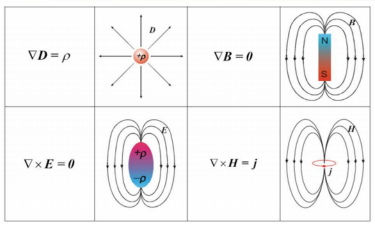
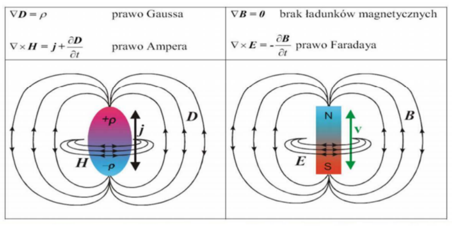
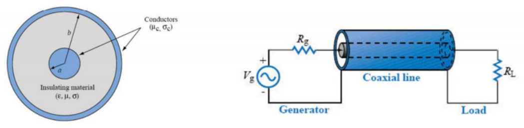
Wykład 1



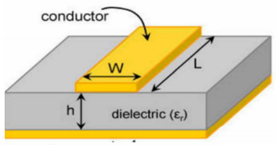
$$\nabla \times E = -\frac{\partial B}{\partial t}$$
$$\nabla \times H = J + \frac{\partial D}{\partial t}$$
$$\nabla \cdot D = \rho$$
$$\nabla \cdot B = 0$$





$$Z_0 = \frac{1}{2\pi}\sqrt{\frac{\mu}{\epsilon}}\ln\frac{D}{d} \approx \frac{138\Omega}{\sqrt{\epsilon_r}}\log_{10}\frac{D}{d}$$

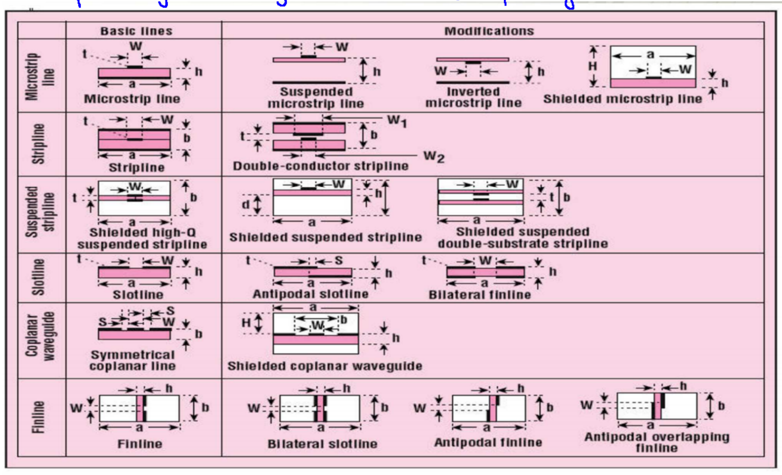
Impedancja środkowa



$$\varepsilon_{eff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\frac{1}{\sqrt{1+12d/w}}$$

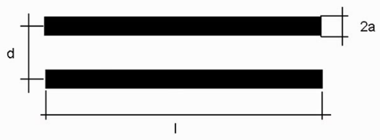
$$Z_f = \begin{cases} \dfrac{60}{\sqrt{\varepsilon_{eff}}}\ln\left(\dfrac{8d}{w} + \dfrac{w}{4d}\right) & \text{dla} \quad W/d \le 1 \\ \dfrac{120\pi}{\sqrt{\varepsilon_{eff}}\,[w/d + 1.393 + 0.667\ln(w/d + 1.444)]} & \text{dla} \quad W/d \ge 1 \end{cases}$$

Przykłady budowy układów mikrofalowych



Warunek kwazistacjonarności

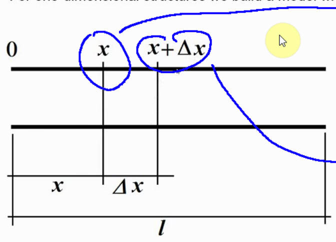
- All physical (real) structures are structures with distributed parameters
- if the quasi-stationary condition is not met $d \ll \lambda$ the structure can only be analyzed with the use of electromagnetic field theory methods



- For one-dimensional structures i.e.: in which one of three dimensions does not fulfill the quasi-stationary condition the analysis with circuit theory can be introduced

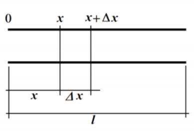
Długość fali jest znacznie większa od długości fizycznej przewodów

- For one-dimensional structures we build a model which we call tranmssion line



wybieramy dowolną wartość

oraz wartość przesuniętą o Δx przy czym zakładamy, że wartość ta jest bardzo mała

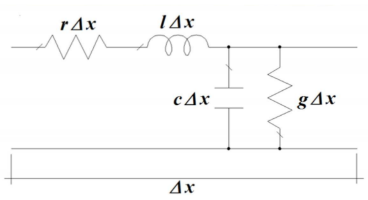


$\Delta x \ll \lambda$

For section Δx we can introduce a model consisting of lumped elements which are:

- inductor L_Δ -which represents magnetic field -Indukcyjność na danej jednostkę długości
- capacitor C_Δ -which represents electric field -Pojemność między liniami transmisyjnymi
- resistor R_Δ -which represents losses in conductors - rezystancja, która reprezentuje straty przewodnika
- conductor G_Δ -which represents losses in dielectrics

reprezentuje straty w dielektryku (izolatorze między liniami)

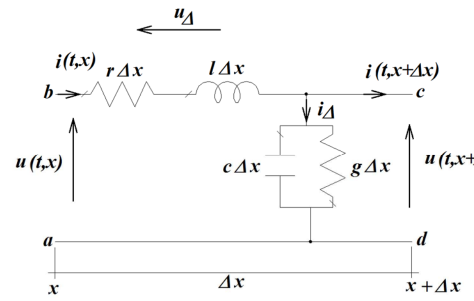


$r\ [\Omega/m] \stackrel{\text{def}}{=} \lim_{\Delta x \to 0}\frac{R_\Delta}{\Delta x}$ $\qquad$ $c\ [F/m] \stackrel{\text{def}}{=} \lim_{\Delta x \to 0}\frac{C_\Delta}{\Delta x}$

$l\ [H/m] \stackrel{\text{def}}{=} \lim_{\Delta x \to 0}\frac{L_\Delta}{\Delta x}$ $\qquad$ $g\ [S/m] \stackrel{\text{def}}{=} \lim_{\Delta x \to 0}\frac{G_\Delta}{\Delta x}$

We create a transmission line by cascading to get $n\Delta x = l$

where n – number of elementary subsections



Now we can apply Kirschofs laws to our model

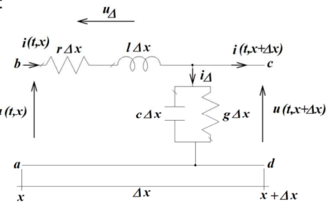
According to II Kirchhoffs law for abcd loop:

$$u(t,x) = u_\Delta + u(t,x+\Delta x)$$

where

$$u_\Delta = r\Delta x\, i(t,x) + l\Delta x\frac{\partial}{\partial t}i(t,x)$$

$$u(t,x) - u(t,x+\Delta x) = r\Delta x\, i(t,x) + l\Delta x\frac{\partial}{\partial t}i(t,x)$$

