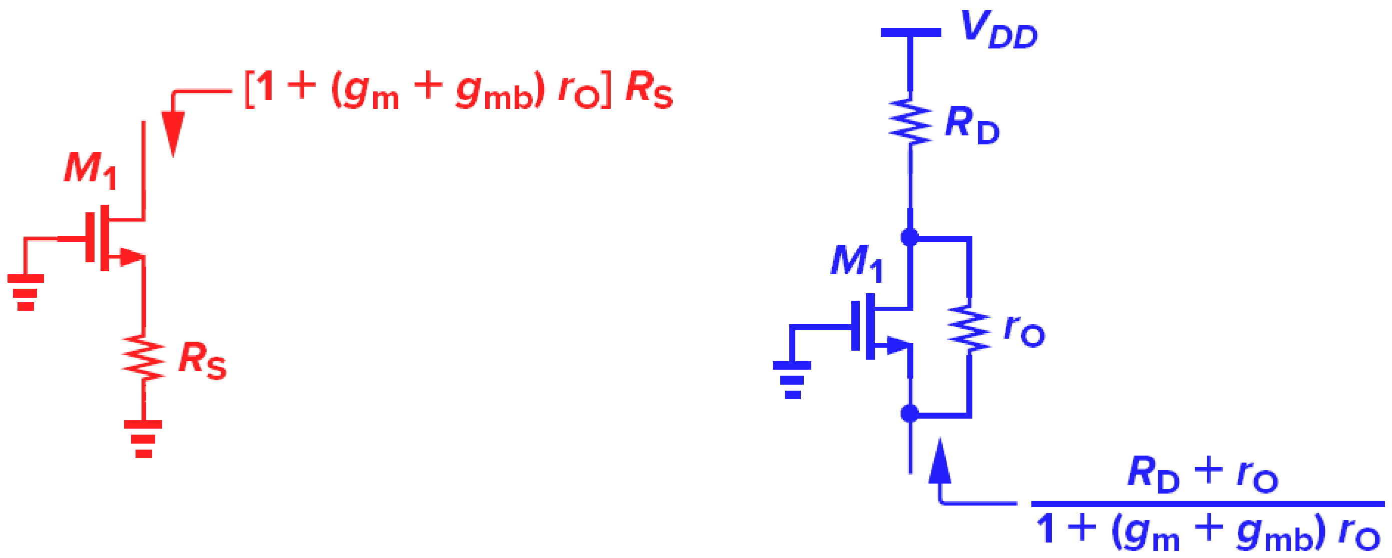
Output resistance - similar circuit to:
- CS with source degeneration,
- current source with the resistor connected to source



$$R_{out} = \{[1 + (g_m + g_{mb})r_O]R_S + r_O\} \| R_D$$

Impedance transformation by a MOSFET

Our analysis of the degenerated CS stage and the CG stage gives another interesting insight. As illustrated in Fig. (below), we loosely say that a transistor transforms its source resistance up and its drain resistance down (when seen at the appropriate terminal).



Impedance transformation by a MOSFET.

CG stage works as „current buffer”:
- low input impedance,
- high output impedance

Cascode

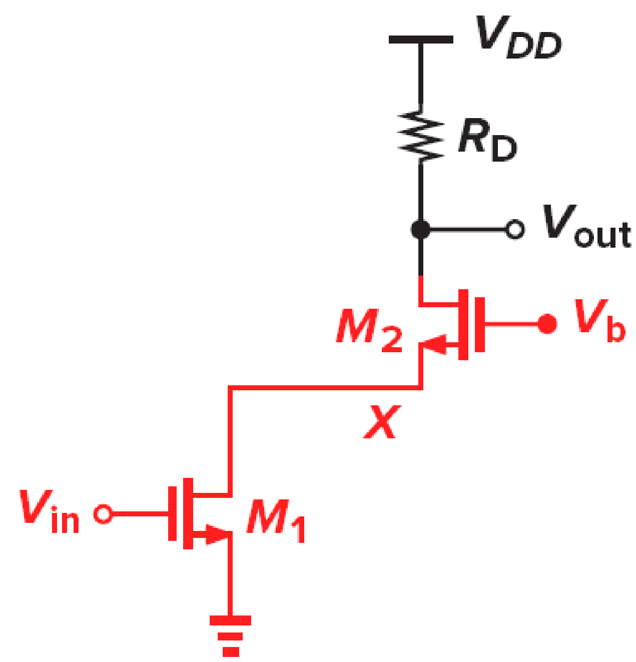
(CS + CG)

Cascode stage - introduction

The input signal of a common-gate stage may be a current. We also know that a transistor in a common-source arrangement converts a voltage signal to a current signal. The cascade of a CS stage and a CG stage is called a "cascode" topology, providing many useful properties.