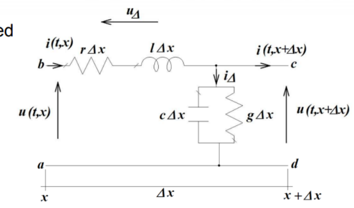


Voltage function u(t,x+Δx) can be expanded in Taylor series around x

$$u(t,x+\Delta x) = u(t,x) + \frac{\partial u(t,x)}{\partial x}\Delta x + \ldots$$

Since Δx is small the higher order terms can be neglected

gdyby Δx nie było małe



Taylors theorem: for function f(x) differentiable on the section (a,b) n+1 times

We can for arbitrary point x of the section write

$$f(x) = f(a) + \frac{x-a}{1!}f'(a) + \frac{(x-a)^2}{2!}f''(a) + \cdots + \frac{(x-a)^n}{n!}f^{(n)}(a) + R_n(x,a)$$

Voltage function u(t,x+Δx) can be expanded in Taylor series around x

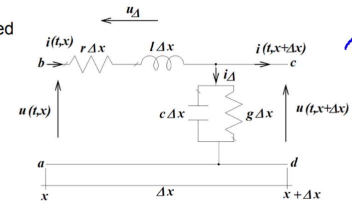
$$u(t,x+\Delta x) = u(t,x) + \frac{\partial u(t,x)}{\partial x}\Delta x + \ldots$$

Since Δx is small the higher order terms can be neglected

$$u(t,x) - u(t,x+\Delta x) = r\Delta x\, i(t,x) + l\Delta x\frac{\partial}{\partial t}i(t,x)$$

$$-\frac{\partial u(t,x)}{\partial x}\Delta x = r\Delta x\, i(t,x) + l\Delta x\frac{\partial}{\partial t}i(t,x)$$

Finally we get $\qquad -\frac{\partial u(t,x)}{\partial x} = ri(t,x) + l\frac{\partial}{\partial t}i(t,x)$

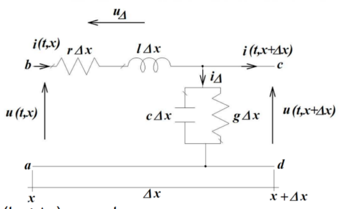


przekształcenie równań Taylora

For c node we can write, according to the I Kirchhoffs law

$$i(t,x) = i_\Delta + i(t,x+\Delta x)$$

$$i_\Delta = g\Delta x u(t,x+\Delta x) + c\Delta x\frac{\partial}{\partial t}u(t,x+\Delta x)$$



Applying Taylor's expansion of a function u(t,x+Δx) we get

$$i_\Delta = g\Delta x u(t,x) + g\frac{\partial}{\partial x}u(t,x)\Delta x^2 + c\Delta x\frac{\partial}{\partial t}u(t,x) + c\frac{\partial^2}{\partial t\partial x}u(t,x)\Delta x^2$$

Terms with Δx² are neglected, hence we get

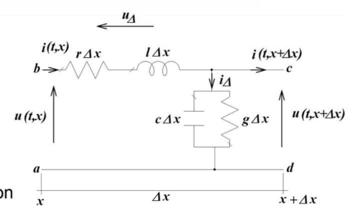
$$i_\Delta = g\Delta x u(t,x) + c\Delta x\frac{\partial}{\partial t}u(t,x)$$

Putting the resulting equetion

$$i_\Delta = g\Delta x u(t,x) + c\Delta x\frac{\partial}{\partial t}u(t,x)$$

To the initial equation

$$i(t,x) = i_\Delta + i(t,x+\Delta x)$$



And applying once again Taylor's expansion of i(t,x+Δx)

$$i(t,x+\Delta x) = i(t,x) + \frac{\partial i(t,x)}{\partial x}\Delta x + \ldots$$

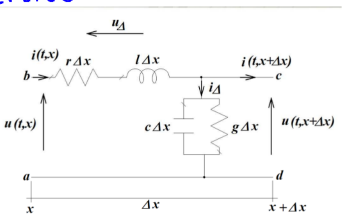
we get

$$-\frac{\partial i(t,x)}{\partial x} = gu(t,x) + c\frac{\partial}{\partial t}u(t,x)$$

Równania Telegrafistów

$$-\frac{\partial u(t,x)}{\partial x} = ri(t,x) + l\frac{\partial}{\partial t}i(t,x)$$

$$-\frac{\partial i(t,x)}{\partial x} = gu(t,x) + c\frac{\partial}{\partial t}u(t,x)$$



- Solution of Telegraphers' equations is difficult wihtout assuming the shape of voltages and currents
- Stady state of transmisison line is possible i.e. under assumption that voltages and currents are sinusoidal

opisują nam sposób propagacji sygnału czyli zachowanie się prądu i napięcia w linii transmisyjnej.

For a given point x we can write

$$u(x,t) = \sqrt{2}U_{sk}(x)\sin(\omega t + \varphi(x)) = Im[\sqrt{2}Ue^{j\omega t}]$$
$$i(x,t) = \sqrt{2}I_{sk}(x)\sin(\omega t + \varphi(x)) = Im[\sqrt{2}Ie^{j\omega t}]$$

> wzory Eulera

where

$U = U_{sk}(x)e^{j\varphi(x)}$
$I = I_{sk}(x)e^{j\varphi(x)}$ are complex rms values of voltage and current

Going back to Telegrapher's equations

$$-\frac{\partial u(t,x)}{\partial x} = ri(t,x) + l\frac{\partial}{\partial t}i(t,x) \qquad -\frac{\partial i(t,x)}{\partial x} = gu(t,x) + c\frac{\partial}{\partial t}u(t,x)$$

$$-Im\left[\sqrt{2}\frac{dU}{dx}e^{j\omega t}\right] = Im[\sqrt{2}rIe^{j\omega t}] + Im[\sqrt{2}j\omega lIe^{j\omega t}]$$

$$-Im\left[\sqrt{2}\frac{dI}{dx}e^{j\omega t}\right] = Im[\sqrt{2}gUe^{j\omega t}] + Im[\sqrt{2}j\omega cUe^{j\omega t}]$$

Oridinary derivatives $\qquad$ Differentiation in time

$$-Im\left[\sqrt{2}\frac{dU}{dx}e^{j\omega t}\right] = Im[\sqrt{2}rIe^{j\omega t}] + Im[\sqrt{2}j\omega lIe^{j\omega t}]$$

$$-Im\left[\sqrt{2}\frac{dI}{dx}e^{j\omega t}\right] = Im[\sqrt{2}gUe^{j\omega t}] + Im[\sqrt{2}j\omega cUe^{j\omega t}]$$

Skiping Im and dividing by $\sqrt{2}e^{j\omega t}$

$$-\frac{dU}{dx} = (r + j\omega l)I$$

$$-\frac{dI}{dx} = (g + j\omega c)U$$

Równania Telegrafistów dla stanu stałego przy wymuszeniu sinusoidalnym

To remember:

- Equations of transmission line in a steady state for sinusoidal excitation
- U, I complex rms values, which are functions of x (distance for the beginning of line)

Introduce new quantity

$$\gamma^2 = (g + j\omega c)(r + j\omega l)$$

zespolona wartość stałej propagacji

Final version of the equations

$$\frac{d^2U}{dx^2} - \gamma^2 U = 0$$

$$\frac{d^2I}{dx^2} - \gamma^2 I = 0$$

These are ordinary differential equations of II order for which the general solutions are known, they are:

$$U = A_1 ch(\gamma x) + B_1 sh(\gamma x)$$

$$I = A_2 ch(\gamma x) + B_2 sh(\gamma x)$$

Since the general solution is the solution of the system of 2 ordinary differential equations of the first order, only two constants are independent