Figure (right) shows the basic configuration: M1 generates a small-signal drain current proportional to the small-signal input voltage, Vin, and M2 simply routes the current to R_D.We call M1 the input device and M2 the cascode device. Note that in this example, M1 and M2 carry equal bias and signal currents.

As we describe the attributes of the circuit in this section, many advantages of the cascode topology over a simple common-source stage become evident. This circuit is also known as the "telescopic" cascode.

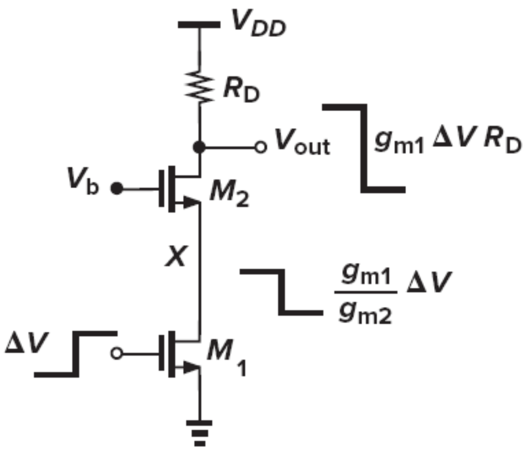


Cascode stage

Cascode stage - qualitative analysis

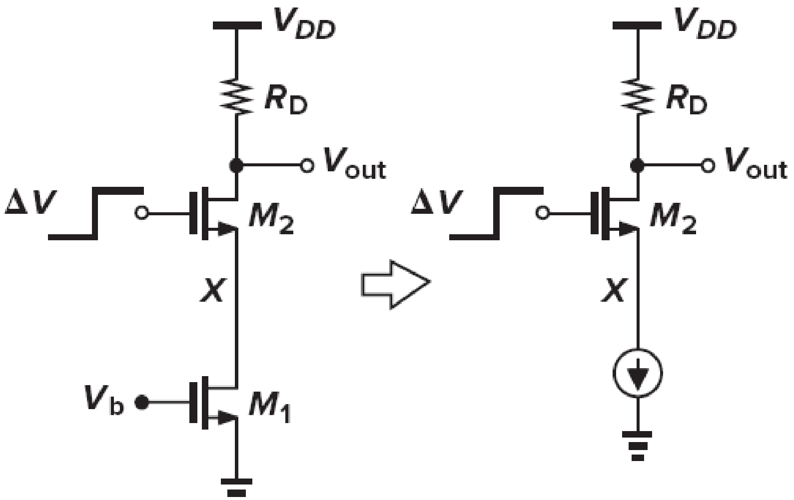
(what happens if the value of Vin or Vb changes by a small amount)

Assume that both transistors are in saturation and $\lambda = \gamma = 0$. If Vin rises by ΔV, then I_{D1} increases by $g_{m1}\Delta V$. This change in current flowsthrough the impedance seen at X, i.e., the impedance seen at the source of M2, which is equal to $1/g_{m2}$. Thus, V_X falls by an amount given by $g_{m1}\Delta V(1/g_{m2})$ [Fig.(a)]. The change in I_{D1} also flows through R_D, producing a drop of $g_{m1}\Delta V$ R_D in Vout - just as in a simple CS stage.



(a)

Now, consider the case where Vin is fixed and Vb increases by ΔV. Since V_{GS1} is constant and $r_{O1} = \infty$, we simplify the circuit as shown in Fig. (b). How do V_X and Vout change here? As far as node X is concerned, M2 operates as a source follower because it senses an input, ΔV, at its gate and generates an output at X. With $\lambda = \gamma = 0$, the small-signal voltage gain of the follower is equal to unity, regardless of the value of R_D (why?). Thus, V_X rises by ΔV. On the other hand, Vout does not change because I_{D2} is equal to I_{D1} and hence remains constant. We say that the voltage gain from Vb to Vout is zero in this case.