Finally we can write:

$$U = Ach(\gamma x) + Bsh(\gamma x)$$

$$I = -\frac{1}{Z_f}[Bch(\gamma x) + Ash(\gamma x)]$$

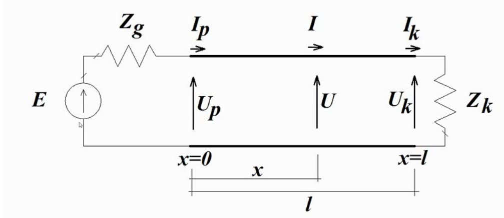
where

$$Z_f = \sqrt{\frac{r + j\omega l}{g + j\omega c}}$$

impedancja falowa

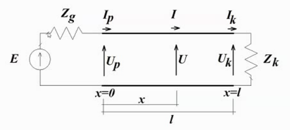
Constants *A* and *B* are calculated form initial conditions

Lets consider a section of the length *l* in a circuit below



$$U = Ach(\gamma x) + Bsh(\gamma x)$$

$$I = -\frac{1}{Z_f}[Bch(\gamma x) + Ash(\gamma x)]$$



For x = 0 → początek

$U = U_p \qquad I = I_p$

Putting to the equations we get

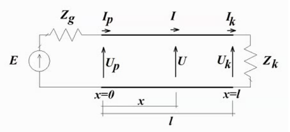
$$U_p = A$$

$$I_p = -\frac{1}{Z_f}B$$

Finally we get the solutions

$$U = U_p ch(\gamma x) - Z_f I_p sh(\gamma x)$$

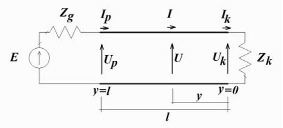
$$I = I_p ch(\gamma x) - \frac{U_p}{Z_f} sh(\gamma x)$$



Similar solutions we can write measuring distance from the end of the line:

$$U = U_k ch(\gamma y) + Z_f I_k sh(\gamma y)$$

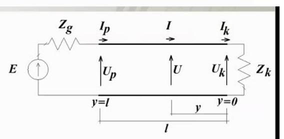
$$I = I_k ch(\gamma y) + \frac{U_k}{Z_f} sh(\gamma y)$$



Physical interpretation of Z_f

$$U = U_k ch(\gamma y) + Z_f I_k sh(\gamma y)$$

$$I = I_k ch(\gamma y) + \frac{U_k}{Z_f} sh(\gamma y)$$



Assuming *y* = *l* we get

$$U_p = U_k ch(\gamma l) + Z_f I_k sh(\gamma l)$$

$$I_p = I_k ch(\gamma l) + \frac{U_k}{Z_f} sh(\gamma l)$$

Assuming, that the line is loaded with impedance Z_k: $\quad U_k = I_k Z_k$

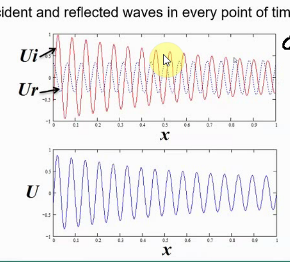
$$U_p = U_k ch(\gamma l) + \frac{Z_f}{Z_k} U_k sh(\gamma l) \qquad I_p = \frac{U_k}{Z_k} ch(\gamma l) + \frac{U_k}{Z_f} sh(\gamma l)$$

$$\longrightarrow \quad U_p = U_k\left(ch(\gamma l) + \frac{Z_f}{Z_k} sh(\gamma l)\right) \qquad I_p = U_k\left(\frac{1}{Z_k} ch(\gamma l) + \frac{1}{Z_f} sh(\gamma l)\right)$$

We can therefore write the voltage and current in a line

$U = U_i + U_r \qquad I = I_i + I_r$

As a sum of incident and reflected waves in every point of time and every point in a line



odbicie sygnału od impedancji obciążenia

Calculating the ratios of voltages and currents of transmitted and reflected waves

$$\frac{U_i}{I_i} = \frac{0.5(U_p + I_p Z_f)\, e^{-\gamma x}}{0.5(I_p + \frac{U_p}{Z_f}) e^{-\gamma x}} = \frac{Z_f\left(I_p + \frac{U_p}{Z_f}\right)}{I_p + \frac{U_p}{Z_f}} = Z_f$$

$$\frac{U_r}{I_r} = \frac{0.5(U_p - I_p Z_f)\, e^{\gamma x}}{0.5(I_p - \frac{U_p}{Z_f}) e^{\gamma x}} = \frac{Z_f\left(\frac{U_p}{Z_f} - I_p\right)}{I_p - \frac{U_p}{Z_f}} = -Z_f$$

Współczynnik odbicia

Reflection coefficient is defined as an amplitude ratio of reflected to the transmitted wave

$$\Gamma_U = \frac{U_r}{U_i} = \frac{U_k - I_k Z_f}{U_k + I_k Z_f} e^{-2\gamma y}$$

Jest to stosunek fali odbitej do fali padającej

Since at the end of a line $y = 0$, $U_k = I_k Z_k$ hence

$$\Gamma_{Uk} = \frac{Z_k - Z_f}{Z_k + Z_f}$$ - the reflection coefficient at the end of a line → współczynnik odbicia na końcu linii

We can express the reflection coefficient with the reflection coefficient at the end of a line

$$\Gamma_U = \Gamma_{Uk} e^{-2\gamma y}$$

Reflection coefficient for current: $\Gamma_I = \frac{I_r}{I_i} = \frac{I_k - \frac{U_k}{Z_f}}{I_k + \frac{U_k}{Z_f}} e^{-2\gamma y} = \frac{I_k Z_f - U_k}{I_k Z_f + U_k} e^{-2\gamma y} = -\Gamma_U$

Taking that equity we assume that $\qquad \Gamma = \Gamma_U$

$$\Gamma = \Gamma_k e^{-2\gamma y}$$

Writing reflection coefficient Γ_k in exponential form

$$\Gamma_k = |\Gamma_k| e^{j\Theta_k}$$

We get

$$\Gamma = |\Gamma_k| e^{j\Theta_k} e^{-2\gamma y} = |\Gamma_k| e^{-2\alpha y} e^{j(\Theta_k - 2\beta y)}$$

$|\Gamma| = |\Gamma_k| e^{-2\alpha y}$ Magnitude of reflection coefficient takes the largest value at the end of a line

Since $\quad \Gamma_k = \frac{Z_k - Z_f}{Z_k + Z_f} \quad$ It can take the following values:

Since $\quad \Gamma_k = \frac{Z_k - Z_f}{Z_k + Z_f} \quad$ It can take the following values:

for Z_k = 0 $\qquad \Gamma = -1$ → cały sygnał jest odbity

for $Z_k = Z_f$ $\qquad \Gamma = 0$ → cała moc sygnału przekazywanego w linii transmisyjnej jest efektywnie przekazywana do obciążenia

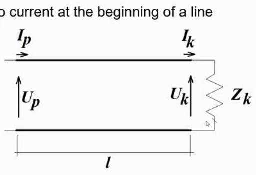
for $Z_k = \infty$ $\qquad \Gamma = 1$

$0 \le |\Gamma| \le 1$

rozwarcie na końcu (całkowite odbicie)

Input impedance – by def. – ratio of voltage to current at the beginning of a line

$$Z_w = \frac{U_p}{I_p}$$



Alternatively we can express input impedance with reflection coefficient

$$Z_w = \frac{U_p}{I_p} = Z_f \frac{Z_k + Z_f\, th(\gamma l)}{Z_f + Z_k\, th(\gamma l)}$$

$$\downarrow$$

$$Z_w = \frac{U_p}{I_p} = Z_f \frac{1 + \Gamma_k e^{-2\gamma l}}{1 - \Gamma_k e^{-2\gamma l}}$$