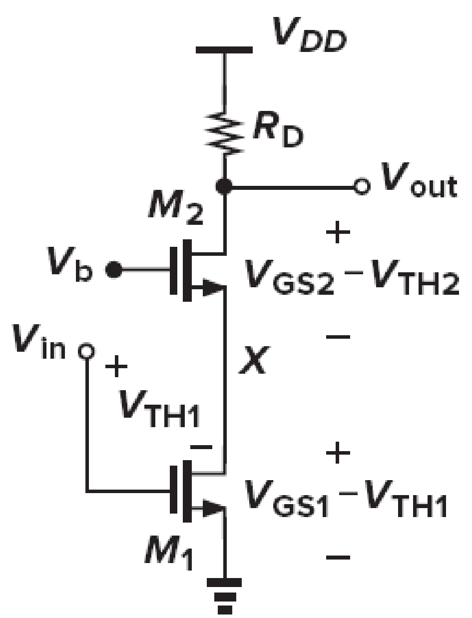
(b)

Cascode stage – study the bias condition
(M1 and M2 must be in saturation)

Let us now study the bias conditions of the cascode, still assuming that $\lambda = \gamma = 0$. For M1 to operate in saturation, we must have $V_X \geq V_{in} - V_{TH1}$. If M1 and M2 are both in saturation, M2 operates as a source follower and V_X is determined primarily by Vb: $V_X = V_b - V_{GS2}$. Thus, $V_b - V_{GS2} \geq V_{in} - V_{TH1}$, and hence $V_b > V_{in} + V_{GS2} - V_{TH1}$ (Fig.). For M2 to be saturated, $V_{out} \geq V_b - V_{TH2}$; that is,

$$V_{out} \geq V_{in} - V_{TH1} + V_{GS2} - V_{TH2}$$

$$= (V_{GS1} - V_{TH1}) + (V_{GS2} - V_{TH2})$$



Allowable voltages in cascode

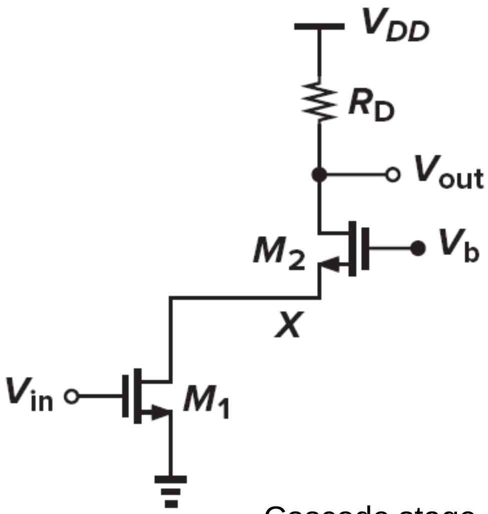
Now, if Vb is chosen to place M1 at the edge of saturation. Consequently, the minimum output level for which both transistors operate in saturation is equal to the overdrive voltage of M1 plus that of M2. In other words, addition of M2 to the circuit reduces the output voltage swing by at least the overdrive voltage of M2. We say that M2 is “stacked” on top of M1. We also loosely say that the minimum output voltage is equal to two overdrives or $2V_{Dsat}$.

Cascode stage – large signal behaviour

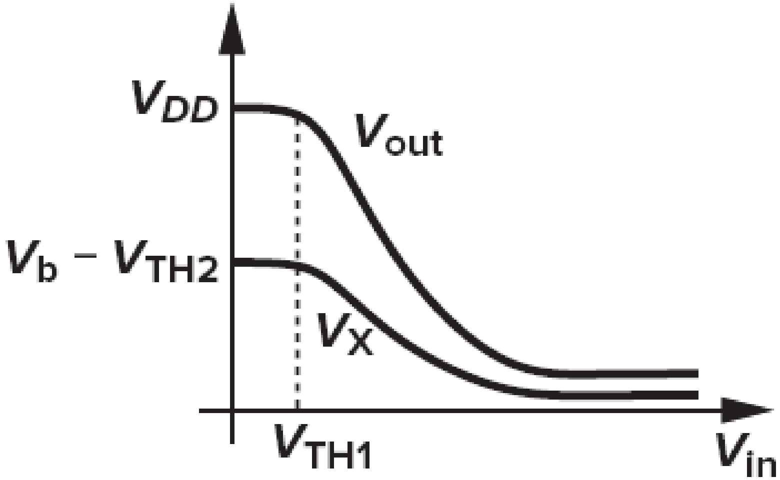
We now analyze the large-signal behavior of the cascode stage shown in Fig. (below - left) as Vin goes fromnzero to V_{DD}. For Vin ≤ V_{TH1}, M1 and M2 are off, Vout = V_{DD}, and $V_X \approx Vb - V_{TH2}$ (if subthreshold conduction is neglected) (Fig.). As Vin exceeds V_{TH1}, M1 begins to draw current, and Vout drops. Since I_{D2} increases, V_{GS2} must increase as well, causing V_X to fall. As Vin assumes sufficiently large values, two effects can occur:

(1) V_X drops below Vin by V_{TH1}, forcing M1 into the triode region;

(2) Vout drops below Vb by V_{TH2}, driving M2 into the triode region. Depending on the device dimensions and the values of R_D and Vb, one effect may occur before the other. For example, if Vb is relatively low, M1 may enter the triode region first. Note that if M2 goes into the deep triode region, V_X and Vout become nearly equal.



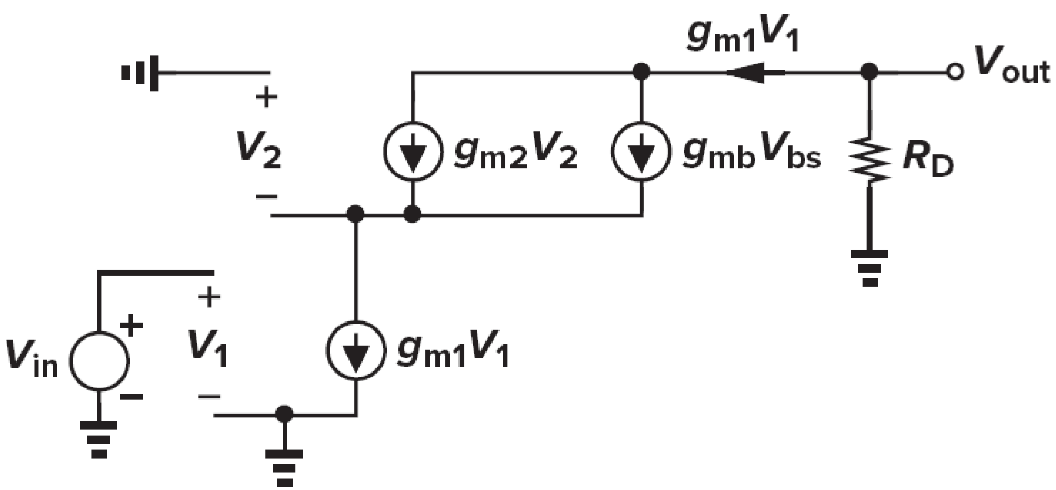
Cascode stage



Input-output characteristic of cascode stage

Cascode stage – small-signal analysis

Let us now consider the small-signal characteristics of a cascode stage, assuming that both transistors operate in saturation. If $\lambda = 0$, the voltage gain is equal to that of a common-source stage because the drain current produced by the input device must flow through the cascode device. Illustrated in the equivalent circuit of Fig. (below), this result is independent of the transconductance and body effect of M2.



Small-signal equivalent circuit of cascode stage.

$V_{out} = - g_m V_1 R_D$

$V_{in} = V_1$

$V_{out}/V_{in} = -g_m R_D$

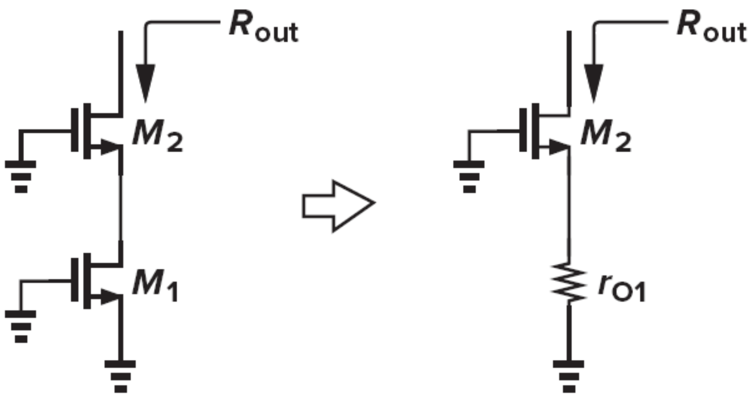
Where is the advantage of cascode stage?