Impedancja wejściowa zależy od impedancji obciążenia jak i długości i parametrów linii

Rozkład napięcia i prądu skutecznego wzdłuż linii

Lets investigate the distribution of an rms values for voltages and currents along the line

From the expressions

$$U = U_i(1 + \Gamma)$$

$$U = 0.5(U_k + I_k Z_f)\, e^{\gamma y}(1 + \Gamma_k e^{-2\gamma l})$$

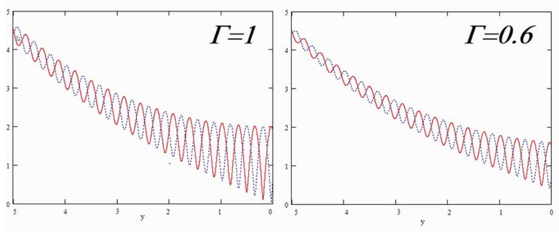
And the expression $U_{ik} = 0.5(U_k + I_k Z_f)$

$$U = U_{ik} e^{\gamma y}(1 + \Gamma_k e^{-2\gamma l})$$

We calculate the magnitude of the expression $\quad |U| = \sqrt{UU^*}$ ← moduł z wartości bezwzględnej

$$|U| = U_{ik} e^{\alpha y}\sqrt{1 + 2\Gamma_k e^{-2\alpha y}\cos(\Theta_k - 2\beta y) + {\Gamma_k}^2 e^{-4\alpha y}}$$

$$|I| = \frac{I_{ik}}{Z_f} e^{\alpha y}\sqrt{1 - 2\Gamma_k e^{-2\alpha y}\cos(\Theta_k - 2\beta y) + {\Gamma_k}^2 e^{-4\alpha y}}$$



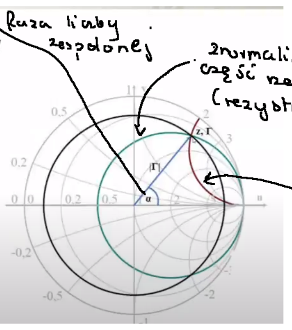
Also

$\Gamma_k e^{-j2\beta l} = p + jq$ faza liaby zespolonej

$|\Gamma_k|^2 = p^2 + q^2$

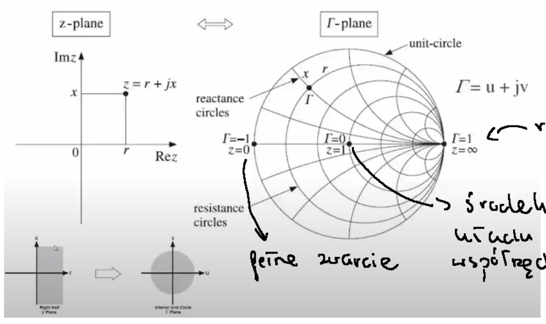
znormalizowana część rzeczywista (rezystancja)

Create concentric circles on a Γ plane



reaktancja

Wykres Smitha to transformacja 2 układów współrzędnych



rozwarcie

środek układu współrzędnych

pełne zwarcie

Opis macierzy linii transmisyjnej

ABCD matrix $\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = A \begin{bmatrix} U_2 \\ -I_2 \end{bmatrix}$

$A = ABCD = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$U_1 = a_{11}U_2 - a_{12}I_2$

$I_1 = a_{21}U_2 - a_{22}I_2$

From transmission line equations

$U_p = U_k ch(\gamma l) + Z_f I_k sh(\gamma l)$

$I_p = I_k ch(\gamma l) + \frac{U_k}{Z_f} sh(\gamma l)$

$U_1 = a_{11}U_2 - a_{12}I_2$

$I_1 = a_{21}U_2 - a_{22}I_2$

$U_p = U_k ch(\gamma l) + Z_f I_k sh(\gamma l)$

$I_p = I_k ch(\gamma l) + \frac{U_k}{Z_f} sh(\gamma l)$

$I_k = -I_2$

$I_p = I_1$

$U_p = U_1$

$U_k = U_2$

$U_1 = a_{11}U_2 - a_{12}I_2$

$I_1 = a_{21}U_2 - a_{22}I_2$

$U_p = U_k ch(\gamma l) + Z_f I_k sh(\gamma l)$

$I_p = I_k ch(\gamma l) + \frac{U_k}{Z_f} sh(\gamma l)$

$I_k = -I_2$

$I_p = I_1$

$U_p = U_1$

$U_k = U_2$

$U_1 = U_2 ch(\gamma l) + Z_f I_2 sh(\gamma l)$

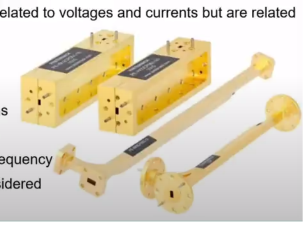
$I_1 = U_2 \frac{1}{Z_f} sh(\gamma l) - I_2 ch(\gamma l)$

$$\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \underbrace{\begin{bmatrix} ch(\gamma l) & Z_f sh(\gamma l) \\ \frac{1}{Z_f} sh(\gamma l) & ch(\gamma l) \end{bmatrix}}_{} \begin{bmatrix} U_2 \\ -I_2 \end{bmatrix}$$

wzór dla modułu ogólnego (również stratnego)
stąd funkcje sh i ch

Użycie Macierzy rozproszonej (Scattering Matrix, Macierz S)

- In electrical circuits we use impedance / admittance description which utilize coefficients of voltage-current equations
- Impedance / admittance matrices may not exist for every passive circuit
- A description with scattering matrix has been introduced
- Scattering parameters are not directly related to voltages and currents but are related to their linear combination
- Every linear and passive circuit can be described with scattering matrix
- Scattering matrix is defined in conditions of ports loaded with given impedances
- Convenient description in microwave frequency range where the power transfer is considered instead of voltage / current description



Let us consider one-port

We define incident parameter $a = \alpha_{11}U + \alpha_{12}I$ (fala padająca)

and reflection parameter $b = \alpha_{21}U + \alpha_{22}I$ (fala odbita)

α_{ij} - Arbitrary constants, can be choosen depending on the considered problem, typically they are chosen is a way to get simple description of power

We assume

$$\alpha_{11} = \alpha_{21} = \frac{1}{2\sqrt{R_0}}, \qquad \alpha_{12} = -\alpha_{22} = \frac{\sqrt{R_0}}{2}$$

współczynniki normalizacji

R_0 - Arbitrary real value called normalization constant, the dimmension is Ω

$a = \alpha_{11}U + \alpha_{12}I$

$b = \alpha_{21}U + \alpha_{22}I$

$\alpha_{11} = \alpha_{21} = \frac{1}{2\sqrt{R_0}}$

$\alpha_{12} = -\alpha_{22} = \frac{\sqrt{R_0}}{2}$

We get

$$a = \frac{1}{2}\left(\frac{U}{\sqrt{R_0}} + I\sqrt{R_0}\right) = \frac{1}{2}\left(\frac{U + R_0 I}{\sqrt{R_0}}\right)$$

$$b = \frac{1}{2}\left(\frac{U}{\sqrt{R_0}} - I\sqrt{R_0}\right) = \frac{1}{2}\left(\frac{U - R_0 I}{\sqrt{R_0}}\right)$$

$$a = \frac{1}{2}\left(\frac{U}{\sqrt{R_0}} + I\sqrt{R_0}\right) = \frac{1}{2}\left(\frac{U + R_0 I}{\sqrt{R_0}}\right)$$

$$b = \frac{1}{2}\left(\frac{U}{\sqrt{R_0}} - I\sqrt{R_0}\right) = \frac{1}{2}\left(\frac{U - R_0 I}{\sqrt{R_0}}\right)$$