Consider the $R_D \to \infty$

Cascode stage – output resistance

An important property of the cascode structure is its high output impedance. As illustrated in Fig. (below), for calculation of Rout , the circuit can be viewed as a common-source stage with a degeneration resistor equal to r_{O1}. Thus,

$$R_{out} = [1 + (g_{m2} + g_{mb2})r_{O2}]r_{O1} + r_{O2}$$



Calculation of output resistance of cascode stage.

Cascode stage – with ideal current load

Homework

Calculate the exact voltage gain of the circuit shown in Fig. 3.67.

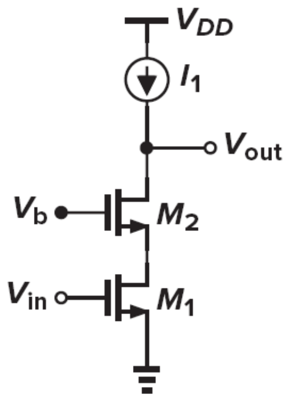


Figure 3.67 Cascode stage with current-source load.

Solution

The actual G_m of the stage is slightly less than g_{m1} because a fraction of the small-signal current produced by M_1 is shunted to ground by r_{O1}. As depicted in Fig. 3.68(a), we short the output node to ac ground and recognize that the impedance seen looking into the source of M_2 is equal to $[1/(g_{m2}+g_{mb2})]||r_{O2}$. Thus,

$$I_{out} = g_{m1}V_{in}\frac{r_{O1}}{r_{O1} + \dfrac{1}{g_{m2}+g_{mb2}}\Bigg|\Bigg|r_{O2}}$$

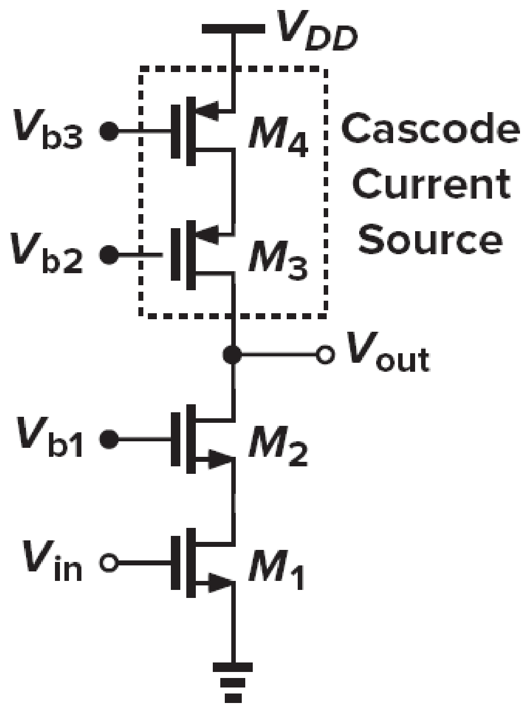
It follows that the overall transconductance is equal to

$$G_m = \frac{g_{m1}r_{O1}[r_{O2}(g_{m2}+g_{mb2})+1]}{r_{O1}r_{O2}(g_{m2}+g_{mb2})+r_{O1}+r_{O2}}$$

and hence the voltage gain is given by

$$\begin{aligned}|A_v| &= G_m R_{out}\\ &= g_{m1}r_{O1}[(g_{m2}+g_{mb2})r_{O2}+1]\end{aligned}$$

Cascode stage – with cascode current load



NMOS cascode amplifier
with PMOS cascode load.