Normalizing voltage and current — Normalizacja

$u = \frac{U}{\sqrt{R_0}}$ $\qquad i = I\sqrt{R_0}$

We get

$a = \frac{1}{2}(u + i)$ (fala padająca) $\qquad b = \frac{1}{2}(u - i)$ (fala odbita)

By adding we get

$u = a + b \qquad i = a - b$

$u = a + b \qquad i = a - b$

Since for transmission line

$U = U_i + U_r$

U_i → napięcie fali padającej

U_r → napięcie fali odbitej

The parameter a – is called incident parameter (incident wave) fala padająca

b – is called reflection parameter (reflected wave) fala odbita

The ratio $\frac{b}{a} = s_{11}$ **is the scattering coefficient of a one-port** → współczynnik odbicia (dokładnie to samo co gamma)

$$s_{11} = \frac{b}{a} = \frac{u-i}{u+i} = \frac{\frac{u}{i}-1}{\frac{u}{i}+1} = \frac{z-1}{z+1}$$

where $z = \frac{Z}{R_0}$ is normalized impedance of a one-port

$$s_{11} = \frac{b}{a} = \frac{u-i}{u+i} = \frac{\frac{u}{i}-1}{\frac{u}{i}+1} = \frac{z-1}{z+1} \Longleftrightarrow \Gamma_k = \frac{Z_k - Z_f}{Z_k + Z_f}$$

One can notice that for a passive one-port, i.e. Re[Z] ≥ 0 absolute value $|s_{11}| \le 1$

co najwyżej jeden bo fala odbita może być najwyżej taka duża jak fala padająca

$$|s_{11}|^2 = \frac{(Re(z-1))^2 + Im^2 z}{(Re(z+1))^2 + Im^2 z}$$

Hence for Re[Z] ≥ 0 absolute value $|s_{11}|^2 \le 1$

therefore also $|s_{11}| \le 1$ – this is the condition of passivity of a one-port necessary and sufficient

Generator impedancji

Let us consider the circuit

$I = \frac{E_g}{Z + Z_g} \qquad U = E_g \frac{Z}{Z + Z_g}$

$R_g = \mathrm{Re}Z_g > 0$

$$a = \frac{1}{2\sqrt{R_0}}(U + IR_0) = \frac{1}{2\sqrt{R_0}}\left(E_g \frac{Z}{Z+Z_g} + \frac{E_g}{Z+Z_g}R_0\right) = \frac{E_g}{2\sqrt{R_0}}\left(\frac{Z + R_0}{Z + Z_g}\right)$$

$$b = \frac{1}{2\sqrt{R_0}}(U - IR_0) = \frac{1}{2\sqrt{R_0}}\left(E_g \frac{Z}{Z+Z_g} - \frac{E_g}{Z+Z_g}R_0\right) = \frac{E_g}{2\sqrt{R_0}}\left(\frac{Z - R_0}{Z + Z_g}\right)$$

normalizing $\frac{E_g}{\sqrt{R_0}} = e_g \qquad \frac{Z}{R_0} = z \qquad \frac{Z_g}{R_0} = z_g$

We get

$a = \frac{e_g}{2}\left(\frac{z+1}{z+z_g}\right)$

$b = \frac{e_g}{2}\left(\frac{z-1}{z+z_g}\right)$

$R_g = \mathrm{Re}Z_g > 0$

If we choose $R_0 = Z_g$, then $z_g = 1$ and

$a = \frac{e_g}{2}$

If also $R_0 = Z$, then

$b = 0$ hence $s_{11} = 0$ In the circuit we have match between the one-port and the source

Calculating the power dissipated in a one-port

$$P = Re\{UI^*\} = Re\left\{u\sqrt{R_0}\left(\frac{i}{\sqrt{R_0}}\right)^*\right\} = Re\{ui^*\}$$

$$P = Re\{(a+b)(a^* - b^*)\} = |a^2| - |b^2|$$

$|a^2|$ → moc fali padającej; $|b^2|$ → moc fali odbitej

$R_g = \mathrm{Re}Z_g > 0$

Taking into account, that

$b = s_{11}a$

$P = |a^2|(1 - |s_{11}|^2)$

$P = |a^2|(1 - |s_{11}|^2)$

Available power equals

$P_{av} = \frac{E_g^2}{4R_0} = \frac{e_g^2}{4}$ Moc dostępna

$R_g = \mathrm{Re}Z_g > 0$

since

$a = \frac{e_g}{2}$

we get

$P_{av} = a^2$

hence $P = |a^2|(1 - |s_{11}|^2) = P_{av}(1 - |s_{11}|^2)$

for $s_{11} = 0$ $P = P_{av}$	for $s_{11} = 1$ $P = 0$

Pełne odbicie

dopasowanie impedancyjne

$P = |a^2| - |b^2|$

We define incident power

$P_i = a^2$ Moc padająca

And reflected power

$P_r = b^2$ Moc odbita

$R_g = \mathrm{Re}Z_g > 0$

hence

$P = |a^2| - |b^2| = P_i - P_r$ Moc dostarczona (~~delivered~~ or accepted)

in matched condition

$P = P_i = P_{av} \qquad P_r = 0$

for $s_{11} = 1$ $\quad P = 0 \quad P_i = P_r$ All the power is reflected from the load

Dwuwrotnik

We define

$$a_1 = \frac{1}{2}\left(\frac{U_1}{\sqrt{R_{01}}} + I_1\sqrt{R_{01}}\right) \qquad a_2 = \frac{1}{2}\left(\frac{U_2}{\sqrt{R_{02}}} + I_2\sqrt{R_{02}}\right)$$

$$b_1 = \frac{1}{2}\left(\frac{U_1}{\sqrt{R_{01}}} - I_1\sqrt{R_{01}}\right) \qquad b_2 = \frac{1}{2}\left(\frac{U_2}{\sqrt{R_{02}}} - I_2\sqrt{R_{02}}\right)$$

normalizing

$u_1 = \frac{U_1}{\sqrt{R_{01}}} \qquad u_2 = \frac{U_2}{\sqrt{R_{02}}}$

$i_1 = I_1\sqrt{R_{01}} \qquad i_2 = I_2\sqrt{R_{02}}$

$a_1 = \frac{1}{2}(u_1 + i_1) \qquad a_2 = \frac{1}{2}(u_2 + i_2)$

$b_1 = \frac{1}{2}(u_1 - i_1) \qquad b_2 = \frac{1}{2}(u_2 - i_2)$

Finally we can define scattering matrix of a two-port

fali padającej w przeciwną na porcie 1, ale również fali padającej w przeciwną na porcie 2.

$b_1 = s_{11}a_1 + s_{12}a_2$

$b_2 = s_{21}a_1 + s_{22}a_2$

jest to superpozycja

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

Scattering matrix connects incident and reflected coefficients of a two-port — Macierz rozpraszania

Components of a matrix are given as follows

$s_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0}$

Macierz S wiąże nam fale padające z falami odbitymi lub falami padającymi

Jeśli pada fala padająca a_1 to tworzy falę odbitą b_1, ale jednocześnie tworzy falę b_2, bo istnieje możliwość, że dwuwrotnik jest w trakcie transmisji z portu 1 na port 2

$s_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0}$

Z=R02

Assumption that $a_2 = 0$ means that $u_2 = -i_2$

$a_2 = \frac{1}{2}(u_2 + i_2)$

hence $\frac{U_2}{\sqrt{R_{02}}} = -I_2\sqrt{R_{02}}$

$\frac{U_2}{-I_2} = R_{02}$

$s_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0}$

Therefore s_{11} is an input reflection coefficient of a two-port

$$s_{11} = \frac{b_1}{a_1} = \frac{z_1 - 1}{z_1 + 1} = \frac{Z_1 - R_{01}}{Z_1 + R_{01}} \qquad z_1 = \frac{Z_1}{R_{01}}$$

From the second equation $\qquad b_2 = s_{21}a_1 + s_{22}a_2$

$s_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0}$ Współczynnik transmisji

$s_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0}$

$$s_{21} = \frac{\frac{1}{2}\left(\frac{U_2}{\sqrt{R_{02}}} - I_2\sqrt{R_{02}}\right)}{\frac{1}{2}\left(\frac{U_1}{\sqrt{R_{01}}} + I_1\sqrt{R_{01}}\right)}$$

Taking ino account that $\quad I_1 = \frac{E_g - U_1}{R_{01}} \qquad I_2 = \frac{-U_2}{R_{02}}$

$s_{21} = 2\frac{U_2}{E_g}\sqrt{\frac{R_{01}}{R_{02}}}$ therefore $s_{21}{}^2 = \frac{P_2}{P_{av}}$ Transducer power gain

wzmocnienie (tłumienie) sygnału przekazywanego od mocy

P_2 power dissipated in the resistor R_{02}

Każdy współczynnik podniesiony do kwadratu informuje nas o mocy

Similarly for excitation of output port

$s_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0} = \frac{z_2 - 1}{z_2 + 1}$ ← współczynnik odbicia po drugiej stronie → aby to zmierzyć należy to obciążyć bezodbiciowo

$s_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0} = 2\frac{U_1}{E_{g2}}\sqrt{\frac{R_{02}}{R_{01}}}$ ← stosunek napięć wejściowych modyfikowany przez rezystancje charakterystyczne na wejściach

$s_{22}{}^2 = \frac{P_{r2}}{P_{av2}}$ Power reflected towards the source

↳ stosunek mocy odbitej do padającej

$s_{12}{}^2 = \frac{P_1}{P_{av2}}$ Reverse transducer power gain

↳ wzmocnienie (tłumienie) mocy odwróconej (z portu 2)

Power delivered to a two-port — moc dostarczona na porcie 1

P_1 - power delivered to the input

P_2 - power delivered to output — moc dostarczona na porcie 2

$P = P_1 + P_2$ - total power delivered to the circuit

$P_1 = |a_1|^2 - |b_1|^2 \qquad P_2 = |a_2|^2 - |b_2|^2$

$$P = P_1 + P_2 = |a_1|^2 - |b_1|^2 + |a_2|^2 - |b_2|^2 = |a_1|^2 + |a_2|^2 - (|b_1|^2 + |b_2|^2)$$

since

$$\boldsymbol{a}^{*t}\boldsymbol{a} = [a_1^* \quad a_2^*]\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = a_1^* a_1 + a_2^* a_2 = |a_1|^2 + |a_2|^2$$

hence $P = P_1 + P_2 = \boldsymbol{a}^{*t}\boldsymbol{a} - \boldsymbol{b}^{*t}\boldsymbol{b}$

Power delivered to a two-port

P_1 - power delivered to the input

P_2 - power delivered to output

$P = P_1 + P_2 = \boldsymbol{a}^{*t}\boldsymbol{a} - \boldsymbol{b}^{*t}\boldsymbol{b}$

Knowing that $\boldsymbol{b} = \boldsymbol{S}\boldsymbol{a}$

$P = \boldsymbol{a}^{*t}\boldsymbol{a} - \boldsymbol{b}^{*t}\boldsymbol{b} = \boldsymbol{a}^{*t}\boldsymbol{a} - \boldsymbol{S}^{*t}\boldsymbol{a}^{*t}\boldsymbol{S}\boldsymbol{a}$

$\boldsymbol{P = a^{*t}(1 - S^{*t}S)a}$

Wyliczenia stratności układu

For lossless two-ports P = 0, hence for a ≠ 0

$\boldsymbol{1 - S^{*t}S = 0}$

$\boldsymbol{S^{*t}S = 1}$ Scattering matrix of a lossless network is unitary ← oznacza to, że macierz jest bezstratna

warunek sprawdzenia bezstratności wielowrotnika

$\boldsymbol{S^{*t}S = 1}$

From unitary condition

$$\boldsymbol{S}^{*t}\boldsymbol{S} = \begin{bmatrix} s_{11}^* & s_{21}^* \\ s_{12}^* & s_{22}^* \end{bmatrix}\begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} = \begin{bmatrix} s_{11}^* s_{11} + s_{21}^* s_{21} & s_{11}^* s_{12} + s_{21}^* s_{22} \\ s_{11} s_{12}^* + s_{22}^* s_{21} & s_{12}^* s_{12} + s_{22}^* s_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

hence

$s_{11}^* s_{11} + s_{21}^* s_{21} = 1$

$|s_{11}|^2 + |s_{21}|^2 = 1$

to co się odbije i to co się przedostanie na drugą stronę sumarycznie muszą dać 1 (oczywiście do kwadratu, bo moce)

similarly

$|s_{12}|^2 + |s_{22}|^2 = 1$

~~od~~ drugiej strony to samo

Technika mikroprocesorowa - wykład II

Linia transmisyjna

$\omega L >> r$, $\omega C >> g$ // traktujemy linię jako bezstratną

$$Z_f = \sqrt{\frac{r + j\omega L}{g + j\omega C}} = \sqrt{\frac{L}{C}}$$

impedancja falowa linii

przestaje zależeć od częstotliwości

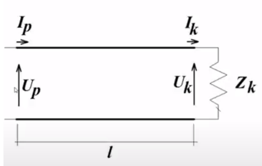
$\alpha = 0$ // część rzeczywista γ → stała propagacji

$\gamma = j\beta = j\omega\sqrt{LC}$

$V_f = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$ - prędkość falowa

! Jeśli napięcie i prąd określają funkcję trygonometryczne sin i cos to oznacza, że linia transmisyjna jest bezstratna, w przeciwnym wypadku występują funkcje hiperboliczne (ch, sh)

Impedancja wejściowa od strony generatora



obciążenie

część urojona γ

$U_p = U_k\cos(\beta l) + jZ_f I_k\sin(\beta l)$

$I_p = I_k\cos(\beta l) + j\frac{U_k}{Z_f}\sin(\beta l)$

$$Z_w = \frac{U_p}{I_p} = Z_f\frac{Z_k + jZ_f\tan(\beta l)}{Z_f + jZ_k\tan(\beta l)}$$

Standing wave distribution

$|U| = U_{ik}e^{\alpha y}\sqrt{1 + 2\Gamma_k e^{-2\alpha y}\cos(\Theta_k - 2\beta y) + \Gamma_k^2 e^{-4\alpha y}}$

$|I| = \frac{I_{ik}}{Z_f}e^{\alpha y}\sqrt{1 - 2\Gamma_k e^{-2\alpha y}\cos(\Theta_k - 2\beta y) + \Gamma_k^2 e^{-4\alpha y}}$