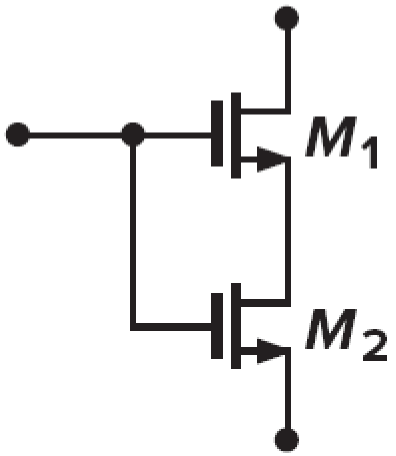
For typical values, we approximate the voltage gain as

$$|A_v| \approx g_{m1}[(g_{m2}r_{O2}r_{O1})\|(g_{m3}r_{O3}r_{O4})]$$

Poor man's cascode

Homework

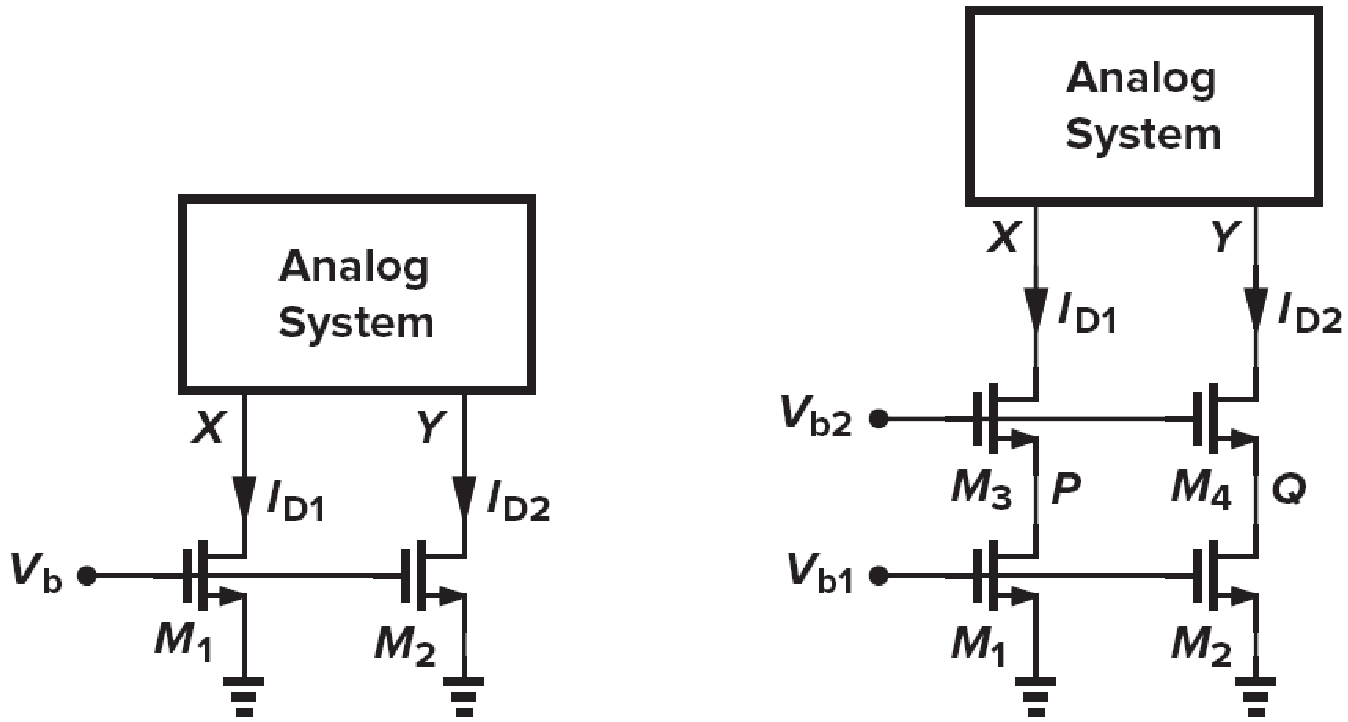
A "minimalist" cascode current source omits the bias voltage necessary for the cascode device. Shown in Fig. (below), this "poor man's cascode" places M2 in the triode region because $V_{GS1} > V_{TH1}$ and $V_{DS2} = V_{GS2} - V_{GS1} < V_{GS2} - V_{TH2}$. In fact, if M1 and M2 have identical dimensions, it can be proved that the structure is equivalent to a single transistor having twice the length - not really a cascode.