$\alpha = 0$

$|U| = U_{ik}\sqrt{1 + 2\Gamma_k\cos(\Theta_k - 2\beta y) + \Gamma_k^2}$

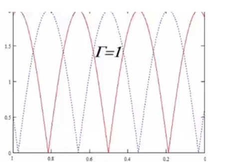
No de

$|I| = \frac{I_{ik}}{Z_f}\sqrt{1 - 2\Gamma_k\cos(\Theta_k - 2\beta y) + \Gamma_k^2}$

Niezależny od $\boxed{y}$ -> rozmiarów (wymiarów linii)

Standing wave distribution

$|U| = U_{ik}\sqrt{1 + 2\Gamma_k\cos(\Theta_k - 2\beta y) + \Gamma_k^2}$



Maxima are located at the distance

$\cos(\Theta_k - 2\beta y) = 1$

$\Theta_k - 2\beta y = 2\pi k$

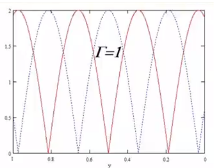
$y_{max} = \frac{1}{2\beta}(\Theta_k - 2\pi k)$ k = 0,1,2,...

$y_{min} = \frac{1}{2\beta}(\Theta_k - \pi(2k+1))$ k = 0,1,2,...

wyliczamy maksimum i minimum wymiarów linii

Standing wave distribution

$|U| = U_{ik}\sqrt{1 + 2\Gamma_k\cos(\Theta_k - 2\beta y) + \Gamma_k^2}$



The distance between minimum and maximum

$y_{max} - y_{min} = \frac{1}{2\beta}(\Theta_k - 2\pi k) - y_{min} = \frac{1}{2\beta}(\Theta_k - \pi(2k+1)) = \frac{\pi}{2\beta} = \frac{\lambda}{4}$

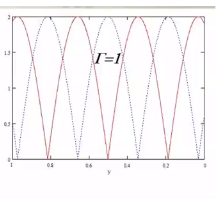
The distance between two consecutive minima

$y_{min\_k} - y_{min\_k+1} = \frac{\lambda}{4} = \frac{V_f}{2f}$ a dla ε_r=1 $y_{min\_k} - y_{min\_k+1} = \frac{\lambda}{4} = \frac{c}{2f}$

Odległości pomiędzy sąsiednimi minimami będą maksymami to połowa długości fali $\frac{\lambda}{2}$

Standing wave distribution

$|U| = U_{ik}\sqrt{1 + 2\Gamma_k\cos(\Theta_k - 2\beta y) + \Gamma_k^2}$



Standing wave ratio współczynnik fali stojącej

$VSWR = \varrho = \frac{1 + |\Gamma_k|e^{-2\alpha y}}{1 - |\Gamma_k|e^{-2\alpha y}}$

$\alpha = 0$

$VSWR = \varrho = \frac{1 + |\Gamma_k|}{1 - |\Gamma_k|} = \frac{U_{max}}{U_{min}}$

Standing wave ratio in lossless line does not depend on y and can be used for measurement of reflection coefficient

1. Transmission line loaded with matched load

$Z_k = Z_f$

Reflection coefficient

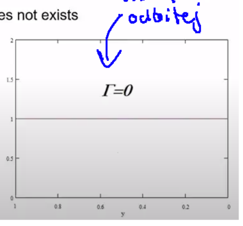
współczynnik odbicia $\Gamma_k = \frac{Z_k - Z_f}{Z_k + Z_f} = 0$ Reflected wave does not exists

brak fali odbitej

Standing wave ratio

$VSWR = \varrho = \frac{1 + |\Gamma_k|}{1 - |\Gamma_k|} = \frac{U_{max}}{U_{min}} = 1$

najniższa wartość jaką można uzyskać



2. Transmission line shorted at the end

$Z_k = 0$ // linia jest zwarta

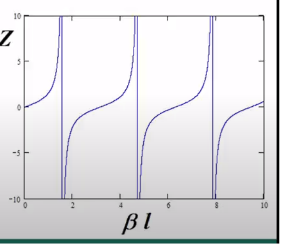
Reflection coefficient

$\Gamma_k = \frac{Z_k - Z_f}{Z_k + Z_f} = -1$

Input impedance

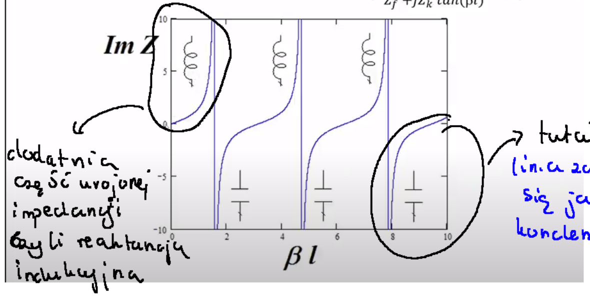
$Z_w = Z_f\frac{Z_k + jZ_f\tan(\beta l)}{Z_f + jZ_k\tan(\beta l)} = jZ_f\tan(\beta l)$

impedancja wejściowa linii bezstratnej



AGH

2. Transmission line shorted at the end $Z_k = 0$ $Z_w = Z_f\frac{Z_k + jZ_f\tan(\beta l)}{Z_f + jZ_k\tan(\beta l)} = jZ_f\tan(\beta l)$

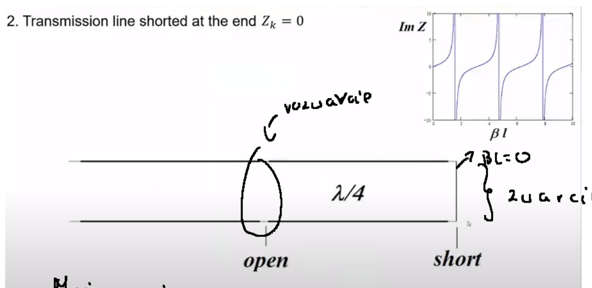


dodatnia część urojonej impedancji czyli reaktancja indukcyjna

czyli linia zachowuje się jak cewka

tutaj linia zachowuje się jak kondensator

2. Transmission line shorted at the end $Z_k = 0$



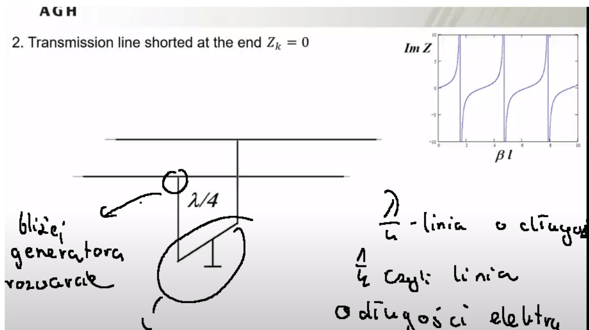
Można więc zrobić w następujący sposób:

- zostawić układ jak wyżej
- odciąć odcinek $\frac{\lambda}{4}$

Działanie będzie identyczne

AGH

2. Transmission line shorted at the end $Z_k = 0$



$\frac{\lambda}{4}$ - linia o długości $\frac{\lambda}{4}$ czyli linia o długości elektrycznej 90° ($\frac{1}{4}$ okresu)

zwarcie do masy (obciążenie zwarte do masy)

- dla prądu stałego zwarcie do masy
- dla częstotliwości, na której pracuje ten układ dostajemy rozwarcie

AGH

2. Transmission line shorted at the end $Z_k = 0$ $Z_w = Z_f\frac{Z_k + jZ_f\tan(\beta l)}{Z_f + jZ_k\tan(\beta l)} = jZ_f\tan(\beta l)$

Standing wave

$|U| = U_{ik}\sqrt{1 + 2\Gamma_k\cos(\Theta_k - 2\beta y) + \Gamma_k^2}$

$|\Gamma_k| = 1$ $\Theta_k = \pi$

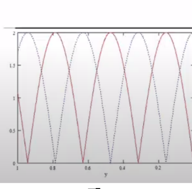
$|U| = U_{ik}\sqrt{2 - 2\cos(2\beta y)}$

Maximum values $|U|_{max} = 2U_{ik}$

are spaced with the distance

$y_{max\_k} = \frac{\pi(2k+1)}{2\beta} = \frac{\lambda}{4}(2k+1)$

Standing wave ratio $VSWR = \frac{U_{max}}{U_{min}} = \infty$



Jeśli linia jest zwarta to współczynnik VSWR dąży do ∞.