Poor man's cascode

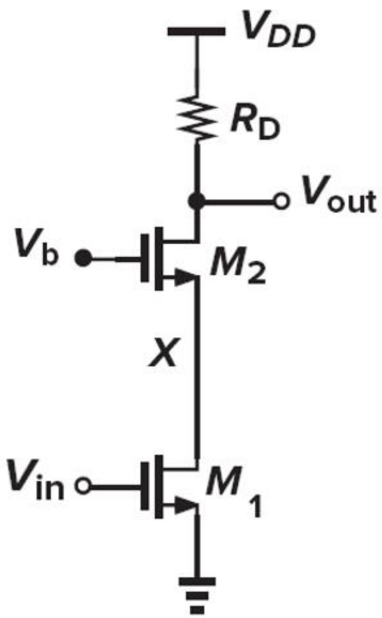
Cascode – shielding property

The high output impedance arises from the fact that if the output-node voltage is changed by V, the resulting change at the source of the cascode device is much less. In a sense, the cascode transistor “shields” the input device from voltage variations at the output.
The shielding property of cascodes proves useful in many circuits.

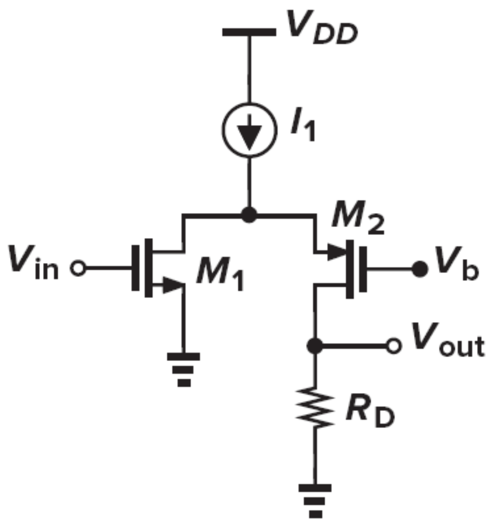


(a) (b)

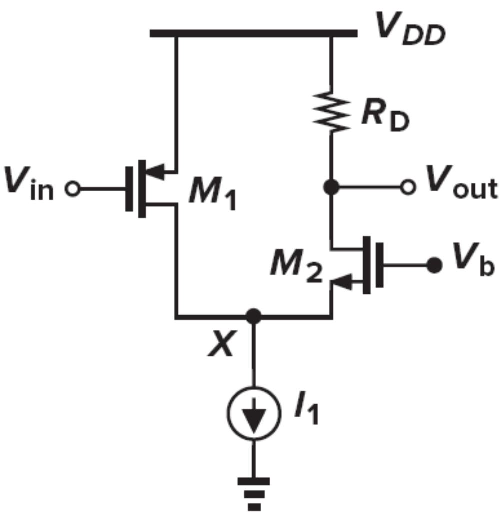
Folded Cascode



Cascode



Folded cascode
with NMOS input



Folded cascode
with PMOS input

Folded cascode

Homework : large signal analysis

It is instructive to examine the large-signal behavior of a folded-cascode stage. Suppose that in Fig. (below-left), Vin decreases from V_{DD} to zero. For Vin > $V_{DD} - |V_{TH1}|$, M1 is off and M2 carries all of I_1, yielding Vout = $V_{DD} - I_1R_D$. For Vin < $V_{DD} - |V_{TH1}|$, M1 turns on in saturation, giving:

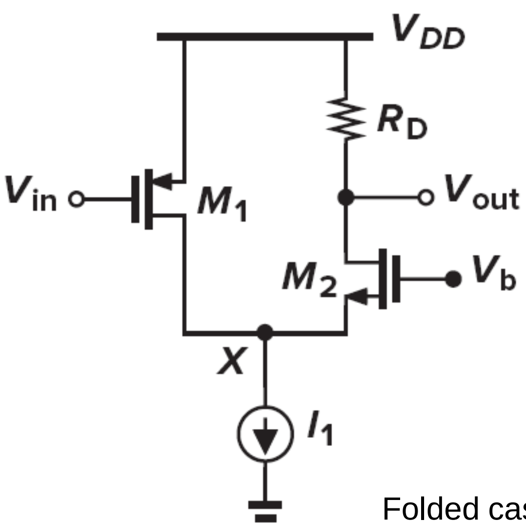
$$I_{D2} = I_1 - \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD} - V_{in} - |V_{TH1}|)^2$$

As Vin drops, I_{D2} decreases further, falling to zero if I_{D1} = I1. This occurs at Vin = Vin_1 if

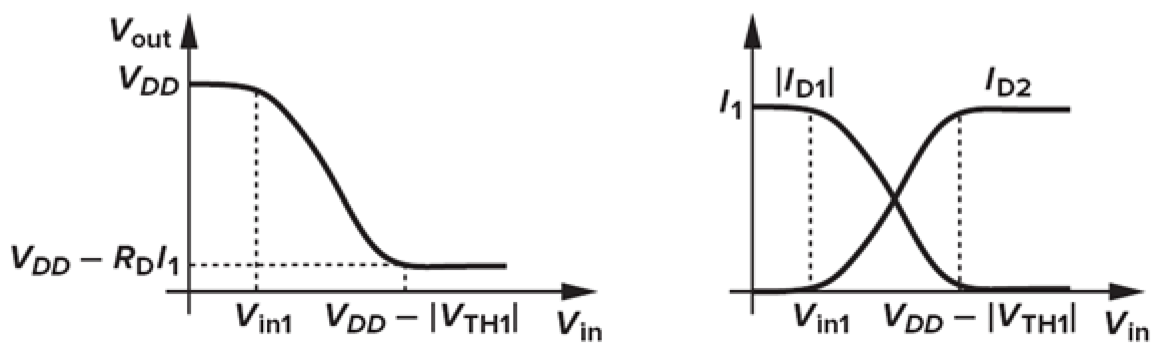
$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD} - V_{in1} - |V_{TH1}|)^2 = I_1$$

$$V_{in1} = V_{DD} - \sqrt{\frac{2I_1}{\mu_p C_{ox}(W/L)_1}} - |V_{TH1}|$$

If Vin falls below this level, I_{D1} tends to be greater than I_1, and M1 enters the triode region so as to ensure $I_{D1} = I_1$. The result is plotted in Fig. (below).

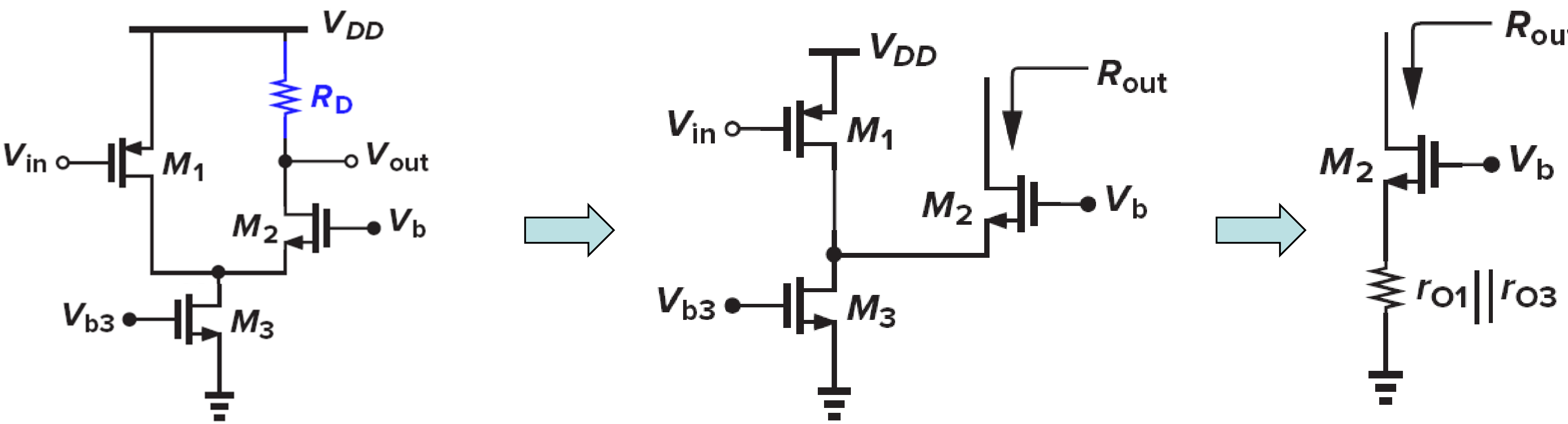


Folded cascode
with PMOS input



Large-signal characteristics of folded cascode.

Folded cascode – output resistance



$$R_{out} = [1 + (g_{m2} + g_{mb2})r_{O2}](r_{O1}\|r_{O3}) + r_{O2}$$

Folded cascode with cascode load

(for high gain)