3. Transmission line open at the end

Linia transmisyjna bezstratna rozwarta na końcu

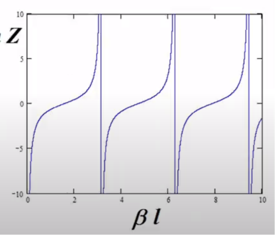
$Z_k = \infty$

Reflection coefficient

$\Gamma_k = \frac{Z_k - Z_f}{Z_k + Z_f} = 1$

Input impedance

$Z_w = Z_f\frac{Z_k + jZ_f\tan(\beta l)}{Z_f + jZ_k\tan(\beta l)} = -jZ_f\cot(\beta l)$



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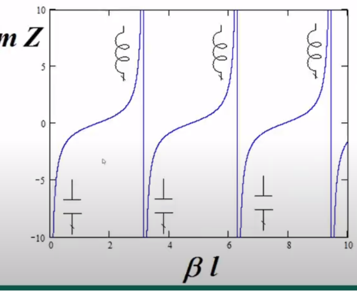
3. Transmission line open at the end $Z_k = \infty$ $Z_w = Z_f\frac{Z_k + jZ_f\tan(\beta l)}{Z_f + jZ_k\tan(\beta l)} = -jZ_f\cot(\beta l)$



To samo zachowanie co wcześniej, ale przesunięte w fazie o $\frac{\pi}{2}$

3. Transmission line open at the end $Z_k = \infty$ $Z_w = Z_f\frac{Z_k + jZ_f\tan(\beta l)}{Z_f + jZ_k\tan(\beta l)} = -jZ_f\cot(\beta l)$

$|U| = U_{ik}\sqrt{1 + 2\Gamma_k\cos(\Theta_k - 2\beta y) + \Gamma_k^2}$

$|\Gamma_k| = 1$ $\Theta_k = 0$

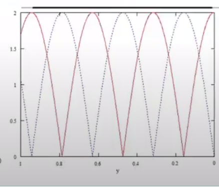
$|U| = U_{ik}\sqrt{2 + 2\cos(2\beta y)}$

Maximum values $|U|_{max} = 2U_{ik}$

are spaced with the distance

$y_{max\_k} - y_{max\_k+1} = \frac{\pi k}{\beta} = k\frac{\lambda}{2}$

Standing wave ratio $VSWR = \frac{U_{max}}{U_{min}} = \infty$



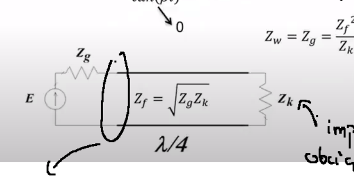
W tym przypadku znów otrzymujemy współczynnik fali stojącej równy ∞

4. Transformer λ/4 (Transformator ćwierćfalowy)

For the line $l = \lambda/4$ long $\beta l = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2}$

$$Z_w = Z_f\frac{Z_k + jZ_f\tan(\beta l)}{Z_f + jZ_k\tan(\beta l)} = Z_f\frac{\frac{Z_k}{\tan(\beta l)} + jZ_f}{\frac{Z_f}{\tan(\beta l)} + jZ_k} = Z_f\frac{Z_f}{Z_k} = \frac{Z_f^2}{Z_k}$$

$Z_w = Z_g = \frac{Z_f^2}{Z_k}$



impedancja obciążenia

impedancja widziana w tym miejscu to to

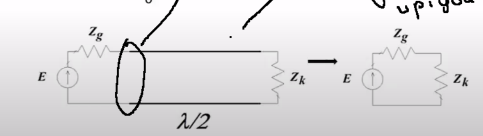
Długość elektryczna elementu - to długość fizyczna elementu odniesiona do długości fali

Długość elektryczna = 90° => $l = \frac{\lambda}{4}$

5. λ/2 section

For the line $l = \lambda/2$ long $\beta l = \frac{2\pi}{\lambda}\frac{\lambda}{2} = \pi$

$Z_w = Z_f\frac{Z_k + jZ_f\tan(\beta l)}{Z_f + jZ_k\tan(\beta l)} = Z_k$



linia "udaje, że jej nie ma" impedancja falowa linii nie ma wpływu na impedancję wejściową

Wykres Smitha

A chart useful for practical calculations, since very simple calculations with the use of input impedance relation take complicated form

Input impedance of a line

$Z_w = Z_0\frac{Z_k + jZ_0\tan(\beta l)}{Z_0 + jZ_k\tan(\beta l)} = Z_0\frac{1 + \Gamma_k e^{-j2\beta l}}{1 - \Gamma_k e^{-j2\beta l}}$

$\Gamma_k = \frac{Z_k - Z_0}{Z_k + Z_0}$

We normalize impedance to Z_0 zazwyczaj 50Ω

$z_k = \frac{Z_k}{Z_0} = r_k + jx_k$ $z_w = \frac{Z_w}{Z_0} = r_w + jx_w$

hence $\Gamma_k = \frac{z_k - 1}{z_k + 1}$ $\Gamma_k e^{-j2\beta l} = \frac{z_w - 1}{z_w + 1}$

wartości znormalizowane

Smith chart

AGH

We find real and imaginary part on the right hand side

$\Gamma_k e^{-j2\beta l} = \frac{z_w - 1}{z_w + 1} = \frac{r_w + jx_w - 1}{r_w + jx_w + 1} = \frac{r_w^2 - 1 + x_w^2}{(r_w + 1)^2 + x_w^2} + j\frac{2x_w}{(r_w + 1)^2 + x_w^2}$

On the other hand we can write

$\Gamma_k e^{-j2\beta l} = p + jq$

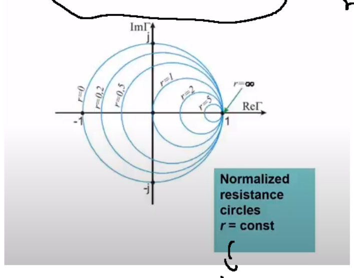
hence

$p = \frac{r_w^2 - 1 + x_w^2}{(r_w + 1)^2 + x_w^2}$ część rzeczywista współczynnika odbicia

$q = \frac{2x_w}{(r_w + 1)^2 + x_w^2}$ część urojona współczynnika odbicia

$\left(p - \frac{r_w}{r_w + 1}\right)^2 + q^2 = \frac{1}{(r_w + 1)^2}$

dla stałego r tworzą się kolejne równania okręgu



stała rezystancja (część rzeczywista)

$(p - 1)^2 + \left(q - \frac{1}{x_w}\right)^2 = \frac{1}{x_w^2}$

przy stałej części urojonej